D 30559 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions only)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Find the quotient and remainder when $-x^4+7x^3-4x^2$ is divided by $x-3$.

2. Write a cubic equation with roots $1, 1+i, 1-i$.

3. Factorise into real linear and quadratic factors : $x^4+x^3+x^2+x+1$.

4. State Rolles Theorem.

5. Verify whether the equation $x^3-3x^2-4x+13=0$ has roots in the interval $(-3,-2)$.

6. Define Euler's function. Find the value of $\varphi(36)$.

7. Find the multiplicative inverse of (14) in $\mathbb{Z}_{15}$.

8. Prove that if p is a prime $(p-1)! \equiv -1 \pmod{p}$.

9. Check whether the relation $\sim$ on R defined by $a \sim b$ if $|a-b| \leq 1$ is an equivalence relation.

10. Let $\sigma, \tau \in S_7$ be given by $\sigma = (1\,3\,5\,6)$ and $\tau = (1\,2)(3\,5\,4\,7)$. Find $\sigma\tau\sigma^{-1}$.

11. Find the order of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 6 & 5 & 2 & 3 & 1 & 7 \end{pmatrix}$.

Turn over

12. Let G be a group. Prove that if $x^2 = e$ for all $x \in G$ then G is abelian.

13. Write the subgroups of S_3.

14. Let G be a group with $a, b \in G$ then show that $O\left(aba^{-1}\right) = O(b)$.

15. Let $G = \mathbb{Z}_{12}$ and $H = 4\,\mathbb{Z}_{12}$. Find all cosets of H.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35.

16. Solve the biquadratic equation $x^4 - 7x^3 + 18x^2 - 22x + 12 = 0$ whose roots are a, b, c and d and if $ab = 6$.

17. Find the limits of roots of the equation $2x^5 - 8x^4 - 11x^3 + 5x^2 + 2x - 11 = 0$.

18. Separate the roots of the equation $3x^4 - 2x^3 - 6x^2 + 6x - 2 = 0$.

19. Let G be a group and $a, b \in G$. Prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$ iff $ab = ba$.

20. Find all cyclic subgroups of $\mathbb{Z}_8$.

21. Check whether the set $\left\{a + b\sqrt[3]{2} \mid a, b \in \mathbb{Q}\right\}$ are subring of the field of real numbers.

22. Show that the cyclic group $<i>$ is isomorphic to $\mathbb{Z}_4$.

23. Let $\varphi : G \rightarrow H$ is an isomorphism of groups. Then show that :

    a) If G is abelian then so is H ; and

    b) If G is cyclic then so is H.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum marks 20.

24. Solve the cubic equation : $2x^3 + 3x^2 + 3x + 1 = 0$.

25. Let G be a group with normal subgroups H and K such that $HK = G$ and $H \cap K = \{e\}$ then show that $G \cong H \times K$.

26. Let G be a cyclic group. Then show that :

    a) If G is infinite then $G \cong \mathbb{Z}$.

    b) If G is finite with $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Fundamental Homomorphism theorem.

(2 × 10 = 20 marks)

D 10666 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.

2. Factorize into linear factors the polynomial $x^4 - 1$.

3. Write a cubic equation with roots $1,\ 1+i,\ 1-i$.

4. State Identity theorem.

5. How many real roots has the equation $x^4 - 4ax + b = 0$.

6. Make addition and multiplication tables for $\mathbb{Z}_2$.

7. Check whether the relation on $\mathbb{R}$ defined by $a \sim b$ if $a - b \in \mathbb{Q}$ is an equivalence relation.

8. Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Compute $\sigma\tau$ and $\tau\sigma$.

9. Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

10. Write a subgroup of $(\mathbb{Z}, +)$.

Turn over

11. Check whether $\mathbb{Z} \times \mathbb{Z}$ is cyclic.

12. Find order of the permutation $(1, 3)(2, 6)(1, 4, 5)$.

13. Let $\Phi : G_1 \to G_2$ be a group homomorphism. Show that $\Phi(e) = e'$ where e and e' are identity elements of G_1 and G_2 respectively.

14. Define a Ring.

15. Give example of an integral domain.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Solve $x^5 - 3x^4 + 4x^3 - 4x + 4$ having the root $1 + i$.

17. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ whose roots are a, b, c with $a + b = 0$.

18. Find an upper limit of the positive roots of the equation $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.

19. Prove that set of all even permutations of S_n is a subgroup of S_n.

20. Define * on $\mathbb{Z}$ by $a * b = a - b$. Check whether $(\mathbb{Z}, *)$ is a group.

21. Check whether $\mathbb{Z}_n$ is cyclic.

22. Draw the subgroup diagram of $\mathbb{Z}_{36}$.

23. Let G_1 and G_2 be groups and let $\Phi : G_1 \to G_2$ be a function such that $\Phi(ab) = \Phi(a)\Phi(b)$ for all $a, b \in G$. Prove that Φ is $1 - 1$ if and only if $\Phi(x) = e$ implies that $x = e$ for all $x \in G_1$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. Examine whether $x^4 - x^3 - x^2 + 19x - 42 = 0$ has integral roots or not.

25. Solve $x^3 - 6x - 6 = 0$ by Cardan's method.

26. Let G be a cyclic group. Show that :

    (a) If G is infinite then $G \cong \mathbb{Z}$.

    (b) If $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Lagrange's theorem.

(2 × 10 = 20 marks)