C 2217 (Pages : 4) Name....................................

Reg. No...............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. What do you mean by a non-linear differential equation ?
2. What are the steps for finding general solution of a non-homogeneous equation $y'' + ay' + by = r(x)$.
3. Find Wronskian of $y_1(x) = e^{-2x}$ and $y_2(x) = e^{-3x}$.
4. What is $L[1]$?
5. Define periodic function.
6. What is unit step function ?
7. State Convolution theorem.
8. Define and give an example of an even function.
9. Give one dimensional wave equation.
10. Write the formula for Runge Kutta method.
11. Give formula for Euler method.
12. Give a formula for an error for Simpson's rule.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find the particular integral for $y'' - 4y' + 3y = 10e^{-2x}$.

14. Solve $\left(D^2 - 2D + 3\right) y = x^3 + \sin x$.

15. Find $W\left[e^{\lambda_1 x}, e^{\lambda_2 x}\right]$.

16. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at} F(t)$.

17. Show that the Laplace transform is a linear operation.

18. Find $L\left[t^2 \cos t\right]$.

19. Using convolution property, find $L^{-1}\left[\frac{1}{s^2(s-a)}\right]$.

20. Find the Fourier series of $f(x) = x^2$, when $-1 < x < 1$ with period 2.

21. Show that $u = \cos 4t \sin 2x$ is a solution of the wave equation.

22. Apply Picard's iteration upto 3 steps to solve $y' = 1 + y^2$ and $y(0) = 1$.

23. Compute $\int_0^1 x^2\, dx$ by the rectangular rule with $h = 0.5$.

24. Solve $\int_1^2 \frac{1}{x}\, dx$ by Trapezoidal rule with $n = 4$ and compare the estimate with the exact value of the integral.

(9 × 2 = 18 marks)

Part C

*Answer any **six** questions.*
Each question carries 5 marks.

25. Solve $x^2 y'' + 7xy' + 13y = 0$.

26. Solve the non-homogeneous equation $y'' - 4y' + 3y = 10e^{-2x}$.

27. Obtain the Fourier cosine series representation of $f(x) = e^x, x \in [0, \pi]$.

28. Find the inverse transform of $\dfrac{s^3 - 4s^2 + 4}{s^2\left(s^2 - 3s + 2\right)}$.

29. Solve $u_x + u_y = 2(x + y)u$.

30. Express the function $f(x) = x^2$, when $-1 < x < 1$ as a Fourier series with period 2.

31. Solve the integral equation $y = 1 - \int_0^t (t - \tau)\, y(\tau)\, d\tau$.

32. Find an approximate value of $\log_e 5$ by calculating $\int_0^5 \frac{dx}{4x + 5}$ by Simpson's rule of integration.

33. Solve by Picard's method $y' - xy = 1$, given $y = 0$ when $x = 2$. Also find $y(2.05)$ correct to four places of decimal.

(6 × 5 = 30 marks)

Part D

*Answer any **two** questions.*
Each question carries 10 marks.

34. (a) Solve $x^2y'' - 4xy' + 6y = 21x^{-4}$.

(b) Solve $\left(D^2 - 2D + 1\right) y = 3x^{3/2}e^x$.

Turn over

35. Find the solution of the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

corresponding to the triangular initial deflection

$$f(x) = \begin{cases} \frac{2k}{l} x, & \text{when } 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l - x), & \text{when } \frac{l}{2} < x < l \end{cases}$$

and the initial velocity zero.

36. Find the Fourier series of $f(x) = \begin{cases} 2, & -2 \le x < 0 \\ x, & 0 \le x < 2 \end{cases}$ in $(-2, 2)$.

(2 × 10 = 20 marks)

C 80917 (Pages : 4) Name.......................................

Reg. No...................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write Euler-Cauchy equation.
2. State the first shifting theorem for Laplace transforms.
3. Define odd function. Give an example.
4. What do you mean by a periodic function ? Give an example.
5. Find $L\left(t+e^{t}\right)$.
6. Find $L^{-1}\left(\dfrac{s}{s^{2}-a^{2}}\right)$.
7. If $L(f(t))$ and $f'(t)$ exists, find $L(f'(t))$.
8. Define Half range Fourier sine series.
9. Write one dimensional wave equation.
10. Write the characteristic equation of the equation $y''+10y'+29y=0$.
11. Write the error estimate the Trapezoidal rule.
12. Find the Wronskian of y_1, y_2 where $y_1=\cos x$, $y_2=\sin x$.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $y'' + y = 0,\ y(0) = 3,\ y(\pi) = -3.$

14. Find a basis of solutions for $x^2 y'' - xy' + y = 0.$

15. Solve $\left(D^2 + w^2\right) y = 0.$

16. Solve $x^2 y'' - 2.5\, xy' - 2y = 0.$

17. Find a particular solution of $y'' - 3y' - 4y = -8e^t \cos 2t.$

18. Show that Laplace transform is a linear operator.

19. Find the Laplace transform of sinh at.

20. Find $L^{-1}\left(\dfrac{1}{(s-1)^4}\right).$

21. Find $L\left(\dfrac{1-e^t}{t}\right).$

22. Find the Fourier series of $f(x) = x - x^2, -\pi < x < \pi, f(x + 2\pi) = f(x).$

23. A town wants to drain and fill a small polluted swamp. The swamp averages 5 feet deep. About how many cubic yards of dirt will it take to fill the area after the swamp is drained.

24. Show that the function $y = e^x \cos y$ is a solution of the two dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve the non-homogeneous equation :

$$y'' - y' - 2y = 10 \cos x.$$

26. Solve the differential equation :

$$\left(D^2 + 4D + 4\right) y = \frac{e^{-2x}}{x^2}.$$

27. Find the inverse transform of $\frac{1}{s(s+1)(s+2)}$.

28. Find $L(t \sin at)$.

29. Solve :

$$y(t) = t^3 + \int_0^1 \sin(t-u)\, y(u)\, du.$$

30. Find the Fourier series for $f(x) = |x|$ is $[-\pi, \pi]$ with $f(x+2\pi) = f(x)$.

31. Find the approximate solution to $y' = 1 + y^2$, $y(0) = 0$.

32. Compare the values of $\int_0^1 x\, dx$ obtained by using Trapezoidal and Simpson's rule.

33. Given $y' = -y$, $y(0) = 1$. Find the value of y' at x, $x = (0.01)(0.01)(0.04)$ by improved Euler method.

(6 × 5 = 30 marks)

Part D

*Answer any **two** questions.*
Each question carries 10 marks.

34. Solve : $x^2 y'' - zxy' + 2y = \left(3x^2 - 6x + 6\right)e^x$

$y(1) = 2 + 3e \qquad y'(1) = 30.$

35. Find the inverse transform of $\frac{1}{s^2}\left(\frac{s+1}{s^2+9}\right)$.

36. Find the Fourier series of $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$.

Hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots = \frac{\pi^2}{12}$.

(2 × 10 = 20 marks)

C 61250 (Pages : 3)

Name...

Reg. No.....................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS–UG)

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. State the Existence and Uniqueness theorem for initial value problem.
2. Define and give an example of an even function.
3. What do you mean by a non-linear differential equation ?
4. Solve $y'' - y' - 2y = 0$.
5. Define a unit step function.
6. State the existence theorem for Laplace transforms.
7. Find $L^{-1}\left(\frac{a}{s^2 - a^2}\right)$.
8. Find $L\left(t^{-1/2}\right)$.
9. Define a rectangular wave.
10. Write the 2-dimensional Poisson equation.
11. Give a formula for an error for Simpson's rule.
12. Write the formula for Runge Kutta method.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the particular integral for $y' + 4y = 8x^2$.
14. Find a basis for the solution of the differential equation $y'' + y = 0$.

Turn over

15. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at}F(t)$.

16. Solve $3y'' - 8y' - 3y = 0, y(-3) = 1, y(3) = \left(\frac{1}{e^2}\right)$.

17. Find $L\left(e^{-at}\cos\beta t\right)$.

18. If $f(x)$ is a periodic function of x of period p, show that $f(ax), a \neq 0$, is a periodic function of x of period $\frac{p}{a}$.

19. Find the Fourier series of $f(x) = x + |x|, -\pi < x < \pi$.

20. Show that $u = e^{-t}\sin x$ is a solution of heat equation.

21. Apply Picards iteration to solve $y' = y - x^2, y(0) = 1$ also find y (0.1) and y (0.2).

22. Evaluate $\int_{-3}^{3} x^4 dx$ using Simpson's rule.

23. What do you mean by convolution ?

24. Evaluate $\int_0^6 \frac{1}{1+x} dx$ by Trapezoidal rule.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Solve $\left(4x^2D^2 + 12xD + 3\right)y = 0$.

26. Find a general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.

27. Find the Laplace transform of $(t-1)^2 u(t-1)$.

28. Find $L^{-1}\left(\frac{4\left(e^{-2s}-2^{e-5s}\right)}{s}\right)$.

29. Solve $u_{xy} = u_x$.

30. Find the Fourier series of $f(x)=\begin{cases}-k, \text{ if } -\pi<x<0; \\ k, \text{ if } 0<x<\pi,\end{cases}$ and $f(x+2\pi)=f(x)$.

31. Given $y'=-y, y(0)=1$. Find the value of y at $x=(0.01)(0.01)(0.04)$ by improved Euler method.

32. Find approximate solution to $y'+y=e^x, y(0)=0$.

33. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Simpson's rule.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $x^2y''-2xy'+2y=x^3\sin x$.

35. Solve the integral equation $y(t)=t+\int_0^t y(\tau)\sin(t-\tau)\,d\tau$.

36. Find the Fourier series $f(x)=\begin{cases}x+x^2, \text{ if } -\pi<x<\pi; \\ \pi^2 \quad , \quad \text{if } x=\pm\pi.\end{cases}$

(2 × 10 = 20 marks)

18/7/18

D 41988 (Pages : 3)

Name.....................................

Reg. No.................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(CUCBCSS—UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What do you mean by fundamental period ?
2. Write the characteristic equation of the differential equation $2y''+10y'+25y=0$.
3. Write the 2 dimensional wave equation.
4. Give an example of a function which is neither odd nor even.
5. State the existence and uniqueness theorem for initial value problem.
6. Find the Laplace transform of $f'(t)$.
7. Find the Wronskian of $e^{\lambda x}$ and $xe^{\lambda x}$.
8. Find $L^{-1}\left(\dfrac{1}{s/a-1}\right)$.
9. Find $L(t+e^{t})$.
10. State the second shifting theorem of Laplace transform.
11. Write the error estimate for Simpson's rule.
12. Write the formula for Runge-Kutta method.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the Fourier cosine series of $f(x)=\pi-x, 0<x<\pi$.
14. Show that $u=e^{-w^2c^2t}\sin wx$ is a solution of heat equation.
15. Apply Picard's iteration to solve $y'=y-x^2, y(0)=1$. Also find y (0, 1) and y (0, 2).
16. By reducing to first order solve $y''+(y')^2=0$.

Turn over

17. Find fundamental set of solutions of $2t^2y'' + 3ty' - y = 0$ $t > 0$. Given that $y_1(t) = t^{-1}$ is a solution.

18. Solve $y'' + 8y' + 16y = 0$.

19. Find the inverse transform of $\frac{4}{(s+1)(s+2)}$.

20. Solve the initial value problem $y'' + y' - 6y = 1$ $y(0) = 0,\ y'(0) = 1$.

21. Find $L\left(te^{-t}\sin t\right)$.

22. Using convolution property find $L^{-1}\left[\frac{1}{s\left(s^2+4^2\right)}\right]$.

23. Use Trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$.

24. Solve the partial differential equation $u_{yy} + 4u = 0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve $\left(x^2D^2 + 3xD + 1\right)y = 0$.

26. Find the general solution of the differential equation $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$.

27. Find the Laplace transform of $e^{-2t}u(t-3)$.

28. Find $L^{-1}\left(\frac{se^{-2s}}{s^2+\pi^2}\right)$.

29. Find the Fourier series expansion of :

$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < 2\pi \end{cases}.$$

30. Solve $xy'' - y' = (3+x)x^2e^x$.

31. Find the solution of $u_x + u_y = 0$ by separating variables.

32. Find approximate solution of $y' + y = e^x$ $y(0) = 0$.

33. Obtain the half-range cosine series of $f(x) = x$ when $0 < x < 2$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the non-homogeneous equation : $y'' - y' - 2y = 10\cos x$.

35. Solve the initial value problem $y'' + 2y' + 2y = r(t)$

$$\text{where } r(t) = \begin{cases} 10\sin 2t & \text{if } 0 < t < \pi \\ 0 & \text{if } t > \pi \end{cases}$$

$y(0) = 1 \quad y'(0) = -5.$

Using Laplace transforms.

36. Find the Fourier series of the functions

$$f(x) = \begin{cases} x + x^2 & -\lambda < x < \pi \\ \pi^2 & x = \pm\pi \end{cases}$$

Deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$.

(2 × 10 = 20 marks)

C 23323 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2017

(CUCBCSS–UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the general solution of $y^{11} + w^2y = 0$.
2. Find $(D^2 + 3D) \cosh 3x$.
3. Write the general form of the Cauchy-Euler equation.
4. Find $L\left(a + bt + ct^2\right)$.
5. $L(f') =$ ———.
   (a) $L(f) - S f(0)$. (b) $L(f) - f(0)$.
   (c) $SL(f) - f(0)$. (d) $SL(f) - f'(0)$.
6. $L^{-1}\left(\frac{1}{s^2 + a^2}\right) =$ ———.
7. Write the second shifting theorem of Laplace transform.
8. Sketch $f(x) = |x|$ for $-\pi < x < \pi$.
9. Find a_0 in the Fourier series expansion of $f(x) = x^2, -\pi < x < \pi$.
10. Write the Picard's iteration formula to find a numerical solution of $y' = f(x, y), y(x_0) = y_0$.
11. Write the one-dimensional wave equation.
12. Find the period of $\cos \pi x$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the general solution of $\left(9D^2 + 6D + 1\right) y = 0$.
14. Solve $x^2y'' - 3xy' + 4y = 0$.
15. Find the Laplace transform of $(t + 1)^2 e^t$.

Turn over

16. Find $L^{-1}\left(\frac{60+6s^2+s^4}{s^7}\right)$.

17. If $f(t)=t$ and $g(t)=e^{at}$ find the convolution $(f*g)(t)$.

18. Find the Fourier cosine series of the function $f(x)=\pi-x$ in $0<x<\pi$.

19. Prove that product of an even function and an odd function is an odd function.

20. Apply Picard's method to solve the initial value problem $y'=x^2+y,\ y(0)=-1$.

21. Solve the partial differential equation $u_{xy}=u$.

22. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule taking $h=1$.

23. Solve the initial value problem $y''+y'=0,\ y(0)=5,\ y'(0)=-3$ using Laplace transform.

24. Derive the Euler's formula to solve the differential equation $y'=f(x,y),\ y(x_0)=y_0$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find the general solution of $y''+2y'+y=2x+x^2$.

26. Find the general solution of $(D^2+3D-4)y=8\cos 2x$.

27. Solve $y''+y=\sec x$ by the method of variation of parameters.

28. Find the inverse Laplace transform of $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}$.

29. Solve the initial value problem by Laplace transform $y'+3y=10\sin t,\ y(0)=0$.

30. Find the Fourier series of the function $f(x)=x^2, -\pi<x<\pi$.

31. Using Laplace transform solve the integral equation $y(t)=1-\int_0^t (t-\tau)\,y(\tau)d\tau$.

32. Find the deflection $u(x,t)$ of a string of length $L=2\pi$ when $c^2=1$, the initial velocity is zero and initial deflection is $0.1(\pi^2-x^2)$.

33. Evaluate $\int_1^7 \frac{dx}{x}$ using Simpson's rule by dividing [1, 7] into 6 equal parts.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the initial value problem $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$, $y(0) = 0$, $y'(0) = 1$.

35. Solve $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0$. Using Laplace transform. Given $y'(0) = 1$.

36. Use improved Euler's method to determine $y(0.2)$ in two steps from $\frac{dy}{dx} = x^2 + y$, given that $y(0) = 1$.

(2 × 10 = 20 marks)

C 25631 (Pages : 2) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2017

(UG—CCSS)

Complementary Course

MM 4C 04—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries a weight of ¼.

I. 1 Find the general solution of $y'' - y = 0$.

2 Find the 'Wronskian' of e^x and xe^x and verify the linear independence.

3 Write the Euler- Cauchy equation.

4 Find the Laplace transform of $f(t) = e^{at}$.

5 Find $L(f)$ from $f(t) = \sin^2(t)$.

6 Define the Dirac delta function.

7 Find the fundamental period of $\cos(2\pi x / k)$.

8 Write the Fourier series of an even function of period 2L.

9 What is one dimensional wave equation ?

10 Write Picard's iteration formula.

11 What is the Lipschitz condition ?

12 State Trapezoidal rule.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *questions.*
Each question carries a weightage of 1.

II. 13 Solve $y'' + 8y' + 16y = 0$.

14 Find the Wronskian of the functions $x^4, x^4 \log x$.

15 Solve $x^2y'' + 7xy' + 13y = 0$.

16 Find the Laplace Transform of e^{a-bt}.

Turn over

17 Find the inverse Laplace transform of $F(s)=\frac{1}{(s+1)^2}$.

18 Find a solution $u(x, y)$ of the partial differential equation $u_{xx} - u = 0$.

19 Solve $u_{yy} = 0$.

20 Use Simpson's rule with $n = 4$ to approximate $\int_0^1 5x^4\, dx$.

21 Find approximate solution to the initial value problem $y' = 1 + y^2, y(0) = 0$.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*

Each question carries a weightage of 2.

III. 22 Solve $y'' + 2y' - 35y = 12e^{5x} + 37 \sin 5x$.

23 Solve $y'' + 2y' + y = e^{-x} \cos x$.

24 Solve the initial value problem $y'' + y' - 6y = 1, y(0) = 0, y'(0) = 1$.

25 Using convolution find the inverse $h(t)$ of $H(s)=\frac{1}{s(s^2+4)}$.

26 Find the Fourier series representing x in the interval $[-\pi, \pi]$.

27 Using Euler's method, find y when $y' + 2xy^2 = 0, y(0) = 1, h = 0.2$.

28 Solve $y' = -0.2xy$; $y(0) = 1$ by the classical Runge-Kutta method (10 steps, $h = 0.2$).

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*

Each question carries a weightage of 4.

IV. 29 Solve $4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3$.

30 Using the Laplace transform, solve $y'' + 6y' + 8y = e^{-3t} - e^{-5t}$, $y(0) = 0, y'(0) = 0$.

31 Find the Fourier series of $f(x)=\begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ x - x \text{ if } \pi/2 < x < 3\pi/2 \end{cases}$.

(2 × 4 = 8 weightage)

C 3992 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2016

(CUCBCSS—UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Objective type.
Answer all **twelve** *questions.*

1. What do you mean by a non linear differential equation ?
2. Write the solution for an homogeneous differential equation with $2 \pm 3i$ as a double root for its auxillary equation.
3. State the Existence and Uniqueness theorem for initial value problem.
4. What is the Shifting property of Laplace transform ?
5. Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$.
6. What is $L\left(t^{-1/2}\right)$?
7. How do you define a sawtooth wave function ?
8. Define and give an example of an odd function.
9. Write the Euler's formula for a fourier series of a periodic function.
10. Write the formula for Runge Kutta method.
11. Does the inital value problem $xy' = 4y, y(0) = 1$ has solution ? Give reason.
12. Give a formula for an error for Simpson's rule.

(12 × 1 = 12 marks)

Part B

Short answer type.
Answer any **nine** *questions.*

13. Find the particular integral for $y' + 4y = 8x^2$.
14. Find a basis for the solution of the differential equation $y' - y = 0$.
15. Find $W\left[e^{\lambda x}, xe^{\lambda x}\right]$.

Turn over

16. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at} F(t)$.
17. Show that the Laplace transform is a linear operation.
18. Find L ($\sin^2 t$).
19. Show that the function $f(x)$ = constant is a periodic function of period p for every positive p.
20. Find the fourier series of $f(x) = -1, -\pi < x < \pi$.
21. Show that $u = \cos 4t \sin 2x$ is a solution of the wave equation.
22. Apply Picard's iteration upto 4 steps to solve $y' = y$ and $y(0) = 1$.
23. Show that the initial value problem $y' = \sqrt{|y|}, y(0) = 0$ does not have a unique solution.
24. What do you mean by Lipschitz condition ?

(9 × 2 = 18 marks)

Part C

Short essay.
Answer any **six** *questions.*

25. Solve $x^2 y'' + 7xy' + 13y = 0$.
26. Verify $y_p = 2x^2 - 6x + 7$ is a solution for $y'' + 3y' + 2y = 4x^2$ and find a general solution.
27. Find the Laplace transform of $f(t) = \begin{Bmatrix} t, 0 < t < 4 \\ 5, t > 4 \end{Bmatrix}$.
28. Find $L^{-1}\left(\dfrac{3s+16}{s^2 - s - 6}\right)$.
29. Solve $u_y = 2xyu$.
30. Find the cosine series of $f(x) = x, 0 < x < L$.
31. Solve the integral equation $y(t) = t + \int_0^t y(\tau) \sin(t - \tau)\, d\tau$.
32. Using Simpson's rule evaluate the integral $\int_1^2 x\, dx$ with $n = 4$ and hence find an upper bound for the error incurred.
33. Apply improved Euler method in 3 steps to solve $y' = y, y(0) = 1$ with $h = 0.1$. Also find the error occurred.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*

34. (a) Solve $x^2y'' - 4xy' + 6y = 21x^{-4}$.

(b) Solve the initial value problem $(D^2 + 4)\,y = -12 \sin 2x$, $y(0) = 1.8$, $y'(0) = 5$.

35. Solve the integral equation $y(t) = t + \int_0^t y(\tau) \sin(t - \tau)\, d\tau$.

36. Find the fourier series of $f(x) = \begin{cases} x^2, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \frac{\pi^2}{4}, & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$.

(2 × 10 = 20 marks)

C 5162

(Pages : 2)

Name.......................................

Reg. No....................................

FOURTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MAY 2016

(UG—CCSS)

Complementary Course

MM 4C 04—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries a weightage of ¼.

1. Is the equation $y'' = \sqrt{y'^2 + 1}$ linear or non-linear ?
2. Is $y = e^{-x}$ a solution of $y'' + y = 0$?
3. Solve $y'' + 7y = 0$.
4. Find the Laplace Transform of $f(t) = t^9$.
5. Define the unit impulse function.
6. Find the inverse Laplace Transform of $F(s) = 2/2s - 3$.
7. Find the fundamental period of $\cos 2\pi x$.
8. Is the following function even or odd or neither $x^2 \cos nx$.
9. What is the 2 dimensional Laplace equation ?
10. Define the Lipschitz condition.
11. What is an initial value problem ?
12. State Simpson's rule.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *questions.*
Each question carries a weightage of 1.

13. Find the Wronskean of the functions $y_1 = x^2$ and $y_2 = x^2 \ln x$.
14. Find the solution of $y'' + 4y' + 4y = 0$.
15. Find the Laplace Transform of $f(t) = \cosh 7t$.

Turn over

16. Find the inverse Laplace Transform of $F(s) = (s-2)^{-5}$.

17. Is $u = \sin ct \sin x$ a solution of the wave equation (with suitable c) ?

18. Solve $u_{xy} = u_x$.

19. Solve $u_y = u$.

20. Show that $f(x, y) = |\sin y| + x$ satisfies the Lipschitz condition with $m = 1$.

21. Apply Euler's method any compute $y_1, y_2, \ldots\ldots, y_5$ with $h = 0.02$, given $y' = (y-x)/(y+x), y(0) = 1$.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries a weightage of 2.

22. Solve $y'' + y' = 2 + 2x + x^2, y(0) = 8, y'(0) = -1$.

23. Solve $x^2y'' + xy' + y = 0$.

24. Find the Laplace Transform of $F(t) = te^{-2t} \sin 2t$.

25. State the convolution theorem and use it to evaluate the inverse $h(t)$ of $H(s) = s(s^2 + a^2)^{-2}$

26. Find the Fourier sine series of $f(x) = \pi - x, 0 < x < \pi$.

27. Using Runge Kutta Method, find y when $x = 0.2$, given $y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$.

28. Evaluate $\int_0^1 \frac{dx}{1 + x^2}$ using Trapezoidal rule, taking $h = 0.25$.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries a weightage of 4.

29. Solve $y'' + y = \sec x$.

30. Solve by the method of Laplace Transforms : $y'' + y = t, y(0) = 1, y'(0) = -2$.

31. Find the Fourier series expansion of $f(x) = \frac{x^2}{2}, -\pi < x < \pi$. Hence show that

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots\ldots = \frac{\pi^2}{6}.$$

(2 × 4 = 8 weightage)

C 81846 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2015

(U.G.–CCSS)

Complementary Course—Mathematics

MM 4C 04—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Unit I

Answer all **twelve** *questions.*

1. Which of the following is not a solution of $y'' - y = 0$?

   (a) e^x. (b) e^{-x}.

   (c) $e^x + e^{-x}$. (d) $1 + e^x$.

2. Find the general solution of $y'' + 9y' + 20y = 0$.

3. Write a pair of basis solutions of $x^2 y'' - 4xy' + 6y = 0$.

4. If $y_1 = e^{2x}$, $y_2 = e^{-x}$ find $w(y_1, y_2)$.

5. Find the Laplace transform of sin wt.

6. Find $L^{-1}\left(\dfrac{60 + 6s^2 + s^4}{s^7}\right)$.

7. $f(x) = x^3 + 2x^2$ is an :

   (a) Even Function. (b) Odd function.

   (c) Neither even nor odd. (d) Either even or odd.

8. Write the one dimensional wave equation.

9. Find the smallest period p of cos πx.

10. Plot the function $f(x) = x|x|$, $-\pi < x < \pi$.

11. Find a solution of the partial different equation $u_{xx} - u = 0$.

12. Write the iteration formula for the Picards methods.

(12 × ¼ = 3 weightage)

Turn over

Unit II

Answer any **nine** *questions.*

13. Apply $(D + 5)^2$ to $\sin 5x + 5x$.

14. Find the general solution of $y'' + 10y' + 25 = 0$.

15. Find two linearly independent solutions of $x^2 y'' - 2{\cdot}5x\, y' - 2 = 0$.

16. Find a particular solution of $y'' - 5y' + 6y = e^x$.

17. Reduce to first order and solve $2xy'' = 3y'$.

18. Find the Laplace transform of $(t+1)^2 e^t$.

19. If $L[f(x)] = F(s)$ prove that

$$L\left(\frac{f(x)}{x}\right) = \int_s^\infty F(p)\, dp.$$

20. Find $L^{-1}\left[\frac{1}{s(1+2s)}\right]$.

21. Show that $u = x^2 + y^2$, $f = 4$ satisfies the Poissons equation.

22. Find the solutions of $u_{xx} + u_{yy} = 0$ by separating the variables.

23. Find a_0 in the Fourier series expansion of $f(x) = \begin{cases} o & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1. \\ o & \text{if } 1 < x < 2 \end{cases}$

24. Find first two approximate solutions $y_1(x)$ and $y_2(x)$ of the initial value problem $y' = x + y$, $y(0) = -1$ using Picard's method.

(9 × 1 = 9 weightage)

Unit III

Answer any **five** *questions.*

25. Solve the initial value problem $y'' + 1{\cdot}5y' - y = 12x^2 - 6x^3 - x^4$, $y(0) = 4, y'(0) = 8$.

26. Using method of variation of parameters solve $y'' + y = \sec x$.

27. Find $t * e^t$ where * denotes convolution.

28. Using method of partial fractions find $L^{-1}\left[\frac{s^2 + 9s - 9}{s^3 - 9s}\right]$.

29. Using convolution find the inverse Laplace transform of $\frac{1}{s\left(s^2+4\right)}$.

30. Solve the integral equation $y(t) = t + \int_0^t y(\tau)\sin(t-\tau)\,d\tau$.

31. Find the Fourier series expansion of $f(x) = x^2,\ -\pi < x < \pi$.

32. Using Simpson's rule with $n = 4$ estimate $\int_0^1 5x^4\,dx$.

(5 × 2 = 10 weightage)

Unit IV

Answer any **two** *questions.*

33. Solve $x^2 y'' - 4xy' + 6y = 21x^{-4}$.

34. Using Runge-Kutta method solve the initial value problem

$y' = x + y,\ y(0) = 0,\ h = 0{\cdot}2$.

35. Find the Fourier series of $f(x) = \begin{cases} \frac{1}{2}(\pi + x), & -\pi \le x < 0 \\ \frac{1}{2}(\pi - x), & 0 \le x < \pi \end{cases}$

(2 × 4 = 8 weightage)

C 62072 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Mathematics (Complementary Course)

MM 4C 04—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part I

Answer **all** *questions.*
Each question carries ¼ weightage.

1. Apply the operator $D^2 + 3D$ on $e^{-x} + e^{2x}$.
2. Find the general solution of $y'' + y = 0$.
3. Verify whether linearly independent : e^{3x} and xe^{3x}.
4. Find the Laplace Transform of e^{a-bt}.
5. Find the inverse Laplace transform of $\frac{1}{s^3}$.
6. Define the unit step function $u(t-a)$.
7. Give an example of a function without a fundamental period.
8. Define an even function.
9. What is the *one* dimensional wave equation ?
10. Give the general form of an IVP of 1^{st} order.
11. What is the Lipschitz condition ?
12. Find an upper bound for error in approximating using Trapezoidal rule, $n = 4$ for $\int_1^2 x^2\,dx$.

(12 × ¼ = 3 weightage)

Turn over

Part II

Answer **all** *questions.*
Each question carries 1 *weightage :*

13. Solve the IVP $9y'' + 6y' + y = 0$, $y(0) = 4$, $y'(0) = \frac{-13}{3}$.

14. Solve $x^2 y'' - 3xy' + 4y = 0$.

15. Solve $y'' + 4y = \sin 3x$.

16. Find the Laplace transform of $t \cos wt$.

17. Find the inverse Laplace transform of $\frac{1}{(s-7)^9}$.

18. Are the following functions even, odd or neither.

(i) $f(x) = x^2$.

(ii) $f(x) = e^{-|x|}$.

(iii) $\sin x + \cos x$.

(iv) $x|x|$.

19. Check whether $u = e^x \sin y$ is a solution of the Laplace equation.

20. Find $y_2(x)$ by Picard's iteration for the IVP $y' = 1 + y^2$, $y(0) = 0$.

21. Compute y_3 by Euler's method with $h = 0.2$ for the IVP $y' = x + y$, $y(0) = 0$.

(9 × 1 = 9 weightage)

Part III

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Solve $y'' + y = \sec x$ by the method of variation of parameters.

23. Using convolution, find the inverse Laplace transform of $\frac{1}{s^2(s-1)}$.

24. Using Trapezoidal rule, evaluate $\int_0^1 e^{-x^2} dx$ with $n = 10$.

25. Solve $u_{xy} = -u_x$.

26. Find the Fourier cosine series of $f(x) = \pi - x, 0 < x < \pi$.

—

27. Use Simpson's Rule with $n = 4$ find $\int_1^2 x \, dx$.

28. Find the inverse Laplace transform of : $\ln\left(1 + \frac{9}{s^2}\right)$.

(5 × 2 = 10 weightage)

Part IV

Answer any **two** *questions.*
Each question carries 4 weightage :

29. Using Laplace transforms, solve the IVP $y'' + 6y' + 8y = e^{-3t} - e^{-5t}$, $y(0) = y'(0) = 0$.

30. Find the Fourier series of :

$$f(x) = \begin{cases} k, & -\pi/2 < x < \pi/2 \\ o, & \pi/2 < x < 3\pi/2 \end{cases}$$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$.

31. Solve $y' = x + y, y(0) = 0$ by the Runge-Kutta method with $h = 0.2$ in five steps.

(2 × 4 = 8 weightage)

C 41459 (Pages : 3) Name.................................

Reg. No.................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

MM 4C 04—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Unit I

(*Answer all* **twelve** *questions.*)

1. Which of the following is not a solution of $y''+y=1$?

   (a) $1+\cos x$. (b) $1+\sin x$.

   (c) $2(1+\cos x)$. (d) $1+\cos x+\sin x$.

2. Find the general solution of $y''-8y=0$.

3. Auxillary equation corresponding to $x^2y''-2.5xy'-2y=0$:

   (a) $m^2-2.5m-2=0$. (b) $m^2-3.5m-2=0$.

   (c) $m^2-1.5m-2=0$. (d) $m^2+3.5m-2=0$.

4. If $y_1=\sin 2x$ and $y_2=\cos 2x$. Find $w(y_1, y_2)$.

5. Find the Laplace transform of $2t+6$.

6. Find $L^{-1}\left[\frac{5s}{s^2-25}\right]$.

7. Define the unit step function $u(t-a)$.

8. $f(x)=x\cos nx$ is :

   (a) An even function. (b) Odd function.

   (c) Neither even nor odd. (d) Either even or odd.

9. Write the general form of the two dimensional Poisson's equation.

10. Solve the partial differential equation $u_{xy}+u_x=0$.

11. Find the smallest positive period p of $\sin 2x$.

12. Sketch the function $f(x)=|x|$ for $-\pi<x<\pi$.

($12 \times \frac{1}{4} = 3$ weightage)

Turn over

Unit II

(*Answer any* **nine** *questions.*)

13. Apply $(D+1)(D-2)$ to xe^{-x}.

14. Find the general solution of $(D^2 - D - 2)\,y = 0$.

15. Find two linear independent solutions of $x^2y'' - 3xy' + 4y = 0$.

16. Verify that $y_p = 2x^2 - 6x + 7$ is a solution of $y'' + 3y' + 2y = 4x^2$.

17. Reduce to first order and solve $y'' = y'$.

18. Find the Laplace transform of $5\,e^{2t}\sinh 2t$.

19. If $L(f(t)) = F(s)$ prove that $L\left[e^{at}f(t)\right] = F(s-a)$.

20. Find $L^{-1}\left[\dfrac{s^2+1}{(s-1)^2}\right]$.

21. Show that $u = 2xy$ satisfies the Laplace's equation.

22. Find the solution of $u_x - u_y = 0$ by separating the variables.

23. Find a_0 in the Fourier series expansion of $f(x) = \begin{bmatrix} -2x, & -\pi < x < 0 \\ 2x, & 0 < x < \pi, \end{bmatrix}$ $p = 2\pi$.

24. Find the first two approximate solutions $y_1(x)$ and $y_2(x)$ of the initial value problem $y' = x + y,\ y(0) = 0$.

(9 × 1 = 9 weightage)

Unit III

(*Answer any* **five** *questions.*)

25. Solve the initial value problem $y'' - 16y' + 13y = 4\,e^{3x},\ y(0) = 2,\ y'(0) = 4$.

26. Using method of variation of parameters solve $y'' + 9y = \csc 3x$.

27. Find $e^t * e^{-t}$ where * denotes the convolution.

28. Using method of partial fractions find $L^{-1}\left[\dfrac{-s-10}{s^2-s-2}\right]$.

29. Find $L^{-1}\left[\frac{w}{s^2(s^2+w^2)}\right]$ by convolution.

30. Solve the integral equation $y(t) = 1 + \int_0^t y(\tau)\,d\tau$.

31. Find the Fourier series expansion of $f(x) = 2x,\ -1 < x < 1,\ p = 2$.

32. Use Trapezoidal rule with $n = 4$ to estimate $\int_1^3 (2x-1)\,dx$.

(5 × 2 = 10 weightage)

Unit IV

(*Answer any* **two** *questions.*)

33. Apply Euler's method to solve $y' = x + y,\ y(0) = 0,\ h = 0.2$.

34. Find the Fourier series of $f(x) = x^2\ \ -\pi < x < \pi$.

35. Solve using Laplace transform $y' + 10y = 10\sin t,\ y(0) = 0$.

(2 × 4 = 8 weightage)