Farshana Sherin. K

D 72400 (Pages : 3) Name.....

Reg. No.....

# THIRD SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2014

(U.G.–CCSS)

Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

## Section A

*Answer* **all** *questions.*

*Each question carries ¼ weightage.*

1. Show with an example that scalar product of vectors is commutative.
2. Find the acceleration of a particle with position vector $\vec{r}(t) = [\sin t, 0, 0]$.
3. If $f = x^2 + y^2 + z^2$, find grad $f$.
4. What is the Cartesian form of $\vec{r}(u,v) = [ua\cos v, ub\sin v, u]$?
5. If $\vec{F}$ = grad $f$, then curl $\vec{F}$ = ———.
6. Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 9$.
7. Verify that $y = e^x + ax^2 + bx + c$ is a solution $y''' = e^x$.
8. Solve $y' = -2xy$.
9. Test for exactness : $-\frac{y}{x^2}dx + \frac{dy}{x} = 0$.
10. Define rank of a matrix.
11. Is the matrix $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ singular *or* non-singular ?
12. State Cayley Hamilton theorem.

(12 × ¼ = 3 weightage)

Turn over

## Section B

*Answer* **all** *questions.*
*Each question carries* 1 *weightage.*

13. Find the angle between $x + y + z = 1$ and $x + 2y + 3z = 6$.

14. Find a parametric representation of the straight line through (4, 2, 0) in the direction of [1, 1, 0].

15. Find the length of the semi cubical parabola $\bar{r}(t) = \left[t, t^{3/2}, 0\right]$ from (0, 0, 0) to (4, 8, 0).

16. Evaluate $\int_C \bar{F}\, od\, \bar{r}, \bar{F}(\bar{r}) = [z, x, y]$, $C : \bar{r}(t) = [\cos t, \sin t, 3t]$ and $0 \le t \le 2\pi$.

17. Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = a^2$.

18. Solve the initial value problem $2\sin 2x \sinh y dx - \cos 2x \cosh y dy = 0, y(0) = 1$.

19. Find an integrating factor : $2\cosh x \cos y dx = \sinh x \sin y dy$.

20. Find the rank of $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 6 & 12 \end{bmatrix}$.

21. Find the eigen values of $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$.

(9 × 1 = 9 weightage)

## Section C

*Answer any* **five** *questions.*
*Each question carries* 2 *weightage.*

22. (i) Find the potential function of $[yz, xz, xy]$.

(ii) Test whether irrotational : $\bar{v} = \left[2y^2, 0, 0\right]$.

23. Test for path independence and if independent, integrate from :
$(0,0,0)$ to $(a,b,c)$ : $\cos(x + yz)[dx + zdy + ydz]$.

24. Evaluate $\iint_S \bar{F} \circ \bar{n}\, dA$ using Gauss divergence theorem :

$\bar{F} = \left[x^3, y^3, z^3\right]$, S is the surface of the sphere $x^2 + y^2 + z^2 = 4$.

25. Solve $y' + y\sin x = e^{\cos x}$.

26. Solve using the transformation $\frac{y}{x} = v : 2xyy' = y^2 - x^2$.

27. Find the rank by reducing to normal form : $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$.

28. Using Cayley Hamilton theorem. Find the inverse of : $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

(5 × 2 = 10 weightage)

## Section D

*Answer any* **two** *questions.*
*Each question carries* **4** *weightage.*

29. State Stokes' theorem and verify it for $\vec{F} = [y^2, z^2, x^2]$. S being the portion of the paraboloid $x^2 + y^2 = z, y \geq 0, z \leq 1$.

30. (i) Solve $y' + 2y = y^2$.

(ii) Find the Orthogonal trajectories of $y = ce^{-x}$.

31. Find the eigen values and eigen vectors of :

$$A = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}.$$

(2 × 4 = 8 weightage)

Faeshana Shereen E.K

D 92282 (Pages : 3) Name.....

Reg. No.....

# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A (Objective Type Questions)

*Answer all* **twelve** *questions.*

1. Write the general form of Bernoulli's differential equation.
2. Find the solution of the differential equation $y' = \frac{-y}{x}$.
3. What is the order of the differential equation $y'' - (y')^3 + 4 = 0$?
4. State Cayley Hamilton theorem.
5. What is the rank of a $(n \times n)$ non-singular matrix ?
6. Write the normal form of the matrix : $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
7. Write the parametric equation of the curve $\frac{x^2}{4} + \frac{y^2}{3} = 1$.
8. Define Irrotational vector.
9. Find curl $v$, where $v = [2y, 5x, 0]$.
10. Find the tangent to the curve $r(t) = ti + t^3 j$ at $(1, 1, 0)$.
11. Define scalar potential of a vector.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

## Part B (Short Answer Type Questions)

*Answer any* **nine** *questions.*

13. Find the orthogonal trajectories of the family of curves $y = ce^{-x}$.
14. Write the condition for the differential equation $Mdx + Ndy = 0$ become exact. What is the form of its solution ?

Turn over

15. Find the integrating factor of the linear differential equation $y' - y = e^{2x}$.

16. Find characteristic roots of the matrix : $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

17. Write the elementary transformations in a matrix.

18. Find the comment of vector $a = [4, 2, 0]$ in the direction of $b = [1, -1, 2]$.

19. Find the directional derivative of $f = xyz$ at the point P $(-1, 1, 3)$ in the direction of $i - 2j + 2k$.

20. Find the unit normal to the level surface $z^2 = 4(x^2 + y^2)$ at the point P $(1, 0, 2)$.

21. Find div $v$, where $v = xyzi + 3zxj + zk$.

22. Define Jacobian.

23. Find value of $\lambda$ if $a = [4, 2, \lambda]$ and $b = [2, -3, 1]$ are orthogonal.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

$(9 \times 2 = 18$ marks)

## Part C (Short Essay Type Questions)

*Answer any* **six** *questions.*

25. Solve the intial value problem $y' + y \tan x = \sin 2x$, $y(0) = 1$.

26. Solve $xy' = y + 3x^4 \cos^2\left(\frac{y}{x}\right)$.

27. Find the eigenvalues and eigenvector corresponding to any one eigenvalue of the matrix : $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$.

28. Use Cayley Hamilton theorem to find $A^{-1}$ and $A^4$, where $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

29. Find the tagential and normal componets of acceleration of an object moving along the curve $r(t) = e^t i + e^{-1} j$.

30. Find tangent to the ellipse $\frac{1}{4}\left(x^2+y^2\right)$ at the point $P\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$.

31. Find the area of the cardioid $r=a(1-\cos\theta)$, $0\le\theta\le 2\pi$.

32. Evaluate the double integral $\iint_R y^2\,dxdy$ where R is the region bounded by the unit circle in the first quadrant.

33. Verify Green's theorem in the plane for the vector $F=\left(y^2-7y\right)i+(2xy+2x)\,j$ and the region bounded by the $x^2+y^2=1$.

(6 × 5 = 30 marks)

## Part D (Essay Type Questions)

*Answer any* **two** *questions.*

34. Test for consistency and solve the following system of equation.

(a)
$$\begin{aligned} x+\ y+\ z+\ 3&=0\\ 3x+\ y-2z+\ 2&=0\\ 2x+4y+7z-13&=0. \end{aligned}$$

(b)
$$\begin{aligned} 3x+26y+\ 2z&=9\\ 5x+\ 3y+\ 7z&=4\\ 7x+\ 2y+10z&=5. \end{aligned}$$

35. (a) Solve the differential equation :

$$2\sin\left(y^2\right)dx+xy\cos\left(y^2\right)dy=0,\ y(2)=\sqrt{\frac{\pi}{2}}.$$

(b) Prove that Curl(gradf) = 0.

36. Verify Stokes's theorem for $F=[y,z,x]$ over the surface of the paraboloid $z=1-\left(x^2+y^2\right)$, $z\ge 0$.

(2 × 10 = 20 marks)

Faeshana Sherin : E.K.

D 92619 (Pages : 3) Name.....

Reg. No.....

# THIRD SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2015

(UG—CCSS)

Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

## Section A

*Answer* **all** *questions.*
*Each weightage ¼.*

1. Show with an example that addition of vectors is commutative.
2. Find the acceleration of a particle with position vector $\vec{r} = [3t, -3t, 2t]$.
3. Find grad $f$ if $f = x^2 + y^2$.
4. What is the Cartesian form of the surface $\vec{r}(u, v) =$ [aucos $v$, busin $v$, $u^2$] ?
5. If $\vec{F}$ = grad $f$, then curl $\vec{F}$ = _____.
6. Find the unit vector normal to the surface $x^2 + y^2 + z^2 = a^2$.
7. Verify that $y = a \cos x + b \sin x$ is a solution of $y'' + y = 0$.
8. Solve $y' = ky$.
9. Test for exactness : $(x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0$.
10. Define rank of a matrix.
11. Is $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ singular or non-singular ? $|D| \neq 0$ $1 \times 5 - 2 \times 3$ $-5 - 6 = 1$ //
12. State Cayley-Hamilton theorem.

(12 × ¼ = 3 weightage)

## Section B

*Answer* **all** *questions.*
*Each weightage* 1.

13. Find the angle between [4, 2, 3] and [1, 1, 0].
14. Find a parametric representation of the straight line through (2, 3, 0) and (5, –1, 0).

Turn over

15. Find the length of the catenary $\vec{r}(t) = [t, \cosh t, 0]$ from $t = 0$ to $t = 1$.

16. If $\vec{F} = [-y, -xy]$ and C is the portion of $x^2 + y^2 = 1$ in the first quadrant, evaluate $\int_C \vec{F} \circ d\vec{r}$.

17. Use Green's Theorem to evaluate the area enclosed by the ellipse $x^2/a^2 + y^2/b^2 = 1$.

18. Solve the Initial Value Problem : $[(x+1)e^x - e^y]\,dx = xe^y\,dy,\ y(1) = 0$.

19. Find an Integrating factor for 2 sin $(y^2)\,dx + xy \cos(y^2)\,dy = 0$.

20. Find the rank of $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix}$.

21. Find the eigen values of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(9 × 1 = 9 weightage)

## Section C

*Answer any* **five** *questions.*
*Each weightage 2.*

22. (i) Find the potential function of $[2x, 4y, 8z]$.

(ii) Test whether $\vec{v} = [y, -x, 0]$ is irrotational.

23. Test for path independence and evaluate if independent the integral from **(0, 0, 0) to $(a, b, c)$** : $2xy^2\,dx + 2x^2y\,dy + dz$.

24. Evaluate $\iint_S \vec{F} \circ \vec{n}\,dA$ using **Gauss Divergence Theorem** : $\vec{F} = [x^2, 0, z^2]$, **S is the box** $|x| \le 1, |y| \le 3, |z| \le 2$.

25. Solve $xy' = 2y + x^3e^x$.

26. Solve using the transformation $y = ux$ : $xy' = y^2 + y$.

27. Find the rank by reducing to normal form : $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$.

28. Find the inverse using Cayley-Hamilton Theorem $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$.

(5 × 2 = 10 weightage)

## Section D

*Answer any* **two** *questions.*
*Each weightage* 4.

29. State Stokes' Theorem and verify it for $\vec{F} = [y, z, x]$, S being the paraboloid $z = 1 - (x^2 + y^2)$, $z \geq 0$.

30. (i) Solve $y' + \frac{y}{3} = \frac{1}{3}(1 - 2x)\, y^4$.

(ii) Find the Orthogonal Trajectories of $y = cx^{3/2}$.

31. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

(2 × 4 = 8 weightage)

D 12425 (Pages : 4) Name.....

Reg. No.....

# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS–UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Section A

*Answer* **all** *questions.*

*Each question carries* 1 *mark.*

1. Verify that $y = cx^3$ is a solution of $xy' = 3y$.
2. Solve $y' = -x/y$.
3. Test for exactness $2xydx + x^2dy = 0$.
4. Define rank of a matrix.
5. Find the characteristic roots of the matrix $A = \begin{bmatrix} -7 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.
6. State the Cayley-Hamilton Theorem.
7. If $\vec{p}, [3, 2, 0]$ and $[-2, 4, 0]$ are in equilibrium, find $\vec{p}$.
8. Find the gradient of the scalar function $f(x, y, z) = xyz$.
9. Illustrate commutativity of vector addition with an example.
10. Define a simply connected domain.
11. State Green's theorem in the plane.
12. Give a parametric representation of the sphere.

(12 × 1 = 12 marks)

Turn over

## Section B

*Answer any* **nine** *questions.*
*Each question carries* 2 *marks.*

13. Solve $(x^2+y)\,dx+(y^2+x)\,dy=0$.

14. Solve $y'-y=4$.

15. What is the characteristic equation of $\begin{bmatrix}1 & 2\\ 7 & 1\end{bmatrix}$?

16. Solve $dy/dx+y/x=x^3$.

17. Using Cayley-Hamilton theorem, find $A^2$ if $A=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$.

18. Find the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}$.

19. Find a parametric representation of the straight line through (4, 2, 0) in the direction of [1, 1, 0].

20. Find a tangent vector and unit tangent vector for $\vec{r}(t)=\left[t,t^3,0\right]$.

21. Show that the form under the integral sign is exact :

$$\int e^{x-y+z^2}\,(dx-dy+2z\,dz).$$

22. Give the standard form of the Bernoulli equation with a suitable example.

23. Using Green's theorem, find the area enclosed by the ellipse :

$$x^2/a^2+y^2/b^2=1.$$

24. Find curl of $\left[y/z,\ x/z,\ -xy/z^2\right]$.

(9 × 2 = 18 marks)

## Section C

*Answer any* **six** *questions.*
*Each question carries 5 marks.*

25. Find an integrating factor and solve :

$$\left(x^2 - 2x + 2y^2\right) dx + 2xy\, dy = 0 .$$

26. Find the Orthogonal Trajectories of $y = ce^{-x}$.

27. Reduce to normal form and find the rank of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

28. Solve : $x + 2y + 3z = 0$ .
$2x + y + 3z = 0$
$3x + 2y + z = 0.$

29. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

30. Find the length of the vector $\vec{r}(t) = [t, \cosh t, 0]$ from $t = 0$ to $t = 1$.

31. (i) Prove that curl (grad $f$) $= \vec{0}$ for any twice differentiable scalar function $f$.

(ii) Prove that div (curl $\vec{v}$) $= 0$ for any vector $\vec{v}$.

32. Use Gauss Divergence Theorem to evaluate :

$$\iint_S \vec{F} \circ \vec{n}\, dA \; ; \vec{F} = \left[x^2, 0, z^2\right] \text{ S is the box } |x| \le 1, |y| \le 3, |z| \le 2 .$$

33. Find the work done by a Force $\vec{F} = \left[y^2, -x^2, 0\right]$ in moving an object along the straight line segment from (0, 0) to (1, 4).

(6 × 5 = 30 marks)

Turn over

## Section D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Find the characteristic roots and any two characteristic vectors of :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

35. (i) Find the tangential and normal acceleration of $\vec{r}(t) = \left[0, 0, 5t^2\right]$.

(ii) Find the directional derivative of $f(x,y) = x^2 + y^2$ at (1, 1) in the direction of [2, – 4].

(iii) Given $\vec{v} = \text{grad}\, f$, find '$f$' if $\vec{v} = \left[\frac{y}{z}, \frac{x}{z}, \frac{-xy}{z^2}\right]$.

36. State Stokes Theorem. Verify it for :

$\vec{F} = \left[z^2, 5x, 0\right]$, S is the square $0 \le x \le 1, 0 \le y \le 1, z = 1$.

(2 × 10 = 20 marks)

C 33538 (Pages : 3)

Name.....

Reg. No.....

# THIRD SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2017

(UG–CCSS)

MM 3C 03—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

## Part A

*Answer* **all** *questions.*
Each question carries ¼ weightage.

1. Give the explicit form of the general first order linear ordinary differential equation.
2. Solve $y' = -ky^2$.
3. Is $2xydx + x^2dy = 0$ exact ?
4. Define rank of a matrix.
5. State True *or* False : $\begin{matrix} 7x & - & 9y & + & 8z & - & 7 & = & 0 \\ 8x & + & 5y & + & 9z & & & = & 0 \end{matrix}$ is a homogeneous system of linear equations.
6. What are the characteristic roots of $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ ?
7. Is the cross product of vectors associative ?
8. Curl $(\nabla f)$ = ————.
9. What is the Laplace's equation ?
10. Give the parametric representation of the cylinder $x^2 + y^2 = 4, -1 \le z \le 1$.
11. $\int_C \vec{F} o d\vec{r}$ is independent of path in a domain D iff. $\vec{F}$ = ————.
12. Give the equation of the catenary.

(12 × ¼ = 3 weightage)

## Part B

*Answer any* **nine** *questions.*
*Each question carries* 1 *weightage.*

13. Solve the IVP $y' = -y/x$, $y(1) = 1$.
14. Show that $e^x$ is an integrating factor of $\sin y\, dx + \cos y\, dy$.
15. Give the Bernoulli's equation.
16. What is the rank the matrix $\begin{bmatrix} 2 & 1 & 3 \\ -2 & -1 & -3 \\ 4 & 0 & 9 \end{bmatrix}$?
17. State a necessary and sufficient condition for the system of linear equations AX = B to be consistent.
18. State Cayley Hamilton theorem.
19. Find the angle between [1 2 3] and [–1 1 2].
20. Define the scalar triple product and evaluate $\left(\hat{j}\,\hat{i}\,\hat{k}\right)$.
21. Show that $\text{div}\left(\text{curl}\,\vec{V}\right) = 0$.
22. Find a unit vector normal to $x^2 + y^2 + z^2 - a^2 = 0$.
23. Find the area of the region bounded by the cardioid $r = a(1 - \cos\theta), 0 \le \theta \le 2\pi$.
24. State Stokes theorem.

(9 × 1 = 9 weightage)

## Part C

*Answer any* **five** *questions.*
*Each question carries* 2 *weightage.*

25. Find an integrating factor and solve $\left(x^2 - 2x + 2y^2\right)dx + 2xy\,dy = 0$.
26. Find the orthogonal trajectories of $y = cx^{3/2}$.
27. Reduce to normal form and hence find the rank of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.
28. Use Cayley Hamilton theorem to find $A^4$ if $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.

29. Show that the form under the integral sign is exact and evaluate

$$\int_{(0,-1,1)}^{(2,4,0)} e^{x-y+z^2}(dx-dy+2zdz).$$

30. Evaluate by Gauss Divergence theorem $\iint_S \vec{F} \circ \vec{n} dA$ where $\vec{F} = [\cos y, \sin x, \cos z]$ and S is $x^2 + y^2 \leq 4, |z| \leq 2$.

31. If $f$ is any scalar function, show that $\nabla(f^n) = nf^{n-1}\nabla f$.

32. Using Green's theorem, evaluate $\oint_C \vec{F} \circ d\vec{r}$, where $\vec{F}$ = grad (sin $x$ cos $y$) and C is the ellipse $25x^2 + 9y^2 = 225$.

(5 × 2 = 10 weightage)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 4 *weightage.*

33. (i) Find the entire length of the hypocycloid $\vec{r}(t) = a\cos^3 t\,\hat{i} + a\sin^3 t\,\hat{j}$.

(ii) Find the directional derivative of F = $xyz$ at P (–1, 1, 3) along $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$.

(iii) Find whether $\vec{v} = [y, -x, o]$ is (a) irrotational ; (b) incompressible ?

34. Solve if possible :

$$\begin{aligned} 2x - y + 3z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2. \end{aligned}$$

(2 × 4 = 8 weightage)

C 31164 (Pages : 4) Name.....

Reg. No.....



# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Section A

*Answer any* **ten** *questions.*
*Each question carries* 1 *mark.*

1. Verify that $y = c \sec x$ is a solution of $y' = y \tan x$.
2. Solve $y' = -ky^2$.
3. Test for exactness : $\sinh x \cos y\, dx - \cosh x \sin y\, dy = 0$.
4. Define rank of a matrix.
5. Find the characteristic roots of $A = \begin{bmatrix} -11 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$.
6. State Cayley-Hamilton Theorem.
7. If $\vec{p}, [1, -1, 3]$ and $[-3, 2, 4]$ are in equilibrium, find $\vec{p}$.
8. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$.
9. Illustrate commutativity of the vector dot product with an example.
10. Define a simply connected domain.
11. State Green's Theorem in the plane.
12. Find the unit normal vector to the sphere $x^2 + y^2 + z^2 = a^2$.

(10 × 1 = 10 marks)

**Turn over**

## Section B

*Answer any* **ten** *questions.*
*Each question carries 2 marks.*

13. Solve $e^y\,dx + (xe^y + 2y)\,dy = 0$.

14. Solve $y' - y = e^{2x}$.

15. Find the characteristic equation of $A = \begin{bmatrix} 8 & -1 \\ 2 & 8 \end{bmatrix}$.

16. Solve $\frac{dy}{dx} + \frac{y}{x} = x$.

17. Using Cayley-Hamilton Theorem, find $A^2$ if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

18. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \end{bmatrix}$.

19. Find a parametric representation of the straight line through (2, 3, 0) and (5, –1, 0).

20. Find a tangent vector and unit tangent vector for $\bar{r}(t) = [2\cos t, 2\sin t, 0]$.

21. Show that the form under the integral sign is exact $\int 2xy^2dx + 2x^2ydy + dz$.

22. Give the standard form of the Bernoulli Equation with an example.

23. Use Green 's Theorem to find the area enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

24. Find the curl of $[2x,\ 4y,\ 8z]$.

(10 × 2 = 20 marks)

## Section C

*Answer any* **six** *questions.*
*Each question carries 5 marks.*

25. Find an integrating factor and solve : $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$.

26. Find the Orthogonal Trajectories of $y = cx^{3/2}$.

27. Reduce to normal form and find the rank of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & -6 & -10 \\ 5 & 8 & -12 & -19 \end{bmatrix}$.

28. Solve the system of equations
$$\begin{aligned} x + 3y - 2z &= 0 \\ 2x - y + 4z &= 0 \\ x - 11y + 14z &= 0. \end{aligned}$$

29. Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

30. Find the length of $\bar{r}(t) = \left[a\cos^3 t, a\sin^3 t, 0\right]$ from $t = 0$ to $t = \frac{\pi}{2}$.

31. (i) Prove that for any twice differentiable scalar function '$f$', curl (grad $f$) $= \vec{0}$.

    (ii) Prove that for any vector $\vec{v}$, div (curl $\vec{v}$) $= 0$.

32. State Gauss Divergence Theorem and use it to evaluate $\iint_S \vec{F} \circ \vec{n}\ dA$, $\vec{F} = \left[x^3,\ y^3,\ z^3\right]$, S is the surface $x^2 + y^2 + z^2 = 16$.

33. Find the work done by $\vec{F} = \left[e^x,\ e^{-y},\ e^z\right]$, $c$ is the curve $[t, t^2, t]$ from $(0, 0, 0)$ to $(1, 1, 1)$.

$(6 \times 5 = 30$ marks)

## Section D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Find the characteristic roots and characteristic vectors of $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$.

35. (i) Find the tangential and normal acceleration of $\bar{r}(t) = [2\cos t, 2\sin t, 3]$.

(ii) Find the directional derivative of $f(x, y, z) = xyz$ at $(-1, 1, 3)$ in the direction of $[1, -2, 2]$.

(iii) If $\vec{v} = \text{grad } f$ find '$f$' for $\vec{v} = [yz, zx, xy]$.

36. State Stoke's Theorem and verify it for $\vec{F} = [x^2 - y^2, 2xy, 0]$, S is the surface of the rectangle $x = 0, y = 0, x = a, y = b$.

$(2 \times 10 = 20$ marks)

**D 51292** (Pages : 3) Name.....

Reg. No.....



# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A (Objective Type)

*Answer* **all the twelve** *questions.*
*Each question carries* 1 *mark.*

1. Write the general form of Bernoulli's differential equation.
2. Solve $y' = \frac{xy}{2}$.
3. What is the degree of the differential equation $x^3y'''y' + 2e^xy'' = 0$.
4. State Cayley-Hamilton theorem.
5. What is the determinant of a 2 × 2 matrix whose rank is 1 ?
6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$ ?
7. Find the resultant of the vectors $p = [4, -2, -3]$, $q = [8, 8, 0]$.
8. Write the parametric representation of a straight line through the point (4, 2, 0) and in the direction of the vector $i + j$.
9. Define gradient of a function.
10. Find the directional derivative of $f = x - y$ at (4, 5) in the direction of $2i + j$.
11. Define a smooth curve.
12. State Green's theorem in plane.

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries* 2 *marks.*

13. Write the condition for the differential equation $Mdx + Ndy = 0$ become exact. What is the form of its solution ?
14. Find an integrating factor for $2xydx + 3x^2dy = 0$ and solve it.

**Turn over**

15. Find the characteristic roots of the matrix $\begin{pmatrix} 4 & 3 & 1 \\ -4 & -3 & -1 \\ 1 & 2 & 5 \end{pmatrix}$.

16. Write the elementary transformations in a matrix.

17. Find the component of $a = [4, 0, -3]$ in the direction of $b = [1, 1, 1]$.

18. Find the arc length parameter for the helix $r(t) = [a\cos t, a\sin t, ct]$.

19. Find div $v$ where $v = x^2 i + y^2 j + z^2 k$.

20. Define Jacobian.

21. Show that $\text{curl}(u+v) = \text{curl}\, u + \text{curl}\, v$.

22. Find $\nabla^2 f$ where $f = e^{2x}\sin 2y$.

23. Show that $\int_{(-1,5)}^{(4,3)} 3z^2 dx + 6xz dz$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

**(9 × 2 = 18 marks)**

## Part C (Short Essay)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Solve $(2x - 4y + 5)y' + (x - 2y + 3) = 0$.

26. Solve $2x \tan y dx + \sec^2 y dy = 0$.

27. Find the eigen values and eigen vectors corresponding to any one eigen value of the matrix :

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

28. Use Cayley-Hamilton theorem to find $A^{-1}$ and $A^3$ where A is $\begin{pmatrix} 1 & 5 \\ 3 & 8 \end{pmatrix}$.

29. Find the speed and tangential acceleration of an object moving along the curve :

$r(t) = \cos ti + \sin 2tj + \cos 2tk$.

30. Find unit normal vectors for the surface $z = \sqrt{(x^2 + y^2)}$ at (6, 8, 10).

31. Find the volume of the region in space bounded by the co-ordinate planes and the surfaces $y = 1 - x^2, z = 1 - x^2$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1+\cos\theta)$.

33. Verify Greens theorem in the plane for $F = [3y^2, x - y^4]$ and the region is the rectangle with vertices (1, 1), (–1, 1), (–1, –1), (1, –1).

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Test for consistency and solve the following system :

(a)
$$\begin{aligned} 2x + y + z &= 5 \\ x - y &= 0 \\ 2x + y - z &= 1 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + 3z &= 14 \\ 2x - y + 5z &= 15 \\ -3x + 2y + 4z &= 13. \end{aligned}$$

35. Solve (a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$ ; (b) Find the angle between $3x + 5y = 0$ and $4x - 2y = 1$.

36. Verify Stoke's theorem for $F = [y^2, -x^2, 0]$ over the circular semi-disk $x^2 + y^2 \le 4, y \ge 0, z = 0$.

(2 × 10 = 20 marks)

**D 71693** (Pages : 3) Name.....

Reg. No.....

# THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A (Objective Type)

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. Write the general form of first order ODE.
2. What do you mean by exact differential equation ?
3. Define dot product of two vectors.
4. State Cayley Hamilton theorem.
5. When will you say two matrices are equivalent ?
6. Define curl of a function.
7. Find the resultant of the vectors $p=[2,4,-5], q=[1,-6,9]$.
8. Define characteristic polynomial of a matrix.
9. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^3+8x=0$.
10. Find the directional derivative of $f=x^2+y^2$ at (1, 1) in the direction of $2i-4j$.
11. Write the general form of Bernoulli differential equation.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries* 2 *marks.*

13. Verify that $\frac{c}{x}$ is a solution of the differential equation $xy'=-y, c$ is a constant and $x\neq 0$.
14. Find the curve through the point (1, 1) in the $xy$-plane having at each of its points the slope $-\frac{y}{x}$.

**Turn over**

15. Solve $2xyy' = y^2 - x^2$.

16. Let $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find the distance between $u$ and $v$.

17. Find the projection of $a = [1, -3, 4]$ in the direction of $b = [3, 4, 7]$.

18. Find the unit tangent vector T to the curve $C = F(t) = \left(t^2, 3t-2, t^3, t^2+5\right)$ in $R^4$ when $t = 2$.

19. Find the component of (1, 1, 3) in the direction of (0, 0, 5).

20. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x) = 2x^3 - 4x + 5$. Find $f(A)$.

21. Show that $\text{curl}(u+v) = \text{curl}\, u + \text{curl}\, v$.

22. Show that every elementary matrix E is invertible, and its inverse is an elementary matrix.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} e^x (\cos y dx - \sin y dy)$ is path independent.

24. Find the length of the curve $r(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.

(9 × 2 = 18 marks)

## Part C (Short Essays)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Find all the curves in $xy$-plane whose tangents pass through the point $(a, b)$.

26. Solve $\cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right)dy = 0$.

27. Find an integrating factor and solve the initial value problem

$\left(e^{x+y} + ye^y\right)dx + \left(xe^y - 1\right)dy = 0, y(0) = -1$.

28. Find the straight line $L_1$ through the point P : (1, 3) in the $xy$-plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

29. Find the volume of the tetrahedron with vertices (0, 0, 0), (1, 2, 0), (3, –3, 0), (1, 1, 5).

30. Show that the integral $\int_c F.dr = \int_c 2xdx + 2ydy + 4zdz$ is path independent in any domain in space and find its value in the integration from A : (0, 0, 0) to B : (2, 2, 2).

31. Describe the region and evaluate $\int_0^1 \int_{x^2}^{x} (1 - 2xy)\, dydx$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1+\cos\theta)$.

33. Verify Greens theorem in the plane for $F = \left[-y^3, x^3\right]$ and the region is the circle $x^2 + y^2 = 25$.

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Let $A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}$.

(a) Find all eigen values of A.

(b) Find a maximal set S of non-zero orthogonal eigenvectors of A.

(c) Find an orthogonal matrix P such that $D = P^{-1}AP$ is diagonal.

35. Solve :

(a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$.

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate the integral by divergence theorem $F = \left[z - y, y^3, 2z^3\right]$, S the surface of $y^2 + z^2 \le 4, -3 \le x \le 3$.

(2 × 10 = 20 marks)

D 91751 p14 – Complementary maths (Pages : 4) Name.....

Reg. No.....



# THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
## NOVEMBER 2020

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

### Part A (Objective Type)

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. What do you mean by a homogeneous equation ?
2. Consider a system of linear equations in $n$ unknowns with augmented matrix $M = [A, B]$. Then, the solution is unique if and only if rank (A).
3. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^2 + 8x = 0$.
4. State Cayley-Hamilton theorem.
5. What is the determinant of a 2 × 2 matrix whose rank is 1 ?
6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$?
7. Define eigen value of a matrix.
8. Define divergence of a vector field.
9. Define gradient of a function.
10. Define the derivative of a vector function.
11. Define a smooth curve.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

**Turn over**

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries 2 marks.*

13. Solve the initial value problem $y' = 3y, y(0) = 5.7$.

14. Find an integrating factor for $2\cosh x \cos y dx = \sinh x \sin y dy$ and solve it.

15. Find the angles of the triangle with vertices $(0,0,0), (1,2,3), (4,-1,3)$.

16. Find $x, y, z, t$ where $3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$.

17. Solve the system :

$$x - 3y = 4$$
$$-2x + 6y = -8.$$

18. Show that circle of radius $a$ has curvature $\frac{1}{a}$.

19. Find a unit normal vector $n$ of the cone of revolution $z^2 = 4\left(x^2 + y^2\right)$ at the point $(1,0,2)$.

20. Find the directional derivative of $f = x^2 + y^2 - z$ at $(1,1,-2)$ in the direction of $(1,1,2)$.

21. Show that $\text{curl}\,(u+v) = \text{curl}\,u + \text{curl}\,v$.

22. Show that $\text{div}\,kv = k\,\text{div}\,v$.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} e^x (\cos y\,dx - \sin y\,dy)$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

$(9 \times 2 = 18$ marks)

## Part C (Short Essay)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Show that the form under integral sign is exact in the plane and evaluate the integral $\int_{(-1,-1)}^{(1,1)} e^{-x^2-y^2}(xdx+ydy)$.

26. Solve $2x\tan y\,dx+\sec^2 ydy=0$.

27. Find the minimal polynomial $m(t)$ of $A=\begin{pmatrix}2 & 2 & -5\\ 3 & 7 & -15\\ 1 & 2 & -4\end{pmatrix}$.

28. Let $A=\begin{pmatrix}3 & -4\\ 2 & -6\end{pmatrix}$. Find all eigen values and corresponding eigen vectors. Find matrices P and D such that P is non-singular and $D = P^{-1}AP$ is diagonal.

29. Let L be the linear transformation on $R^2$ that reflects each point P across the line $y=kx$, where $k>0$.

    (a) Show that $v_1=(k,1)$ and $v_2=(1,-k)$ are eigenvectors of L.

    (b) Show that L is diagonalizable, and find a diagonal representation D.

30. Find the straight line $L_1$ through the point $P:(1,3)$ in the $xy$-plane and perpendicular to the straight line $L_2: x-2y+2=0$.

31. Evaluate the double integral $\iint_R y^2\,dxdy$ where R is the region bounded by the unit circle in the first quadrant.

32. Solve $2x\tan ydx+\sec^2 ydy=0$.

33. Verify Greens theorem in the plane for $F=\left[-y^3,x^3\right]$ and the region is the circle $x^2+y^2=25$.

(6 × 5 = 30 marks)

**Turn over**

## Part D

*Answer any* **two** *questions.*
*Each question carries* **10** *marks.*

34. Test for consistency and solve the following system :

(a)
$$x_1 + x_2 - 2x_3 + 4x_4 = 5$$
$$2x_1 + 2x_2 - 3x_3 + x_4 = 3$$
$$3x_1 + 3x_2 - 4x_3 - 2x_4 = 1.$$

(b)
$$x + 2y + z = 3$$
$$2x + 5y - z = -4$$
$$3x - 2y - z = 5.$$

35. Solve :

(a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0,\ y(2) = \sqrt{\frac{\pi}{2}}.$

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate $\iint_S (7xi - zk)\cdot ndA$ over the sphere $S: x^2 + y^2 + z^2 = 4$ by

(a) Divergence theorem.

(b) Directly.

(2 × 10 = 20 marks)

D 11860 (Pages : 4) Name.....

Reg. No.....

# THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 3C 03—MATHEMATICS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

## Part A (Objective Type)

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. Define ordinary differential equation.
2. The solution of the differential equation $y' - 1 + y^2$ is ————.
3. The degree of the differential equation $\frac{dy}{dx} = -2 \sin x \cos x$ is ————.
4. The rank of the matrix $A = \begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix}$ is ————.
5. State True *or* False : The following two matrices are equivalent :

$$\begin{bmatrix} 5 & 5 & -5 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix}.$$

6. The characteristic matrix of the matrix $\begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$ is ————.
7. Find the resultant of the vectors $\mathbf{p} = [4, -2, -3]$, $\mathbf{q} = [8, 8, 1]$, and $\mathbf{u} = [-12, -6, 2]$.
8. For the vectors $\mathbf{a} = [1, 3, 2]$, $\mathbf{b} = [2, 0, -5]$, and $\mathbf{c} = [4, -2, 1]$ , find $(\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$.

**Turn over**

9. The vector function $\mathbf{r}(t) = (4 + t)\,\mathbf{i} + (2 + t)\,\mathbf{j}$ represents ————.

10. Let $\vec{r}(t) = 5t^2\,\vec{k}$ be the position vector of a moving particle, where $t \geq 0$ is time. Then the acceleration vector of the moving particle is ————.

11. If $\vec{v} = e^x(\cos y\mathbf{i} + \sin y\mathbf{j})$ then div $\vec{v}$ = ————.

12. When we say that a vector valued function is conservative ?

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries* 2 *marks.*

13. Verify that $y = e^{4x}$ is a solution of the differential equation $\frac{dy}{dx} = 4y$.

14. Solve the initial value problem $2xy' - 3y = 0$ ; $y(1) = 4$.

15. Show that the equation :

$$ydx + xdy = 0$$

is exact and solve it.

16. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ to its normal form.

17. Solve completely the system of equations :

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0.$$

18. Find the eigen values of :

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

19. Prove that $(\mathbf{u} + \mathbf{v})' = \mathbf{u}' + \mathbf{v}'$.

20. Find $\frac{df}{ds}$ in the direction of the vector $4i + 4j - 2k$ at the point (1, 1, 2) if $f(x, y, z) = x^2 + y^2 - z$.

21. Show that $\mathbf{F}(x, y) = (\cos y + y \cos x)\,\mathbf{i} + (\sin x - x \sin y)\,\mathbf{j}$ is a conservative vector field.

22. Find the unit tangent vector at a point $t$ to the curve $\mathbf{r} = a \cos t\,\mathbf{i} + a \sin t\,\mathbf{j}$.

23. Find unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

24. Verify that $w = x^2 - y^2$ satisfies Laplace's equation $\nabla^2 w = 0$.

(9 × 2 = 18 marks)

## Part C (Short Essays)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Write in the linear form and then solve $\sin 2x \dfrac{dy}{dx} = y + \tan x$.

26. Determine the rank of the following matrix, by reducing to echelon form :

$$\begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}.$$

27. Show that the system of equations :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4 \end{aligned}$$

is consistent and hence solve them.

28. Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

29. Using Cayley-Hamilton theorem find the inverse of $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$.

**Turn over**

30. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ with

$\mathbf{F}(r) = 5z\mathbf{i} + xy\mathbf{j} + x^2z\mathbf{k}$

along the straight - line segment C : $t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

31. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the divergence theorem where $\mathbf{F} = [x^2, 0, z^2]$ and S is the surface of the box given by the inequalities $|x| \le 1, |y| \le 3, |z| \le 2$.

32. Evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ where $\mathbf{F} = \left[e^{2y}, e^{-2z}, e^{2x}\right]$, and

S : $\mathbf{r} = [3\cos u, 3\sin u, v]$, $0 \le u \le \frac{1}{2}\pi$, $0 \le v \le 2$.

33. Apply Green's theorem to evaluate $\oint_C (2x^2 - y^2)\, dx + (x^2 + y^2)\, dy$, where C is the boundary of the area enclosed by the $x$-axis and the upper half off the circle $x^2 + y^2 = a^2$.

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$.

35. Investigate for what values of $a$, $b$ the system of equations :

$$x + y + 2z = 2$$
$$2x - y + 3z = 10$$
$$5x - y + az = b$$

have unique solution.

36. Calculate the line integral $\oint_C \mathbf{F}.\mathbf{r}'(s)\, ds$ using Stoke's theorem, where $\mathbf{F} = [-5y, 4x, z]$, and C is the circle $x^2 + y^2 = 4$, $z = 1$.

(2 × 10 = 20 marks)