D 91713 (Pages : 4) Name.................................

Reg. No.................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Evaluate $\int_0^{\pi/6} \tan 2x \, dx$.

2. Define an alternating series.

3. Find the vertices of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

4. Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

5. Evaluate $\frac{d}{dt}\left(\tan h\sqrt{1+t^2}\right)$.

6. Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

7. Find the directive of the parabola $y^2 = 10x$.

8. Define absolute convergence.

9. Find y if $\ln y = 3t + 5$.

10. Find the parametric equation of the circle $x^2 + y^2 = a^2$.

Turn over

11. Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola, ellipse or hyperbola.

12. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. For what values of x do the series $\sum_{n=0}^{\infty} n!\, x^n$ converges.

14. Evaluate $\frac{d}{dx} \ln_{10}(3x+1)$.

15. Evaluate $\int \frac{\log_2 x}{x}\, dx$.

16. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$.

17. Find $\frac{dy}{dx}$ if $y = x^x, x > 0$.

18. Graph the set of points whose polar co-ordinates satisfy the conditions $1 \le r \le 2$ and $0 \le 0 \le \frac{\pi}{2}$.

19. Examine whether the series :

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{k!} + \ldots$$ converges.

20. Evaluate $\int_0^{\ln 2} 4e^x \sin h x \; dx$.

21. Show that ln x grows slaver than x as $x \to \infty$.

22. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.

23. Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

24. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta} d\theta$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. The standard parametrization of the circle of radius 1 centered at the point (0, 1) in the *xy*-plane is $x = \cos t, y = 1 + \sin t, 0 \le t \le 2\pi$. Use the parametrization to find the area of the surface swept out by revolving the circle about the *x*-axis.

26. Find the centroid of the first quadrant of the astroid $x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$.

27. Show that $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$.

28. Using Integral test show that the *p*-series :

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots + \frac{1}{n^p} + \ldots$$ converges if $p > 1$ and diverges if $p \le 1$.

29. Find the Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

30. Find the length of the Cardioid $r = 1 - \cos\theta$.

31. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

32. Graph the curve $r^2 = 4\cos\theta$.

33. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

(6 × 5 = 30 marks)

Part D

*Answer any **two** questions.*
Each question carries **10** *marks.*

34. Find the area of the region in the plane enclosed by the Cardioid $r = 2(1+\cos\theta)$.

35. Show that the Maclaurin's series for $\cos x$ converges to $\cos x$ for every value of x.

36. Using Integral test examine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(2 × 10 = 20 marks)

D 71655 (Pages : 4) Name.................................

Reg. No..............................

THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find $\lim\limits_{x \to 3} \dfrac{x+3}{x^2-9}$.

2. Find $\dfrac{d}{dx} \ln (x^2+3)$.

3. Find $\lim\limits_{x \to 0} \dfrac{3x - \sin x}{x}$.

4. Give an example of a sequence which has no upper bound.

5. Find a formula for the n^{th} term of the sequence $1, -4, 9, -16, 25, \ldots$

6. Find $\dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \ldots$

7. Write a parametrization of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

8. $\lim\limits_{n \to \infty} \dfrac{\ln n}{n} = \ldots$

Turn over

9. If $\sum_{n=1}^{\infty} a_n$ converges, then a_n converges to ...

10. Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$, and $\sum b_n$ converges, then $\sum a_n$...

11. A series $\sum a_n$ is said to be absolutely convergent if ....

12. $\frac{d}{dx} a^x = \ldots$

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find $\lim_{x \to 0^+} x \cot x$.

14. Evaluate $\int_0^1 \sinh^2 x dx$.

15. Find $\lim_{n \to \infty} \frac{\cos n}{n}$.

16. Let $a_n = \begin{cases} \frac{n}{2n}, & n \text{ odd}; \\ \frac{1}{2^n}, & n \text{ even}. \end{cases}$ Does $\sum a_n$ converge ?

17. For what values of x do the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converges ?

18. Find the center and radius of the conic section $x^2 + 4x + y^2 = 12$.

19. Locate the vertices of an ellipse of eccentricity 0.8 whose foci lie at the points $(0, \pm 7)$.

20. Determine the conic section from the equation $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$.

21. Graph the sets of points whose polar co-ordinates satisfy the condition $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$.

22. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

23. Find the directrix of the parabola $r = \dfrac{5}{2 + 2\cos\theta}$.

24. Determine if the sequence $a_n = \dfrac{3n+1}{n+1}$ is non-decreasing and if it is bounded from above.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that $\lim\limits_{x \to 0^+} (1+x)^{1/x} = e$.

26. Does the sequence whose n^{th} term is $a_n = \left(\dfrac{n+1}{n-1}\right)^n$ converge ? If so, find $\lim\limits_{n \to \infty} a_n$.

27. Find a formula for the n^{th} partial sum of the series $2 + \dfrac{2}{3} + \dfrac{2}{9} + \dfrac{2}{27} + \ldots + \dfrac{2}{3^{n-1}}$ and ude it to find the serie's sum if the series converges.

28. Find the surface area generated by revolving the curves $x = t + \sqrt{2}, y = \dfrac{t^2}{2} + \sqrt{2}t, -\sqrt{2} \leq t \leq \sqrt{2}$ about y-axis.

29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2 \sin 2\theta$.

30. Find the Maclaurin series for the function $f(x) = xe^x$.

31. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges or diverges.

32. Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges ?

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

35. Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where if anywhere, does the series converges to $1/x$?

36. Find the length of the curve $x = t^2/2,\ y = \frac{(2t+1)^{3/2}}{3},\ 1 \le t \le 4$.

(2 × 10 = 20 marks)

D 51243 (Pages : 4) Name..............................

Reg. No..............................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Core Course

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. Find $\frac{d}{dx} \ln 2x$.
2. Define a sequence.
3. Find least upper bound of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots, \frac{n}{n+1}$.
4. Find a formula for n^{th} term of the sequence $1, 5, 9, 13, 17, \ldots.$
5. State Sandwich theorem for sequences.
6. If $|r| < 1$ the series $a + ar + ar^2 + \ldots + ar^{n-1} + \ldots.$ converges to.............
7. Define conditional convergence of a series.
8. Write a parametrization of the circle $x^2 + y^2 = 1$.
9. $\lim_{n \to \infty} \sqrt[n]{n} =$ ....
10. Write the polar form of the parabola $y^2 = Qax$.
11. Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$..........
12. If $\sum |a_n|$ is convergent, then $\sum a_n$ is ....

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Find $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\, d\theta$.

14. Find k if $e^{2k} = 10$.

15. Find $\int_0^{\ln 2} e^{3x}\, dx$.

16. Show that $\lim_{n\to\infty} \frac{1}{n} = 0$.

17. For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge ?

18. Find the series for $f'(x)$ and $f''(x)$ if $f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, -1 < x < 1$.

19. Find the focus and directrix of the parabola $y^2 = 10x$.

20. Find the eccentricity of the hyperbola $x^2 - y^2 = 1$.

21. Determine the conic section from the equation $xy - y^2 - 5y + 1 = 0$.

22. Graph the sets of points whose polar co-ordinates satisfy the conditions $-3 \le r \le 2$ and $\theta = \pi/2$.

23. Replace the polar equation $r^2 = 4r\cos\theta$ by equivalent Cartesian equation.

24. Find the equation for the hyperbola with eccentricity $3/2$ and directrix $x = 2$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Solve the initial value problem $e^y \dfrac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0$.

26. Show that $(-1)^{n+1} \dfrac{n-1}{n}$ diverges.

27. Find a formula for the n^{th} partial sum of the series $\dfrac{1}{2 \cdot 3} + \dfrac{1}{3 \cdot 4} + \dfrac{1}{4 \cdot 5} + .. + \dfrac{1}{(n+1)(n+2)} + ...$ and use it to find the series sum if it converges.

28. Identify the function $f(x) = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + ..., -1 \le x \le 1$.

29. The x and y axes are rotated through an angle of $\pi/4$ radians about the origin. Find an equation for the hyperbola $2xy = 9$ in the new co-ordinates.

30. Find the surface area generated by revolving the curves $x = \cos t, y = 2 + \sin t, 0 \le t < 2\pi$ about x-axis.

31. Show that $(1/2, 3\pi/2)$ lies on the curve $r = -\sin(\theta/3)$.

32. Determine whether the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ converges or diverges.

33. Check whether $\sum_{n=2}^{\infty} \dfrac{1 + n \ln n}{n^2 + 5}$ converges or diverges.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. The series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ converges to $\sin x$ for all x.

    (a) Find the first six terms of the series for $\cos x$. For what values of x should the series converge ?

    (b) By replacing by $2x$ in the series for $\sin x$, find a series that converges to $\sin 2x$ for all x.

35. Find the Taylor series and Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

36. Find the length of the curve curve $x = 8\cos t + 8t \sin t$, $y = 8\sin t - 8t \cos t$, $0 \le t \le \pi/2$.

(2 × 10 = 20 marks)

C 31126 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours | Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. The product rule for natural logarithm is ————.

2. $\lim_{x \to 0} \frac{3x - \sin x}{x} =$ ————.

3. The Hyperbolic cosecant is defined as ————.

4. Let $\{a_n\}$ be a sequence of real numbers. If $a_n \to L$ and if f is a function that is continuous at L and defined at all a_n, then ————.

5. The series $\sum_{n=1}^{\infty} n^2$ diverges because ————.

6. Suppose that $a_n > 0$ and $b_n > 0$ for all $\geq N$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges then ————.

7. The first two terms in the Maclaurin series expansion of $f(x) = xe^x$ is ————.

8. The first two terms in the expansion of $f(x) = \frac{1}{3} x \cos x$ is ————.

9. The remainder of order n of $R_n(x)$ in Taylor's Formula is ————.

10. The eccentricity of the conic section $r = \frac{6}{2 + \cos\theta}$ is ————.

11. The standard form of Hyperbola if $e = 3$ and vertices $(0, \pm 1)$ is ————.

12. The foci of ellipse . $9x^2 + 10y^2 = 90$ is ————.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Define Hyperbolic function and Exponential function.

14. Define natural logarithm. Give examples.

15. Find $\lim_{x \to 0} + \sqrt{x}$ in x.

16. Let $\sum a_n$, $\sum c_n$ and $\sum d_n$ be series with non negative terms and suppose that for some integer N, $d_n \le a_n \le c_n, \forall n \ge N$. Then write the conditions for which the series $\sum a_n$ converges and diverges ?

17. Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ converges or diverges ?

18. Determine whether the Alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ converges or diverges ?

19. Define Power series representation of a function about the point $x = a$.

20. Find the power series representation of $f(x) = \sin x$ about $x = 0$.

21. Define the radius of convergence of a power series.

22. Define eccentricity e of a conic section. Give examples.

23. Write the polar equation of an ellipse.

24. Sketch the circle $r = 6 \sin \theta$.

(9 × 2 = 18 marks)

Part C (Short Answer Type)

Answer any **six** *questions.*

25. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$. converge or diverge?

26. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n + 5}{3^n}$.

27. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\cdot 3^n}$ converge or diverge ?

28. Expand $f(x) = x^4 + x^2 + 1$ as Taylor series about a point $a = -2$.

29. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty}x^n$.

30. Discuss about the convergence of Taylor series. Give examples.

31. Find the eccentricity and directrix of the parabola $r = \frac{25}{10 - 5\cos\theta}$. Also sketch the conic.

32. Identify the conic section and hence find the centre, vertex, foci, asymptotes of $x^2 + y^2 - 2x - 2y = 0$.

33. Find the polar equation of : (i) $r\sin\theta = 2, e = 1/2$; (ii) $r\sin\theta = -6, e = 1/3$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Determine whether the series

    (i) $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$ converge ?

    (ii) Does the series $\sum_{n=1}^{\infty}\frac{(n+1)(n+2)}{n!}$ converge ?

35. Find the values of x for which the replacement for $\sin x$ with an error of magnitude no greater than 3×10^{-4} is possible where $\sin x = x - \frac{x^3}{3!} + \ldots\ldots$

36. Describe about polar co-ordinates and polar equation of a conic. Sketch the region defined by the polar co-ordinate inequalities

    (i) $0 \le r \le 6\cos\theta$.

    (ii) $-4\sin\theta \le r \le 0$.

(2 × 10 = 20 marks)

D 12389 **(Pages : 3)** **Name..**

Reg. No......................................

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS–UG)

Mathematics—Core Course

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. Evaluate : $\lim_{x \to 0} \frac{3x - \sin x}{x}$.
2. Define absolute convergence.
3. Find the focus of the parabola $y^2 = 10x$.
4. Evaluate $\int \coth 5x \, dx$.
5. Find the Taylor polynomial of order 1 generated by $f(x) = \ln x$ at $a = 1$.
6. Write the parametric equations of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
7. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.
8. Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola, ellipse or hyperbola.
9. Evaluate $\frac{d}{dx} \log_{10}(3x+1)$.
10. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.
11. Show that x^2 grows faster than $\ln x$ as $x \to \infty$.
12. State Leibniz's theorem for an alternating series.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Graph the set of points whose polar co-ordinates satisfy the conditions $r \le 0$ and $\theta = \frac{\pi}{4}$.

14. For what values of x do the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1} \text{ converges.}$$

15. Find k if $e^{2k} = 10$.

16. Find the Maclaurin's series for $f(x) = \frac{1}{1+x}$.

17. Find an equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.

18. Evaluate $\int 2^{\sin x} \cos x \, dx$.

19. Examine whether the series :

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{k!} + \ldots \text{ converges.}$$

20. Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola ellipse or hyperbola.

21. Prove that the alternating series :

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \text{ converges.}$$

22. Examine whether $\sum_{n=1}^{\infty} n^2$ converges or diverges.

23. Prove that $e^{x + \ln 2} = 2e^x$.

24. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x-1}}{x}$.

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries **5** *marks.*

25. Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

26. Find the tangent to the right-hand hyperbola branch $x = \sec t, y = \tan t, \frac{-\pi}{2} < t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$ where $t = \frac{\pi}{4}$.

27. Graph the curve $r = 1 - \cos\theta$.

28. Find the Maclaurin's series for $f(x) = \sin 3x$.

29. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$.

30. Find the Taylor polynomial generated by :

$f(x) = e^x$ at $x = 0$.

31. Evaluate $\int_0^1 \sinh^2 x\,dx$.

32. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

33. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the initial value problem :

$$e^y \frac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0.$$

35. Find the length of the cardioid $r = 1 - \cos\theta$.

36. Using Integral test show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \ldots + \frac{1}{n^p} + \ldots$ converges if $p > 1$ and diverges if $p \le 1$.

(2 × 10 = 20 marks)