C 21296 **(Pages : 4)** Name.....................................

Reg. No.................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *twelve questions.*
Each question carries 1 *mark.*

1. State the fundamental theorem of theory of equations.
2. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, write the equation whose roots are $-\alpha, -\beta, -\gamma$.
3. Find the number of real roots of $x^4 - 1 = 0$.
4. Write the standard form of a cubic equation.
5. Find the rank of $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
6. If A and B are non-singular square matrices of order 5, find the rank of AB.
7. Find the number of solutions of the equation $x + 2y = 3$.
8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 2 \\ a & 6 & b \end{bmatrix}$ and the system of homogeneous linear equations AX = 0 has a non-zero solution, find the value of b.
9. Find the characteristic roots of $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.
10. Find the parametric equations of the line through the point (3, – 4, – 1) parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the angle between the planes $x + y = 1$, $2x + y - 2z = 2$.

12. Find the unit tangent vector to the curve $\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j}$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. If α, β, γ are the roots of the equation $2x^3 + x^2 - 2x - 1 = 0$, find the value of $\alpha + \beta + \gamma$.

14. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

15. Show that the equation $x^4 + 4x^2 + 5x - 6 = 0$ has exactly one positive root.

16. Show that the rank of a matrix, every element of which is unity is 1.

17. Find the normal form of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

18. Find the values of λ so that the system of equations $\lambda x + y = 0$, $x + \lambda y = 0$ has zero solution only.

19. Prove that the characteristic roots of triangular matrix are the same as its diagonal elements.

20. Show that if λ is a characteristic root of a matrix A, then $\lambda + k$ is a characteristic root of the matrix $A + kI$.

21. Find the spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

22. If $\mathbf{r}(t) = (3 \cos t)\,\mathbf{i} + (3 \sin t)\,\mathbf{j} + t^2\mathbf{k}$ is the position vector of a particle in space at time t, at what times, if any, are the body's velocity and acceleration orthogonal ?

23. If u is a differentiable vector function of t of constant magnitude, prove that $\mathbf{u} \cdot \frac{du}{dt} = 0$.

24. Show that the curvature of a straight line is zero.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in A.P.

26. If α, β, γ are the roots of the equation $x^3 + 3x^2 + 6x + 1 = 0$, find the value of $\left(\alpha^2 + 1\right)\left(\beta^2 + 1\right)\left(\gamma^2 + 1\right)$.

27. Obtain the real root of the equation $x^3 - 15x = 126$ by Cardan's method.

28. Reducing to the normal form, find the rank of $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

29. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in normal form.

30. Test for consistency and solve the system of equations :

$$5x + 3y + 7z - 4 = 0$$

$$3x + 26y + 2z - 9 = 0$$

$$7x + 2y + 10z - 5 = 0.$$

31. If A is a non-singular matrix, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

32. Find the distance from the point (1, 1, 5) to the line $x = 1 + t, y = 3 - t, z = 2t$.

33. Obtain the curvature of a circle of radius a.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

35. Find the characteristic roots and the corresponding characteristic vectors for the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

36. Find the binormal vector and torsion for the space curve $\mathbf{r}(t) = (3 \sin t)\,\mathbf{i} + (3 \cos t)\,\mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)

C 2216 **(Pages : 4)** **Name..................................**

Reg. No...............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION

APRIL 2021

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*

Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\alpha^2, \beta^2, \gamma^2$.

2. State Descarte's rule of signs.

3. Find a cubic equation, two of whose roots are given by $1, 3 + 2i$.

4. What do you mean by reciprocal equation of first type ? Give example.

5. What is the rank of the identity matrix of order 101 ?

6. If $A = \left[a_{i,j}\right]$ is an $m \times n$ matrix and $a_{i,j} = 7$ for all i, j then rank of A is ——————.

7. A system of m homogeneous linear equations in n unknowns has only trivial solution if ——————.

8. For what values of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If the number of variables in a non-homogeneous system AX = B is n then the system possesses a unique solution if ——————.

10. Find the parametric equation of a line through $P(1, 1, 1)$ and parallel to the z-axis.

11. Find the unit tangent vector of the helix $r(t) = (\cos t + t \sin t)\, i + (\sin t - t \cos t\ j)\, j + tk, t > 0$.

12. Write equations relating spherical and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Solve $6x^3 - 11x^2 - 3x + 2 = 0$. Given that the roots are in harmonic progression.

14. Find the equation whose roots are the roots of $x^3 + 3x^2 - 2x - 4 = 0$ increased by 5.

15. If α, β, γ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $(\beta - \alpha)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

16. If $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$. Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non-singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$.

21. Find the velocity and acceleration vectors of $r(t) = (t+1)\,i + \left(t^2 - 1\right) j + 2tk$ at $t = 1$.

22. Find the rectangular co-ordinates of the centre of the sphere $r^2 + z^2 = 4r\cos\theta + 6r\sin\theta + 2z$.

23. Evaluate $\int_0^{\pi} (\cos ti + j - 2tk)\, dt$.

24. Find the principal unit normal for the circular motion $r(t) = (\cos 2t)\, i + (\sin 2t)\, j$.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of the transpose of a matrix is equal to the rank of the same matrix.

29. Find the rank of $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{pmatrix}$.

30. Using matrix method solve :

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2.$$

31. Find the point in which the line $x = 1 + 2t, y = 1 + 5t, z = 3t$ intersects the plane $x + y + z = 2$.

32. Find the distance from the point $S(2, 1, 3)$ to the line $x = 2 + 2t, y = -1 + 6t, z = 3$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$.

35. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$ and hence evaluate A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = \left(\frac{t^3}{3}\right) i + \left(\frac{t^2}{2}\right)$.

(2 × 10 = 20 marks)

C 80916 (Pages : 3) Name.....................................

Reg. No.....................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *the* **twelve** *questions.*
Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.
2. State the Fundamental theorem of algebra.
3. Find a cubic equation, two of whose roots are given by 1, 3 + 2*i*.
4. What do you mean by reciprocal equation. Give example.
5. What is the rank of the identity matrix of order *n*.
6. If $A = \left[a_{i,j}\right]$ is an m × n matrix and $a_{i,j} = 7$ for all i, j then rank of A is ————.
7. A system of m homogeneous linear equations AX = 0 in n unknowns has only trivial solution if ————.
8. For what values of *a* the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.
9. If the number of variables in a non homogeneous system AX = B is *n* then the system possesses a unique solution if ————.
10. Find the parametric equation of a line through P(3,-4,-l) and parallel to the vector $i + j + k$.
11. Find the unit tangent vector of the helix $r(t) = \cos t i + \sin t j + t k$.
12. Write equations relating rectangular and cylindrical coordinates.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Solve $4x^3 - 24x^2 + 23x + 18 = 0$. Given that the roots are in arithmatic progression.
14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking second term.

Turn over

15. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$.

16. If $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$ Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)i + (3\sin t)j + t^2 k$.

22. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

23. Evaluate $\int_0^1 \left(t^3 i + 7j + (t+1)k\right) dt$.

24. Find the unit tangent vector for the circular motion $r(t) = (\cos 2t)i + (\sin 2t)j$.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of a non singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

30. Using matrix method solve,

$$x + 2y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = 4$$

31. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ intersects the plane $2x - y + 3z = 6$.

32. Find the distance from the point S (0, 0, 1, 2) to the line $x = 4t, y = -2t, z = 2t$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$.

35. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{pmatrix}$.

36. Find the binormal vector and torsion for the space curve $r(t) = (3\sin t)i + (3\cos t)j + 4tk$.

(2 × 10 = 20 marks)

C 61216 (Pages : 4) Name.................................

Reg. No..................................

FOURTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCBCSS—UG)

Common Course for LRP

MA T4 B04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ find the value of $\sum \alpha^2$.

2. Define a reciprocal equation.

3. State Descarte's rule of signs.

4. If α, β, γ are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

5. Rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$ is ... .

6. If A is a non-zero column matrix and B is a non-zero row matrix then rank (AB) is .... .

7. The system AX = 0 in n unknowns has a non-trivial solution if ———.

8. For what value of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If A is an n-rowed non-singular matrix, X and B are $n \times 1$ matrices, then the system of equations AX = B has ——— solution.

10. Find the parametric equation of a line through the point (3, – 4, –1) and parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the unit vector tangent to the curve $r(t) = (2\cos t)\,\mathbf{i} + (2\sin t)\,\mathbf{j} + \sqrt{5t}\,\mathbf{k}, 0 \le t \le \pi$.

12. Write the equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $x^3 - 12x^2 + 39x^2 - 28 = 0$ whose roots are in arithmetic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

15. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\sum \alpha^2\beta\gamma$.

16. If $A = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 2 & 1 \end{bmatrix}$, then rank of AB is :

17. Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18. Show that corresponding to a characteristic vector X of a square matrix A, there exist one and only one characteristic root.

19. If A is non-singular, prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

20. Show that the characteristic roots of a Hermitian matrices are all real.

21. Find the velocity and acceleration vectors of $r(t) = (t + 1)\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}$ at $t = 1$.

22. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

23. Evaluate $\int_0^{\pi} \left((\cos t)\,\mathbf{i} + \mathbf{j} - (2t)\,\mathbf{k}\right) dt$.

24. Find the curvature of $r(t) = t\,\mathbf{i} + (\ln \cos t)\,\mathbf{j}, -\pi/2 < t < \pi/2$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any **six** questions.*
Each question carries 5 marks.

25. If α, β, γ are roots of $x^3 - x - 1 = 0$, find the equation whose roots are

$\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the values of $\Sigma\left(\frac{1+\alpha}{1-\alpha}\right)$.

26. If α, β, γ are roots of $x^2 + qx + r = 0$, find the equation whose roots are $(\beta-\gamma)^2, (\gamma-\alpha)^2, (\alpha-\beta)^2$.

27. Solve the equation $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$.

28. Reduce the matrix A to its normal form and hence find the rank of A where :

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

29. Show that rank (AA′) = rank (A).

30. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

31. Find the point where the line $x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

32. Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

33. Using matrix method solve the equations :

$$x + y + z = 6$$

$$x - y - z = 2$$

$$2x + y - z = 1.$$

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any two questions.
Each question carries 10 marks.

34. Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

35. Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence obtain A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = (3 \sin t)\, \mathbf{i} + (3 \cos t)\, \mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)

16/7/18 physics D 41960

D 41959 **(Pages : 4)** **Name..................................**

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 4B 04—THEORY OF EQUATIONS MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

2. State the Fundamental theorem of algebra.

3. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of :

   $(\beta-\gamma)(\gamma-\alpha) + (\gamma-\alpha)(\alpha-\beta) + (\alpha-\beta)(\beta-\gamma)$.

4. What do you mean by reciprocal equation of second type ? Give example.

5. What is the rank of the identity matrix of order 20 ?

6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 7$ for all i, j then rank of A is ————.

7. A system of m homogeneous linear equations in n unknowns has only trivial solution if ————.

8. For what values of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If the number of variables in a non-homogeneous system AX = B is n then the system possesses a unique solution if ————.

10. Find the parametric equation of a line through P (3, – 4, – 1) and parallel to the vector $i + j + k$.

11. Find the unit vector tangent to the curve $r(t) = \left(\cos^3 t\right) j + \left(\sin^3 j\right) k, 0 \le t \le \frac{\pi}{2}$.

12. Write equations relating rectangular and cylindrical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

*Answer any **nine** questions.*
Each question carries 2 marks.

13. Solve $4x^3 - 24x^2 + 23x + 18 = 0$. Given that the roots are in arithmatic progression.

14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking second term.

15. If α, β, γ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $(\beta - \alpha)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

16. If $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$. Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non-singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$.

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)\,i + (3\sin t)\,j + t^2 k$.

22. Find a Cartesian equation for the surface $z = r^2$. And identify the surface.

23. Evaluate $\int_{-\pi/4}^{\pi/4} \left[(\sin t)\,i + (1 + \cos t)\,j + \left(\sec^2 t\right) k\right] dt$.

24. Find the normal vector for $r(t) = (a\cos t)\,i + (a\sin t)\,j + bk$.

(9 × 2 = 18 marks)

Part C (Short Essays)

*Answer any **six** questions.*
Each question carries 5 marks.

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

30. Using matrix method solve :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4. \end{aligned}$$

31. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ intersects the plane $2x - y + 3z = 6$.

32. Find the distance from the point S (0, 0, 1, 2) to the line $x = 4t, y = -2t, z = 2t$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries **10** *marks.*

34. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

35. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{pmatrix}$ and hence evaluate A^{-1}.

36. Find the binormal vector and torsion for the space curve $r(t) = (e^t \cos t)\, i + (e^t \sin t)\, j + 2k$.

(2 × 10 = 20 marks)

C 23294 (Pages : 4) Name..............................

Reg. No..............................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2017

(CUCBCSS—UG)

Mathematics

MAT 4B 04—THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*

1. If $2+\sqrt{3}$ is a root of $x^4 - 2x^3 - 22x^2 + 62x - 15 = 0$, without solving the equation completely, state the other root.

2. If $\alpha, \beta, \gamma \ldots$ are the roots of $f(x) = 0$, then what is the equation whose roots are $-\alpha, -\beta, -\gamma \ldots$?

3. If α and β are the roots of $lx^2 + mx + n = 0$ find $\alpha^2 + \beta^2$.

4. Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$.

5. What is the rank of a non-singular matrix of order n ?

6. Find the row reduced Echelon form of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

7. Find the characteristic root of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

8. State Cayley-Hamilton theorem.

9. Fill in the blanks :

   The characteristic roots of a diagonal matrix are the same as its ————————.

10. Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$, $2x + y - 2z = 5$.

11. Find a Cartesian equation for the surface $z = r^2$ and identify the surface.

12. Find the unit tangent vector of :

    $r(t) = (2\cos t)i + (2\sin t)j + \sqrt{5}\, t\, k$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer **all** *questions.*

13. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

14. Solve $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, given that the roots are in A.P.

15. Form an equation whose roots are increased by 2 of the equation $2x^3 + 3x^2 - x - 1 = 0$.

16. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$.

17. Compute the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

18. State Sylvestee's Law of Nullity.

19. Show the characteristic roots of a Hermitian matrix are all real.

20. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

21. Find the principal unit normal vector $\dot{N}$ for a curve $r(k) = (2t+3)i + (5-t^2)j$.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*

22. Find the rational roots of the equation $2x^3 - 3x^2 - 11x + 6 = 0$.

23. Solve the equation $x^3 - 7x^2 + 36 = 0$, given that the difference between two of its roots is 5.

24. Solve the reciprocal equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

25. Reduce to the normal form and find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

26. Obtain the row reduced echelon form to find the rank of $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$.

27. Solve the system of equations :

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0.$$

28. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem.

29. The vector $r(t) = (2\cos t)i + (3\sin t)j + 4t\,k$ gives the position of a moving body at time t. Find the body's speed and acceleration. When $t = \pi/2$ find speed.

30. Find curvature for the helix :

$$r(t) = (a\cos t)i + (a\sin t)j + btk\,; a, b \geq 0, a^2 + b^2 \neq 0.$$

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*

31. (a) Discuss the nature of roots of the equation :

$x^9 + 5x^8 - x^3 + 7x + 2 = 0$ using Descarte's rule of signs.

(b) Solve $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$, which is a reciprocal equation of second type.

32. (a) Find characteristic vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ corresponding to any one characteristic root.

(b) Obtain the inverse of the above matrix using Cayley-Hamilton theorem.

33. (a) Find the length of the indicated portion of the curve :

$$r(t) = t\,i + \frac{2}{3}t^{3/2}k \,;\, 0 \le t \le 8.$$

(b) Show that the curvature of a circle of radius a is $\frac{1}{a}$.

(2 × 10 = 20 marks)

C 3964 **(Pages : 4)** Name...

Reg. No.....................................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2016

(CUCBCSS—UG)

Core Course—Mathematics

MAT AB 04—THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *twelve questions.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$ find the equation whose roots are $\alpha - 1, \beta - 1, \gamma - 1$.
2. State Descarte's rule of signs.
3. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ find the value of $\sum \alpha^2$.
4. If α, β, γ .... are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$....
5. What is the rank of a unit matrix of order n ?
6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 4$, for all i, j then rank of A is ————.
7. A system of m homogeneous linear equations $AX = 0$ in n unknowns has only trivial solution if ————.
8. For what value of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.
9. If the number of variables in a non-homogeneous system $AX = B$ is n, then the system possesses a unique solution if ————.
10. Find the parametric equation of a line through the point $(-2, 0, 4)$ and parallel to the vector $2\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$.
11. Find the unit vector tangent to the curve $r(t) = t\boldsymbol{i} + (2/3)t^{3/2}\,\boldsymbol{k}$.
12. Write the equations relating rectangular and spherical co-ordinates.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Solve $8x^3 - 14x^2 + 7x - 1 = 0$ whose roots are in geometric progression.

14. Find the equation whose roots are the roots of $x^3 + 3x^2 - 2x - 4 = 0$ increased by 3.

15. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\sum \alpha^2 \beta$.

16. If $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then rank of A^2 is :

17. Under what condition the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 3.

18. Show that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

19. If α is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is a characteristic root of *adj* A.

20. Show that the characteristic roots of a Hermitian matrices are all real.

21. Find the velocity and acceleration vectors of $r(t) = e^t \, \boldsymbol{i} + \frac{2}{9} e^{2t} \, \boldsymbol{j}$ at $t = \ln 3$.

22. Find the equation for the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical co-ordinates.

23. Evaluate $\int_0^1 \left((3t^2) \, \boldsymbol{i} + 2\boldsymbol{j} - (t-3) \, \boldsymbol{k} \right) dt$.

24. Fine the curvature of $r(t) = (a \cos t) \, \boldsymbol{i} + (a \sin t) \, \boldsymbol{j}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. If α, β, γ are roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$. Hence write down the values of $\sum\left(\frac{1+\alpha}{1-\alpha}\right)$.

26. If α, β, γ are roots of $x^2 + qx + r = 0$, find the equation whose roots are $(\beta-\gamma)^2, (\gamma-\alpha)^2, (\alpha-\beta)^2$.

27. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

28. For the matrix A, find non-singular matrices P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

29. Prove that rank of a non-singular matrix is equal to the rank of its reciprocal matrix.

30. Using matrix method solve the equations :

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11.$$

31. Find the point where the line $x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

32. Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

33. Find the latent roots and latent vectors of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

35. Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1}.

36. Find the binormal vector and torsion for the space curve

$$r(t) = (a\cos t)\,\boldsymbol{i} + (a\sin t)\,\boldsymbol{j} + bt\boldsymbol{k}, a, b \geq 0, a^2 + b^2 = 1.$$

(2 × 10 = 20 marks)