

D 30177

(Pages : 4)

Name.....

Reg. No.....

**FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022**

Mathematics

MAT 5B 05—VECTOR CALCULUS

(2017—2018 Admissions)

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all questions.

Each question carries 1 mark.

- The domain of $z = \sqrt{1 - x^2 - y^2}$ is _____.
- $\lim_{(x,y) \rightarrow (0,0)} \frac{y}{2x^2 + 1} =$ _____.
- Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.
- Find $\frac{\partial f}{\partial x}$ if $f(x, y) = \sqrt{x^2 + y^2}$.
- Define local maximum of a function of two variables.
- Evaluate $\int_0^1 \int_0^2 xy(x - y) dx dy$.
- If R is a simple polar region whose boundaries are the rays $\theta = \alpha$ and $\theta = \beta$ and the curves $r = r_1(\theta)$ and $r = r_2(\theta)$ and if $f(r, \theta)$ is continuous on R, then $\iint_R f(r, \theta) dA =$ _____.

Turn over

8. $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx.$
9. Define Scalar field.
10. Give a parametrization of the cylinder $x^2 + (y - 3)^2 = 9, 0 \leq z \leq 5.$
11. Find curl \mathbf{F} where $\mathbf{F} = x^2z\mathbf{i} - 2y^3z^2\mathbf{j} + xy^2z\mathbf{k}.$
12. When a vector field is solenoidal ?

(12 × 1 = 12 marks)

Section B

*Answer any ten questions.
Each question carries 4 marks.*

13. Find all first and second order partial derivatives of the function $f(x, y) = x \cos y + ye^x.$
14. Find the linearization of $f(x, y) = x^2 + y^2 + 1$ at the point $(0, 0).$
15. Evaluate $\iint_D (x + y) dx dy$ where D is the domain in the first quadrant of the circle $x^2 + y^2 = 9$
16. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $P_0(1, 2, 4).$
17. Evaluate $\iiint_V \frac{1}{(x + y + z + 1)^3} dx dy dz,$ where V is the volume bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1.$
18. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $|\mathbf{r}| = r,$ then show that $\nabla \left(\frac{1}{r} \right) = -\frac{\mathbf{r}}{r^3}.$

19. Find the work done in moving a particle once round a circle C in the xy -plane : the circle has centre at the origin at radius 3 and the force field is given by $\mathbf{F} = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$.
20. Find the work done by the conservative field $\mathbf{F} = xz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla(xyz)$ along any smooth curve C joining the points $(-1, 3, 9)$ to $(1, 6, -4)$.
21. Using Green's theorem, evaluate the integral $\oint_C xydy - y^2dx$,
where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.
22. Find *unit normal* to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.
23. If $v = f\left(\frac{x}{z}, \frac{y}{z}\right)$, show that $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z} = 0$.
24. Find the Centroid of the solid (with density given by $\delta = 1$) enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy -plane.
25. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.
26. Use Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F} = xz\mathbf{i} + xy\mathbf{j} + 3xz\mathbf{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed counterclockwise as viewed from above.

(10 × 4 = 40 marks)

Section C

*Answer any six questions.
Each question carries 7 marks.*

27. Show that $f(x, y) = \begin{cases} \frac{4x^2y}{x^3 + y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

is continuous at every point except the origin.

Turn over

28. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
29. Find the volume of the upper region D cut from solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$.
30. Evaluate $\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2} dx dy$ by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over an appropriate region in the uv -plane.
31. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin and the point.
32. Show that $ydx + xdy + 4dz$ is exact, and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$ over the line segment from $(1, 1, 1)$ to $(2, 3, -1)$.
33. Using Green's theorem in the plane for $\oint_C (xy dx + x^2 dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.
34. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2$, $z \geq 0$, by the cylinder $x^2 + y^2 = 1$.
35. Evaluate the integral $I = \int_C (3x^2 dx + 2yz dy + y^2 dz)$ from A: $(0, 1, 2)$ to B: $(1, -1, 7)$ by showing that \mathbf{F} has a potential.

(6 × 7 = 42 marks)

Section D*Answer any two questions.**Each question carries 13 marks.*

36. If $u = f(r)$ and $x = r \cos \theta$, $y = r \sin \theta$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.
37. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the point on the ellipses that lie closest to and farthest from the origin.
38. Verify the Divergence Theorem for the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

(2 × 13 = 26 marks)

D 10230

(Pages : 3)

Name.....

Reg. No.....

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Part A

Answer all questions.
Each question carries 1 mark.

1. Find the domain and range of $z = \sqrt{25 - x^2 - y^2}$.
2. Evaluate $\lim_{(x,y) \rightarrow (1,-1)} \frac{1+x-y}{2-x+y}$.
3. Define gradient of a scalar function.
4. Compute the divergence of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$.
5. Define solenoidal vector.
6. What do you mean by directional derivative.
7. Write the component test for the differential $M(x, y, z) dx + N(x, y, z) dy + P(x, y, z) dz$ to be exact.
8. Find du if $u = \arcsin \frac{x}{y}$.
9. Fill in the blanks : If \vec{f} and \vec{g} are irrotational vector point functions, then $\nabla \cdot (\vec{f} \times \vec{g}) = \dots$
10. State the normal form of Green's theorem in the plane.
11. Fill in the blanks : If \vec{a} is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then of $\nabla(\vec{r} \cdot \vec{a}) = \dots$
12. State Stoke's theorem.

(12 × 1 = 12 marks)

Part B

Answer any ten questions.
Each question carries 4 marks.

13. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

Turn over

14. Find the vector normal to the surface $\phi(x, y, z) = xyz$ at $(1, -1, 1)$.
15. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (π, π, π) from $\sin(x + y) + \sin(y + z) + \sin(x + z) = 0$.
16. Prove that $\nabla(r^n) = nr^{n-2}\vec{r}$.
17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.
18. Evaluate $\int_1^2 \int_3^4 \frac{1}{(x+y)^2} dx dy$.
19. Linearize the function $f(x, y, z) = xy + yz + zx$ at $(1, 1, 1)$.
20. Find the directional derivative of $f(x, y, z) = xy$ at $(1, 2)$.
21. Evaluate $\iint_R (xy) dx dy$ where R is the positive quadrant of the circle of radius a centred at the origin.
22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix $x = \cos t, y = \sin t, z = t; 0 \leq t \leq \pi/2$.
23. Test whether $\vec{f} = (yz)\vec{i} + (xz)\vec{j} + (xy)\vec{k}$ is conservative or not.
24. Prove that $\text{div}(\text{curl}\vec{f}) = 0$.
25. Verify whether the differential $(e^x \cos y + yz) dx + (xz - e^x \sin y) dy + (xy + z) dz$ is exact or not.
26. If S is a closed surface enclosing a volume V then prove that $\iint_S \text{curl}\vec{f} \cdot \hat{n} dS = 0$.

(10 × 4 = 40 marks)

Part C

Answer any **six** question.
Each question carries 7 marks.

27. Using double integrals prove that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$.
28. Evaluate the line integral $\int_C y dx + x dy$ where C is the boundary of the square $x = 0, x = 1, y = 0$ and $y = 1$.

29. Find the work done by the force field $\vec{f} = 3xy\vec{i} - 58\vec{j} + 10x\vec{k}$ along the space curve $C: \vec{r} = (t^2 + 1)\vec{i} + 2t^2\vec{j} + t^3\vec{k}$ where $0 \leq t \leq 2\pi$.
30. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.
31. Evaluate the volume bounded by $y = x^2, x = y^2$ and the planes $z = 0$ and $z = 3$.
32. Evaluate the area enclosed by the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.
33. Find the Local extreme values of $f(x, y) = x^2 + y^2 + xy + 3x - 3y + 4$.
34. Evaluate the line integral $\int_C \vec{f} \cdot d\vec{r}$ where C is the boundary of the triangle with vertices $(0, 0, 0), (1, 0, 0), (1, 1, 0)$.
35. Show that $\vec{f} = y\sin z\vec{i} + x\sin z\vec{j} + xy\cos z\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

*Answer any two question.
Each question carries 13 marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$ through the surface of the cube cut from the first Octant by the planes $x = y = z = 1$.
- (b) Evaluate $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.
37. Verify Stoke's Theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the rectangular region bounded by $x = 0, x = a, y = 0, y = a$.
38. Verify the Tangential form of Green's theorem in the plane for the vector Field $\vec{f} = (x - y)\vec{i} + x\vec{j}$ over the region bounded by the unit circle $x^2 + y^2 = 1$.

(2 × 13 = 26 marks)

D 90230

(Pages : 4)

Name.....
Reg. No.

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all questions.
Each question carries 1 mark.

1. Find the domain and range of $u(x, y) = \sin xy$.
2. Evaluate $\text{Lt} \left[\frac{3-x+y}{4+x-2y} \right]$
 $(x, y) \rightarrow (1, 2)$
3. If \vec{a} is a constant vector, $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\vec{\nabla}(\vec{r} \cdot \vec{a}) = \text{_____}$.
4. When do we say a vector is Solenoidal ?
5. What do you mean by irrotational vector ?
6. Find the total differential of $\ln(xyz)$.
7. What is the linearization of the function $f(x, y, z)$ at the point (x_0, y_0, z_0) ?
8. Write the condition for the differential form $P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz$ to be exact ?
9. State the normal form of Green's theorem in the plane.
10. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and $r = |\vec{r}|$, then $r^n \vec{r}$ is solenoidal if $n = \text{_____}$.

Turn over

11. State Gauss Divergence theorem.
12. If \vec{f} and \vec{g} are irrotational vector functions then $\text{div}(\vec{f} \times \vec{g}) = \underline{\hspace{2cm}}$.

(12 × 1 = 12 marks)

Section B

*Answer at least eight questions.
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 48.*

13. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \left[\frac{x-y}{x+y} \right]$.
14. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the pt (π, π, π) from $\sin(x+y) + \sin(y+z) + \sin(z+x) = 0$.
15. Find the outward unit normal vector to the surface $\phi(x, y, z) = 3x^2y - y^3z^2$ at $(1, -2, -1)$.
16. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.
17. Find the total derivative of $u(x, y, z) = xyz$ with respect to t where $x = t + 1, y = t^2 + 1, z = t^3 + 1$.
18. Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$?
19. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at the pt $(1, 2, 3)$ in the direction of $3\hat{i} + 4\hat{j} + 5\hat{k}$.
20. Using double integrals, obtain the area of the Lemniscate $r^2 = 4 \cos 2\theta$.
21. Using triple integrals find the average value of $f(x, y, z) = xyz$ over the boundary of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.

22. Evaluate $\int_0^2 \int_0^x \frac{dx dy}{x^2 + y^2}$.
23. Find the circulation of $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the unit circle centered at the origin.
24. Show that the vector field $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is conservative.
25. If $g(x, y, z)$ has continuous 2nd order partial derivatives, show that ∇g is irrotational.
26. State the tangential form of Green's theorem in the plane. Also state its generalization in space.

(8 × 6 = 48 marks)

Section C

Answer at least five questions.

Each question carries 9 marks.

All questions can be attended.

Overall Ceiling 45.

27. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ where C is the rectangle in xy plane bounded by $x = 0, x = a, y = 0$ and $y = b$.
28. If $\vec{f}(t)$ is a differentiable vector function of a scalar variable t , then prove that :

$$\frac{d}{dt} [\vec{f} \cdot \vec{f}', \vec{f}''] = [\vec{f} \cdot \vec{f}'' \vec{f}''']$$

29. Test the continuity of $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$ at the origin.
30. Show that the work done by the Force field $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is independent of the path joining the points $(-1, 3, 9)$ and $(1, 6, -4)$. Also find the work done along any smooth curve joining $(-1, 3, 9)$ and $(1, 6, -4)$.

Turn over

31. Find the local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
32. Using triple integral find the volume of the ellipsoide $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
33. Evaluate the area of the region in the xy plane enclosed by $x^2 + y^2 = 4$, $y = 1$ and $y = \sqrt{3}x$.
34. Integrate the function $f(x, y, z) = xyz$ over the surface of the cube cut from the 1st octant by the planes $x = 1$, $y = 1$ and $z = 1$.
35. Evaluate $\int_{(1,0,0)}^{(0,1,1)} \sin y \cos x dx + \cos y \sin x dy + dz$.

(5 × 9 = 45 marks)

Section D

*Answer any one question.
The question carries 15 marks.*

36. Verify Gauss Divergence theorem for $\vec{F} = x\hat{i} + xy\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.
37. (a) Verify the tangential form of Green's theorem for $\vec{F} = (x - y)\hat{i} + x\hat{j}$ and the Region R bounded by the unit circle $x^2 + y^2 = 1$.
- (b) State the Fundamental theorem of line integrations.
38. (a) Evaluate $\int_C \vec{f} \cdot d\vec{r}$, if $\vec{f} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$, C is the boundary of the triangle with vertices (0,0,0), (1,0,0) and (1,1,0).
- (b) For any closed surfaces enclosing of a volume V prove that $\iint_S \text{curl } \vec{f} \cdot \hat{n} dS = 0$.

(1 × 15 = 15 marks)

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Part A

*Answer all the twelve questions.**Each question carries 1 mark.*

1. Evaluate $\lim_{(x,y) \rightarrow (1,3)} \frac{x+1}{4-y}$.
2. Find the domain and range of $z = \sqrt{1-x^2-y^2}$.
3. Find the gradient of $\phi(x, y, z) = x^2 + y^2 + z^2$.
4. Compute the divergence of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$.
5. Define directional derivative of a function.
6. What do you mean by a conservative vector field?
7. Give a very brief description of linearization of a function of two variables.
8. Find du if $u = e^{x^2+y^2+z^2}$.
9. Fill in the blanks : If \vec{f} and \vec{g} are differentiable vector point functions, then
 $\nabla \cdot (\vec{f} \times \vec{g}) = \dots\dots\dots$
10. State the tangential form of Green's theorem in the plane.

Turn over

11. Fill in the blanks : If \vec{a} is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $\nabla \times (\vec{a} \times \vec{r}) = \dots\dots\dots$
12. State Stokes theorem mentioning all the assumptions involved in it explicitly.

(12 × 1 = 12 marks)

Part B*Answer any ten questions.**Each question carries 4 marks.*

13. Find the vector normal to the surface $\phi(x, y, z) = x^2y - 2y^2z^3$ at $(1, -1, 2)$.
14. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.
15. If $x^2 + y^2 + z^2 + ye^x z + z \cos y = 0$ then, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the origin.
16. Prove that $\nabla(r^n) = nr^{n-2}\vec{r}$.
17. Find the total derivative of $u = xy + z$ with respect to t if $x = \cos t$, $y = \sin t$ and $z = t$.
18. Compute the average value of the function $f(x, y) = x \cos(xy)$ over the rectangular region $0 \leq x \leq \pi$, $0 \leq y \leq 1$.
19. Linearize the function $f(x, y) = \sin(\pi xy^2)$ at $(1, 1)$.
20. Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at $(2, 0)$ in the direction of $3\vec{i} - 4\vec{j}$.
21. Find the velocity and acceleration vectors of $r(t) = (3 \cos t)\vec{i} + (3 \sin t)\vec{j} + t^2\vec{k}$.
22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix
 $x = \cos t$, $y = \sin t$, $z = t$; $0 \leq t \leq \pi/2$.

23. Test whether the vector $\vec{f} = (e^x \cos y + yz)\vec{i} + (xz - e^x \sin y)\vec{j} + (xy + z)\vec{k}$ is conservative or not.
24. If the sides and angles in a triangle vary in such a way that its circum-radius R remains a constant, then show that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$.
25. Verify whether the differential $ydx + xdy + 4dz$ is exact or not.
26. Show that $\vec{f} \times \vec{g}$ is solenoidal if \vec{f} and \vec{g} are irrotational.

(10 × 4 = 40 marks)

Part C

*Answer any six questions.
Each question carries 7 marks.*

27. Evaluate $\int_0^\pi \int_y^\pi \frac{\sin x}{x} dy dx$.
28. If \vec{f} is a differentiable vector function of t , differentiable at least 3 times, prove that $\frac{d}{dt} [\vec{f}, \vec{f}', \vec{f}''] = [\vec{f}', \vec{f}'', \vec{f}''']$.
29. Find the work done by the force field $\vec{f} = z\vec{i} + x\vec{j} + y\vec{k}$ along the boundary of the curve $C: \vec{r} = \cos t \vec{i} + \sin t \vec{j} + 3t \vec{k}$ where $0 \leq t \leq 2\pi$.
30. Test the continuity of $f(x, y)$ defined by $f(x, y) = \frac{xy}{x^2 + y^2}, (x, y) \neq (0, 0)$ and $f(x, y) = 0, (x, y) = (0, 0)$.
31. Find the equation to the tangent plane and normal line to the surface $f(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.
32. Evaluate the area enclosed by the Lemniscate $r^2 = 4 \cos 2\theta$ using double integrals.

Turn over

33. Find the Local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.
34. Evaluate the volume of the region bounded by $x^2 + y^2 = 4$, $y + z = 3$, $z = 0$.
35. Show that $\vec{f} = y \sin z \vec{i} + x \sin z \vec{j} + xy \cos z \vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

*Answer any two questions.
Each question carries 13 marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy \vec{i} + yz \vec{j} + xz \vec{k}$ through the surface of the cube cut from the first octant by the planes $x = y = z = 1$.
- (b) If S is a closed surface enclosing a volume V, then prove that $\int_S \vec{r} \cdot n dS = 3V$.
37. (a) Evaluate the surface integral $\int_S \vec{f} \cdot n dS$ where $\vec{f} = y \vec{i} + x \vec{j} + z^2 \vec{k}$ over the cylindrical surface S given by $x^2 + y^2 = a^2$, $z = 0$, $z = h$.
- (b) Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -2, 2)$.
38. (a) Find the value of $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.
- (b) In what direction from the point $(2, 1, -1)$ the directional derivative of $\phi(x, y, z) = x^2 yz^3$ is maximum and find the magnitude of this maximum.

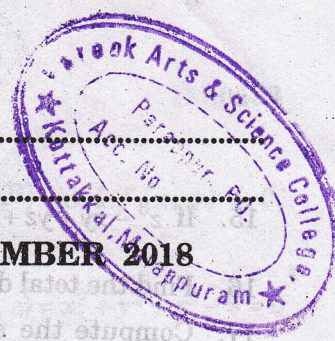
(2 × 13 = 26 marks)

D 50599

(Pages : 3)

Name.....

Reg. No.....



FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS-UG)

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Part A

Answer all the twelve questions.

Each question carries 1 mark.

1. Find the domain and range of $f(x, y) = \sin xy$.

2. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{x}{y}$.

3. Find the gradient of $\phi(x, y, z) = xyz$.

4. Compute the divergence of $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$.

5. In what direction the directional derivative of a function becomes maximum ?

6. What do you mean by an irrotational vector ?

7. What is the linearization of the function $f(x, y, z)$ at the point (1, 2, 3) ?

8. Find the total differential of u if $u = \ln(x^2 + y^2 + z^2)$.

9. Fill in the blanks : If \vec{f} and \vec{g} are irrotational vector point functions, then $\nabla \cdot (\vec{f} \times \vec{g}) = \dots$

10. State the Normal form of Green's theorem in the plane.

11. Fill in the blanks : If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $r^n\vec{r}$ is solenoidal if $n = \dots$

12. State Gauss's Divergence theorem mentioning all the assumptions involved in it explicitly.

(12 × 1 = 12 marks)

Part B

Answer any ten questions.

Each question carries 4 marks.

13. Find the vector normal to the surface $\phi(x, y, z) = x^3 + y^3 + z^3$ at (1, -1, 1).

14. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{y-x}{y+x}$.

Turnover

15. If $z^3 - xy + yz + y^3 - 2 = 0$ then, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 1, 1)$.
16. Find the total derivative of $u = x^3 + y^3$ with respect to t if $x = \cos t, y = \sin t$.
17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.
18. Linearize the function $f(x, y, z) = x^2 - xy + 3\sin z$ at $(2, 1, 0)$.
19. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $(1, 2, 3)$ in the direction of $3\vec{i} + 4\vec{j} + 5\vec{k}$.
20. Find the circulation of $\vec{f} = (x, y)\vec{i} + x\vec{j}$ around the unit circle centered at the origin.
21. Find the value of λ which makes the following vector $\vec{f} = (\lambda xy - z^3)\vec{i} + ((\lambda - 2)x^2)\vec{j} + ((1 - \lambda)xz^2)\vec{k}$ is irrotational.
22. Verify whether the differential $e^x \cos y dx + (xz - e^x \sin y) dy + (xy + z) dz$ is exact or not.
23. If \vec{f} and \vec{g} are irrotational, then show that $\vec{f} \times \vec{g}$ is solenoidal.
24. Evaluate $\int_0^{2x} \int_{x^2}^{2x} (4x + 2) dy dx$ by changing the order of integration.
25. Prove that if a function $f(x, y)$ is differentiable at the origin, then it is continuous at the origin.
26. Evaluate the integral $\int_c xy dy - y^2 dx$ where c is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

(10 × 4 = 40 marks)

Part C

Answer any **six** questions.
Each question carries 7 marks.

27. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.
28. Find $\nabla f(r)$, if $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.
29. Show that the work done by the force field $\vec{f} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is independent of the path joining the points $(-1, 3, 9)$ and $(1, 6, -4)$.

30. Test the continuity of $f(x, y)$ defined by $f(x, y) = \frac{x^2 + y^2}{x + y}$, $(x, y) \neq (0, 0)$ and $f(x, y) = 0$, $(x, y) = (0, 0)$.
31. Find the equation to the tangent plane and normal line to the surface $f = (x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.
32. Evaluate the area enclosed by the region $x^2 + y^2 = 4$, $y = 1$, $y = \sqrt{3x}$ in the (x, y) plane, using double integrals.
33. Find the Local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
34. Evaluate the volume of the ellipsoide $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
35. Show that $\vec{f} = (x + 2y + 4z)\vec{i} + (2x - 3y - z)\vec{j} + (4x - y - 2z)\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any two questions.
Each question carries 13 marks.

36. (a) Verify Gauss's divergence theorem for $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere of radius a centered at the origin.
(b) State the Fundamental theorem of line integration.
37. (a) Evaluate the line integral $\int_C \vec{f} \cdot d\vec{r}$ where C is the boundary of the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$.
(b) Find (Curl Curl) \vec{f} , if $\vec{f} = x^2y\vec{i} - 2xz\vec{j} + 2yz\vec{k}$.
38. (a) If S is a closed surface enclosing a volume V then prove that $\iint_S \text{curl } \vec{f} \cdot \hat{n} dS = 0$.
(b) Verify Green's theorem for $\int_C (xy + y^2) dx + x^2 dy$, where C is the curve joining $y = x^2$ and $y = x$.

(2 × 13 = 26 marks)

**FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY)
EXAMINATION, NOVEMBER 2017**

(UG-CCSS)

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :

- 1 Find the curl of $F(x, y) = (x^2 - y)i + (xy - y^2)j$.
- 2 Examine whether $F = yzi + zxj + xyk$ is conservative.
- 3 If \vec{F} is a field defined on D and $\vec{F} = \nabla f$ for some scalar function f on D then f is called a ——— of \vec{F} .
- 4 Find the gradient field of $g(x, y, z) = e^z - \ln(x^2 + y^2)$.
- 5 Define gradient field of a differentiable function.
- 6 Define critical point.
- 7 Say True or False :
If a function $f(x, y)$ is differentiable at (x_0, y_0) then f is continuous at (x_0, y_0) .
- 8 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 + 3xy + y - 1$ at $(4, -5)$.
- 9 $\lim_{(x,y) \rightarrow (3,4)} \sqrt{x^2 + y^2} - 1 = \underline{\hspace{2cm}}$.
- 10 If $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ then $f(3, 0, 4) = \underline{\hspace{2cm}}$.
- 11 If $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$ then $\lim_{t \rightarrow \frac{\pi}{4}} \vec{r}(t) = \underline{\hspace{2cm}}$.
- 12 Find the parametric equation for the line through P $(1, 2, -1)$ and Q $(-1, 0, 1)$.

(12 × ¼ = 3 weightage)

II. Answer *all* nine questions :

- 13 Find the curl of $F = (x^2 - y)i + 4zj + x^2k$.
- 14 State Green's theorem (normal form).

Turn over

15 Evaluate $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (x+y+z) dy dx dz$.

16 Write an equivalent integral for $\int_0^1 \int_2^{4-2x} dy dx$ with the order of integration reversed.

17 Find the direction in which $f(x, y) = x^2 + xy + y^2$ increases most rapidly at $p_0(-1, 1)$.

18 If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$ find $\left(\frac{\partial w}{\partial y}\right)_z$.

19 If $w = xy + \frac{e^y}{y^2 + 1}$ find $\frac{\partial^2 w}{\partial x \partial y}$.

20 Write the range of the function $f(x, y) = xy$.

21 Find the length of one turns of the helix $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$.

(9 × 1 = 9 weightage)

III. Answer any five questions :

22 Evaluate $\iint_R e^{x^2+y^2} dy dx$ where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

23 Find the local extreme values of the function $f(x, y) = xy - y^2 - x^2 - 2x - 2y + 4$.

24 Find $\frac{dw}{dt}$ at $t = \pi$. Given $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$.

25 Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$.

26 Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is discontinuous at the origin.

27 Find the torsion for the space curve $\vec{r}(t) = (3\sin t)i + (3\cos t)j + 4tk$.

28 Find T and N for the plane curve $\vec{r}(t) = ti + (\ln \cos t)j$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \geq 0$, by the cylinder $x^2 + y^2 = 1$.

30 Find the work done by $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curve $\vec{r}(t) = ti + t^2j + t^3k, 0 \leq t \leq 1$ from $(0, 0, 0)$ to $(1, 1, 1)$.

31 Find an upper bound for the magnitude of the error E in the approximation $f(x, y) \approx L(x, y)$ over the rectangle R . Given $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3, P_0(3, 2), R: |x - 3| \leq 0.1, |y - 2| \leq 0.1$.

(2 × 4 = 8 weightage)

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Name.....

Reg. No.....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS



Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the twelve questions.

Each question carries 1 mark.

1. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} \right)$.
2. Find the domain and range of the function $f(x, y) = \sqrt{1 - x^2 - y^2}$.
3. State Euler's mixed derivative theorem.
4. Find the total derivative of $u = e^{x^2 + y^2 + z^2}$.
5. Define the gradient of a scalar function.
6. When is a vector becoming irrotational?
7. Give the definition of divergence of a vector function.
8. State the tangential form of Greens theorem.
9. Find the length of one turn of the helix $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + t \vec{k}$ when $t \in [0, 2\pi]$.
10. What is directional derivative?
11. If $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$, prove that $\int_C \vec{r} \cdot d\vec{r} = 0$ over any smooth closed curve C in the plane.
12. Prove that for a closed surface S enclosing a volume V, $\int_S \vec{r} \cdot \vec{n} \, dS = 3V$ where \vec{n} is a positive oriented normal vector to the surface S.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any ten out of fourteen questions.

Each question carries 4 marks.

13. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$.
14. Find the partial derivatives of z w.r.t. x and y respectively at $(0, 0, 0)$ if $x^2 + y^2 + z^2 + ye^{xz} + z \cos y = 0$
15. In a triangle ABC, the sides and angles vary in such a way that the circumradius remains constant. Prove that $\frac{a}{\cos A} + \frac{b}{\cos B} + \frac{c}{\cos C} = 0$.
16. If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin u$, verify that $x \frac{\partial z}{\partial v} + y \frac{\partial z}{\partial u} = e^{2u} \frac{\partial z}{\partial y}$.
17. Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at $(2, 1, 0)$.
18. Find the flux of $\vec{F} = (x - y)\vec{i} + x\vec{j}$ across the circle $x^2 + y^2 = 1$ in the xy -plane.
19. Evaluate the work done by the force $\vec{F} = (y - x^2)\vec{i} + (z - y^2)\vec{j} + (x - y^2)\vec{k}$ along the curve $C: \vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ from $(0, 0, 0)$ to $(1, 1, 1)$.
20. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\nabla(r^n) = nr^{n-2}\vec{r}$.
21. Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at $(0, 0)$ in the direction of $3\vec{i} - 4\vec{j}$.
22. Find the divergence of $\vec{f} = x^2\vec{i} - 2y^3z^2\vec{j} + xy^2z\vec{k}$ at $(1, -1, 1)$.
23. Test whether the vector $\vec{v} = x(y - z)\vec{i} + y(z - x)\vec{j} + z(x - y)\vec{k}$ is solenoidal or not.
24. With usual notations, show that $\nabla \times \left(\frac{\vec{r}}{r} \right) = \vec{0}$ when $r \neq 0$.

25. Whenever \vec{f} and \vec{g} are irrotational, show that $\vec{f} \times \vec{g}$ is solenoidal.
26. Evaluate $\int_{\mathcal{R}} f(x, y) dA$ for over the rectangular region given by $\mathcal{R} : 0 \leq x \leq 2, -1 \leq y \leq 1$.

(10 × 4 = 40 marks)

Section C

Answer any six out of nine questions.

Each question carries 7 marks.

27. Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy$.
28. Change the order of integration and evaluate $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x} dy dx$.
29. If \vec{f} is a differentiable vector function of t , prove that $\frac{d}{dt} [\vec{f}, \vec{f}', \vec{f}''] = [\vec{f}', \vec{f}'', \vec{f}''']$.
30. Verify Green's theorem in the case of $\int_C (y - \sin x) dx + \cos x dy$ where C is the triangle in the xy -plane formed by the points $(0, 0)$, $(\pi/2, 0)$ and $(\pi/2, 1)$.
31. Test whether $\vec{f} = (e^x \cos y + yz)\vec{i} + (zx - e^x \sin y)\vec{j} + (xy + z)\vec{k}$ is conservative or not. If it is conservative, find its scalar potential.
32. If $z = f(x, y)$ where $x = e^u + e^{-u}$ and $y = e^{-u} - e^u$, verify that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$.
33. Compute the local extremum of $f(x, y) = x^2 + y^2 - 4y + 9$, if any.
34. Using triple integral find the volume of the prism bounded by the $y = 0$, $y = x$, $x = 1$ and $x + y + z = 3$.
35. Compute $\int_S (y^2 z^2 \vec{i} + x^2 z^2 \vec{j} + z^2 x^2 \vec{k}) \cdot \vec{n} dS$ where S is the upper half of $x^2 + y^2 + z^2 = 1$.

(6 × 7 = 42 marks)

Turn over

Section D

Answer any two out of three questions.

Each question carries 13 marks.

36. (a) State Stoke's theorem.

(b) Evaluate $\int_C \vec{f} \cdot d\vec{r}$ by Stokes theorem for $\vec{f} = y^2 \vec{i} + x^2 \vec{j} - (x+z) \vec{k}$ with C as the boundary of the triangle formed by the origin and the points (1, 0, 0) and (1, 1, 0).

37. Verify both forms of Green's theorem for $\vec{f} = (x-y) \vec{i} + x \vec{j}$ over the region R bounded by the unit circle $x^2 + y^2 = 1$.

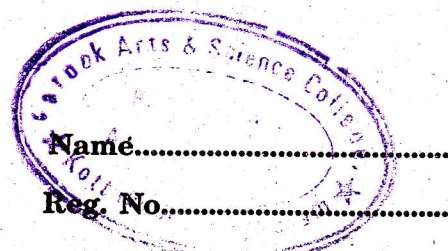
38. (a) Evaluate $\int_0^1 \int_{y/2}^{y/2+1} \frac{2x-y}{2} dx dy$ by applying the transformation $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$.

(b) Find the average value of $f(x, y, z) = x \cos(xy)$ over the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1$.

(2 × 13 = 26 marks)

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FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, NOVEMBER 2016

(UG-CCSS)

Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :

- 1 Curvature of a straight line is _____.
- 2 Find the parametric equation for the line through the points P(-3,2,-3) and Q(1,-1,4).
- 3 The domain of the function $w = \sin(x y)$ is the entire function. Then range = _____.
- 4 If $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$ then $\lim_{t \rightarrow \frac{\pi}{4}} \vec{r}(t) =$ _____.
- 5 Vector equation for the line through $p_0(x_0, y_0, z_0)$ and parallel to \vec{v} is $\overline{p_0p} =$ _____.
- 6 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 + 3xy + y - 1$ at (4, -5).
- 7 $\lim_{(x, y) \rightarrow \left(0, \frac{\pi}{4}\right)} \sec x \tan x =$ _____.
- 8 Find the gradient of $g(x, y) = y - x^2$ at (-1,0).
- 9 Define critical point.
- 10 Find the gradient field of $f(x, y, z) = xyz$.
- 11 If F is a field defined on D and $\vec{F} = \nabla f$ for some scalar function f on D then f is called a _____ of F .
- 12 Examine whether $F = yz i + zx j + xy k$ is conservative.

(12 × ¼ = 3 weightage)

Turn over

II. Answer any *nine* questions :

13 Show that $\vec{u}(t) = (\sin t) i + (\cos t) j + \sqrt{3} k$ is orthogonal to its derivative.

14 Evaluate $\int_0^1 [t^3 i + 7j + (t+1)k] dt$.

15 Find the parametric equation for the line that is tangent to the curve :

$$\vec{r}(t) = (a \sin t) i + (a \cos t) j + b t k, t_0 = 2\pi.$$

16 Find the unit tangent vector of the curve :

$$\vec{r}(t) = (\cos^3 t) j + (\sin^3 t) k, 0 \leq t \leq \frac{\pi}{2}.$$

17 Write the range of the function :

$$f(x, y) = 4x^2 + 9y^2.$$

18 If $w = xy^2 + x^2y^3 + x^3y^4$. Verify that $w_{xy} = w_{yx}$.

19 If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$, find $\left(\frac{\partial w}{\partial z}\right)_x$.

20 Find ∇f at $(1, 1, 1)$ of $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$.

21 Evaluate $\int_0^3 \int_0^2 (4 - y^2) dy dx$.

22. Find the divergence of $F(x, y) = (x^2 - y) i + (xy - y^2) j$.

23 State Green's theorem (Normal form).

24 State divergence theorem.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

25 Find the distance from the point $(2, 1, 3)$ to the line $x = 2 + 2t, y = 1 + 6t, z = 3$.

26 Find the plane through the points $(1, 1, -1), (2, 0, 2), (0, -2, 1)$.

27 Prove that the curvature of a circle of radius a is $\frac{1}{a}$.

28 Find the Torsion for the helix :

$$\vec{r}(t) = (a \cos t) i + (a \sin t) j + b t k, a, b \geq 0, a^2 + b^2 \neq 0.$$

- 29 Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.
- 30 Find the linearization of $f(x, y, z) = x^2 + y^2 + z^2$ at $(0, 1, 0)$.
- 31 A fluid's velocity field is $F = xi + zj + yk$. Find the flow along the helix :
- $$\vec{r}(t) = (\cos t)i + (\sin t)j + tk, \quad 0 \leq t \leq \pi/2.$$
- 32 Evaluate $\iint_R e^{x^2+y^2} dy dx$ where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

(5 × 2 = 10 weightage)

IV. Answer both the questions :

- 33 Find the point of intersection of the lines $x = 2t + 1, y = 3t + 2, z = 4t + 3$ and $x = s + 2, y = 2s + 4, z = -4s - 1$ and then find the plane determined by these lines.
- 34 Find a potential function f for the field $F = 2xi + 3yj + 4zk$.

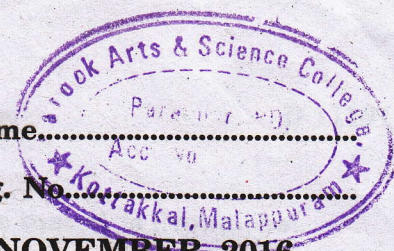
(2 × 4 = 8 weightage)

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Name.....

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FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the twelve questions.

Each question carries 1 mark.

1. Evaluate $\lim_{(x, y, z) \rightarrow (1, 0, 1)} \frac{e^{x+z}}{z^2 + \cos \sqrt{xy}}$.
2. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the tangent to this parabola at $(1, 2, 5)$.
3. Find the domain and range of the function $f(x, y) = \log_e (y^2 - x^2)$.
4. Find du if $u = \log_e (xyz)$. Find the gradient of $f(x, y, z) = xyz$.
5. Find the normal vector to the surface $x^2y - 2y^2z^3 + 17 = 0$ at the point $(1, -1, 2)$.
6. Calculate divergence of $\vec{f} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$.
7. Give the definition of Curl \vec{f} .
8. Distinguish between flux and circulation of a vector \vec{f} across/around a plane curve C .
9. What do you mean by scalar potential ?
10. Evaluate $\int_C 2xy^2 dS$ over the unit circle C in the first quadrant.
11. Find the maximum value the directional derivative of $f(x, y, z) = x^2yz^3$ at $(2, 1, -1)$.
12. State Gauss divergence theorem.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any ten out of fourteen questions.

Each question carries 4 marks.

13. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$.
14. Find the partial derivatives of z w.r.t x and y respectively at :
 (π, π, π) if $\sin(x+y) + \sin(y+z) + \sin(z+x) = 0$.
15. Find $\frac{du}{dt}$ if $u = x^3 + y^3$, $x = a \cos t$ and $y = b \sin t$.
16. If $H = f(y-z, z-x, x-y)$, verify that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$.
17. Find the linearization of $f(x, y) = x^2 - xy + y^2/2$ at $(3, 2)$.
18. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.
19. Find the circulation of the field $\vec{F} = (x-y)\vec{i} + x\vec{j}$ around the circle $x^2 + y^2 = 1$.
20. Evaluate the work done by the force $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$ in moving a particle along the curve $C: \vec{r}(t) = 3\cos t\vec{i} + 3\sin t\vec{j}$ where $0 \leq \theta \leq 2\pi$.
21. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find $\vec{\nabla} \left(\frac{1}{r} \right)$.
22. Find the directional derivative of $f(x, y) = 2x^2 + 3y^2 + z^2$ at $(2, 1, 3)$ in the direction of $\vec{i} - 2\vec{k}$.
23. Find the value of n so that $r^n \vec{r}$ is solenoidal.
24. If \vec{a} is a constant vector, show that $\vec{\nabla} \cdot (\vec{r} \times \vec{a}) = 0$.

25. If $\psi(x, y, z)$ has continuous second order partial derivatives, prove that $\vec{\psi}$ is irrotational.

26. Evaluate $\int_1^2 \int_3^4 \frac{dx dy}{(x+y)^2}$.

(10 × 4 = 40 marks)

Section C

Answer any six out of nine questions.
Each question carries 7 marks.

27. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$.

28. Change the order of integration and evaluate $\int_0^1 \int_y^1 e^{x^2} dx dy$.

29. Evaluate $\int_C y dx + 2x dy$ where C is boundary of the square in the xy-plane given by $0 \leq x \leq 1, 0 \leq y \leq 1$.

30. Show that the differential form $y dx + x dy + 4 dz$ is exact and hence evaluate $\int_{(1,1,1)}^{(2,3,-1)} y dx + x dy + 4 dz$.

31. If $u = f\left(\frac{y-x}{yx}, \frac{z-x}{zx}\right)$, verify that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.

32. Find Curl Curl \vec{f} if $\vec{f} = x^2 y \vec{i} + 2xz \vec{j} + 2yz \vec{k}$.

33. Compute the local extremum of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$, if any.

34. Using triple integral find the volume of the tetrahedron with vertices $(0, 0, 0)$; $(1, 1, 0)$; $(0, 1, 0)$ and $(1, 1, 1)$.

Turn over

35. Compute $\iint_S (2x + 3z)\vec{i} - (xz + y)\vec{j} + (y^2 + 2z)\vec{k} \cdot \vec{n} \, dS$ where S is the surface of the sphere with

radius 3 units and centre at $(3, 1, -2)$.

(6 × 7 = 42 marks)

Section D

Answer any two out of three questions.

Each question carries 13 marks.

36. (a) State Stoke's theorem.

(b) Employing Stokes theorem, evaluate for $\int_C \vec{f} \cdot d\vec{r}$ $\vec{f} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ with C as

the boundary of the unit sphere in the upper half with centre at the origin.

37. Verify Gauss divergence theorem for $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = 1$.

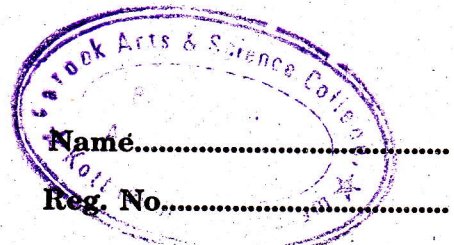
38. (a) Applying the triple integral, find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(2 × 13 = 26 marks)

D 11545

(Pages : 3)



**FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, NOVEMBER 2016**

(UG-CCSS)

Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :

- 1 Curvature of a straight line is _____.
- 2 Find the parametric equation for the line through the points $P(-3, 2, -3)$ and $Q(1, -1, 4)$.
- 3 The domain of the function $w = \sin(x y)$ is the entire function. Then range = _____.
- 4 If $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$ then $\lim_{t \rightarrow \frac{\pi}{4}} \vec{r}(t) =$ _____.
- 5 Vector equation for the line through $p_0(x_0, y_0, z_0)$ and parallel to \vec{v} is $\overline{p_0p} =$ _____.
- 6 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 + 3xy + y - 1$ at $(4, -5)$.
- 7 $\lim_{(x, y) \rightarrow \left(0, \frac{\pi}{4}\right)} \sec x \tan x =$ _____.
- 8 Find the gradient of $g(x, y) = y - x^2$ at $(-1, 0)$.
- 9 Define critical point.
- 10 Find the gradient field of $f(x, y, z) = xyz$.
- 11 If F is a field defined on D and $\vec{F} = \nabla f$ for some scalar function f on D then f is called a _____ of F .
- 12 Examine whether $F = yz i + zx j + xy k$ is conservative.

(12 \times $\frac{1}{4}$ = 3 weightage)

Turn over

II. Answer any *nine* questions :

13 Show that $\vec{u}(t) = (\sin t) i + (\cos t) j + \sqrt{3} k$ is orthogonal to its derivative.

14 Evaluate $\int_0^1 [t^3 i + 7j + (t+1)k] dt$.

15 Find the parametric equation for the line that is tangent to the curve :

$$\vec{r}(t) = (a \sin t) i + (a \cos t) j + b t k, t_0 = 2\pi.$$

16 Find the unit tangent vector of the curve :

$$\vec{r}(t) = (\cos^3 t) j + (\sin^3 t) k, 0 \leq t \leq \frac{\pi}{2}.$$

17 Write the range of the function :

$$f(x, y) = 4x^2 + 9y^2.$$

18 If $w = xy^2 + x^2y^3 + x^3y^4$. Verify that $w_{xy} = w_{yx}$.

19 If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$, find $\left(\frac{\partial w}{\partial z}\right)_x$.

20 Find ∇f at $(1, 1, 1)$ of $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$.

21 Evaluate $\int_0^3 \int_0^2 (4 - y^2) dy dx$.

22. Find the divergence of $F(x, y) = (x^2 - y) i + (xy - y^2) j$.

23 State Green's theorem (Normal form).

24 State divergence theorem.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

25 Find the distance from the point $(2, 1, 3)$ to the line $x = 2 + 2t, y = 1 + 6t, z = 3$.

26 Find the plane through the points $(1, 1, -1), (2, 0, 2), (0, -2, 1)$.

27 Prove that the curvature of a circle of radius a is $\frac{1}{a}$.

28 Find the Torsion for the helix :

$$\vec{r}(t) = (a \cos t) i + (a \sin t) j + b t k, a, b \geq 0, a^2 + b^2 \neq 0.$$

- 29 Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.
- 30 Find the linearization of $f(x, y, z) = x^2 + y^2 + z^2$ at $(0, 1, 0)$.
- 31 A fluid's velocity field is $F = xi + zj + yk$. Find the flow along the helix :
$$\vec{r}(t) = (\cos t)i + (\sin t)j + tk, \quad 0 \leq t \leq \pi/2.$$
- 32 Evaluate $\iint_R e^{x^2+y^2} dy dx$ where R is the semi-circular region bounded by the x -axis and the curve $y = \sqrt{1-x^2}$.

(5 × 2 = 10 weightage)

IV. Answer both the questions :

- 33 Find the point of intersection of the lines $x = 2t + 1, y = 3t + 2, z = 4t + 3$ and $x = s + 2, y = 2s + 4, z = -4s - 1$ and then find the plane determined by these lines.
- 34 Find a potential function f for the field $F = 2xi + 3yj + 4zk$.

(2 × 4 = 8 weightage)

D 90905

(Pages : 3)

Name.....

Reg. No.....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(UG—CCSS)

Core Course—Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions.

1 Find the curl of $F(x, y) = (x^2 - y)i + (xy - y^2)j$.

2 Show that $F = (2x - 3)i - zj + (\cos z)k$ is not conservative.

3 Define gradient field of differentiable function.

4 Define saddle point.

5 $\lim_{(x, y) \rightarrow (3, 4)} \sqrt{x^2 + y^2} - 1 = \underline{\hspace{2cm}}$.

6 Find the parametric equation for the line through the point $(-2, 0, 4)$ and parallel to $\vec{V} = 2i + 4j - 2k$.

7 Write the equation of the planes through $P_0(x_0, y_0, z_0)$ and normal to $\vec{n} = Ai + Bj + Ck$.

8 A vector function $\vec{r}(t)$ is continuous at a point $t = t_0$ in its domain if $\lim_{t \rightarrow t_0} \vec{r}(t) = \underline{\hspace{2cm}}$.

9 Find the velocity of a moving particle at $t = 1$ when position vector is $\vec{r}(t) = (t + 1)i + (t^2 - 1)j$.

10 The curvature of a circle of radius a is $\underline{\hspace{2cm}}$.

11 Find the gradient of $f(x, y) = y - x$ at $(2, 1)$.

12 Domain of the function $w = \sqrt{x^2 + y^2 + z^2}$ is the entire space. then range = $\underline{\hspace{2cm}}$.

(12 × ¼ = 3 weightage)

Turn over

II. Answer any *nine* questions.

13 State divergence Theorem.

14. Find the divergence of $F(x, y) = (x^2 - y^2)i + (xy - y^2)j$.

15 State Tangential form of Green's theorem.

16 Evaluate $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx$.

17 Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decreases most rapidly.

18 Find $\left(\frac{\partial w}{\partial y}\right)_z$ if $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$.

19 If $w = \ln(2x + 3y)$ verify that $w_{xy} = w_{yx}$.

20 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = (x^2 - 1)(y + 2)$.

21. Find the unit tangent vector of the helix $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$.

22 Find the acceleration of a moving particle at $t = 1$ whose position vector is $\vec{r}(t) = (t + 1)i + (t^2 - 1)j$.

23 Find the angle between the planes $x + y = 1$ and $2x + y - 2z = 2$.

24 Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions.

25 Find the flux of $F = xyi + yzj + xzk$ out ward through the surface of the cube cut from the first octant by the planes $x = 1, y = 1$ and $z = 1$.

26 Verify normal form of Green's theorem for the field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C: \vec{r}(t) = (\cos t)i + (\sin t)j, 0 \leq t \leq 2\pi$.

27 Find the work done by $F = xyi + yj - yzk$ over the curve $\vec{r}(t) = ti + t^2j + tk$, $0 \leq t \leq 1$.

28 Evaluate $\int_C (x + y) ds$ where C is the straight line segment $x = t$, $y = 1 - t$, $z = 0$ from $(0, 1, 0)$ to $(1, 0, 0)$.

29 Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$.

30 Find the derivative of $f(x, y) = xe^y + \cos(x, y)$ at $(2, 0)$ in the direction of $A = 3i - 4j$.

31 Find the torsion for the space curve $\vec{r}(t) = (3 \sin t)i + (3 \cot t)j + 4tk$.

32. Find the distance from the point $s(0, 0, 2)$ to the line $x = 4t$, $y = -2t$, $z = 2t$.

(5 × 2 = 10 weightage)

IV. Answer *both* the questions.

33 Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} (ydx + xdy + 4dz)$.

34 Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

(2 × 4 = 8 weightage)

D 70940

(Pages : 3)

Name.....

Reg. No.....



FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.-CCSS)

Core Course—Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 Weightage

I. Answer all questions :

- 1 Plane through $P_0(x_0, y_0, z_0)$ and normal to $\vec{m} = Ai + Bj + Ck$ is _____.
- 2 Find the parametric equation for the line through the points $P(-3, 2, -3)$ and $Q(1, -1, 4)$.
- 3 Vector equation for the line through $P_0(x_0, y_0, z_0)$ and parallel to \vec{v} is $\vec{r} = \vec{P_0} + \lambda \vec{v}$.
- 4 A vector function $\vec{r}(t)$ is continuous at a point $t = t_0$ in its domain if $\lim_{t \rightarrow t_0} \vec{r}(t) = \vec{r}(t_0)$.
- 5 Domain of the function $w = \sin(xy)$ is the entire plane. Then range = _____.
- 6 $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} = \frac{5}{2}$.
- 7 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = (x^2 - 1)(y + 2)$.
- 8 Find the gradient of $g(x, y) = y - x^2$ at $(-1, 0)$.
- 9 The curl of a vector field $\vec{F} = Mi + Nj$ at the point (x, y) is _____.
- 10 Curvature of a straight line is _____.
- 11 Define Saddle point.
- 12 Examine whether $F = yi + (x + z)j - yk$ conservative.

(12 × ¼ = 3 weightage)

Turn over

II. Answer all *nine* questions :

- 13 Find the angle between the planes :

$$3x - 6y - 2z = 15 \text{ and } 2x + y - 2z = 5.$$

- 14 Find the spherical co-ordinate equation for the sphere :

$$x^2 + y^2 + (z-1)^2 = 1.$$

- 15 Show that $\vec{u}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + \sqrt{3}\mathbf{k}$ is orthogonal to its derivative.

- 16 Find the equation for the plane through $P_0(0, 2, -1)$ and normal to $\vec{n} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$.

- 17 Find the acceleration of a moving particle at $t = 1$ whose position vector is

$$\vec{r}(t) = (t+1)\mathbf{i} + (t^2 - 1)\mathbf{j}.$$

- 18 Find the parametric equation for the line that is tangent to the curve :

$$\vec{r}(t) = (a \sin t)\mathbf{i} + (a \cos t)\mathbf{j} + bt\mathbf{k} \text{ at } t_0 = 2\pi.$$

- 19 If $t_0 = 0$ find the arc length parameter along the helix $\vec{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$.

- 20 Write the range of the function $f(x, y) = xy$.

- 21 State Stoke's theorem.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

- 22 Find T and N for the plane curve :

$$\vec{r}(t) = (2t+3)\mathbf{i} + (5-t^2)\mathbf{j}.$$

- 23 Find the point where the line $x = 1 + 2t, y = 1 + 5t, z = 3t$ intersects the plane $x + y + z = 2$.

- 24 Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

- 25 Find the curvature for the space curve $\vec{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + 2t\mathbf{k}$.

- 26 Calculate the outward flux of the field $\mathbf{F}(x, y) = x\mathbf{i} + y^2\mathbf{j}$ across the square bounded by the lines $x = \pm 1, y = \pm 1$.

27 Evaluate $\int_C (xy + y + z) dz$ along the curve $\vec{r}(t) = 2ti + tj + (2 - 2t)k, 0 \leq t \leq 1$.

28 Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$.

(5 × 2 = 10 weightage)

IV. Answer any two questions :

29 Find the plane determined by the intersecting lines :

$$L_1 : x = -1 + t, y = 2 + t, z = 1 - t, -\infty < t < \infty$$

$$L_2 : x - 1 - 4s, y = 1 + 2s, z = 2 - 2s, -\infty < s < \infty$$

30 Find an upper bound for the magnitude of the error E in the approximation :

$f(x, y, z) \approx L(x, y, z)$ over the rectangle R. Given $f(x, y, z) = xz - 3yz + 2$ at $P_0(1, 1, 2)$.

$$R : |x - 1| \leq 0.01, |y - 1| \leq 0.01, |z - 2| \leq 0.02.$$

31 Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative and find a potential function for it.

(2 × 4 = 8 weightage)

D 50722

(Pages : 3)

Name.....

Reg. No.....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER, 2013

(UG-CCSS)

Mathematics (Core Course)

MM 5B 05—VECTOR CALCULUS

Time : Three Hours

Maximum : 30 weightage

I. Answer all the *twelve* questions :

1 Find the parametric equation of the line through the points P (-3, 2, -3) and Q (1, -1, 4).

2 Find the angle between the vectors $\bar{A} = i - 2j - 2k$, $\bar{B} = 6i + 3j + 2k$.

3 Find a vector perpendicular to both $\bar{A} = 2i + j + k$ and $\bar{B} = -4i + 3j + k$.

4 Find the equation of the plane through $P_0 (-3, 0, 7)$ and perpendicular to $\bar{n} = 5i + 2j - \bar{k}$.

5 The equation $x = y^2 - z^2$ represents the surface of a _____

(a) Ellipsoid.

(b) Cylinder.

(c) Cone.

(d) Hyperbolic paraboloid.

6 Find the equation of the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.

7 Find the unit tangent vector to the helix $\bar{r}(t) = \cos t i + \sin t j + tk$.

8 Find the domain and range of the function $w = \sqrt{x^2 + y^2 + z^2}$.

9 Find $\lim_{\substack{(x,y) \rightarrow (1,1) \\ x \neq 1}} \left[\frac{xy - y - 2x + 2}{x - 1} \right]$.

10 If $w = x^2 + y^2 - z + \sin t$ and $x + y = t$, find $\left(\frac{\partial w}{\partial y} \right)$ and $\left(\frac{\partial w}{\partial x} \right)$.

Turn over

11 Find the gradient of $g(x, y, z) = e^z - \ln(x^2 + y^2)$

12 State the Fubini's theorem (first form).

(12 × ¼ = 3 weightage)

II. Answer *all* the nine questions :

13 Find the point where line $x = \frac{8}{3} + 2t$, $y = -2t$, $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

14 Find the spherical and cylindrical equation of the hemisphere $x^2 + y^2 + (z - 1)^2 = 1$, $z \leq 1$.

15 Show that $u(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + \sqrt{3} \mathbf{k}$ has a constant length and is orthogonal to its derivative.

16 Show that the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ satisfies the Laplace's equation.

17 Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $\bar{A} = 3\mathbf{i} - 4\mathbf{j}$.

18 Find the saddle point if any of the function $f(x, y) = x^2 + xy + 3x + 2y + 5$.

19 Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy plane bounded by the x -axis, the line $y = x$ and the line $x = 1$.

20 Find the work done by $\bar{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ over the curve $\bar{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq 1$.

21 Evaluate $\int_0^1 \int_0^{1-z} \int_0^2 dx dy dz$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions from seven :

22 Find the unit tangent vector, normal vector and binormal for the curve

$$\bar{r}(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j} + 3\mathbf{k}.$$

23 Find the linearization of $f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$ at the point (3, 2)

- 24 Find the derivative of $f(x, y, z) = \ln(2x + 3y + 6z)$ at $p(-1, -1, 1)$ in the direction of $\bar{A} = 2i + 3j + 6k$.
- 25 Find the average value of $F(x, y, z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the co-ordinate planes and the planes $x = 1, y = 1$ and $z = 1$.
- 26 Show that $\bar{F} = (y + z)i + (x + z)j + (x + y)k$ forms a conservative force field and find its potential function.
- 27 Apply Green's theorem to evaluate $\oint_C (y^2 dx + x^2 dy)$ where C is the triangle bounded by $x = 0, x + y = 1, y = 0$.
- 28 Integrate $g(x, y, z) = xyz$ over the surface of the cube cut-off by the first octant by $x = 1, y = 1, z = 1$.

(5 × 2 = 10 weightage)

IV. Answer any two questions :

- 29 Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
- 30 Use Taylor's theorem for $f(x, y)$ to find a quadratic and cubic approximation of $f(x, y) = x e^y$ at origin.
- 31 Use Stoke's theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ if $\bar{F} = xzi + xyj + 3xzk$ where C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed in counterclockwise sense.

(2 × 4 = 8 weightage)