C 4161 **(Pages : 4)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the minimum value of $f(x) = \cos x$, on $[-\pi/2, \pi/2]$.
2. Evaluate $\lim_{x \to \infty}\left(5 + \frac{1}{x}\right)$.
3. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$.
4. Evaluate $\int_1^{32} x^{-6/5}\, dx$.
5. Evaluate the sum $\sum_{k=1}^{2} \frac{6k}{k+1}$.
6. Suppose that f is integrable and that $\int_1^2 f(x)\, dx = -4$, $\int_1^5 f(x)\, dx = 6$. Evaluate $\int_2^5 f(x)\, dx$.
7. How do you define and calculate the area of the region between the graphs of two continuous functions ?
8. How do you define and calculate the length of the graph of a smooth function over a closed interval ?
9. How do you define and calculate the area of the surface swept out by revolving the graph of a smooth function $y = f(x)$, $a \le x \le b$, about the x-axis ?

Turn over

10. What is the moment about the origin of a thin road along the x-axis with density function $\delta(x)$?

11. Define the work done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$.

12. State Hooke's Law for springs.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. State the Max-Min Theorem for Continuous Functions.

14. Verify Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$, in the interval $[0, 1]$.

15. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.

16. Evaluate $\int_{-4}^{4} |x| \, dx$.

17. Using substitution evaluate the integral $\int_0^3 \sqrt{y+1} \, dy$.

18. Find the area of the region enclosed by the line $y = 2$ and curve $y = x^2 - 2$.

19. The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

20. Set up an integral for the length of the curve $y = x^2$, in the interval $-1 \le x \le 2$.

21. Set up an integral for the area of the surface generated by revolving the curve $y = \tan x, 0 \le x \le \pi/4$; about x-axis.

22. Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

23. Find the work done by a force of $F(x) = 1/x^2$ N along the x-axis from $x = 1\,m$ to $x = 10\,m$.

24. What is the Center of Mass of a thin plate covering a region in the xy-plane ?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Given $f'(x)=(x-1)^2(x+2)^2$.

    (a) What are the critical points of f?

    (b) On what intervals is f increasing or decreasing ?

26. Find the asymptotes of the curve :

    $$y=\frac{x+3}{x+2}.$$

27. State and prove Rolle's Theorem.

28. Find two positive numbers whose sum is 20 and whose product is as large as possible.

29. Find the area of the region between the x-axis and the graph of $f(x)=x^3-x^2-2x, -1\le x\le 2$.

30. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m ona side. Find the volume of the pyramid.

31. Find the length of the curve $y=\frac{4\sqrt{2}}{3}x^{3/2}-1, \quad 0\le x\le 1$.

32. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$.

33. Find the moment about the x-axis of a wire of constant density that lies along the curve $y=\sqrt{x}$ from $x=0$ to $x=2$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Sketch the Graph of $y = (x-2)^3 + 1.$ Include the co-ordinates of inflection point in the graph.

(5 marks)

(b) Find the intervals on which $g(x) = -x^3 + 12x + 5, \ -3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

(5 marks)

35. (a) If *f* is continuous at every point of $[a, b]$ and F is any antiderivative of *f* on $[a, b]$, then prove that

$$\int_a^b f(x)\,dx = \mathrm{F}(b) - \mathrm{F}(a).$$

(5 marks)

(b) A surveyor, standing 30ft from the base of a building, measures the angle of elevation to the top of the building to be 75°. How accurately must the angle be measured for the percentage error in estimating the height of the building to be less than 4 % ?

(5 marks)

36. (a) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, \ 1 \leq x \leq 2,$ about the x-axis.

(5 marks)

(b) Find the center of mass of a thin plate of constant density δ covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis.

(5 marks)

[2 × 10 = 20 marks]

C 81805 (Pages : 4) Name..................................

Reg. No..............................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2020

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the minimum value of $f(x) = x^2 - 1$ on $[-1, 2]$.

2. Find the critical points of $f(x)$ if $f'(x) = (x-1)(x+2)(x-3)$.

3. Find $\lim\limits_{x \to \infty} \dfrac{2x+3}{5x+7}$.

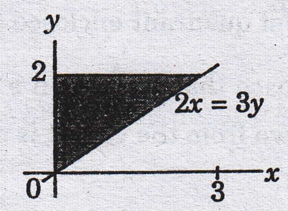
4. Write the sum $\sum\limits_{k=-1}^{1} \dfrac{(-1)^k}{k+2}$ without sigma notation.

5. Evaluate $\int\limits_{-3}^{0} [-g(x)]\,dx$ if $\int\limits_{-3}^{0} g(t)\,dt = \sqrt{2}$.

6. State Fundamental Theorem of Calculus.

7. Evaluate $\int\limits_{0}^{1} \left(x^2 + \sqrt{x}\right) dx$.

8. Set up an integral to find the volume of the solid generated by revolving the shaded region about the y-axis :



Turn over

9. Set up an integral to find the area of the shaded region of the figure in Question 8.

10. Set up an integral to find the length of the curve $y = x^{3/2}, 0 \le x \le 4$.

11. Set up an integral to find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le 1/2$ about the x-axis.

12. Find the work done by a force $F(x) = x^2$ N along the x-axis from $x = 1$ m to $x = 3$ m.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the absolute maximum value of $f(x) = x^{4/3}$ on [– 1, 8].

14. If $f'(x) = 0$ at each point of an interval I, prove that $f(x) = c$ for all x in I, where c is a constant.

15. Find $f(2)$, if $f(1) = 0$ and $f'(x) = 2x$ for all x.

16. Evaluate $\sum_{k=1}^{5} k(3k+5)$.

17. State Rolle's Theorem.

18. Find $\frac{dy}{dx}$ if $y = \int_1^{x^4} \sqrt{u}\,du$.

19. Evaluate $\int_0^1 t^3 (1+t^4)^3 \, dt$.

20. Find the area of the region in the first quadrant enclosed by the curves $x = y^2$ and $x = y^3$.

21. A pyramid 3 m. high has a square base that is 3 m. on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

22. Find the volume of the solid generated by revolving the region bounded by the curve

$x = \sqrt{2 \sin 2y}, 0 \le y \le \pi/2$ and the line $x = 0$ about the y-axis.

23. Find the length of the curve $y^2 + 2y = 2x + 1$, from $(-1, -1)$ to $(7, 3)$.

24. Find the work required to compress a spring from its natural length of 0.75 ft if the force constant is $k = 16$ lb/ft.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. State and prove Mean Value Theorem.

26. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$ using Sandwich Theorem.

27. Find the value of c in the Mean Value Theorem for $f(x) = x^2$ in $[0, 2]$.

28. What is the smallest perimeter possible for a rectangle whose area is 16 in^2.

29. Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

30. Find the area of the region enclosed by the curves $y = \frac{x^2}{4}$ and the lines $y = x, y = 1$.

31. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

32. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

33. Find the center of mass of a wire of constant density δ shaped like a semicircle of radius a.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (i) Find the intervals on which $h(x) = -x^3 + 2x^2$ is increasing and decreasing. Identify the local extreme values, if any, of $h(x)$, saying where they are taken on. Which of the extreme values are absolute ?

(ii) Graph the function $y = \frac{x^3 - 1}{x}$.

35. (i) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$, about the x-axis.

(ii) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the initial line $y = x - 2$.

36. (i) Find the center of mass of a thin plate of constant density δ covering the region bounded by the parabola $y = 4 - x^2$ and below by the x-axis.

(ii) A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m.

(a) Find the force constant k.

(b) How much work will it take to stretch the spring 2 m. beyond its natural length ?

(c) How far will a 45-N force stretch the spring ?

(2 × 10 = 20 marks)

C 62588

(Pages : 4)

Name.....................................

Reg. No..................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, MAY 2019

B.Sc.—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{x \to -\infty} \frac{x^5}{x^6 + 2}$.

2. Find absolute extrema of $y = x^2$ on (0, 2).

3. Find dy if $y = 6 \cos x^5$.

4. Find the interval in which $y = x^3$ is concave up.

5. Suppose $\int_2^3 f(x)\,dx = -6$. Find $-\int_3^2 f(x)\,dx$.

6. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(c_k^{\,2} - 3c_k\right) \Delta x_k$ as an integral, where P is the partition of $[-7, 5]$.

7. Define average value of a function f on $[a, b]$.

8. Find all possible functions with derivative $y' = x^2$.

9. Evaluate $\int_0^{\pi/2} \sin^2 x dx$.

10. Write the sum $\sum_{k=1}^{5} k(3k+5)$ without sigma notation.

Turn over

11. State Mean Value Theorem.

12. Evaluate $\int_{1}^{10} x^{-2/3}\, dx$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find $\lim_{t \to -\infty} \frac{2 - t + \sin t}{t + \cos t}$.

14. Evaluate $\sum_{k=1}^{9} k^3$.

15. Find the average value of the function $f(t) = \sin t$ on $[0, 2\pi]$.

16. Express the solution of the initial value problem $\frac{dy}{dx} = \tan x,\ y(1) = 5$ as an integral.

17. Find $\frac{dy}{dx} \int_{0}^{\sqrt{x}} \cos t\, dt$.

18. Find absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.

19. Suppose that f is continuous and that $\int_{0}^{5} f(z)\, dz = 3$ and $\int_{0}^{6} f(z)\, dz = 7$. Find $\int_{5}^{6} f(z)\, dz$.

20. The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

21. Find the work done by the force $F(x) = \frac{1}{x^2}$ N along x-axis from $x = 1$ m to $x = 10$ m.

22. Find $\lim_{x \to \infty} \frac{\sin 2x}{x}$.

23. Evaluate $\int_0^{\pi/3} 2\sec^2 x dx$.

24. Find the interval in which $f(x) = -x^2 - 3x + 3$ is increasing and decreasing.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find asymptotes of the graph of $f(x) = \frac{x^2 - 3}{2x - 4}$.

26. Find $\lim_{x \to \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7}$.

27. If b, c and d are constants, for what value of b will be the curve $y = x^3 + bx^2 + cx + d$ have a point of inflection at $x = 1$?

28. Suppose that $f(-1) = 3$ and that $f'(x) = 0$ for all $x \in R$. Must $f(x) = 3$? Give reasons for your answer.

29. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

30. Find the area of the region enclosed by $x = 2y^2, x = 0$ and $y = 3$.

31. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

32. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

33. Show that center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the center of mass of a thin plate of constant density δ covering the legion bounded above by parabola $y = 4 - x^2$ and below by x-axis.

35. A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m.

    (a) Find the force constant k.

    (b) How much work will it take to stretch the spring 2 m. beyond its natural length ?

    (c) How far will a 45 N force stretch the spring ?

36. What values of a and b make $f(x) = x^3 + ax^2 + bx$ have,

    (a) A local maximum at $x = -1$ and a local minimum at $x = 3$.

    (b) A local minimum at $x = 4$ and a point of inflection at $x = 1$?

(2 × 10 = 20 marks)

D 43194 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS—UG)

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Absolute maximum of the function $y = x^2$ on $(0, 2]$ is ....
2. Find dy if $y = x^5 + 37x$.
3. Find the interval in which the function $y = x^3$ is concave up.
4. Suppose that $\int_1^4 f(x)\,dx = -2$, evaluate $\int_4^1 f(x)\,dx$.
5. A partition's longest subinterval is called ————.
6. Find $\lim\limits_{x \to -\infty} \frac{\pi\sqrt{3}}{x^2}$.
7. Express the limit of Riemann sums $\lim\limits_{\|P\| \to 0} \sum_{k=1}^{n} \left(3c_k^2 - 2c_k + 5\right) \Delta x_k$ as an integral if P denotes a partition of the interval $[-1, 3]$.
8. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.
9. Define critical point of a function.
10. Evaluate $\int 5\sec x \tan x \, dx$.
11. State Rolls' Theorem.
12. Define point of inflection.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Evaluate $\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

14. Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

15. Find the interval in which $f(t) = -t^2 - 3t + 3$ is increasing and decreasing.

16. Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

17. Suppose $\int_1^x f(t)\, dt = x^2 - 2x + 1$. Find $f(x)$.

18. Evaluate $\sum_{k=1}^{4} \left(k^2 - 3k\right)$.

19. Give an example of a function with no Riemann integral. Explain.

20. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2}\, dx$.

22. Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.

23. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

24. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}$, $x > 0$. Express $\int_1^3 \frac{\sin 2x}{x}\, dx$ in terms of F.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find the linearization of $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\, dt$ at $x = 1$.

26. Find the area of the region between the curve $y = x^2$ and the x-axis on the interval $[0, b]$.

27. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

28. A rectangle is to be inscribed in a circle of radius 2. What is the largest area the rectangle can have, and what are its dimensions ?

29. Show that functions with zero derivatives are constant.

30. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \le x \le 4$, about the x-axis.

31. Show that if f is continuous on $[a, b, a \neq b$, and if $\int_a^b f(x)\, dx = 0$, then $f(x) = 0$ atleast once in $[a, b]$.

32. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by x-axis and the line $y = x - 2$.

33. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$, about the x-axis.

(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Find the curve through the point $(1, 1)$ whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$.

    (b) How many such curves are there ?

35. Find the length of the curve $y = (1/3)\left(x^2 + 2\right)^{3/2}$ from $x = 0$ to $x = 3$.

36. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5y^2}, x = 0, y = -1, y = 1$ about x-axis.

(2 × 10 = 20 marks)

C 24738 (Pages : 4) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS—UG)

Core Course—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Find dy if $y = \frac{2x}{1+x^2}$.

2. A function with a continuous first derivative is said to be ————.

3. Suppose that $\int_1^3 f(x)\,dx = 6$. Find $\int_1^3 f(u)\,du$.

4. If f is smooth in $[a, b]$ then the length of the curve $y = f(x)$ from a to b is $L =$ ————.

5. Find the intervals in which the function f is increasing given $f'(x) = x(x-1)$.

6. The radius r of a circle increases from $r_0 = 10m$ to $10.1m$. Estimate the increase in the circle's area A by calculating dA.

7. Evaluate $\int_0^1 \left(x^2 + \sqrt{x}\right) dx$.

8. Write the sum without sigma notation and then evaluate the sum $\sum_{k=1}^{4} \cos k\pi$.

9. State Rolle's Theorem.

10. What are the critical points of f given $f'(x) = x^{-1/3}(x+2)$.

Turn over

11. Evaluate $\lim_{x\to\infty} \frac{\sin 2x}{x}$.

12. Find the linearization of $f(x)=\sqrt{1+x}$ at $x=0$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find the absolute maximum and minimum values of $f(x)=-\frac{1}{x}, -2\le x\le -1$.

14. Evaluate $\int_0^{\pi/4} \tan x \sec^2 x\, dx$.

15. Find the volume of the solid generated by revolving the region bounded by the line $y=0$ and the curve $y=x-x^2$.

16. Suppose that f is continuous and that $\int_0^3 f(x)\,dx=3$ and $\int_0^4 f(x)\,dx=7$. Find $\int_4^3 f(x)\,dx$.

17. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

18. Find the average value of $f(x)=x^2-1$ on $\left(0,\sqrt{3}\right)$.

19. Evaluate $\sum_{k=1}^{7}(-2k)$.

20. Find $\frac{dy}{dx}$ if $y=\int_1^{x^2}\cos t\, dt$.

21. Show that if f is continuous on $[a, b]\ a \neq b$ and if $\int_a^b f(x)dx = 0$ then $f(x) = 0$ at least once in $[a, b]$.

22. Evaluate $\frac{d}{dt}\int_0^{t^4} \sqrt{u}\, du$.

23. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$.

24. Express the solution of the following initial value problem as an integral :

Differential equation : $\frac{dy}{dx} = \tan x$

Initial condition : $y(1) = 5$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find the lateral surface area generated by revolving $xy = 1,\ 1 \leq y \leq 2$ about the y-axis.

26. About how accurately should we measure the radius r of a sphere to calculate the surface area $S = 4\pi r^2$ within 1% of its true value.

27. Evaluate the length of the curve $x = \sqrt{1 - y^2}, -\frac{1}{2} \leq y \leq \frac{1}{2}$.

28. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}, 1 \leq y \leq 4$ about the y-axis.

29. Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.

30. Find the intervals on which the function $h(x) = -x^3 + 2x^2$ is increasing and decreasing.

31. Find the length of the curve $x = \sin y, 0 \leq y \leq \pi$.

32. Find the area of the region enclosed by the curve $y = x^2 - 2$ and the line $y = 2$.

33. Find the value of local maxima and minima of $f(x) = x^2 - 4, -2 \leq x \leq 2$ and 2*ay* where they are assumed.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \leq x \leq 2$ about the x-axis.

35. State and prove the Fundamental Theorem of calculus.

36. Find the centre of mass of a thin plate of constant density δ covering the region bounded by the parabola $y = 4 - x^2$ and below by the x-axis.

(2 × 10 = 20 marks)

C 82976 (Pages : 4) Name.....................................

Reg. No.....................................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2015

(CUCBCSS—UG)

Core Course—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Evaluate $\lim_{x\to\infty} \frac{5x^2+8x-3}{3x^2+2}$.

2. Find the intervals in which the function f is increasing given $f^1(x) = x^{-1/3}(x+3)$.

3. State the Mean Value Theorem.

4. What are the critical points of f given $f'(x) = (x-1)(x+2)(x-3)$.

5. Find dy if $y = \sin 3x$.

6. Evaluate $\int_0^4 \left(3x - \frac{x^3}{4}\right) dx$.

7. The length of the longest subinterval of a partition is called its ———.

8. Write the sums without sigma notation and then evaluate the sum $\sum_{k=1}^{2} \frac{6k}{k+1}$.

9. If $\int_0^3 f(x)\,dx = 5$ find $\int_0^3 \sqrt{2}\, f(x)\,dx$.

10. A function with a continuous first derivative is said to be ———.

11. The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circle's area A by calculating dA.

12. If f is smooth in $[a, b]$ then the length of the curve $y = f(x)$ from a to b is L = ———.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find the work done by a force of $F(x) = \frac{1}{x^2}$ N along the x-axis from $x = 1$m to $x = 10$m.

14. Find the absolute maximum and minimum values of $f(x) = 4 - x^2, -3 \le x \le 1$.

15. Evaluate $\int_0^{2\pi} \frac{\cos z}{\sqrt{4 + 3\sin z}} dz$.

16. Find the volume of the solid generated by revolving the region bounded by the lines $y = 0$, $x = 2$ and the curve $y = x^3$.

17. Evaluate $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$.

18. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x)\, dx = 0$ then $f(x) = 0$ at least once in $[a, b]$.

19. Evaluate $\sum_{k=1}^{6} (3 - k^2)$.

20. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 3$.

21. Find the average value of $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$.

22. About how accurately should we measure the radius r of a sphere to calculate the surface area $S = 4\pi r^2$ within 1% of its true value.

23. Find the length of the curve $x = \sin y$, $0 \le y \le \pi$.

24. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 marks.

25. Find the length of the curve $y = \tan x$, $\frac{-\pi}{3} \le x \le 0$.

26. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

27. Find the area of the region enclosed by the curve $y = 2x - x^2$ and the line $y = -3$.

28. Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \le x \le 4$ about the y-axis.

29. Find the asymptotes of the curve $y = \frac{x^2 - 3}{2x - 4}$.

30. Express the solution of the following initial value problem as an integral :

Differential equation : $\frac{dy}{dx} = \tan x$.

Initial condition : $y(1) = 5$.

31. Find the intervals on which the function $g(t) = -t^2 - 3t + 3$ is increasing and decreasing.

32. Find the local maxima and local minima of $g(x) = -x^3 + 12x + 5$, $-3 \le x \le 3$.

33. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$.

(6 × 5 = 30 marks)

Part D

*Answer any **two** questions.*
Each question carries 10 marks.

34. Show that the centre of mass of a straight, thin strip or rod of constant density has half way between its two ends.

35. A rectangle is to be inscribed in a semi-circle of radius 2. What is the largest area then rectangle can have and what are its dimensions ?

36. Find the area of the region between the curve $y = 4 - x^2, 0 \le x \le 3$ and the x-axis.

$(2 \times 10 = 20$ marks)

C 5597 (Pages : 4) Name..............................

Reg. No..............................

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2016

(CUCBCSS—UG)

Core Course—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *the twelve questions.*
Each question carries 1 mark.

1. Find the linearization of $f(x) = \cos x$ at $x = \dfrac{\pi}{2}$.

2. Evaluate $\int_0^{\frac{\pi}{3}} 2\sec^2 x\, dx$.

3. The length of the largest sub-interval of a partition is called its ————.

4. Evaluate $\lim_{x \to -\infty} \dfrac{2x^2 - 3}{7x + 4}$.

5. What are the critical points of f given $f^1(x) = (x-1)^2 (x+2)$.

6. State the Mean Value Theorem.

7. Find dy if $y = x^5 + 37x$.

8. Write the sums without sigma notation and then evaluate the sum $\sum_{k=1}^{3} (-1)^{k+1} \sin \dfrac{\pi}{k}$.

9. Suppose that $\int_2^3 f(x)\, dx = 4$. Find $\int_2^3 -f(x)\, dx$.

10. Find the intervals in which the function f is increasing given $f^1(x) = (x-1)^2 (x+2)$.

Turn over

11. Evaluate $\int_{1}^{32} x^{-6/5}\, dx$.

12. Evaluate $\lim_{x \to \infty} \frac{2x+3}{5x+7}$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Suppose that f is continuous and that $\int_{0}^{3} f(z)\, dz = 3$ and $\int_{0}^{4} f(z)\, dz = 7$. Find $\int_{3}^{4} f(z)\, dz$.

14. Find the volume of the solid generated by revolving the region bounded by the line $y = 0$ and the curve $y = x - x^2$.

15. Find the average value of $f(x) = -3x^2 - 1$ on $[0, 1]$.

16. Evaluate $\int_{-\pi/4}^{0} \tan x \sec^2 x\, dx$.

17. Evaluate $\frac{d}{dt} \int_{0}^{t^4} \sqrt{u}\, du$.

18. Find the absolute maximum and minimum values of $f(x) = -x - 4, -4 \le x \le 1$.

19. Evaluate $\sum_{k=1}^{10} k^2$.

20. Find $\frac{dy}{dx}$ if $y = \int_{1}^{x^2} \cos t\, dt$.

21. Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.

22. The radius r of a circle increases from $r_0 = 10\, m$ to $10.1\, m$. Estimate the increase in the circle's area A by calculating dA.

23. Find the work done by a force of $F(x) = \frac{1}{x^2}$ N along the x–axis is from $x = 1\, m$ to $x = 10\, m$.

24. Find the function $f(x)$ whose derivative is series and whose graph passes through the point (0, 2).

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 marks.

25. Find the value of local maxima and minima of $g(x) = x^2 - 4, -2 \le x \le 2$ and say where they are assumed.

26. Find the surface area of the solid generated by revolving $y = \tan x, 0 \le x \le \frac{\pi}{4}$ about the x - axis.

27. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

28. Find the intervals on which the function $f(x) = 3x^2 - 4x^3$ is increasing and decreasing.

29. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

30. Find the asymptotes of the curve $y = \frac{x^2 - 3}{2x - 4}$.

31. Find the length of the curve $x = \sin y, 0 \le y \le \pi$.

32. Express the solution of the following initial value problem as an integral :

Differential equation	:	$\frac{dy}{dx} = \tan x$
Initial condition	:	$y(1) = 5$

33. About how accurately should we measure the radius r of a sphere to calculate the surface area $s = 4\pi r^2$ within 1 % of its true value.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the x-axis.

35. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, 0 \le x \le 1$.

36. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

(2 × 10 = 20 marks)