D 73128 (Pages : 4) Name.....................................

Reg. No.................................

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. At what points are the function $f(x)=\frac{1}{x-2}-3x$ continuous ?

2. Define an equivalence relation.

3. Find $n(A\times B)$ for the sets $A=\{1,2\}$ and $B=\{a,b,c\}$.

4. Let $f:R\to R$ be defined by $f(x)=x^2+2x$. Find $f\circ f(2)$.

5. Find the domain of the function $f(x)=\frac{1}{x-2}$.

6. The graph of $y=x^2$ is shifted 2 units left and 2 units up, write the equation for the new graph.

7. Let $S=\{1,2,3\}$. Find the number of elements of power set of S.

8. Find the number of constant functions from A into B.

9. Translate these statement "$\forall x\,(C(x)\to F(x))$" into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

10. Consider the sets $A=\{1,2,3,4\}$ and $B=\{3,4,5,6,7\}$. Find $A\oplus B$.

11. If $A\subseteq B$ and $A\cap B=A$, then $A\cup B=$.........

12. Suppose $f:A\to B$ is a constant function. When will f be one-to one ?

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Evaluate $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

14. Check whether the function $f(x) = x^2 - 2$ is bijective.

15. Find x and y if $(y-2, 2x+1) = (x-1, y+2)$.

16. Let $P(x)$ denote the statement "$x > 3$." What are the truth values of $P(4)$ and $P(2)$?

17. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Find the number of functions from :

    (a) A into B ; (b) B into A.

18. Let $X = \{1, 2, 3, 4\}$. Determine whether the relation $\{(2,3), (1,4), (2,1), (3,2), (4,4)\}$ from X to X is a function.

19. If $\lim_{x \to -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \to -2} f(x)$ and $\lim_{x \to -2} \frac{f(x)}{x}$.

20. Define $f(3)$ in a way that extends $f(x) = \frac{x^2 - 9}{x - 3}$ to be continuous at $x = 3$.

21. For what vales of a is $f(x) = \begin{cases} x^2 - 1, & x < 3; \\ ax, & x \geq 3; \end{cases}$ continuous at every x ?

22. Consider the functions $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find a formula defining the composition function : (a) $f \circ g$; (b) $g \circ f$.

23. Prove $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

24. Give examples of relations R on $A = \{1, 2, 3\}$ having the stated property :

    (a) R is both symmetric and antisymmetric ;

    (b) R is neither symmetric nor antisymmetric.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

26. Let $V = \{1, 2, 3, 4\}$ and let $f = \{(1, 3), (2, 1), (3, 4), (4, 3)\}$ and $g = \{(1, 2), (2, 3), (3, 1), (4, 1)\}$. Find :
(a) $f \circ g$; (b) $g \circ f$; (c) $f \circ f$.

27. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

28. Show that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

29. Let R be the relation on positive integers defined by the equation $x + 3y = 12$:

    (a) Write R as a set of ordered pairs.

    (b) Find : (i) Domain of R ; (ii) Range of R ; (iii) R^{-1}.

    (c) Find the composition relation $R \circ R$.

30. Suppose ζ is a collection of relations S on a set A and let T be the intersection of the relations S, that is, $T = \cap(S : S \in \zeta)$. Prove that if every S is transitive, then T is transitive.

31. Use quantifiers and predicates to express the fact that $\lim_{x \to a} f(x)$ does not exist.

32. Prove that a function $f : A \rightarrow B$ is invertible if and only if f is objective.

33. Let $A=\{1,2,3\}, B=\{a,b,c\}, C=\{x,y,z\}$. Consider the following relation R from A to B relation S from B to $C: R=\{(1,b),(2,a),(2,c)\}$ and $S=\{(a,y),(b,x),(c,y),(c,z)\}$.

(a) Find the composition relation $R \circ S$.

(b) Find the matrices M_R, M_S and $M_{R \circ S}$ of the respective relations R, S and $R \circ S$; and compare $M_{R \circ S}$ to the product $M_R M_S$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Let $f(x)=\begin{cases}\sqrt{1-x^2}, & 0 \le x < 1 ; \\ 1, & 1 \le x < 2 ; \\ 2, & x = 2\end{cases}$.

(a) What are the domain and range of f?

(b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist ?

(c) At what points does only the left-hand limit exists ?

(d) At what points does only the right-hand limit exists ?

35. Let $S=\{1,2,3,\ldots,19,20\}$. Let $\equiv$ be the equivalence relation on S defined by congruence modulo 7.

(a) Find the quotient set $S/\equiv$.

(b) Find a system of equivalence class representatives consisting of even integers.

36. Prove that "If n is a positive integer, then n is odd if and only if n^2 is odd.

(2 × 10 = 20 marks)

D 52725 (Pages : 4) Name.....................................

Reg. No.....................................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

Core Course (Mathematics)

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*

1. Find the number of elements in the power set of {positive divisors of 6}.
2. Find $A \oplus B$ for $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$.
3. What is symmetric relation ?
4. Let f and g be functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$ then $f \circ g$ is :
5. Find the domain of the real valued function $f(x) = \sqrt{25 - x^2}$.
6. What is the cardinal number of the set $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$?
7. Check whether the function $h(t) = |t^3|$ is even, odd or neither.
8. The graph of $y = x^2$ is shifted 1 unit to the right and 4 units down. Write equation of the new graph.
9. Find $\lim_{x \to \pi} \sin(x - \sin x)$.
10. State the contrapositive of the implication :

    "If it snows tonight, then I will stay at home."
11. Define a Tautology.
12. What is the truth value of $\forall x\, p(x)$ where $p(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers ?

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. If A = {1, 2}, B = {a, b, c} and C = {c, d}, find (A × B) ∩ (A × C).

14. Let R be the relation R = {(1, b), (2, a), (2, c)} and S be the relation S = {(a, y), (b, x), (c, y), (c, z)}. Find R ∘ S.

15. Find x and y if $(y - 2, 2x + 1) = (x - 1, y + 2)$.

16. Suppose f : A → B is a constant function when will f be :

    (a) one-to-one ; (b) onto.

17. Let A = {a, b} and B = {1, 2, 3}, find the number of functions from :

    (a) A into B ; (b) B into A.

18. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\begin{cases}3x-1 & \text{if } x>3\\ x^2-2 & \text{if } -2\le x\le 3\\ 2x+3 & \text{if } x<-2.\end{cases}$

    Find (a) $f(-1)$; (b) $f(2)$.

19. For the function $f(x)=\begin{cases}0 & x<0\\ 1 & x\ge 0\end{cases}$ find $\lim\limits_{x\to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim\limits_{x\to 4}\dfrac{4x-x^2}{2-\sqrt{x}}$.

21. If $2-x^2\le g(x)\le 2\cos x$ for all x, find $\lim\limits_{x\to 0} g(x)$.

22. Let p and q be the proposition "The election is decided" and "The votes have been counted" respectively. Express the compound proposition $\neg q\vee(\neg p\wedge q)$ as an English sentence.

23. Show that $\neg(p\vee q)$ and $\neg p\wedge\neg q$ are logically equivalent.

24. What is the negation of the statement $\forall x\left(x^2>x\right)$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any **six** questions.*

25. Let A be a set of non-zero integers, and let ~ be a relation on A × A defined by (a,b) ~ (c,d) whenever $ad = bc$. Prove that ~ is an equivalence relation.

26. Let A = {1, 2, 3, 4}, consider the following relation R on A.

    R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}.

    (a) Draw its directed graph ; (b) Find $R^2 = R \circ R$.

27. Consider the functions $f : A \to B$ and $f : B \to C$ prove the following :

    (a) If f and g are one-to-one then $f \circ g$ is one-to-one.

    (b) If f and g are onto functions then $g \circ f$ is an onto function.

28. Let R_1 and R_2 be relations on a set A represented by the matrix :

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

    Find the matrix representing (i) $R_1 \cup R_2$; (ii) $R_1 \cap R_2$.

29. Find the continuous extension to $x = 3$ of the function $f(x) = \dfrac{x^2 - 9}{x - 3}$.

30. Let $f(x) = \begin{cases} 0 & x \le 0 \\ \sin 1/x & x > 0. \end{cases}$

    (a) Does $\lim_{x \to 0^+} f(x)$ exist ? If so what is it ? If not, why not ?

    (b) Does $\lim_{x \to 0^-} f(x)$ exist ? If so what is it ? If not, why not ?

    (c) Does $\lim_{x \to 0} f(x)$ exist ? If so what is it ? If not, why not ?

31. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

32. Express the statements "All lions are fierce", "some lions do not drink coffee", "some fierce creature do not drink coffee" using predicates and quantifiers, assuming the domain consists of all creatures.

33. Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then n is odd".

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. Let A = {1, 2, 3, 4}. Consider the relations R and S on A given by R = {(1,1), (1,2), (2,3), (3,1), (3,3)} ; S = {(1, 2), (1, 3), (2, 1), (3, 3)}.

    Find the matrices (a) $M_{R\cap S}$; (b) $M_{R\cup S}$; (c) M_{R^c} ; (d) $M_{R\circ S}$; (e) M_{S^2}.

35. If $\lim\limits_{x\to c} f(x) = L$ and $\lim\limits_{x\to c} g(x) = M$, prove that $\lim\limits_{x\to c} \left(f(x)+g(x)\right) = L + M$.

36. Show that $p\vee(q\wedge r)$ and $(p\vee q)\wedge(p\vee r)$ are logically equivalent.

(2 × 10 = 20 marks)

C 33293 (Pages : 4) Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find the number of elements in the power set of {days of the week}.
2. Find A – B for the sets A = {1, 2, 3, 4} , B = {3, 4, 5, 6, 7}
3. Give an example of relation R on A = {1, 2, 3} which is transitive but $R \cup R^{-1}$ is not transitive.
4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 2x$. Then $(f \circ f)(2)$ is :
5. Find the domain of the real valued function $f(x) = 1/_{x-2}$.
6. Define a denumerable set.
7. If the graph of a function is symmetric about the origin, then the function is an ———.
8. The graph of $y = x^2$ is shifted 2 units to the left and 2 units up, write the equation of the new graph.
9. Find $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.
10. Write the negation of "This is a boring course".
11. What is the truth value of $\forall x \left(x^2 \geq x\right)$ if the domain consists of all real numbers.
12. State which rule of inference is the basis of the argument "It is below freezing now. Therefore it is either below freezing or raining now".

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. If A = $\{a, b, c, d\}$ and B = $\{y, z\}$. Find A × B and B × A.

14. Let R be a relation on A = {1, 2, 3} defined by R = {(1, 1), (1, 2), (2, 3), (3, 1), (3, 2)}. Find R^c and R^{-1}.

15. Find all partitions of S = {1, 2, 3}.

16. Let S = {–1, 0, 2, 5} find f (S) where $f(x) = \left[\frac{x}{5}\right]$.

17. Find the inverse of the function $f(x) = \frac{2x-3}{5x-7}$.

18. Let the functions f and g be defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find (a) $f \circ g$. ; and (b) $g \circ f$.

19. For the function $f(x) = \begin{cases} 0 & x \le 0 \\ \sin 1/x & x > 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

21. If $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to 0} g(x) = -5$ find $\lim_{x \to 0} \frac{2f(x) - g(x)}{(f(x) + 7)^{2/3}}$.

22. Determine whether these biconditions are true *or* false :
    (a) 2 + 2 = 4 if and only if 1 + 1 = 2.
    (b) 0 > 1 if and only if 2 > 1.

23. Compare the terms Tautology and Contradiction.

24. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Let R_1 and R_2 be relations on a set A represented by the matrices :

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Find the matrices representing :

(a) $R_1 \cup R_2$. (ii) $R_1 \cap R_2$.

26. Suppose $\mathscr{C}$ is a collection of relations S on a set A and let T be the intersection of relations S, that is $T = \bigcap\{S \mid S \in \mathscr{C}\}$. Prove that if S is transitive, then T is transitive.

27. Show that P × P is denumerable, where P is the set of all positive integers.

28. Let R be the relation on P defined by the equation $x + 3y = 12$.

(a) Write R as a set of ordered pairs.

(b) Find (i) Domain of R ; (ii) Range of R ; and (c) R^{-1}.

29. Find the continuous extension to $x = 2$ of the function $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$.

30. Show that if $\lim\limits_{x \to c} |f(x)| = 0$ then $\lim\limits_{x \to c} |f(x)| = 0$.

31. Show that $\forall x \, (P(x) \wedge Q(x))$ and $\forall x \, P(x) \wedge \forall x \, Q(x)$ are logically equivalent.

32. Show that the hypothesis $(p \wedge q) \vee r$ and $r \to s$ imply the conclusion $p \vee s$.

33. Prove that "If n is an integer and n^2 is odd then n is odd".

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Consider the following five relations on the set A = {1, 2, 3, 4} :

$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$; $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$;

$R_3 = \{(1, 3), (2, 1)\}$; $R_4 = \varnothing$, empty relation ; $R_5 = A \times A$.

Determine which of the relations are :

(a) Reflexive. (b) Symmetric.

(c) Antisymmetric. (d) Transitive.

35. Let $f(x) = \begin{cases} 3 - x & x < 2 \\ \dfrac{x}{2} + 1 & x > 2 \end{cases}$

(a) Find $\lim\limits_{x \to 2^+} f(x)$ and $\lim\limits_{x \to 2^-} f(x)$.

(b) Does $\lim\limits_{x \to 2} f(x)$ exist ? If so what is it ? If not, why not ?

(c) Find $\lim\limits_{x \to 4^-} f(x)$ and $\lim\limits_{x \to 4} f(x)$.

(d) Does $\lim\limits_{x \to 4} f(x)$ exist ? If so what is it ? If not, why not ?

36. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(2 × 10 = 20 marks)

D 13791 (Pages : 3) Name............................

Reg. No............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Core Course—Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 mark.

1. Find symmetric difference of the sets $A=\{1,2,3,4,5\}$; $B=\{4,5,6,7\}$.
2. Find $A\times B$ if $A=\{x,y\}$ and $B=\{1,2,3\}$.
3. Define a symmetric relation on a set A.
4. Let $f:R\to R$ be defined by $f(x)=2x-3$. Find a formula for $f^{-1}(x)$.
5. Represent the relation $R=\{(1,1),(1,2)(1,3)(3,4)\}$ on $\{1,2,3,4\}$ using a matrix.
6. Let $f:R\to R$ and $g:R\to R$ defined by $f(x)=x^3$ and $g(x)=2x+9$ find $f\circ g(x)$.
7. Give an example of a function which is continuous at every value of x.
8. Find $\lim_{x\to 1}\dfrac{x^2+x-2}{x^2-x}$.
9. Discuss the behaviour of $f(x)=\dfrac{1}{x^2}$ near $x=0$.
10. Write the converse of the statement 'If the home team wins, then it is raining'.
11. Write the truth table for the proposition $p\leftrightarrow q$.
12. What is the truth value of $\exists x\, p(x)$ where $p(x)$ is the statement '$x^2\geq 16$' and the domain consists of positive integers not exceeding 4.

(12 × 1 = 12 marks)

Turn over

Section B

Answer **all** *questions.*
Each question carries 2 *marks.*

13. Determine the power set $P(A)$ of $A = (a, b, c, d)$.

14. Let $S = (1, 2, 3, \ldots. 9)$. Write a partition of S.

15. Let $R = \{(1,3)(1,4)(3,2)(3,3)(3,4)\}$ be a relation on $A = \{1,2,3,4\}$. Find $R \circ R$.

16. Let $f : A \to B$, $g : B \to C$ be two functions prove if f and g are one-to-one, then the composition $g \circ f$ is one-to-one.

17. Define a countable set and give an example.

18. Write down the conditions for a function $f(x)$ is continuous at $x = c$.

19. State the Sandwich theorem.

20. Write the negation of the statement :

    "There is an honest politician".

21. Write De Morgan's laws of propositions.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

22. Show $a \equiv b \pmod 5$ is an equivalence relation on the set of all integers.

23. Let $A = \{1, 2\}$, $B = \{a, b, c\}$, $C = \{c, d\}$. Find $(A \times B) \cap (A \times C)$ and $A \times (B \cap C)$.

24. Let $f : A \to B, g : B \to C$, $h : C \to D$. Prove $(f \circ g) \circ h = f \circ (g \circ h)$.

25. Let $A = \{a, b\}$ and $B = \{1, 2, 3\}$. Find the number of functions (1) from A into B ; (2) from B into A.

26. Prove that $\lim_{x \to 2} f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}.$$

27. At what points the function

$$y = \frac{1}{x-2} - 3x \text{ is continuous.}$$

28. Write the converse, contrapositive and inverse of the conditional statement "If it snows today, I will ski tomorrow".

29. Construct a tenth table for the compound proposition $(p \vee q) \oplus (p \wedge q)$.

30. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

31. (a) Let $A = \{a, b, c, d\}$. Give a relation on A which is (1) Reflexive ; (2) Symmetric ; (3) Antisymmetric.

(b) Define partial ordering relation on a set S. Show that the relation $\leq$ on the set R of real numbers is a partial ordering.

32. (a) Prove that a function $f : A \rightarrow B$ us invertible iff f is one-to-one and onto.

(b) Define recursively defined functions and give an example.

33. (a) Translate into logical expression using predicates and quantifiers. "Someone in your class has visited Mexico". Domain consists of all students in your class.

(b) Show $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

(c) Write the negation of the statement :

'All Americans eat cheeseburgers'.

(2 × 10 = 20 marks)

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D 74380 **(Pages : 4)** Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(CUCBCSS-U.G.)

Complementary Course—Mathematics

MAT IC 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer all **twelve** *questions.*

1. Evaluate $\underset{x \to -5}{\text{Lt}} \dfrac{x^2+3x-10}{x+5}$.

2. At what points are the function $y = \dfrac{\cos x}{x}$ continuous ?

3. Find the slope of $f(x) = x^2+1$ at (3, 7).

4. Find the derivative of $y = x^2$ using the definition of derivative.

5. Find the second derivative of $y = \dfrac{1}{3x^2} - \dfrac{5}{2x}$.

6. How fast does the area of a circle change with respect to the diameter when the diameter is 8 m ?

7. Find the critical points of $f(x) = \dfrac{2x^3-3x^2}{6}$.

8. Graph the parabola $y = x^2$.

9. Find $\underset{x \to \infty}{\text{Lt}} \dfrac{2x+3}{5x+7}$.

10. Evaluate the sum of the first 20 cubes.

11. State the mean value theorem for definite integrals.

12. Find the intersection points of $f(x) = 2-x^2$ and $g(x) = -x$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer all **nine** *questions.*

13. If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}, -1 \le x \le 1$, find $\underset{x\to 0}{\text{Lt}}\, f(x)$.

14. Prove that $\underset{x\to 1}{\text{Lt}}\, f(x) = 1$ if $f(x) = \begin{cases} x^2, x \ne 1 \\ 2, \; x = 1 \end{cases}$.

15. Suppose $\underset{x\to c}{\text{Lt}}\, f(x) = 5$ and $\underset{x\to c}{\text{Lt}}\, g(x) = -2$. Find :

    (i) $\underset{x\to c}{\text{Lt}}\left[f(x)+3g(x)\right]$; and (ii) $\underset{x\to c}{\text{Lt}}\, \dfrac{f(x)}{f(x)-g(x)}$.

16. Find the derivative of $y = \dfrac{1}{\left(x^2-1\right)\left(x^2+x+1\right)}$.

17. Find the equation of the tangent to the curve $y = x^3 - 4x + 1$ at (2, 1).

18. Find $\underset{x\to 0}{\text{Lt}}\, \dfrac{8x^2}{\cos x - 1}$.

19. Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

20. Graph the function $y = \dfrac{1}{2x+4}$.

21. Find the area of the region enclosed by $y = x^2 - 2$ and $y = 2$.

22. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through (0, 2).

23. Find the derivatives of all orders of $y = x^5/120$.

24. State both parts of the fundamental theorem of calculus.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*

25. Show that $y = \sin\left(1/x\right)$ has no limit point as x approaches zero from either side. Also sketch the graph of this function.

26. Evaluate $\underset{x \to 1}{\text{Lt}} \frac{x-1}{\sqrt{x+3}-2}$.

27. The curve $y = ax^2 + bx + c$ passes through (1, 2) and is tangent to $y = x$ at the origin. Find a, b, c.

28. State and prove the product rule for derivatives. Use it to find the derivative of $y = (x^2+1)(x^3+3)$.

29. Find the intervals on which $f(x) = \frac{x^2-3}{x-2}$, $x \neq 2$ is increasing and decreasing. Identify local extrema if they exist.

30. Define average value of an integrable function over a closed interval. Find the average value of $f(x) = -3x^2 - 1$ on [0,1]. where in the given interval does $f(x)$ assume its average value.

31. Show that $\underset{x \to 0+}{\text{Lt}} (1+x)^{1/x} = e$.

32. An object is dropped from the top of a 100 m high tower. Its height above ground after 't' seconds is $(100 - 4.9\,t^2)\,m$. How fast is it falling 2 seconds after it is dropped ?

33. Find the derivative $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\, dt$ by (i) evaluating the integral and differentiating the result ; and (ii) by differentiating the integral directly.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*

34. (i) Find the area of the region enclosed by the curves $x + 4y^2 = 4$ and $x + y^4 = 1$ for $x \geq 0$.

(ii) Find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 0, x = 2$ about the x-axis.

35. (i) Graph the function $y = x^4 - 4x^3 + 10$ by finding the first and second derivative.

(ii) Evaluate $\underset{x \to 0}{\text{Lt}} \frac{x^2}{\ln(\sec x)}$.

(iii) Evaluate $\sum_{k=1}^{4} \cos k\pi$.

36. (i) Let $f(x)=\begin{cases}3-x, x<2\\ \frac{x}{2}+1, x>2\end{cases}$. Find :

(a) $\underset{x\to 2+}{\text{Lt}} f(x)$ and $\underset{x\to 2-}{\text{Lt}} f(x)$.

(b) Does $\underset{x\to 2}{\text{Lt}} f(x)$ exist ? why or why not ?

(c) $\underset{x\to 4-}{\text{Lt}} f(x)$ and $\underset{x\to 4+}{\text{Lt}} f(x)$.

(d) Does $\underset{x\to 4}{\text{Lt}} f(x)$ exist ? Why or why not ?

(ii) Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

(2 × 10 = 20 marks)

D 73679 (Pages : 3) Name..................................

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FIRST SEMESTER B.Sc DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2014

(U.G.-CUCSS)

Core Course—Mathematics

MM 1B 01—FOUNDATIONS OF MATHEMATICS

(2010 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer all **twelve** *questions.*
Each bunch of four questions carries 1 *weightage.*

1. If $R = \{(1, a)\,(2, d)\,(3, a)\}$ and $S = \{(a, z), (d, z), (c, y)\}$ find $R_o s$.

2. Let $A = \{(1, 2, 3, 4)\}$ and R be a relation on A given by $R = \{(1, 3)\,(1, 4)\,(3, 2)(3, 3), (3, 4)\}$ Find matrix representation M_R of R.

3. What is the cardinality of the set $\{\{a\}, b, \{a, b\}\}$

4. If $A_i = \{\wedge, \wedge + 1, \wedge + 2.....\}$. Find $\bigcup_{\wedge=1}^{\infty} A_i$.

5. $f : R \to R$, $g : R \to R$ be defined as

   $f(x) = x^2$, $g(x) = x + 4$ find fog $^{(x)}$.

6. If $f = \begin{cases} 3x - 1 & \text{if} \quad x > 3 \\ x^2 - 2 & \text{if} \quad -2 \le x \le 3 \\ 2x + 3 & \text{if} \quad x < -2 \end{cases}$

   find $f(2)$.

7. Define characteristic function of A.

8. What is the inverse of $p \to q$.

9. Whether the statement '$x + y = z$' a proposition ?

10. State the rule of inference 'simplification rule'.

Turn over

11. P (x) is the statement '$x = x^2$'. If the domain consists of integers, what is the truth value of P (–1).
12. What is the contra positive of the statement 'If it is raining, then the home team wins'.

(12 × ¼ = 3 weightage)

Part B

Answer all **nine** *questions.*
Each question carries 1 *weightage.*

13. Find A × B if $A = \{a, b, c\}$ $B = \{x, y\}$.
14. If $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7, 8, 9, 10\}$ find $(A \cap B) \cup C$.
15. Find 26 (mod 7) and 25 (mod 5).
16. Let $A = \{1, 2, 3, 4\}$ and f is defined on A by $f = \{(1, 3)\,(2, 1)\,(3, 4)\,(4, 3)\}$. Find fof.
17. Write the De Morgain's laws of propositions.
18. Write the truth table for biconditional statement $p \rightarrow q$.
19. Define tautology. Show $p \vee \neg p$ is a tautology.
20. Let Q (x) be the statement '$x + 1 > 2x$'. If the domain consists of integers what is the truth value of $\forall x\, Q(x)$?
21. Which rule of inference is used in the following argument ?

    Jerry is a mathematics major and computer science major. Therefore Jerry is a mathematics major.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Show that the relation $a \equiv b \pmod 5$ is an equivalence relation.
23. Let $f : R \rightarrow R$ defined by $f(x) = 2x + 4$ show f is one-one and onto.
24. Find a formula for the inverse g^{-1} of the function $f(x) = \dfrac{x-2}{x-3}$.
25. Show $(p \rightarrow q) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.
26. Construct a truth table for the statement $[p \wedge (p \rightarrow q)] \rightarrow q$.
27. Express the mathematical statement using predicates, quantifiers and logical connectives.

    'The difference of a real number and itself is zero'

28. Translate the following quantifications into English statement if the domain consists of all real number '$\forall x \forall y ((x<0) \wedge (y<0)) \rightarrow xy>0))$'

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions.*
Each question carries 4 weightage.

29. (i) If $f: A \rightarrow B$, $g: B \rightarrow C$ and if f and g are one-to-one, prove that the composition gof is one-to-one.
    (ii) If f and g and onto, then prove gof is onto function.
30. (i) Write the converse, inverse and contra positive of the conditional statement 'If it snows, then I will stay at home'.
    (ii) Use truth table to verify absorption laws $p \vee (p \wedge q) = p$ and $p \wedge (p \vee q) = p$.
31. (a) Prove that the following statements are equivalent :
        (i) n is even.
        (ii) $n-1$ is odd.
        (iii) n^2 is even.
    (b) Give a counter example to show that the statement "Every positive integers is the sum of the squares of two integers" is false.

(2 × 4 = 8 weightage)

51 Maths

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(Pages : 3)

Name.....................................

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FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2014

(UG—CCSS)

Core Course

Mathematics

MM IB 01—FOUNDATIONS OF MATHEMATICS

(2010 Admission Onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A (Objective Type Questions)

Answer all **twelve** *questions.*
Each bunch of **four** *questions carries* 1 *weightage.*

1. Find x and y if $(y-2, 2x+1) = (x-1, y+2)$.
2. Find R^{-1} if $R = \{(1, y)(1, z), (3, y)\}$.
3. $A = \{1, 2, 3, 4\}$, R is a relation on A given by $R = \{(1,1)(2,2)(2,3)(3,2)(4,2)(4,4)\}$. Is R symmetric ?
4. What is the cardinality of the set $\{a, \{a\}, \{a, \{a\}\}\}$?
5. If $f : R \to R$, $g : R \to R$ be defined as $f(x) = x^2$, $g(x) = x+4$ find $gof(x)$.
6. Fill in the blanks , $n! =$ ———— $(n-1)!$ if $n > 0$.
7. If $A = \{a, b\}$, $B = \{4, 5, 6\}$, find the number of functions from A into B.
8. Write the converse of the statement $p \to q$.
9. Write the truth table for $p \to q$.
10. What is the negation of the statement $\forall x\, p(x)$.
11. State the rule of inference 'Modus ponens'.
12. What is the truth value of the statement $\exists x\,(2x = 3x)$ if the domain consists of all integers.

(12 × ¼ = 3 weightage)

Turn over

Part B (Short Answer Type Questions)

*Answer all **nine** questions .*
Each question carries 1 weightage.

13. Find A × B and B × A if A = {1, 2}, B = { a, b}.

14. Let $A_i = \{1, 2, 3 \dots i\}$. Find $\bigcup_{i=1}^{n} A_i$.

15. Find floor function of (1) $\lfloor 7 \cdot 5 \rfloor$ (2) $\lfloor -2 \cdot 5 \rfloor$.

16. Let A = {1, 2, 3, 4} f and g are two functions on A defined by

    $f = \{(1,3)\,(2,1)\,(3,4)\,(4,3)\}$

    $g = \{(1,2)\,(2,3)\,(3,1)\,(4,1)\}$ find $f \circ g$.

17. What is the negation of the statement

    'There is an honest politician'.

18. What is the truth value of $\exists x\, p(x)$ where $p(x) : x^2 > 10$ and the domain consists of {1, 2, 3,4}.

19. Define Tautology. Show $p \wedge \neg p$ is not a tautology.

20. Give a proof by contra position 'if n is an integer and $3n + 2$ is odd, then n is odd'.

21. Which rule of inference is used in the following argument :

    "Alice is a Mathematics major. Therefore Alice is either a Mathematics major or a Computer Science major".

(9 × 1 = 9 weightage)

Part C (Short Essay Questions)

*Answer any **five** questions.*
Each question carries 2 weightages.

22. Show that the relation $a \equiv b \pmod{n}$ is an equivalence relation.

23. Find a formula for inverse of $g(x) = \dfrac{2x-3}{5x-7}$.

24. Let $f : \mathrm{R} \to \mathrm{R}$ defined by $f(x) = 2x - 3$ show f is one-to-one and onto.

25. Show $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

26. Express the mathematical statement using predicates, quantifiers and logical connectives.

'The product of two negative real numbers is positive'.

27. Translate the following quantifications into English statement if the domain consists of all real numbers

' $\exists x \, \forall y \, (xy = y)$'.

28. Construct a truth table for the compound proposition $(p \to q) \wedge (\neg p \to q)$.

(5 × 2 = 10 weightage)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Let $f : A \to B$, $g : B \to C$ be two functions prove :

(a) If gof is one-to one then f is one-to-one.

(b) If gof is onto then g is onto.

30. (a) Write the contra positive, inverse and converse of

'If it is rainy, then the pool will be closed'.

(b) Which rule of inference is used in the argument ?

If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore if I go swimming, then I will sunburn'.

31. (a) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

(b) Give a proof by contradiction to prove 'if n is an integer and $n^3 + 5$ is odd, then n is even.

(2 × 4 = 8 weightage)

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FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(CUCBCSS-U.G)

Core Course—Mathematics

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions*

1. If $U = \{1, 2, \ldots. 9\}, A = \{1, 2, 3, 4, 5\}, B = \{4, 5, 6, 7\}$ show $B \setminus A = B \cap A^c$.

2. Write DeMorgan's Laws for sets A and B.

3. Let $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$ and

   $S = \{(b, x), (b, z), (c, y), (d, z)\}$ be two relations, find $R \circ S$.

4. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,2), (2, 1), (2,2), (3, 3), (4, 4)\}$ be a relation on A. Is R antisymmetric ?

5. Let $S = \{1, 2, 3, \ldots. 8, 9\}$. Write a partition of S.

6. Give an example of a partial ordering relation on the set of real numbers.

7. Find $\underset{x \to -3}{\text{Lt}} \dfrac{x+3}{x^2+4x+3}$.

8. State sandwich theorem for limits.

9. Find $\underset{x \to \infty}{Lt} \dfrac{2x+3}{5x+7}$.

10. Write the converse of the statement "If it raining, then the home team wins".

11. What is the truth value of $\forall x\, p(x)$, where $p(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers not exceeding 4.

12. Show that $p \vee \neg p$ is a tautology.

(12 × 1 = 12 marks)

Turr
Turn over

$A \setminus B = \{1, 6\}$
$B \setminus A = \{7\}$
$A \oplus B = \{1, 6, 7\}$

Part B

Answer any **nine** *questions.*

13. If $A = \{1, 2, 5, 6\}$, $B = \{2, 5, 7\}$, $C = \{1, 3, 5, 7, 9\}$ find $A \oplus B$ and $A \oplus C$.

14. Write the numbers of elements in the power set of the set {positive divisors of 12}.

15. Let A {1, 2}, B {a, b, c}, C = {c, d}. Find $(A \times B) \cap (A \times C)$ and $A \times (B \cap C)$. What is your observation about the answers.

16. Let $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ be the matrix representation of a relation. Find the matrix representation of the relation RoR.

17. Let the functions f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find the formula defining the composite functions fog $f \circ g$ and $g \circ f$.

18. Let $f : R \to R$ defined by $f(x) = 4x + 5$. Find a formula for f^{-1}.

19. Let $A_m = \{m, 2m, 3m, \ldots\}$ where $m \in P$; the positive multiples of m. Find $A_3 \cap A_5$

20. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.

21. Discuss the behaviour of $f(x) = \frac{1}{x^2}$ near $x = 0$.

22. Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

23. Let $p(x)$ be the statement "x can speak Russian". and let $Q(x)$ be the statement "x knows computer language C++". Express the statement" there is a student at your school who can speak Russian and who knows C++" using quantifiers and logical connectives. Domain consists of all students in your school.

24. What is the negation of the statement "All Americans eat cheese burgers".

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*

25 Show that the relation $x \equiv y \pmod{m}$ is an equivalence relation on the set of integers ; $m > 1$.

26. Let R_1 and R_2 be relations on a set A represented by matrices $M_{R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Find the matrices that represent (i) $R_1 \cap R_2$; (ii) $R_1 \cup R_2$; (iii) R_1 0 R_2.

27. Let $f : A \to B$ and $g : B \to C$, prove if $g \circ f$ is one-to one, than f is one-to-one.

28. Show that the set Q of rational numbers is denumerable.

29. Discuss the continuity of $f(x) = |x|$

30. Prove that $\lim_{x \to 2} f(x) = 4$ if

$$f(x) \begin{cases} x^2 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

31. Prove $(p \to q) \wedge (p \to r)$ is logically equivalent to $p \to (q \wedge r)$

32. Translate the statement $\exists p\,(F(p) \wedge B(p)) \to \exists j\, L(j)$ into English where F (*p*) is "printers *p* is out of service", B (*p*) is "printers *p* is busy" and L (*j*) is " printers *j* œ *j* is lost".

33. Use De Morgan's Laws to find the negation of the following statements :

(a) Jan is rich and busy.

(b) James walks or takes a bus to college.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*

34. Let R be an equivalence relation on a set S. $[a]$ denote equivalence class of a in S under R. Prove the following :

    (a) for each $a \in S$, $a \in [a]$

    (b) $[a] = [b]$ if and only if $(a, b) \in R$.

    (c) If $[a] \neq [b]$ then $[a]$ and $[b]$ are disjoint.

35. (a) Let $f: A \to B$, $g: B \to C$, $h: C \to D$. Then prove $h \circ (g \circ f) = (h \circ g) \circ f$.

    (b) Show the set of positive integers and set of even positive integers have the same cardinality.

36 (a) Write the converse, inverse and cantrapositive of the statement "If you drive more than 400 miles, you will need to buy gasoline'.

   (b) Constenct a truth table for the compound preposition $(p \vee q) \to p \oplus q$.

(2 × 10 = 20 marks)

D 32464 (Pages : 3) Name..................................

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FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2013

(CCSS)

Mathematics

MM 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of **four** *questions carries 1 weightage.*

1. If $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$, the symmetric difference $A \oplus B$.
2. Fill up : $A \cup \bar{A} =$ ———.
3. Let $A_i = \{1, 2, 3 \ldots i\}$ find $\bigcap_{i=1}^{\infty} A_i$.
4. Let $f : R \to R$; $g : R \to R$ defined by $f(x) = x^2$ and $g(x) = x + 4$. Find *gof.*
5. Define a countable set.
6. Give an example of a partition of S if $S = \{1, 2, 3, 4\}$.
7. Is the statement 2 + 2 = 3, a proposition ?
8. What is the converse of the statement $p \to q$.
9. State which rule of inference is used in the following argument "It is below freezing and raining now. Therefore it is below freezing".
10. Give an example of a complete bipartite graph.
11. Draw Petersen Graph.
12. Define a regular graph.

(12 × ¼ = 3 weightage)

Turn over

Part B (Short Answer Type Questions)

Answer **all** *nine questions.*
Each question carries 1 weightage.

13. Let $A = \{1, 2, 3, 4\}$, R be the relation on A $R = \{(1,1)(1,2)(2,1)(2,2)(3,3)(4,4)\}$. Is R reflexive ?
14. State De Morgan's laws.
15. Define equivalence relation R on a set S.
16. Is identity function on a set A is onto. Justify your answer.
17. Define Fibonacci sequence.
18. Write the truth table for $p \vee \neg p$.
19. Express using predicates and quantifness. 'The product of two negative real numbers is positive.
20. Draw all graphs on 1, 2, 3 points.
21. Draw K_4 and $K_4 - V$ where V is a point of K_4. Identity $K_4 - V$.

(9 × 1 = 9 weightage)

Part C (Short Essay Questions)

Answer any **five** *questions.*
Each question carries 2 weights.

22. Show that the relation $x \equiv y \pmod 5$ is an equivalence relation on Z.
23. Let $f : A \to B$, $g : B \to C$ are invertible functions. Show $gof : A \to C$ is invertible.
24. Find a formula for inverse of f if $f(x) = \dfrac{2x+4}{7x-3}$.
25. Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.
26. Translate into an English statement :

    (a) $\exists x \forall y (x + y = y)$;

    (b) $\exists x \exists y ((x^2 > y) \wedge x < y))$;

    where the domain consists of all real numbers.
27. Prove in any graph G the number of points of odd degree is even.
28. Prove isomorphism preserves the degree of vertices.

(5 × 2 = 10 weightage)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Let A be the set of non-zero integers and let $\approx$ be the relation on A × A defined by $(a, b) \approx (c, d)$ whenever $ad = bc$ show $\approx$ is an equivalence relation. Also if $A = \{1, 2, 3, ... 14, 15\}$. Find the equivalence class of (3, 2).

30. (a) Prove that if n is a positive integers, then n is even if and only if $7n + 4$ is even.

    (b) Give a proof by contradiction of the theorem 'if $3n + 2$ is odd, then n is odd'.

31. (a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

    (b) Draw G and its complement $\overline{G}$, where G is the complete graph K_3.

(2 × 4 = 8 weightage)

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Name.....................................

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FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2012

(CCSS)

Mathematics—Core Course

MM 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours

Maximum Weightage : 30

Part A (Objective Type Questions)

Answer all **twelve** *questions. Each bunch of* **four** *questions carries* 1 *Weightage.*

1. Find the number of subsets of the set $A = \{x \mid x \text{ is a day of the week}\}$.
2. If $|A| = 24, |B| = 69$ and $|A \cup B| = 81$, find $|A \cap B|$.
3. Find $(A \cup B) \cap (A \cup B^C)$.
4. State the DeMorgan's law of sets.

(4 × ¼ = 1 Weightage)

5. Determine whether $R = \{(1,3), (2,1)\}$ is anti-symmetric if $A = \{1,2,3,4\}$.
6. How many functions can be defined from a finite set A to a finite set B with $|A|$ and $|B|$ elements respectively?
7. Define characteristic function of a set A.
8. Whether the statement "Do not litter" is a proposition?

(4 × ¼ = 1 Weightage)

9. Write the contrapositive of the statement $p \rightarrow q$?
10. When will you say that two propositions p and q are logically equivalent?
11. Give an example of a propositional function.
12. What is the truth value of the quantification $\exists x Q(x)$ if $Q(x)$ denotes the statement $x = x + 1$ and the universe of discourse is the set of real numbers?

(4 × ¼ = 1 Weightage)

Part B (Short Answer Type Questions).

Answer all **nine** *questions.*
Each question carries 1 *Weightage.*

13. If $A = \{a, b, c, d, e\}, B = \{a, b, d, j, g\}, C = \{b, c, e, g, h\}$, find $(A \oplus C) \setminus B$.
14. Find x and y, if $(x + 2, 4) = (5, 2x + y)$.
15. Check whether the relation $x > y$ on the set of natural numbers is anti-symmetric?

Turn over

16. Define a recursive function to obtain the successive terms of the Fibonacci Series.
17. State the converse of the implication "If it snows tonight, then I will stay at home"
18. Show that $\neg(\neg p)$ and p are logically equivalent.
19. Use quantifiers to express the following statement:
"Every student in this class knows to speak Hindi"
20. Determine the truth value of the statement $\exists x \forall y \neq 0 \exists (xy = 1)$ if the universe of discourse of each variable is the set of real numbers.

(9 × 1 = 9 Weightage)

Part C (Short Essay Questions).

Answer any **five** *questions. Each question carries* 2 *Weights.*

21. Suppose R is a partial order on a set A. Show that R^{-1} is also a partial order on A.
22. Let A be a set of nonzero integers and let $\approx$ be the relation on A x A defined as follows: $(a, b) \approx (c, d)$ whenever ad = bc. Prove that $\approx$ is an equivalence relation.
23. Suppose $\mathbb{C}$ is a collection of relations S on a set A and let T be the intersection of the relations S, i.e., $T = \cap (S : S \in \mathbb{C})$. Prove that if every S is transitive then T is transitive.
24. Consider the formula $f(x) = x^2$.
    a. Find the largest interval D such that $f: D \rightarrow R$ is a one-to-one function.
    b. Find the smallest target set T such that $f: R \rightarrow T$ is an onto function.
25. Show that $(p \vee q) \rightarrow (p \wedge q)$ is a tautology.
26. Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.
27. Determine whether $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x (P(x) \wedge Q(x))$ are logically equivalent.

(5 × 2 = 10 Weightage)

Part D (Essay Questions).

Answer any **two** *questions. Each question carries* 4 *Weightage.*

28. Consider functions $f: A \rightarrow B$ and $g: B \rightarrow C$. Prove the following:
    a) If f and g are one-to-one, then $g o f$ is one-to-one.
    b) If f and g are onto functions, then $g o f$ is an onto function.
    c) If $g o f$ is one-to-one, then f is one-to-one.
    d) If $g o f$ is onto, then g is onto.

29. Consider the following five relations:

    1. Relation ≤ (less than or equal) on the set Z of integers.
    2. Set inclusion ⊆ on a collection C of sets.
    3. Relation ⊥ (perpendicular) on the set L of lines in the plane.
    4. Relation ∥ (parallel) on the set L of lines in the plane.
    5. Relation | of divisibility on the set P of positive integers.

    Determine which of the relations are: (a) reflexive, (b) symmetric, (c) anti symmetric, (d) transitive.

30. Establish the following logical equivalences where A is a proposition not involving any quantifiers.

    a. $(\forall x P(x)) \vee A \Leftrightarrow \forall x(P(x) \vee A)$
    b. $(\exists x P(x)) \vee A \Leftrightarrow \exists x(P(x) \vee A)$
    c. $(\forall x P(x)) \wedge A \Leftrightarrow \forall x(P(x) \wedge A)$
    d. $(\exists x P(x)) \wedge A \Leftrightarrow \exists x(P(x) \wedge A)$

(2 × 4 = 8 Weightage)

D 52725 (Pages : 4) Name..............................

Reg. No..............................

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

Core Course (Mathematics)

MAT 1B 01—FOUNDATIONS OF MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all twelve** *questions.*

1. Find the number of elements in the power set of {positive divisors of 6}.
2. Find $A \oplus B$ for A = {1, 2, 3} , B = {1, 2, 3}.
3. What is symmetric relation ?
4. Let *f* and *g* be functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$ then $f \circ g$ is :
5. Find the domain of the real valued function $f(x) = \sqrt{25 - x^2}$.
6. What is the cardinal number of the set $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$?
7. Check whether the function $h(t) = |t^3|$ is even, odd or neither.
8. The graph of $y = x^2$ is shifted 1 unit to the right and 4 units down. Write equation of the new graph.
9. Find $\lim_{x \to \pi} \sin(x - \sin x)$.
10. State the contrapositive of the implication :

    "If it snows tonight, then I will stay at home."
11. Define a Tautology.
12. What is the truth value of $\forall x\, p(x)$ where $p(x)$ is the statement "$x^2 < 10$" and the domain consists of positive integers ?

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. If A = {1, 2}, B = {*a*, *b*, *c*} and C = {c, *d*}, find (A × B) ∩ (A × C).

14. Let R be the relation R = {(1, *b*), (2, *a*), (2, *c*)} and S be the relation S = {(*a*, *y*), (*b*, *x*), (*c*, *y*), (*c*, *z*)}. Find R ∘ S.

15. Find *x* and *y* if (*y* – 2, 2*x* +1) = (*x* – 1, *y* + 2).

16. Suppose *f* : A → B is a constant function when will *f* be :

(a) one-to-one ; (b) onto.

17. Let A = {*a*, *b*} and B = {1, 2, 3}, find the number of functions from :

(a) A into B ; (b) B into A.

18. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\begin{cases}3x-1 & \text{if } x>3\\ x^2-2 & \text{if } -2\le x\le 3\\ 2x+3 & \text{if } x<-2.\end{cases}$

Find (a) *f* (–1) ; (b) *f* (2).

19. For the function $f(x)=\begin{cases}0 & x<0\\ 1 & x\ge 0\end{cases}$ find $\lim_{x\to 0} f(x)$ or explain why they do not exist.

20. Evaluate $\lim_{x\to 4}\dfrac{4x-x^2}{2-\sqrt{x}}$.

21. If $2-x^2\le g(x)\le 2\cos x$ for all *x*, find $\lim_{x\to 0} g(x)$.

22. Let *p* and *q* be the proposition "The election is decided" and "The votes have been counted" respectively. Express the compound proposition $\neg q\vee(\neg p\wedge q)$ as an English sentence.

23. Show that $\neg(p\vee q)$ and $\neg p\wedge\neg q$ are logically equivalent.

24. What is the negation of the statement $\forall x\left(x^2>x\right)$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Let A be a set of non-zero integers, and let ~ be a relation on A × A defined by (a,b) ~ (c,d) whenever $ad = bc$. Prove that ~ is an equivalence relation.

26. Let A = {1, 2, 3, 4}, consider the following relation R on A.

    R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}.

    (a) Draw its directed graph ; (b) Find $R^2 = R \circ R$.

27. Consider the functions $f : A \to B$ and $f : B \to C$ prove the following :

    (a) If f and g are one-to-one then $f \circ g$ is one-to-one.

    (b) If f and g are onto functions then $g \circ f$ is an onto function.

28. Let R_1 and R_2 be relations on a set A represented by the matrix :

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the matrix representing (i) $R_1 \cup R_2$; (ii) $R_1 \cap R_2$.

29. Find the continuous extension to $x = 3$ of the function $f(x) = \dfrac{x^2 - 9}{x - 3}$.

30. Let $f(x) = \begin{cases} 0 & x \le 0 \\ \sin \frac{1}{x} & x > 0. \end{cases}$

    (a) Does $\lim_{x \to 0^+} f(x)$ exist ? If so what is it ? If not, why not ?

    (b) Does $\lim_{x \to 0^-} f(x)$ exist ? If so what is it ? If not, why not ?

    (c) Does $\lim_{x \to 0} f(x)$ exist ? If so what is it ? If not, why not ?

31. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

32. Express the statements "All lions are fierce", "some lions do not drink coffee", "some fierce creature do not drink coffee" using predicates and quantifiers, assuming the domain consists of all creatures.

33. Give a proof by contradiction of the theorem "If 3 n + 2 is odd, then n is odd".

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Let A = {1, 2, 3, 4}. Consider the relations R and S on A given by R = {(1,1), (1,2), (2,3), (3,1), (3,3)} ; S = {(1, 2), (1, 3), (2, 1), (3, 3)}.

Find the matrices (a) $M_{R \cap S}$; (b) $M_{R \cup S}$; (c) M_{R^c} ; (d) $M_{R \circ S}$; (e) M_{S^2} .

35. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, prove that $\lim_{x \to c} (f(x) + g(x)) = L + M$.

36. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

(2 × 10 = 20 marks)