

C 40185

(Pages : 4)

Name.....

Reg. No.....

**SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
MARCH 2023**

Mathematics

MAT 6B 09—REAL ANALYSIS

(2017—2018 Admissions)

Time : Three Hours

Maximum : 120 Marks

Part A

Answer all questions.

Each question carries 1 mark.

1. Give an example for a continuous function which is not bounded.
2. State Preservation of Intervals Theorem.
3. Define Lipschitz itz function.
4. State Boundedness theorem.
5. Find $\| P \|$ if $P = (0, 1, 1.5, 2, 3.4, 5)$ in a partition of $[0, 5]$.
6. State Lebesgue integrability criterion.
7. Define uniform convergence of a sequence of functions.
8. $\lim_{n \rightarrow \infty} \frac{\sin (nx + n)}{n} =$
9. Give an example for an improper integral of second kind.
10. Cauchy principal value of $\int_{-1}^1 \frac{1}{x} dx =$
11. Fill in the blanks : $\Gamma (1) =$ _____.
12. Define Beta function.

(12 × 1 = 12 marks)

Turn over

Part B

*Answer any ten question.
Each question carries 4 marks.*

13. Let I be an interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $a, b \in I$ and if $k \in \mathbb{R}$ satisfies $f(a) < k < f(b)$, then prove that there exists $c \in I$ between a and b such that $f(c) = k$.
14. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Then prove that $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.
15. If $f : A \rightarrow \mathbb{R}$ is uniformly continuous on a subset A of \mathbb{R} and if (x_n) is a cauchy sequence in A then prove that $(f(x_n))$ is a cauchy sequence in \mathbb{R} .
16. Show that $f(x) = x^2$ on $A = [0, b]$, $b > 0$ satisfies a Lipschitz condition.
17. Suppose that $f, g \in \mathfrak{R}[a, b]$ and $k \in \mathbb{R}$. Prove that $kf \in \mathfrak{R}[a, b]$ and $\int_a^b kf = k \int_a^b f$.
18. Let $G(x) = \frac{1}{n}$ for $x = \frac{1}{n}$ ($n \in \mathbb{N}$) and $G(x) = 0$ elsewhere on $[0, 1]$. Show that $G \in \mathfrak{R}[0, 1]$.
19. State and prove composition theorem for Riemann Integrable functions.
20. Let (f_n) be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Suppose that (f_n) converges uniformly on A to f . Then prove that $\|f_n - f\|_A \rightarrow 0$.
21. Discuss the uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.
22. State and prove Weierstrass M-Test for a series of functions.
23. Test the convergence of $\int_0^{\infty} \frac{1}{x^2} dx$.
24. Show that $\Gamma(n+1) = n!$ when n is a positive integer.

25. Show that $\beta(m, n) = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} dy$.

26. Evaluate $\int_0^1 x^7 (1-x)^8 dx$.

(10 × 4 = 40 marks)

Part C

*Answer any six question.
Each question carries 7 marks.*

27. State and Prove Maximum-Minimum Theorem.
28. State and prove Continuous extension theorem.
29. Let $I = [a, b]$ be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $\epsilon > 0$, then prove that there exists a step function $S_{\epsilon} : I \rightarrow \mathbb{R}$ such that $|f(x) - S_{\epsilon}(x)| < \epsilon$ for all $x \in I$.
30. If $f \in \mathcal{R}[a, b]$ then prove that the value of the integral is uniquely determined.
31. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded on $[a, b]$.
32. State and prove Cauchy Criterion for Riemann Integrability.
33. State and prove Taylor's Theorem with the Remainder.
34. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $\forall m, n > 0$.

35. Show that $\int_0^{\frac{\pi}{2}} \sin^{(p-1)} \theta \cos^{(q-1)} \theta d\theta = \frac{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q}{2}\right)}{2\Gamma\left(\frac{p+q}{2}\right)}$.

(6 × 7 = 42 marks)

Turn over

Part D

*Answer any two question.
Each question carries 13 marks.*

36. a) State and prove Location of roots theorem.
b) Test the uniform continuity of $f(x) = \sqrt{x}$ on $[0, 2]$.
37. a) If f is continuous on $[a, b]$, then Prove that the indefinite integral $F(z) = \int_a^z f$ for $z \in [a, b]$ is differentiable on $[a, b]$ and $F'(x) = f(x)$ for all $x \in [a, b]$.
b) Show that Dirichlet function is not Riemann Integrable.
38. a) Express the integral $\int_0^{\frac{\pi}{2}} \sin^p x \cos^q x dx$ in terms of Beta function.
b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^5 x dx$.

(2 × 13 = 26 marks)

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(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 09—REAL ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours

Maximum : 120 Marks

Part A*Answer all questions.**Each question carries 1 mark.*

1. Define uniform continuity of a function.
2. State Weierstrass approximation theorem.
3. Find $\|P\|$ if $P = \{0, 2, 3, 4\}$ in a partition of $[0, 4]$.
4. Give an example for a function which is not Riemann integrable.
5. Define step function.
6. State Lebesgue integrability criterion.
7. Define uniform convergence of a series of functions.
8. $\lim_{n \rightarrow \infty} \frac{x^2 + nx}{n}$.
9. Write an example for an absolutely convergent improper integral.
10. Cauchy principal value of $\int_{-\infty}^{\infty} x dx =$.
11. Define Beta function.
12. Fill in the blanks : $\Gamma(3) =$ _____.

(12 × 1 = 12 marks)

Part B*Answer any ten questions.**Each question carries 4 marks.*

13. Let $I = [a, b]$ be a closed bounded interval and $f : I \rightarrow \mathbb{R}$ be continuous on I . If $k \in \mathbb{R}$ is any number satisfying $\inf f(I) \leq k \leq \sup f(I)$ then prove that there exists a number $c \in I$ such that $f(c) = k$.
14. State and prove Preservation of Intervals theorem.

Turn over

15. Show by an example that every uniformly continuous function need not be a Lipschitz function.
16. If $\phi : [a, b] \rightarrow \mathbb{R}$ is a step function, prove that $\phi \in \mathfrak{R}[a, b]$.
17. Suppose that $f, g \in \mathfrak{R}[a, b]$. Prove that $fg \in \mathfrak{R}[a, b]$.
18. State the substitution theorem of Riemann integration. Use it to evaluate $\int_0^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$.
19. Let (f_n) be a sequence of bounded functions of $A \subseteq \mathbb{R}$. Suppose that $\|f_n - f\|_A \rightarrow 0$. Then prove that (f_n) converges uniformly on A to f .
20. If f_n is continuous of $D \subseteq \mathbb{R}$ to \mathbb{R} for each $n \in \mathbb{N}$ and if $\sum f_n$ converges to f uniformly on D , then prove that f is continuous on D .
21. State and prove Weierstrass M-Test for a series of functions.
22. Discuss the uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.
23. Test the convergence of $\int_0^{\infty} \frac{1}{x^2} dx$.
24. Show that $\Gamma(n + 1) = n!$ when n is a positive integer.
25. Show that $\beta(m, n) = \beta(n, m)$.
26. Evaluate $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$.

(10 × 4 = 40 marks)

Part C

*Answer any six questions.
Each question carries 7 marks.*

27. Let $I = [a, b]$ be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Then prove that f is bounded on I .
28. State and prove Uniform Continuity Theorem.
29. State and prove Continuous Extension Theorem.
30. If $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$ then prove that $f \in \mathfrak{R}[a, b]$.
31. Discuss the convergence of the sequence $(f_n(x))$ where $f_n(x) = \frac{x^n}{x^n + 1}$, $x \in [0, 2]$.
32. State and prove Taylor's Theorem with the Remainder.

33. Let $f \in \mathfrak{R}[a, b]$ and let f be continuous at a point $c \in [a, b]$. Prove that the indefinite integral

$$F(z) = \int_a^z f \text{ for } z \in [a, b] \text{ is differentiable at } c \text{ and } F'(c) = f(c).$$

34. Show that $\Gamma\left(\frac{P}{2}\right)\Gamma\left(\frac{P+1}{2}\right) = \frac{\sqrt{\pi}}{2^{P-1}}\Gamma(P)$.

35. Evaluate the integral $\int_0^1 x^2(1-\sqrt{x})dx$.

(6 × 7 = 42 marks)

Part D

*Answer any two questions.
Each question carries 13 marks.*

36. (a) State and prove Maximum Minimum Theorem.

(b) Test the uniform continuity of $f(x) = \sqrt{x}$ on $[0, 2]$.

37. (a) Let $f : [a, b] \rightarrow \mathbb{R}$ and $c \in (a, b)$. Prove that $f \in \mathfrak{R}[a, b]$ if and only if its restriction to $[a, c]$ and $[c, b]$ are both Riemann integrable. In this case show that $\int_a^b f = \int_a^c f + \int_c^b f$.

(b) If $f \in \mathfrak{R}[a, b]$ and if $[c, d] \subseteq [a, b]$ then prove that the restriction of f to $[c, d]$ is in $\mathfrak{R}[c, d]$.

38. (a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \forall m, n > 0$.

(b) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

(2 × 13 = 26 marks)

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(Pages : 4)

Name.....

Reg. No.....

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS-UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all questions.

Each carries 1 mark.

1. Define absolute maximum of a real valued function $f : A \rightarrow \mathbb{R}$, $A \subseteq \mathbb{R}$.
2. Give an example of a continuous function. Why unbounded eventhough its domain is a bounded set in \mathbb{R} .
3. State the location of Roots theorem.
4. Define uniform continuity of a function.
5. Find $\|p\|$ if $p = \{0, 0.2, 0.5, 0.9, 1.5, 2\}$ is a partition of $[0, 2]$.
6. Let F, G be differentiable on $[a, b]$ and $F' = f$ and $G' = g$ both belongs to $\mathbb{R} [a, b]$.
Then $\int_a^b fG =$ _____.
7. Give an example of the improper integral of the 3rd kind.
8. The radius of convergence of the power series $\sum \frac{x^n}{n}$ is _____.
9. State the Weierstrass M-test for the uniform convergence of a series of functions.
10. When do we say a series of functions is absolutely convergent ?
11. $\lim_{n \rightarrow \infty} \left[\frac{\cos(nx + n)}{n} \right] =$ _____.
12. State the relation between Beta function and Gamma function.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any ten questions.
Each carries 4 marks.

13. (a) What do you mean by Lipschitz condition?
(b) What is its geometrical interpretation?
14. Let $f_n(x) = x^n(1-x)$, $x \in A = [0, 1]$. Prove that $f_n(x)$ converges to '0' uniformly on A.
15. Discuss the convergence of $\int_1^{\infty} \frac{\ln x \, dx}{x+u}$, where $u > 0$.
16. Evaluate $\int_1^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$.
17. State the boundedness theorem on Riemann integral. Justify the converse by an example.
18. State continuous extension theorem. Use it to show that $f(x) = x \sin\left(\frac{1}{x}\right)$ is uniformly continuous on $(0, b]$, $b > 0$.
19. Show by an example that every uniformly continuous function need not be Lipschitz function.
20. Determine the uniform convergence of $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ for $|x| \leq a$.
21. Distinguish between pointwise and uniform convergence of a sequence. Write the relation between them. Also write the necessary and sufficient condition for sequence (f_n) fail to converge uniformly on $A_0 \subset \mathbb{R}$ to f .
22. If $f \in \mathbb{R}[a, b]$, then P.T. $|f| \in \mathbb{R}[a, b]$.
23. State the limit comparison test for the convergence of improper integrals. Test the convergence of $\int_1^{\infty} \frac{dx}{1+x^2}$.

24. Prove that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$.
25. Prove that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$, $m, n > 0$.
26. Prove that $\beta(m, n) = 2 \int_0^{\pi/2} (\sin x)^{2m-1} (\cos x)^{2n-1} dx$.

(10 × 4 = 40 marks)

Section C

Answer any six questions.
Each carries 7 marks.

27. P.T. the image of a closed and bounded interval under a continuous mapping is a closed and bounded interval.
28. State and prove Bolzano's intermediate value theorem.
29. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous $[a, b]$, then P.T. $f \in R_{[a, b]}$.
30. State and prove uniform continuity theorem.
31. P.T. a sequence of bounded functions (f_n) on $A \subseteq \mathbb{R}$ converges uniformly to f on A iff $\|f_n - f\|_A \rightarrow 0$.
32. Discuss the convergence of $(f_n(x))$, where $f_n(x) = x^n$, $x \in \mathbb{R}$, $n \in \mathbb{N}$.
33. Prove that the uniform limit of a sequence of continuous function on $A \subseteq \mathbb{R}$ is continuous on A .
34. Prove that $\frac{\beta(m, n+1)}{n} = \frac{\beta(m+1, n)}{m} = \frac{\beta(m, n)}{m+n}$.
35. Evaluate $\int_0^1 (x \ln x)^3 dx$.

(6 × 7 = 42 marks)

Turn over

Section D

Answer any two questions.
Each carries 13 marks.

36. (a) State and prove Maximum-Minimum theorem.
(b) Show by an example that none of the conditions of the Maximum-Minimum theorem can be relaxed.
37. (a) If (x_n) is a Cauchy sequence on $A \subseteq \mathbb{R}$, then P.T. $(f(x_n))$ is a Cauchy sequence in \mathbb{R} , when $f: A \rightarrow \mathbb{R}$ is uniformly continuous on $A \subseteq \mathbb{R}$.
(b) Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, 1)$.

38. (a) Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$.

(b) If $I = \int_0^{\infty} \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$, S.T. $\int_n^{\infty} \frac{1}{1-x} = \frac{\pi}{\sin n\pi}$ for $0 < n < 1$.

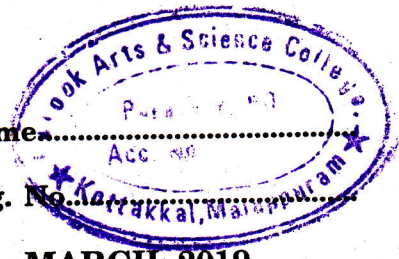
(2 × 13 = 26 marks)

C 60048

(Pages : 3)

Name.....

Reg. No.....



SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the twelve questions.

Each question carries 1 mark.

1. Give an example of a real valued function with set of real numbers as domain which is nowhere continuous.
2. Define uniform continuous function.
3. State Maximum-Minimum Theorem.
4. Define a tagged partition of a closed and bounded interval in \mathbb{R} .
5. Let $f(x) = x^2$ for $x \in [0, 4]$. Calculate the Riemann sum corresponding to the partition $P = (0, 1, 2, 4)$ with the tags at the right end points of the subintervals.
6. State Fundamental Theorem of Calculus (Second form).
7. Define uniform norm of a bounded function.
8. Give an example to show that pointwise convergence of sequence of functions need not implies uniform convergence.
9. State Cauchy Criterion for the uniform convergence of a series of functions.
10. Define improper integral of the second kind.
11. Show that $B(m, n) = B(n, m)$.
12. Define Gamma function.

(12 × 1 = 12 marks)

Section B

Answer any ten out of fourteen questions.

Each question carries 4 marks.

13. Let I be a closed and bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $k \in \mathbb{R}$ is any number satisfying $\inf f(I) \leq k \leq \sup f(I)$, then show that there exists a number $c \in I$ such that $f(c) = k$.

Turn over

14. Show that the equation $x = \cos x$ has a solution in the interval $[0, \pi]$.
15. If $f : A \rightarrow \mathbb{R}$ is uniformly continuous on a subset A of \mathbb{R} and if (x_n) is a Cauchy sequence in A , then show that $(f(x_n))$ is a Cauchy sequence in \mathbb{R} .
16. If f and g are uniformly continuous on subset A of \mathbb{R} , show that $f + g$ is uniformly continuous on A .
17. Consider the function h defined by $h(x) = x + 1$ for $x \in [0, 1]$ rational and $h(x) = 0$ for $x \in [0, 1]$ irrational. Show that h is not Riemann integrable.
18. State and prove the Mean Value Theorem for Integrals.
19. State Lebesgue's Integrability Criterion. Using this discuss the Riemann integrability of any step function on $[a, b]$.
20. Let F and G be differentiable on $[a, b]$ and let $f = F'$ and $g = G'$ belongs to $R[a, b]$, then show that
- $$\int_a^b f G = [FG]_a^b - \int_a^b F g.$$
21. Show that $\lim(x/(x+n)) = 0$ for all $x \in \mathbb{R}, x \geq 0$. Also show that the convergence is not uniform on the interval $[0, \infty)$.
22. Discuss the convergence and uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.
23. Show that $\int_1^{\infty} \frac{1}{x^{4/3}} dx$ converges.
24. Show that $\int_0^1 \frac{\sin x}{\sqrt{x}} dx$ converges.
25. Show that $\Gamma(1/2) = \sqrt{\pi}$.

(10 × 4 = 40 marks)

Section C

*Answer any six out of nine questions.
Each question carries 7 marks.*

26. State and prove Boundedness Theorem.
27. If $f(x) = x$ and $g(x) = \sin x$, show that both f and g are uniformly continuous of \mathbb{R} , but that their product fg is not uniformly continuous on \mathbb{R} .
28. If $f \rightarrow \mathbb{R}$ is monotone on $[a, b]$, then show that $f \in R[a, b]$.

29. State and prove Fundamental Theorem of Calculus (First Form).
30. Let (f_n) be a sequence of bounded functions on $A \in \mathbb{R}$. Then show that this sequence converges uniformly on A to a bounded function f if and only if for each $\varepsilon > 0$ there is a number $H(\varepsilon)$ in \mathbb{N} such that for all $m, n \geq H(\varepsilon)$ then $\|f_m - f_n\| \leq \varepsilon$.
31. Show that the uniform limit of a sequence of continuous real valued functions on a subset of real numbers is continuous.
32. Show that the improper integral $\int_0^1 \frac{1}{x} dx$ diverges.
33. Let f be a non-increasing function on $[1, \infty)$ such that $f(x) \geq 0$ ($1 \leq x < \infty$). Then show that $\sum_1^\infty f(n)$ will diverge if $\int_1^\infty f(x) dx$ diverges.
34. Derive a relation between Beta and Gamma function.

(6 × 7 = 42 marks)

Section D

Answer any two out of three questions.

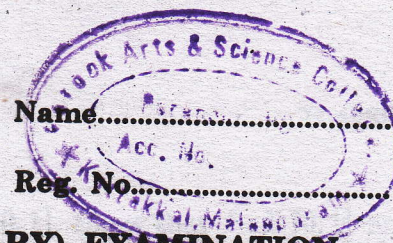
Each question carries 13 marks.

35. Show that a continuous function on a closed and bounded interval I can be approximated arbitrarily closely by step function.
36. Let $f: [a, b] \rightarrow \mathbb{R}$ and let $c \in (a, b)$. Then show that $f \in \mathbb{R}[a, b]$ if and only if its restriction to $[a, c]$ and $[c, b]$ are both Riemann integrable. In this case show that $\int_a^b f = \int_a^c f + \int_c^b f$.
37. (a) Let $f_n(x) := 1/(1+x)^n$ for $x \in [0, 1]$. Find the pointwise limit f of the sequence f_n on $[0, 1]$. Does f_n converge uniformly to f on $[0, 1]$?
- (b) Let (c_n) be a decreasing sequence of positive numbers. If $\sum c_n \sin nx$ is uniformly convergent, then show that $\lim (nc_n) = 0$.

(2 × 13 = 26 marks)

D 40899

(Pages : 3)



**SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION
MARCH 2018**

(CCSS)

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all twelve questions.

Each question carries $\frac{1}{4}$ weightage.

1. State Bolzano's intermediate value theorem.
2. Given an example of a continuous function which need not bounded on the domain set A.
3. Define Lipschitz function on a set $A \subseteq \mathbb{R}$.
4. Given an example of a function which is Riemann integrable in $[0, 1]$.
5. If $\psi : [a, b] \rightarrow \mathbb{R}$ takes only a finite number of distinct values ; then 'is ψ a step function' ?
6. Is it true that $h(x) = x, \forall x \in [0, 1]$ belongs to $R[0, 1]$.
7. If $f_n(x) = \frac{x}{n}$, find $\lim f_n(x)$.
8. Does $g_n(x) = x^n ; x \in [0, 1]$ converges uniformly ?
9. Let $f_n(x) = \frac{1}{x^2 + n^2}$. Does $\sum f_n$ converges uniformly.
10. Define beta function.
11. Find $\sqrt[n]{n}$, where $n \in \mathbb{N}$.
12. Define an improper integral.

(12 \times $\frac{1}{4}$ = 3 weightage)

Turn over

Part B

Answer all nine questions.

Each question carries 1 weightage.

13. State preservation theorem of integrals.
14. What is uniform Continuity theorem ?
15. If $f, g \in R[a, b]$, then show that $f + g \in R[a, b]$.
16. If $\varphi : [a, b] \rightarrow R$ is a step function, then prove that $\varphi \in R[a, b]$.
17. Show that $\lim \left[\frac{x^2 + nx}{n} \right] = x, \forall x \in R$.
18. State Cauchy's criterion for uniform convergence of series.
19. Test the convergence of the integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^p} dx$.
20. Find $\sqrt{n+1} - \sqrt{n}$, where n is a natural number.
21. Prove that $\int_0^1 \frac{1}{x}$ diverges.

(9 × 1 = 9 weightage)

Part C

Answer any five questions.

Each question carries 2 weightage.

22. If $f : A \rightarrow R$ is uniformly continuous on $A \subseteq R$ and if (x_n) is a Cauchy sequence in A , then prove that $(f(x_n))$ is a Cauchy sequence in R .
23. Check whether $\sin \frac{1}{x}$ and $x \sin \frac{1}{x}$ are uniformly continuous on $[a, b] \forall b > 0$.

24. Let $f(x) = 1$ for $x = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and $f(x) = 0$ elsewhere on $[0, 1]$. Show that $f \in R[0, 1]$ and

$$\int_0^1 f = 0.$$

25. If $f: [a, b] \rightarrow \mathbb{R}$ is monotonic on $[a, b]$, then prove that $f \in R[a, b]$.

26. State and prove first form of fundamental theorem of Calculus.

27. Show that, if (f_n) and (g_n) converge uniformly on the set A to f, g respectively, then $f_n + g_n$ converges to $f + g$ on A .

28. Show that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

(5 × 2 = 10 weightage)

Part D

Answer any two questions.

Each question carries 4 weightage.

29. State and prove additivity theorem for Riemann integrals.

30. (a) State and prove second form of fundamental theorem of calculus.

(b) Show that $f_n(x) = x^2 e^{-nx}$ converges uniformly on $(0, \infty)$.

31. Show that $B(m, n) = \frac{\sqrt{m} \sqrt{n}}{\sqrt{m+n}}$ and hence evaluate $B(3, 4)$.

(2 × 4 = 8 weightage)

D 40041

(Pages : 4)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Part A

*Answer all the twelve questions.
Each question carries 1 mark.*

1. State the Location of Roots Theorem.
2. Give an example of a continuous function on $A = (0, \infty)$ which is not a uniformly continuous function on $A = (0, \infty)$.
3. State the Preservation of Intervals Theorem.
4. State the Weierstrass Approximation Theorem.
5. Define the Norm of a Partition of a closed and bounded interval.
6. State the Boundedness Theorem on Riemann integration.
7. Give the Lebesgue Integrability criterion of a function.
8. State the Substitution Theorem for Riemann Integration.
9. For what value of r the series $\sum_{n=1}^{\infty} r^n \sin nx$ uniformly convergent.
10. Fill in the blanks : $\lim \left(\frac{(x^2 + nx)}{n} \right) = \dots$
11. Fill in the blanks : $\beta(1/2, 1/2) = \dots$
12. Fill in the blanks : $\int_0^{\infty} \sqrt{x} e^{-x} dx = \dots$

(12 × 1 = 12 marks)

Turn over

Part B

*Answer any ten questions.
Each question carries 4 marks.*

13. State the Boundedness Theorem. Show by an example that the boundedness theorem fails if the Interval is not bounded.
14. Let $I = [a, b]$, be a closed and bounded Interval . If $f : I \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then prove that $f(I)$ is a closed and bounded interval.
15. Show by an example that the continuous image of an open interval need not be an open interval.
16. State the "sequential criteria" for the uniform continuity of a function. Apply this result to test the uniform continuity of $f(x) = 1/x$ on $(0, 1)$.
17. Use the location of Roots Theorem to show that the equation $x^3 - x - 1 = 0$ has a root in $(1, 2)$.
18. If $f \in \mathbb{R}[a, b]$ and $|f(x)| \leq M, \forall x \in [a, b]$, then prove that $\left| \int_a^b f \right| \leq M(b-a)$.
19. If $\phi : [a, b] \rightarrow \mathbb{R}$ is a Step function, then prove that $\phi \in \mathbb{R}[a, b]$.
20. If F and G are differentiable functions on $[a, b]$ and $f = F', g = G' \in \mathbb{R}[a, b]$, then prove that

$$\int_a^b fG = [FG]_a^b - \int_a^b Fg.$$
21. Define uniform convergence of a sequence of functions. Test uniform convergence of the sequence $(x/n); x \in \mathbb{R}, n \in \mathbb{N}$.
22. Define uniform norm of a bounded function. Find $\|f_n - f\|_A$, if $(f_n)(x) = (x^n; x \in A = [0, 1])$ and $f(x) = 0$ for $0 \leq x < 1$; $f(x) = 1$ for $x = 1$.
23. Define uniform convergence of a series of function. Show that the Uniform limit of a series of continuous functions is continuous .
24. Show that $\int_{x=a}^b \frac{1}{(x-a)^p} dx$ converges , if $p < 1$; and diverges if $p \geq 1$.

25. Define the Cauchy Principal value of the integral $\int_{x=-\infty}^{\infty} f(x) dx$. Find it if $f(x) = 1/x$.
26. Define Beta function . Prove that $\beta(m, n) = \beta(n, m), \forall m, n > 0$.

(10 × 4 = 40 marks)

Part C

*Answer any six questions.
Each question carries 7 marks.*

27. State and prove the Intermediate Value Theorem.
28. State and prove the Continuous Extension Theorem.
29. State and prove the Composition Theorem of Riemann integration.
30. Define Riemann integral of a function. If $f \in R[a, b]$, then prove that the value of the Riemann integral of f is uniquely determined.
31. Prove that a sequence of bounded functions on A converges uniformly on A if and only if $\|f_n - f\|_A \rightarrow 0$.
32. Test the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^p}$.
33. (a) Distinguish between absolute and conditional convergence of an improper integral.
- (b) Discuss the convergence of the Improper integral $\int_{x=\pi}^{\infty} \frac{\sin x}{x} dx$.
34. Evaluate $\int_a^b (x-a)^{m-1} (b-x)^{n-1} dx; m, n > 1$.
35. Express the integral $\int_0^1 (x)^p (1-x^q)^n dx; p, q, n > 0$ in terms of Gamma function.

(6 × 7 = 42 marks)

Turn over

Part D

*Answer any two questions.
Each question carries 13 marks.*

36. (a) Let $I = [a, b]$, be a closed and bounded Interval . If $f : I \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then prove that f is bounded on $I = [a, b]$.

(b) Prove that $\beta(p, 1-p) = \int_{x=0}^{\infty} \frac{x^{p-1}}{(1+x)} dx$.

37. (a) State and prove Squeeze Theorem of Riemann Integration.

(b) Evaluate $\left(\frac{(\sin nx)}{1+nx} \right); x \geq 0$.

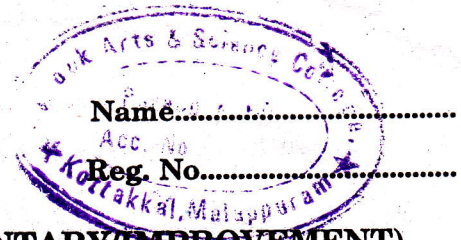
38. (a) State and prove the Weierstrass-M-test for uniform convergence of a Series of functions.

(b) Show that $\beta(m, n) = \int_{x=0}^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$.

(2 × 13 = 26 marks)

C-21562

(Pages : 2)



**SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, MARCH 2017**

(UG-CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all twelve questions.

Each question carries $\frac{1}{4}$ weightage.

1. Give an example of a function such that the continuous image of an open interval need not be an open interval.
2. State boundedness theorem for functions.
3. Define uniform continuity of a function on a set of $A \subseteq \mathbb{R}$.
4. Justify the statement "An unbounded function cannot be Riemann integrable".
5. If $I = [0, 4]$, calculate the norm of the partition $P = (0, 0.5, 2.5, 3.5, 4)$.
6. Is it true that $f(x) = \text{sgn } x$ on $[-1, 1]$ belongs to $R[-1, 1]$?
7. Does $f_n = \frac{x}{n}$ converges uniformly on \mathbb{R} .
8. If (f_n) and (g_n) be sequences converge uniformly to f and g on A respectively, find limit of $(f_n g_n)$ on A .
9. Let $g_n(x) = \sin\left(\frac{x}{n^2}\right)$. Does $\sum f_n$ converges uniformly on \mathbb{R} .
10. Define Gamma function.
11. State the relation between Beta and Gamma functions.
12. Define beta function using trigonometric functions.

(12 \times $\frac{1}{4}$ = 3 weightage)

Part B

Answer all questions.

Each question carries 1 weightage.

13. State Bolzano-intermediate value theorem.
14. If $f: A \rightarrow \mathbb{R}$ is a Lipschitz's function, then prove that f is uniformly continuous.
15. Show that every constant functions on $[a, b]$ is in $R[a, b]$.

Turn over

16. State Cauchy's criterion for a function on $[a, b]$ belongs to $R [a, b]$.
17. Show that $\lim \left[\frac{1}{n} \sin (nx + n) \right] = 0, \forall x \in R$.
18. Give an example of a sequence of a continuous functions that converges non-uniformly to a continuous limit.
19. Test the convergence of $\int_0^1 \frac{1}{x^2} dx$.
20. Show that $\sqrt{1} = 1$.
21. What is the relation between $B(m, n)$ and $B(n, m)$?

(9 × 1 = 9 weightage)

Part C

*Answer any five questions.
Each question carries 2 weightage.*

22. Let I be a closed and bounded interval and let $f: I \rightarrow R$ be continuous on I . Prove that $f(I)$ is a closed bounded interval.
23. Prove that a function f is uniformly continuous on $[a, b]$ if and only if it can be defined at the end points a and b such that the extended function is continuous on $[a, b]$.
24. If $f \in R [a, b]$, then prove that f is bounded.
25. Consider the function h defined by $h(x) = x + 1$ for all $x \in [0, 1]$ rationals and $h(x) = 0$, for all $x \in [0, 1]$ irrationals. Then prove that h is not Riemann integrable.
26. Check the sequence $G(x) = x^n(1-x), \forall x \in [0, 1]$ converges uniformly on $[0, 1]$.
27. Prove that $\sqrt{1/2} = \sqrt{\pi}$.
28. Define Cauchy principal value of $\int_{-\infty}^{\infty} f(x) dx$ and hence evaluate $\int_{-\infty}^{\infty} \sin t dt$.

(5 × 2 = 10 weightage)

Part D

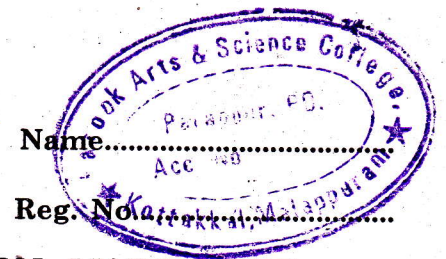
*Answer any two questions:
Each question carries 4 weightage.*

29. State and prove location of root theorem.
30. (a) If $f: [a, b] \rightarrow R$ is monotone on $[a, b]$, then prove that $f \in R [a, b]$.
- (b) Check the series $\sum \frac{\cos(n\theta)}{n^p}$ converges uniformly for all real value of θ and $p > 1$.
31. (a) Derive duplication formula for gamma function.
- (b) Prove that $\int_0^a (a-x)^{m-1} x^{n-1} dx = a^{m+n-1} \beta(m, n)$.

(2 × 4 = 8 weightage)

C 21070

(Pages : 4)



SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

*Answer all questions.
Each question carries 1 mark.*

1. State boundedness theorem.
2. Define absolute Minimum of a function.
3. State the preservation of interval theorem.
4. Give an example of a continuous function on $(0, \infty)$ which has neither absolute maximum nor absolute minimum.
5. Give an example of a continuous function on $A \subseteq \mathbb{R}$, which is not uniformly continuous on A .
6. Find $\|p\|$ if $p = \{0, 0.3, 0.6, 1, 1.5, 2\}$ is a partition of the set $[0, 2]$.
7. Give an example of the 2nd kind improper integral.
8. The Radius of convergence of the power series $\sum \frac{x^n}{n!}$ is _____.
9. Define uniform convergence of a series of functions.
10. $\lim_{n \rightarrow \infty} \left(\frac{\sin(nx+n)}{n} \right) =$ _____.
11. What do you mean by uniform norm of a bounded function for $A \subseteq \mathbb{R}$.
12. Find $\sqrt{\frac{5}{2}}$.

(12 × 1 = 12 marks)

Section B

*Answer any ten questions.
Each question carries 4 marks.*

13. Let $I = [a, b]$, Let $f : I \rightarrow \mathbb{R}$ be continuous. If $K \in \mathbb{R}$ is any number satisfying

$$\inf f(I) \leq K \leq \sup f(I),$$

then prove that there exist a number $C \in I$ such that $f(C) = K$.

Turn over

14. (a) Define Lipschitz function.
 (b) If f is a Lipschitz function on $A \subseteq \mathbb{R}$, then prove that f is uniformly continuous on A .
15. Show that the function f defined by

$$f(x) = \left. \begin{array}{l} 1, \text{ if } x \text{ is rational} \\ 0, \text{ when } x \text{ is irrational} \end{array} \right\} \text{rational}$$

has no Riemann Integral over $[0,1]$.

16. Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$.

17. State the substitution theorem of Riemann Integration. Use it to evaluate $\int_1^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$.

18. State continuous extension theorem. Use it to show that $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $(0, b]$, $b > 0$.

19. Determine the uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.

20. State and prove the Weierstrus M-Test for a series of functions.

21. If F and G are differentiable on $[a, b]$ and let $f = F'$ and $g = G'$ belongs to $R[a, b]$ then P.T.

$$\left[\int_a^b fG = FG \right]_a^b - \int_a^b Fg.$$

22. State Lebesgue's Integrability Criterion for Riemann Integrability. Use it to show that every step function on $[a, b]$ is Riemann Integrable.

23. Test the convergence of $\int_1^{\infty} \frac{\ln x}{x^2} dx$.

24. Evaluate $\int_2^{\infty} \frac{(x+3)dx}{(x-1)(x^2+1)}$.

25. Prove that $\sqrt{n+1} = n\sqrt{n}$.

26. Prove that $\sqrt{n} = \int_0^{\infty} e^{-y^2} y^{2n-1} dy$.

(10 × 4 = 40 marks)

Section C

Answer any six questions.

Each question carries 7 marks.

27. State and prove Intermediate value theorem.
28. If $f \in R[a, b]$, then prove that f is bounded on $[a, b]$.
29. If $f : [a, b] \rightarrow R$ is monotone on $[a, b]$, P. T. $f \in R[a, b]$.
30. If $f : A \rightarrow R$ is uniformly continuous on $A \subseteq R$ and if (x_n) is a Cauchy sequence in A , then prove that $f(x_n)$ is a Cauchy sequence in R .
31. Discuss the convergence of the sequence $(f_n(x))$, where $f_n(x) = \frac{x^n}{x^n + 1}$, $x \in [0, 2]$.
32. State the necessary and sufficient condition for sequence (f_n) to fail to converge uniformly on $A \subseteq R$ to f . Use it to test the uniform convergence of $(f_n(x))$, where $f_n(x) = \frac{x}{n}$, $x \in R$.
33. If (f_n) and (g_n) are uniformly convergent sequences on $A \subseteq R$, is it imply that $(f_n)(g_n)$ is uniformly convergent on A ? Justify by an example.
34. State and prove the product theorem on Riemann Integration.
35. Prove that $\beta(m, n) = \frac{\sqrt{m} \cdot \sqrt{n}}{\sqrt{m+n}}$, $\forall m, n > 0$.

(6 × 7 = 42 marks)

Section D

Answer any two questions.

Each question carries 13 marks.

36. (a) State and prove the Cauchy criterion for uniform convergence of a sequence of functions.
(b) Discuss the convergence of $f_n(x) = x^4$, $x \in [0, 1]$.
37. (a) State and prove uniform continuity theorem.
(b) Test the uniform continuity of $f(x) = \sqrt{x}$, $x \in [0, 2]$.

Turn over

38. (a) Express the integral $\int_0^{\pi/2} \sin^p x \cos^q x dx$ in terms of Beta function ; Hence find the value of

$$\int_0^{\pi/2} \sin^5 x \cos^6 x dx.$$

(b) Evaluate $\int_0^1 x^m \left(\ln \frac{1}{x} \right)^n dx.$

(2 × 13 = 26 marks)

C 1742

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2016

(UG-CCSS)

Core Course—Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all questions.

1. When we say that f is uniform continuous ?
2. State maximum minimum theorem.

3. If $f(x) \leq g(x) \forall x \in [a, b]$ then what is the relation connecting $\int_a^b f$ and $\int_a^b g$.

4. Find the norm of the partition $P = \{0, 0.5, 2, 3.5, 4\}$ of $[0, 4]$.
5. Give an example of a function f which is not Riemann integrable.

6. By the first form of fundamental theorem of calculus, show that $\int_a^b x dx = \frac{1}{2}(b^2 - a^2)$.

7. Define uniform convergent sequence.
8. State Weierstrass M test for uniform convergence of a sequence.

9. Find CPV of $\int_{-\infty}^{\infty} x dx$.

10. Define absolute convergence of $\int_a^{\infty} f(x) dx, f \in R[a, s]$.

11. Define B (m, n) ?

12. State a relationship between Beta and Gamma functions.

(12 \times $\frac{1}{4}$ = 3 weightage)

Turn over

Part B (Answer all Questions)

13. State Boundedness theorem.
14. If f is a uniformly continuous function defined on A and if (x_n) is a Cauchy sequence in A then prove that $f(x_n)$ is a Cauchy sequence in \mathbb{R} .
15. Give an example of a step function.
16. If $h(x) = x, x \in [0,1]$ then prove that $h \in [0,1]$.
17. If $f \in \mathbb{R}[a, b], k \in \mathbb{R}$ prove that $kf \in \mathbb{R}[a, b]$.
18. Find $\lim \frac{x^2 + nx}{n}, x \in \mathbb{R}$.
19. Show that $\int_1^{\infty} \frac{1}{x^2} dx = 1$.
20. Define conditional convergence.
21. Find $\int_0^2 (8 - x^3)^{-1/3} dx$.

(9 × 1 = 9 weightage)

Part C

Answer any five Questions.

22. If $f: [a, b] \rightarrow \mathbb{R}$ is a continuous function and if $f(a) < k < f(b)$ then prove that there exists c in (a, b) such that $f(c) = k$.
23. Show that $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$ has five roots in the interval $[0, 7]$.
24. Show that every uniformly continuous function need not be Lipschitz function.
25. If f is a monotone function then prove that $f \in \mathbb{R}[a, b]$.
26. If f is a positive non increasing function on $[1, \infty]$ then prove that $\sum_{n=1}^{\infty} f(x) dx$ converges if

$$\int_1^{\infty} f(x) dx \text{ converges.}$$

27. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

28. Prove that $\int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$.

(5 × 2 = 10 weightage)

Part D

Answer any two Questions.

29. Explain bisection method to find the root of a function.
30. State and prove Cauchy's criterion for a function $f \in R [a, b]$.
31. Prove that (a) $B(m, n) = B(n, m)$ (b) $\Gamma \frac{1}{2} = \sqrt{\pi}$.

(2 × 4 = 8 weightage)

C 80025

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all questions.

1. State Maximum Minimum theorem.
2. Give an example of a uniform continuous function.
3. Define norm of a partition.
4. When is a function $f : [a, b] \rightarrow \mathbb{R}$ is Riemann integrable ?
5. Define primitive of f .

6. If $J = J = [c, d] \subset [a, b]$ and $\varphi_j(x) = \begin{cases} 1, & x \in J \\ 0, & x \in [a, b] - J \end{cases}$. Find $\int_a^b \varphi_j$?

7. Show that $\frac{x}{n} = 0, \forall x \in \mathbb{R}$.

8. Evaluate $\lim \frac{nx}{1 + (nx)^2}$.

9. Find the radius of convergence of $\sum x^n$.

10. Define improper integral.

11. State True or false. "If f is continuous on $[1, \infty)$ and if $\int_1^{\infty} f(x) dx$ converges then $\lim_{x \rightarrow \infty} f(x) = 0$."

12. What is $B(m, n)$?

(12 × ¼ = 3 weightage)

Turn over

Part B

Answer all questions.

13. Define Lipschitz function.
14. State and prove Bolzano's intermediate value theorem.
15. Show that every constant function on $[a, b]$ is $R[a, b]$.
16. Using First form of fundamental theorem prove that $\int_a^b x dx = \frac{1}{2}(b^2 - a^2)$.
17. If $f_n(x) = x^n, x \in [0, 1]$, check whether $f_n(x)$ converges to zero uniformly.
18. If (f_n) is a sequence of continuous functions converging uniformly to f , prove that f is continuous.
19. Show that $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$ diverges.
20. Define Gamma function.
21. Show that $B(1, n) = 1/n$.

(9 × 1 = 9 weightage)

Part C

Answer any five questions.

22. If f and g are uniform continuous on a subset A of \mathbb{R} , then prove that $f + g$ is uniformly continuous on A .
23. Show that $f(x) = xe^x - 2$ has a root in $[0, 1]$.
24. If $f \in R[a, b]$ then prove that f is bounded on $[a, b]$.
25. State and prove Weierstrass M test for the series.
26. If $\int_a^{\infty} |f(x)| dx$ converges absolutely then prove that $\int_a^{\infty} f(x) dx$ converges.

27. Show that $\int_0^1 \frac{1}{x} dx$ diverges.

28. Prove the recurrence formula for gamma function.

(5 × 2 = 10 weightage)

Part D

Answer any two questions.

29. State and prove Squeeze theorem. Also prove that a step function $\varphi: [a, b] \rightarrow \mathbb{R}$ is in $R[a, b]$.

30. State and prove Cauchy's criterion for uniform convergence.

31. Show that $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ and $\int_0^\infty \frac{1}{x^2 + \sqrt{x}} dx$ are convergent.

(2 × 4 = 8 weightage)

C 60108

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2014

(U.G.-CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all twelve questions.

1. Give an example to a function such that every point of \mathbb{R} is both an absolute maximum and an absolute minimum for this function.
2. Define Uniform continuity of a function in an interval I .
3. State the location of Roots theorem.
4. Find the norm of the partition :
 $p := (0, 1, 1.5, 2, 3.4, 4)$.
5. State True or False :
"If $f \in R[a, b]$, then f is bounded on $[a, b]$ ".
6. Give an example to a function which is not Riemann integrable on any interval.
7. If $f_n(x) = \log(x/n)$, show that $\lim(f_n(x)) = 0$.
8. State a necessary and sufficient condition for uniform convergence of $\{f_n\}$, a sequence of bounded functions on $A \subseteq \mathbb{R}$.
9. State Cauchy criterion for the uniform convergence of $\sum f_n(x)$ on $D \subseteq \mathbb{R}$.
10. Give an example of an improper integral.
11. Define Gamma function.
12. State True or False :

$$\beta(m, n) = \frac{\sqrt{m} + \sqrt{n}}{\sqrt{mn}}$$

(12 × ¼ = 3 weightage)

Turn over

Part B*Answer all nine questions.*

13. Define Absolute maximum and Absolute minimum of :

$$f : A \rightarrow \mathbb{R}; A \subseteq \mathbb{R}.$$

14. State intermediate value theorem.

15. Show that $f(x) = k$ is Riemann integrable on $[a, b]$.

16. If f and g are in $\mathbb{R}[a, b]$, show that $f + g \in \mathbb{R}[a, b]$.

17. Use fundamental theorem of calculus to evaluate $\int_a^b \frac{1}{x^2 + 1} dx$.

18. Let $g_n(x) = \frac{e^{-nx}}{n}$ for $n \geq 0, n \in \mathbb{N}$. Examine the relation between $\lim(g_n)$ and $\lim(g'_n)$.

19. Show that $\sum \frac{\cos \theta}{np}$ is uniformly convergent for all real values of $\theta, p > 1$.

20. Prove that $\int_0^1 \frac{1}{x} dx$ diverges.

21. If $\int_a^\infty f$ and $\int_a^\infty g$ both converges, show that $\int_a^b (f + g)$ converges.

(9 × 1 = 9 weightage)

Part C*Answer any five from seven questions.*

22. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Prove that $f(I)$ is a closed bounded interval.

23. Define a Lipschitz function. Also prove that a Lipschitz function $f : A \rightarrow \mathbb{R}$ is uniformly continuous on A .

24. Consider the function $f(x) = \begin{cases} x+1; & \text{when } x \text{ is rational} \\ \theta; & \text{when } x \text{ is irrational} \end{cases}$ is not Riemann integrable in $[0, 1]$.

25. If $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$ prove that $f \in \mathbb{R}[a, b]$.

26. Show that $\sum \frac{(-1)^n x^{2n}}{n^p (1+x^{2n})}$ converges absolutely and uniformly for all real x if $p > 1$.

27. Define Cauchy principal value of $\int_{-\infty}^{+\infty} f(x) dx$. Also evaluate it for $\int_{-\infty}^{+\infty} \sin t dt$.

28. Prove that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$.

(5 × 2 = 10 weightage)

Part D

Answer any two from three questions.

29. (a) Let I be an interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Prove that $f(I)$ is an interval.

(b) Prove that $\sin x$ is uniformly continuous on $[0, \infty]$.

30. (a) If $f \in R[a, b]$ and k is real number prove that kf is in $R[a, b]$.

(b) If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then prove that $f \in R[a, b]$.

31. (a) State and prove Weirstrass M test.

(b) Prove that $\int_0^a (a-x)^{m-1} x^{n-1} dx = a^{m+n-1} \beta(m, n)$.

(2 × 4 = 8 weightage)

C 40394

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer all *twelve* questions :

- 1 State True or False " $f(x) = \frac{1}{x}$ has neither an absolute maximum nor an absolute minimum on the set $(0, \infty)$ ".
- 2 Does $4 \sin x = x$ has a positive solution in $\left(\frac{\pi}{2}, \pi\right)$.
- 3 Give an example to a uniformly continuous function on $[0, b]$; $b > 0$.
- 4 Find the norm of the partition $p := (0, 2, 3, 4)$.
- 5 Give an example to function which is Riemann integrable in $[a, b]$.
- 6 State True or False "Every continuous function is integrable".
- 7 Define point-wise convergence of a sequence of function $\{f_n(n)\}$.
- 8 Show that $f_n(x) = \frac{1}{x+n}$ in uniformly convergent in $[0, b]$; $b > 0$.
- 9 Define uniform norm of a bounded function ϕ on $A \subseteq \mathbb{R}$ and $\phi : A \rightarrow \mathbb{R}$.
- 10 Define an improper integral.
- 11 Define Beta function.
- 12 If n is a positive integer value of $\sqrt{n+1}$ is _____.

(12 × ¼ = 3 weightage)

Turn over

Part B*Answer all nine questions.*

- 13 Define absolute maximum and absolute minimum of $f : A \rightarrow \mathbb{R} ; A \subseteq \mathbb{R}$.
- 14 State maximum–minimum theorem.
- 15 Show that the function $f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$ is not integrable on any interval.
- 16 If $f(x) \leq g(x) \forall x \in [a, b]$, then show that $\int_a^b f(x) dx \leq \int_a^b g(x) dx$.
- 17 Use fundamental theorem of calculus to evaluate $\int_a^b x dx$.
- 18 Show that $G(x) = x^n(1-x)$ for $x \in A := [0, 1]$ converges uniformly to $g(x) = 0$.
- 19 Show that $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ is uniformly convergent in $[-1, 1]$.
- 20 Examine the convergence of $\int_1^{\infty} \frac{dx}{\sqrt{x}}$.
- 21 Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ as a Beta function.

(9 × 1 = 9 weightage)

Part C*Answer any five questions from 7.*

- 22 State and prove Intermediate value theorem.
- 23 Define a Lipschitz function. Also prove that a Lipschitz function $f : A \rightarrow \mathbb{R}$ is uniformly continuous on A .

- 24 Show that $f(x) = x^2$ is Riemann integrable on $[0, k]$.
- 25 If $f : [a, b] \rightarrow \mathbb{R}$ is monotonic on $[a, b]$ then prove that $f \in \mathcal{R}[a, b]$.
- 26 State and prove Weierstrass M-test.
- 27 If $\int_a^\infty |f(x)| dx$ converges, show that $\int_a^\infty f(x) dx$ converges.
- 28 Using Beta functions prove that :

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}.$$

(5 × 2 = 10 weightage)

Part D*Answer any two from four questions.*

- 29 (a) Let $I := [a, b]$ be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Prove that f is bounded on I .
- (b) Define a step function and give an example.
- 30 Prove that if f is Riemann integrable on $[a, b]$ then it is bounded on $[a, b]$.
- 31 (a) Test for uniform convergence $\sum_1^\infty \frac{\sin nx}{n^p}$ for $p > 1$.
- (b) Prove that $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\sqrt{n}}{a^n}$; where $a > 0, n > 0$.

(2 × 4 = 8 weightage)