C 4185 **(Pages : 3)** **Name.....**

Reg. No.....

SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one** *word.*
Each question carries 1 *mark.*

Name the following :

1. The coefficient of $\frac{(it)^r}{r!}$ in the expansion of characteristic function.
2. The discrete distribution having memoryless property.
3. The distribution of $\frac{X_1}{X_2}$ where X_1 and X_2 are independent gamma variables with parameters n_1 and n_2 respectively.

Fill up the blanks :

4. If X and Y are two independent variables, the conditional distribution of X given $Y = y, f(x \mid y) =$ ————————.
5. If $X \sim B(n, p)$, the distribution of $y = n - X$ is ————————.
6. If $X \sim N(\mu, \sigma^2)$, the points of inflexion of normal curve are ————————.
7. The variance of the rectangular distribution $f(x) = \frac{1}{b-a}; a \le x \le b$ is equal to ————————.

Write true or false :

8. If X, Y and Z are three random variables, then $\text{cov}(X+Y, Z) = \text{cov}(X, Z) + \text{cov}(Y, Z)$.
9. For a geometric distribution mean is always less than the variance.
10. The existence of variances of the random variables is not necessary for applying weak law of large numbers.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in one sentence each.*
Each question carries 2 *marks.*

11. Define mathematical expectation of a random variable.
12. What are the properties of moment generating function ?
13. Define conditional variance.
14. Define joint raw moments for the bivariate distribution.
15. Define geometric distribution.
16. If a random variable $X \sim N(40,5^2)$, find $P(32 < X \leq 50)$.
17. Define convergence in probability.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. State and prove the addition theorem of expectation.
19. What are the physical conditions for which binomial distribution is used ?
20. Show that in a Poisson distribution with unit mean, mean deviation about mean is $\frac{2}{e}$ times the standard deviation.
21. Define beta distributions of Type I and Type II. Give the relation between them.
22. State and prove Bernoulli's weak law of large numbers.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. What is the expectation of the number of failures before the first success in an infinite series of independent trials with constant probability p of success in each trial ?
24. Two random variables X and Y have the following joint probability density function :

$$f(x,y) = \begin{cases} 2-x-y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the covariance between X and Y.

25. Find the m.g.f. of the random variables whose moments are (i) $\mu'_r = (r+1)!2^r$ and (ii) $\mu'_r = r!$

26. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used and (ii) some demand is refused.

27. In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution ?

28. Let X_i assume the values + i and – i with equal probabilities, show that law of large numbers cannot be applied to the independent variables $X_1, X_2, \ldots$

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Prove that characteristic function is uniformly continuous.

30. Derive Poisson distribution as a limiting case of binomial distribution.

31. Explain the properties of normal distribution.

32. State and prove the Chebychev's inequality.

(2 × 10 = 20 marks)

C 81823 (Pages : 4) Name.....

Reg. No.....

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**
Each question carries 1 *mark.*

Name the following :

1. $E\left[X - E(X)\right]^r$ of a random variable X.

2. For two random variables X and Y, $E\left[(x - \bar{x})(y - \bar{y})\right]$.

3. If $X \sim N(0,1)$, then the distribution of the square of X.

Fill up the blanks :

4. X and Y are independent random variables with

   $f_1(x) = e^{-x}, x > 0$ and $f_2(x) = e^{-y}, y > 0$, then $f(x/y) =$ ————————.

5. The mode of a Poisson random variable with $E(X) = 5$ is ————————.

6. X is the number turns up when a fair die is tossed, X follows ———————— distribution.

7. If X follow exponential distribution with mean 0.5, then $P(X > 4) =$ ————————.

Write True *or* False :

8. Fourth cumulant k_4 of a random variable X is μ_4.

9. The mean and standard deviation of X following Poisson distribution are same.

10. Tchebycheve's inequality exists only for the random variable with finite variance.

(10 × 1 = 10 marks)

Section B

Answer **all** *questions in* **one sentence.**
Each question carries 2 *marks.*

11. For two random variables, prove that $\text{Cov}(X+Y, X-Y) = V(X) - V(Y)$.

12. Find $E(X)$, X denotes the square of the number shown by a fair coin when it is tossed.

13. Define Bernoulli distribution.

14. $Y \sim N(1,1)$, find (i) $P(Y>0)$; (ii) $P(Y<2)$.

15. If X follow rectangular distribution over [2, 6], find the mean of X.

16. Define Cauchy distribution.

17. State Central Limit theorem.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each one carries 4 *marks.*

18. State and prove the addition theorem on expectation for two random variables X and Y.

19. Given the joint p.m.f. of X and Y, $f(x,y) = \frac{x+2y}{18}$, $x = 1, 2$; $y = 1, 2$. Find $E(X^2Y)$.

20. For two random variables X and Y, prove that $E(E(X/Y)) = E(X)$.

21. For a random variable X and for two constants 'a' and 'b', prove that $M_{aX+b}(t) = e^{bt} M_X(at)$.

22. If X follows follow discrete uniform distribution over {1, 2, 3...n}, find V (X).

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each one carries 6 *marks.*

23. For two random variables X and Y, the joint p.m.f. $f(x, y) = \frac{x^2 + 2y}{22}, x = 1, 2; y = 1, 2$. Find the condition probability distributions of (i) X/Y = 1 ; and (ii) Y/X = 2.

24. Prove that $-1 \le r_{xy} \le 1$, where r_{xy} is Pearson's co-efficient of correlation between any *two* random variables X and Y.

25. State and prove the lack of memory property of geometric distribution.

26. Prove that $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda \frac{d}{d\lambda}\mu_r$, where $\mu_{r-1}, \mu_r, \mu_{r+1}$ are the central moments of Poisson distribution with parameter λ.

27. If X is exponential random variable with $f(x) = \lambda e^{-\lambda x}, x > 0$, show that $Y = 1 - e^{-\lambda x}$ follow rectangular distribution over [0, 1].

28. Define convergence in probability and convergence in distribution. If X denotes the total number of successes in n Bernoulli trials with probability of success p, prove that $\frac{X}{n}$ converges to p in probability.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each one carries 10 *marks.*

29. Let X and Y are two random variables with joint pdf $f(x, y) = 8xy$, $0 < x < y < 1$ and $f(x, y) = 0$, elsewhere. Find Correlation between X and Y.

30. (i) Define binomial distribution.

(ii) If X following binomial distribution with parameters n and p, (a) obtain the m.g.f. of X ; (b) show that $\text{Cov}\left(\frac{X}{n}, \frac{n - X}{n}\right) = \frac{-pq}{n}$.

31. For a random variable following $N(\mu, \sigma)$, show that (i) $E(X) = \mu$; (ii) $V(X) = \sigma^2$; and

(iii) $M_x(t) = e^{\mu t + \frac{t^2\sigma^2}{2}}$.

32. State and prove weak law of large numbers. Also prove that a sequence of independent and identically distributed random variables $\{X_n\}$ obeys weak law of large numbers.

(2 × 10 = 20 marks)

C 62652 (Pages : 4)

Name.....

Reg. No.....

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
MAY 2019

B.Sc. Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**
Each question carries 1 *mark.*

Name the following :

1. The second central moment of a random variable X.
2. $E\left[e^{itX}\right]$ of a random variable X.
3. Probability distribution of the random variable X denoting the number of failures before the first success in an experiment with only two possible results success and failure.

Fill up the blanks :

4. Pearson's co-efficient of correlation between two random variables satisfying the linear relation $2x + 3y - 5 = 0$ perfectly is ————.
5. Probability density function of X following U [2, 5] = ————.
6. X and Y are two random variables with bivariate m.g.f. function, $M_{X,Y}(t_1, t_2)$ is expressed in terms of expectation as ————.
7. The range of variation of beta distribution of second kind is ————.

Write True *or* False :

8. m.g.f. exists for all the random variables.
9. Normal distribution is symmetric about its median.
10. Central limit theorem discusses the convergence of sum of random variables to normal distribution.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence.**
Each question carries 2 *marks.*

11. Find the mean of a random variable X with first moment about 4 is given as 7.

12. $f(x, y)$ is the joint p.d.f. of two continuous random variables X and Y. Write any *two* of its properties.

13. Write the mean and variance of a binomial distribution with parameters $(15, 0.3)$.

14. Define covariance of X and Y.

15. If $p(x, y) = \frac{(2x + y)}{10}$, where $x = 0.1$ and $y = 1.2$ is a joint p.m.f. of X and Y, write the p.m.f. of X.

16. State Bernoulli's law of large numbers.

17. Define lognormal distribution.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. First three raw moments of X are – 1 , 55 and – 62.5. Obtain co-efficient of skewness based on moments.

19. State and prove Cauchy-Schwartz inequality.

20. For a random variable X, $P(X = 3) = 2P(X = 4) = P(X = 5)$. Find V (X).

21. Obtain the m.g.f. of X following Poisson distribution with parameter λ.

22. The repair time of a machine follows exponential distribution with an average of 2 hours. What is the probability that the repair time will be more than 2 hours ?

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. $f(x, y) = e^{-x-y}, x > 0, y > 0$ be the joint p.d.f. of (X, Y). Find the bivariate m.g.f. of (X, Y) and hence show that X and Y are independent.

24. Show that $E(XY) = E(X)\,E(Y)$ need not imply X and Y are independent.

25. For two random variables X and Y, the joint p.d.f.

$$f(x, y) = 2 - x - y\,;\ 0 \le x \le 1, 0 \le y \le 1$$
$$= 0, \text{ otherwise}$$

. Find E (X) and E (Y).

26. If $X \sim N(\mu_1, \sigma_1), Y \sim N(\mu_2, \sigma_2)$, obtain the distribution of $aX + bY$ when X and Y are independent.

27. If $X \sim N(12, 4)$. Obtain (i) $P(X \le 20)$; (ii) $P(0 \le X \le 24)$; and (iii) $P(|X - 12| \ge 8)$.

28. State and prove Tchebychev's inequality.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Let X and Y are two random variables with joint p.d.f. $f(x, y) = x + y\,;\ 0 < x < 1\,;\ 0 < y < 1$. Compute the coefficient of correlation between X and Y.

30. Obtain the m.g.f. of X following binomial distribution with parameters n and p. Hence state and prove the additive property of binomial distribution. If X and Y are independent binomial random variables with parameters (6, 0.5) and (4, 0.5) respectively, calculate $P(X + Y \ge 3)$.

31. If $X \sim (\mu, \sigma)$, prove that (a) $\frac{X-\mu}{\sigma} \sim N(0,1)$; (b) The quartile deviation of X is $0.6745\,\sigma$.

32. State and prove Weak law of large numbers. Examine whether WLLN hold good for the sequence of random variables $\{X_i\}, i = 1, 2, \ldots$ where $P\left(X_i = \pm\sqrt{2i-1}\right) = 0.5$.

(2 × 10 = 20 marks)

D 43254 (Pages : 3) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS)

Complementary Course

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**
Each question carries 1 *mark.*

Name the following :

1. The moments of a random variable X about origin.
2. The probability distribution in which mean is equal to its variance.
3. The distribution of $\frac{x_1 - x_2}{\sqrt{2}}$ where $X_1 \sim N(1,1)$ and $X_2 \sim N(1,1)$.

Fill up the blanks :

4. If two variables X and Y are independent, then $E(XY) =$ ————.
5. The maximum height of the normal curve lies at the point ————.
6. The mode of the geometric distribution $f(x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, \ldots$ is ————.
7. If $X \sim N(12.5, 12.25)$ and $Y \sim N(8.5, 6.25)$, the variable X + Y is distributed as ————.

Write True *or* False :

8. If X and Y are two random variables, then the covariance between the variables $aX + b$ and $cY + d$ is equal to covariance between X and Y.
9. For a binomial distribution mean is always less than the variance.
10. Convergence in probability is also known as weak convergence.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 *marks.*

11. Define variance of a random variable.

12. Give the properties of characteristic function.

13. Define conditional expectation.

14. Define joint central moments for the bivariate distribution.

15. Define negative binomial distribution.

16. If a random variable $X \sim N\left(40, 5^2\right)$, find $P\left(45 \leq X \leq 50\right)$.

17. Define weak convergence.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. Define moment generating function of a random variable. Prove that it does not exist always.

19. Give the properties of normal distribution.

20. If X and Y are independent Poisson variates, show that the conditional distribution of $\left(X|X+Y\right)$ is binomial.

21. State the weak law of large numbers and central limit theorem.

22. If $E\left(X\right)=5, V\left(X\right)=3$ and if $P\left[|X-5|<h\right] \geq 0.99$, find the value of h.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

24. X_1 and X_2 have a bivariate distribution given by $p(x_1, x_2) = \frac{x_1 + 3x_2}{24}$; where $(x_1, x_2) = (1, 1), (1, 2), (2, 1), (2, 2)$. Find the conditional mean and conditional variance of X_1 given $X_2 = 2$.

25. Let the random variable X assumes the value 'x' with the probability law $P(X = x) = q^{x-1}p$; $x = 1, 2, 3.....$ and $q = 1 - p$. Find the m.g.f, of X and hence find its mean and variance.

26. The mean and variance of a binomial distribution are $\frac{8}{3}$ and $\frac{16}{9}$. Find (i) $P(X = 1)$ and (ii) $P(X \leq 1)$.

27. Assuming that the height of students is distributed as $N(\mu, \sigma^2)$. Out of a large number of students, 5% are under 72 inches and 10% are below 60 inches. Find the values of μ and σ.

28. Examine whether the weak law of large numbers holds $\{X_k\}$ of independent random variables defined as follows :

$P[X_k = \pm 2^k] = 2^{-(2k+1)}$ and $P[X_k = 0] = 1 - 2^{-2k}$.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Let X and Y be two random variables, prove that :

(i) $E(X) = E\{E(X|Y)\}$ and

(ii) $V(X) = E\{V(X|Y)\} + V\{E(X|Y)\}$.

30. State and prove the recurrence relation for central moments for a binomial distribution.

31. Derive the m.g.f. of a normal distribution with parameters μ and σ^2.

32. State and prove the Chebychev's inequality.

(2 × 10 = 20 marks)

C 24785 (Pages : 3) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS—UG)

Complementary Course

STS 2C 02—PROBABILITY DISTRIBUTION

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**

Each question carries 1 *mark.*

Name the following :

1. For a random variable X , the function $E\left[e^{itX}\right]$.
2. For two random variables X and Y, a function denoting $P(X = x, Y = y)$.
3. If $X \sim N(\mu, \sigma)$, then the distribution of the square of X.

Fill up the blanks :

4. Correlation between two random variables X and Y is $\frac{1}{4}$, Cov (X, Y) is $\frac{1}{36}$, then the product of standard deviations of X and Y is ————.
5. X is a Poisson random variable with $E(X) = 5$. $E\left(X^2\right)$ is ————.
6. X is the number shown when a fair die is tossed, X follows ———— distribution.
7. If $X \sim N(4, 2)$, then $P(X > 4) =$ ————.

Write True *or* False :

8. Second cumulant k_2 of a random variable X is $V(X)$.
9. The mean of a Binomial random variable is less than its variance.
10. In some situations Tchebycheve's inequality may give a negative lower bound.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 *marks.*

11. State any *four* properties of expectation.
12. Obtain the expected number of heads obtained in three tosses of a fair coin.
13. Define conditional expectation.
14. $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$, obtain the distribution of $Y = aX + bY$, where X and Y are independent.
15. Define rectangular distribution
16. Define Pareto distribution.
17. State Central Limit theorem.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. Given the pdf of X, $f(x) = k\left(1 - x^2\right), 0 < x < 1$. Prove that $E(X) = \frac{3}{8}$.
19. Obtain the mgf of the random variable X with p.d.f. $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$.
20. Obtain the mgf of Poisson random variable with a parameter λ.
21. If X follows gamma distribution with one parameter p, obtain V(X).
22. Define convergence in probability and convergence in distribution.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. For two random variables X and Y, the joint p.d.f.

$f(x, y) = 2 - x - y ; 0 \le x \le 1, 0 \le y \le 1$
$= 0$, otherwise

. Find Cov (X, Y).

24. State and prove Cauchy-Schwartz inequality. Use it to prove that $-1 \leq r_{xy} \leq 1$, where r_{xy} is Pearson's coefficient of correlation between any two random variables X and Y.

25. State and prove the lack of memory property of exponential distribution.

26. Prove that $\mu_{r+1} = pq\left[\frac{d}{dp}\mu_r + nr\mu_{r-1}\right]$, where $\mu_{r-1}, \mu_r, \mu_{r+1}$ are the central moments of binomial distribution with parameters n and p.

27. If $X \sim N(\mu, \sigma)$, show that the mean deviation about the mean is $\sqrt{\frac{2}{\pi}}\sigma$.

28. State and prove Tchebycheve's inequality.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries **10** *marks.*

29. Let X and Y are two random variables with joint pmf $f(x, y) = \frac{x+y}{21} x = 1, 2, 3 ; y = 1, 2.$

Find :

(i) Correlation between X and Y

(ii) $V(X/Y = 2)$.

30. (i) For a geometric random variable with parameter p, obtain mean, variance and m.g.f.

(ii) If X and Y are two independent geometric random variables with same parameter p, show that the conditional distribution of X given $X + Y = z$ follows uniform distribution.

31. (i) Define beta distribution of first and second kind. Obtain their means.

(ii) If X follow beta distribution of first kind with parameters p and q, show that $Y = X/(1 - X)$ follow beta distribution of second kind.

32. State and prove Weak law of large numbers. For a sequence of random variables (X_k). Given that $P\left(X_k = -2^k\right) = 2^{-(2k+1)} = P\left(X_k = 2^k\right), P(X_k = 0) = 1 - 2^{-(2k+1)}$, Examine if the law of large numbers holds for this sequence.

(2 × 10 = 20 marks)

C 25925

(Pages : 4)

Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2017

(UG–CCSS)

Complementary Course

ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. The joint distribution function of two variables X and Y is defined as :

   (a) $P(X = x, Y = y)$.

   (b) $P(X \leq x, Y \leq y)$.

   (c) $P(X \leq \infty, Y \leq \infty)$.

   (d) $P(0 \leq X \leq \infty, 0 \leq Y \leq \infty)$.

2. If $f(x, y)$ represents the joint p.d.f. two continuous random variables, then $\int_{R_y} f(x,y)\,dy$ represents :

   (a) Marginal distribution of X.

   (b) Marginal distribution of Y.

   (c) Conditional distribution of X.

   (d) Conditional distribution of Y.

3. If X and Y are independent random variables, then :

   (a) Cov (X, Y) = E (X) · E (Y).

   (b) Cov (X, Y) = E (XY).

   (c) E (X Y) = E (X) · E (Y).

   (d) E (X Y) = 0.

4. $P(X \leq a, Y \leq b) = P(X < a, Y < b)$, when the variables are :

   (a) Continuous.

   (b) Discrete.

   (c) Independent.

   (d) Dependent.

5. The regression coefficient of Y on X is given by :

   (a) $\frac{\text{cov}(X,Y)}{V(Y)}$.

   (b) $\frac{\text{cov}(X,V)}{SD(Y)}$.

   (c) $\frac{\text{cov}(X,Y)}{SDX}$.

   (d) $\frac{\text{cov}(X,Y)}{V(X)}$.

Turn over

6. The distribution function F (x) of a rectangular variate X defined on $[a, b]$ is :

(a) $\frac{1}{b-a}$.

(b) $\frac{a-x}{b-a}$.

(c) $\frac{x-a}{b-a}$.

(d) $\frac{x-a}{x-b}$.

7. For a Binomial distribution mean = 6 and variance = 4. The values of the parameters are :

(a) $n=9 \quad p=\frac{2}{3}$.

(b) $n=18 \quad p=\frac{1}{3}$.

(c) $n=18 \quad p=\frac{2}{3}$.

(d) $n=9 \quad p=1/3$.

8. The coefficient of variation of a Poisson distribution with mean 16 is :

(a) 4%.

(b) 16%.

(c) 25%.

(d) 40%.

9. The mode of a geometric distribution $P(x)=\left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, \ldots$ is :

(a) One.

(b) Zero.

(c) $\frac{1}{2}$.

(d) Do not exist.

10. The points of inflexion of a Normal distribution are at :

(a) $\pm\frac{\mu}{\sigma}$.

(b) $\mu \pm 3\sigma$.

(c) $\mu \pm 2\sigma$.

(d) $\mu \pm \sigma$.

11. Lindeberg–Levy form of CLT is applicable to a sequence of r.vs which are :

(a) Independent.

(b) Identically distributed.

(c) Having common mean and variance.

(d) All the above.

12. The mean and SD of X are 8 and 3 respectively. Then $P(|X-8|>6)$ is :

(a) $\leq\frac{1}{2}$.

(b) $\leq\frac{1}{4}$.

(c) $\geq\frac{1}{2}$.

(d) $\geq\frac{1}{4}$.

(12 × ¼ = 3 weightage)

Part B (Short Answer Type Questions)

Answer **all nine** *questions.*
Each question carries 1 *weightage.*

13. Define conditional expectation of $X / Y = y$.

14. Define stochastic independence of r.v's.

15. If X is a uniform variate which takes values 1, 2, 3, . . . , n, find E (x).

16. The m.g.f. of a distribution is $\left(\cdot 4e^{t} + \cdot 6\right)^{12}$. Identify the distribution.

17. If $X \sim N(0.1)$, find 'C' such that $P(|X| > C) = 0.05$.

18. State the additive property of Poisson variates.

19. What is the relation between a Normal variate and a lognormal variate ?

20. Define Pareto distribution.

21. Define convergence in probability.

(9 × 1 = 9 weightage)

Part C (Short Essay Type Questions)

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. If $f(x,y) = 2 - x - y \quad 0 \le x \le 1, \quad 0 \le y \le 1$, find the conditional distribution of X given Y = y.

23. Find the mean and variance of a geometric distribution $P(x) = q^{x} p \; ; x = 0,1,2,3, \ldots$

24. A Poisson variate X is such that P (X = 1) = 2 . P (X = 2), find P (X = 0).

25. Given $P(X = x, Y = y) = \frac{x + y}{21} \; ; x = 1,2,3 \qquad y = 1,2.$ Examine whether X and Y are independent.

26. Find the mode of the normal distribution.

27. Find the m.g.f. of an exponential variate. Hence find its mean and variance.

28. State Weak Law of large numbers.

(5 × 2 = 10 weightage)

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. (a) State the important properties of Normal distribution.

    (b) The marks obtained by 100 students in an Examination is found to follow Normal distribution. If 30 of them got less than 40 marks and 37 of them got more than 50 marks, find the mean and S.D.

30. If $f(x,y) = 2; 0 < x < y < 1$, find the conditional mean and variance of X / Y.

31. (a) State and prove Chebychev's inequality.

    (b) If $X \sim B\left(24, \frac{1}{3}\right)$, obtain $P(5 \leq x \leq 11)$ using Chebychev's inequality.

(2 × 4 = 8 weightage)

C 82145 (Pages : 4) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, APRIL/MAY 2015

(UG—CCSS)

Complementary Course—Statistics

ST 2C 02—PROBABILITY DISTRIBUTION

Time : Three Hours Maximum : 30 Weightage

I. Answer all *twelve* question.

1 As $x \to -\infty$, the joint cumulative distribution function $F(x, y)$ of a bivariate random variable (X, Y) becomes :

(a) Zero.

(b) One.

(c) A number between zero and one.

(d) None of these.

2 The heights of fathers and their sons form bivariate variables which are :

(a) Discrete variables. (b) Continuous variables.

(c) Pseudo variables. (d) None of these.

3 If X and Y are independent discrete variables, then $P\{X \leq x, Y \leq y\}$ is equal to :

(a) $P\{X < x, Y < y\}$. (b) $P\{X = x, Y = y\}$.

(c) $P\{X = x\} \cdot P\{Y = y\}$. (d) $P\{X \leq x\} \cdot P\{Y \leq y\}$.

4 For any bivariate distribution, which of the following is true ?

(a) $\mu_{02} = \mu_{20}$. (b) $\mu_{11} = \mu_{22}$.

(c) $\mu_{10} = \mu_{01}$. (d) $\mu_{12} = \mu_{21}$.

5 $E\{XY|Y = 1\} =$

(a) $E(X)$. (b) $E(X|Y = 1)$.

(c) $E(XY)$. (d) $E(X) \cdot E(Y)$.

Turn over

6 When $n = 1$, the binomial distribution $B(n, p)$ reduces to ——— distribution.

(a) Standard binomial. (b) Pseudo binomial.

(c) Point binomial. (d) Poisson binomial.

7 The Poisson distribution $P(\lambda)$ is leptokurtic for :

(a) $\lambda > 1$ only. (b) All values of λ.

(c) $\lambda < 1$ only. (d) $\lambda \neq 1$.

8 The moment generating function of geometric distribution with parameter p is :

(a) $p\left(1 - pe^{-t}\right)^{-1}$. (b) $p\left(1 - pe^{t}\right)^{-1}$.

(c) $p\left[1 - (1-p)e^{-t}\right]^{-1}$. (d) $p\left[1 - (1-p)e^{t}\right]^{-1}$.

9 For large values of λ, the gamma distribution $\gamma(\lambda)$ tends to :

(a) Uniform. (b) Exponential.

(c) Normal. (d) Cauchy.

10 Standard deviation of standard exponential distribution is :

(a) 1. (b) $\frac{1}{2}$.

(c) $\frac{1}{4}$. (d) None of these.

11 If X is a standard normal variate, then $P(X > 5)$ equals :

(a) 0.6. (b) 0.45.

(c) 0.25. (d) None of these.

12 If X follows Pareto distribution, then $P(X = 0.5)$ is :

(a) 0.5. (b) 1.

(c) 0. (d) None of these.

(12 × ¼ = 3 weightage)

II. Short Answer Type questions. Answer all *nine* questions :—

13 Define marginal probability density function.

14 Define conditional distribution function of a continuous random variable Y given X.

15 Define conditional variance of a random variable X given Y.

16 Define joint central moments of a bivariate distribution.

17 Define a degenerate random variable.

18 Obtain second raw moment of discrete uniform distribution.

19 Find the moment generating function of a Bernoulli distribution.

20 Define rectangular distribution.

21 Define Cauchy distribution.

(9 × 1 = 9 weightage)

III. Short essay *or* paragraph questions. Answer any *five* questions :

22 If X and Y are random variables with joint probability density function :

$$f(x, y) = \begin{cases} e^{-(x+y)}, & 0 < x, y < \infty \\ 0 & , \text{elsewhere,} \end{cases}$$

find $P(X > 1)$.

23 If X, Y, Z are independent and identically distributed random variables, show that $E(Y - Z)^2 = 2\,\text{Var}(X)$.

24 Let $p(x, y) = \frac{1}{n^2}$, for $x = 1, 2, \ldots, n$ and $y = 1, 2, \ldots, n$. Verify whether X and Y are independent.

25 Establish Renovsky formula.

26 Derive the moment generating function fo a Poisson variate. Hence obtain its first four central moments.

27 Obtain the median of normal distribution.

28 State and prove Bernoulli's law of large numbers.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 Let the joint probability density function of (X, Y) be :

$$f(x, y) = \begin{cases} 3xy, 0 \le x \le 1; 0 \le y \le 1; 0 \le x + y \le 1 \\ 0 \quad \text{, elsewhere.} \end{cases}$$

Find (i) $E(Y|X = x)$; and (ii) Cor (X, Y).

30 (a) If X and Y are independent Poisson variates, obtain the conditional distribution of X given X + Y.

(b) Find the r^{th} central moment of normal distribution.

31 (a) State and prove Chebyshev's inequality.

(b) Briefly explain weak law of large numbers.

(2 × 4 = 8 weightage)

C 6167 (Pages : 4) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, MAY 2016

(UG—CCSS)

Statistics—Complementary Course

ST 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer all *twelve* questions.

1 As $x \to \infty$, the cumulative distribution function $F_{XY}(x, y)$ of a bivariate random variable becomes :

(a) $P(Y = y)$. (b) $P(Y \leq y)$.

(c) $P(Y > y)$. (d) None of these.

2 If (X, Y) is a bivariate continuous random variable, which of the following statements is/are true ?

I : $P\{X \leq a, Y \leq b\} \geq P\{X < a, Y \leq b\}$.

II : $P\{X \leq a, Y < b\} \geq P\{X < a, Y < b\}$.

(a) I only. (b) II only.

(c) Both I and II. (d) Neither I nor II.

3 If X and Y are independent continuous random variables with finite E(X), E(Y) and E(XY), then $E\{[X - E(X)][Y - E(Y)]\}$.

(a) $= 0$. (b) > 0.

(c) < 0. (d) $= E(XY)$.

4 If μ_{rs} denote the $(r, s)^{th}$ product central moments of a bivariate random variable (X, Y), then :

(a) $\mu_{11} = 0$. (b) $\mu_{11} > 0$.

(c) $\mu_{10} = 0$. (d) $\mu_{10} > 0$.

Turn over

5 $E(XY|X=x)=$

(a) $E(Y)$.

(b) $E(XY)$.

(c) $E(Y|X=x)$.

(d) $x\,E(Y|X=x)$.

6 The outcome of an experiment classified as success S or failure F will follow a Bernoulli distribution if and only if :

(a) $P(S)=0$.

(b) $P(S)=0.5$.

(c) $P(S)=1$.

(d) $P(S)$ remain constant in all trials.

7 If X and Y are independent Poisson variates such that $X \sim P(1)$ and $Y \sim P(2)$, then $P(X+Y>3)$ is :

(a) $4.5\,e^{-3}$.

(b) $1-4.5\,e^{-3}$.

(c) $8.5\,e^{-3}$..

(d) $1-8.5\,e^{-3}$.

8 If X follows discrete uniform distribution over $[1, 11]$, then $E(X^2)=$

(a) 6.

(b) $\frac{6}{11}$.

(c) 46.

(d) $\frac{46}{11}$.

9 If Z is a standard normal variate with distribution function $F(z)$, then for any $z>0$, $F(z)+F(-z)$ is :

(a) $=1$.

(b) <1.

(c) >1.

(d) ≥ 1.

10 If $X \sim N(\mu, \sigma)$, then $P\{\mu-\sigma<X<\mu+\sigma\}=$

(a) 0.5.

(b) 0.6826.

(c) 0.9454.

(d) 0.9973.

11 If X follows exponential distribution with mean θ, then its variance is :

(a) θ. (b) θ^2.

(c) $\sqrt{\theta}$. (d) $\frac{2}{\theta^2}$.

12 If X follows beta distribution of second kind the distribution of $Y = (1+X)^{-1}$ is :

(a) Beta distribution of first kind.

(b) Beta distribution of second kind.

(c) Cauchy distribution.

(d) t-distribution.

(12 × ¼ = 3 weightage)

II. Short answer Type questions. Answer all *nine* questions :

13 Define marginal density function.

14 Define conditional distribution function.

15 Define product moment about origin of a bivariate random variable (X, Y).

16 Define conditional variance.

17 State the additive property of binomial distribution.

18 Obtain mean of geometric distribution.

19 Find the characteristic function of rectangular distribution over (–1, 1).

20 State the characteristic property of exponential distribution.

21 Define convergence in probability.

(9 × 1 = 9 weightage)

III. Short Essay *or* Paragraph questions. Answer any *five* questions.

22 If $f(x, y) = kx\,e^{-y}$, $0 < x < 2$, $y > 0$ is a probability density function, find the value of k ?

23 Let (X, Y) has probability function : $P(X = x, Y = y) = \frac{1}{32}(x^2 + y)$, for $x = 0, 1, 2, 3$ and $y = 0, 1$. Find the marginal distribution of X and Y.

24 Obtain an expression for variance of a random variable X in terms of conditional variance.

25 Derive mean deviation about mean of binomial distribution.

26 Derive a recurrence relation for central moments of the Poisson distribution.

27 Derive the characteristic function of normal distribution.

28 State and establish the reproductive property of gamma distribution.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 Let (X, Y) has probability density function :

$$g(x, y) = \begin{cases} 2 - x - y, 0 \le x \le 1, 0 \le y \le 1 \\ 0, \quad \text{, elsewhere.} \end{cases}$$

Find :

(a) Conditional density functions.

(b) Cov (X, Y).

30 Distinguish between beta distribution of first kind and beta distribution of second kind. Obtain mean and variance in each case.

31 (a) Explain weak law of large numbers.

(b) State and establish Windberg-Levy form of CLT.

(2 × 4 = 8 weightage)

C 5646 (Pages : 3) Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2016

(CUCBCSS–UG)

(Complementary Course)

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in one word.*
Each question carries 1 *mark.*

Name the following :

1. The moments of a random variable X about E (X).
2. The distribution of X, where $\log_e X$ follows normal distribution.
3. The random variable with only two possible values 0 and 1 with respective probabilities q and p.

Fill up the blanks :

4. For the constants a and b and for any two random variables X and Y, Cov[X + a,Y + b] is ————.
5. X is a Poisson random variable with mean 5. Then V(X) = ————.
6. X and Y are two random variables with bivariate distribution function, $F_{xy}(x, y)$, then the value of $F_{xy}(-\infty, y)$ is ————.
7. In the expansion of $M_x(t)$ the coefficient of $\frac{t^r}{r!}$ is ————.

Write True *or* False :

8. Coefficient of correlation between two random variables X and Y is $\frac{\text{Cov}(X,Y)}{V(X)\,V(Y)}$.
9. The mean and standard deviation of a Poisson random variable are same.
10. For a sequence of random variables, variances of the random variables should exit to apply WLLN.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in one sentence each.*
Each one carries 2 *marks.*

11. State Cauchy-Schwartz inequality.
12. Define conditional expectation.
13. Find the mode of a binomial distribution with parameters (10, 0.4)
14. Define the bivariate mgf of X and Y.
15. Find c, if $p(x,y) = c(2x + 3y)$, where $x = 0,1$ and $y = 1,2$ is a joint p.m.f.
16. Define standard normal distribution.
17. Define Pareto distribution.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each one carries 4 *marks.*

18. For two random variables X and Y, Show that E (X) = E [E (X / Y)].
19. $f(x, y) = e^{-x-y}$, $x > 0, y > 0$ be the joint p.d.f. of (X,Y). Find the bivariate m.g.f. of (X,Y).
20. Obtain the characteristic function of $X \sim B(n, p)$
21. Express Gamma distribution as sum of independent exponential random variables.
22. Define convergence in probability and convergence in distribution.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each one carries 6 *marks.*

23. For two random variables X and Y, the joint p.d.f. :

$f(x,y) = 2 - x - y\,;\ 0 \le x \le 1, 0 \le y \le 1$
$= 0$, otherwise.

Find Cov (X, Y).

24. X is a random variable with continuous distribution. Show that the distribution of $Y = F(x)$, where $F(x)$ is the distribution function of X is U [0, 1].

25. Prove that $\mu_{r+1} \; r\lambda\mu_{r-1} + \lambda\frac{d}{d\lambda}\mu_r$, where $\mu_{r-1}, \mu_r, \mu_{r+1}$ are the central moments of Poisson distribution with parameter λ.

26. If $X \sim N(\mu, \sigma)$, prove that the odd order central moments are zero.

27. If $X \sim N(\mu, \sigma)$, show that $Y = X^2$ follow gamma distribution with parameters $\left(\frac{1}{2}, \frac{1}{2}\right)$.

28. State and prove Bernoulli's law of large numbers.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each one carries 10 *marks.*

29. Let X and Y are two random variables with joint p.d.f. $f(x, y) = 2 ; 0 < x < y < 1$. Find
    (i) Correlation between X and Y. (ii) V (X/Y = y).

30. Under some limiting conditions such as $n \to \infty$ and $p \to 0$ such that $np = \lambda$ (finite) prove that binomial distribution, B (n,p) tends to Poisson P (λ).

31. $X \sim N(\mu, \sigma)$, prove that the $(2k)^{th}$ central moment $\mu_{2k} = \sigma^2 (2k-1)\mu_{2(k-1)}$ for $k = 1, 2, 3....$.

32. State and prove Tchebychev's inequality. For a random variable with p.d.f. $f(x) = e^{-x} ; x \geq 0$, use Tchebychev's inequality to find a lower bound to the probability $P(|X-1|<2)$.

(2 × 10 = 20 marks)