C 20212 **(Pages : 3)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. Define vertex of a convex set.
2. Find the convex hull of $\{x_1, x_2\} \subset \mathbb{R}^2$.
3. Define objective function.
4. While solving an LPP by graphical method, when does one conclude that there exist infinitely many points in the feasible region at which the objective function attains optimum ?
5. Define a slack variable.
6. Write the matrix form of general LPP.
7. What is meant by feasible solution to an LPP ?
8. What is meant by optimal solution to an LPP ?
9. When do you say that a transportation problem is unbalanced ?
10. State the necessary and sufficient condition for the existence of a feasible solution to general transportation problem.
11. Write the number of basic variables of the general transportation problem at any stage of feasible solution.
12. In assignment problem, what is the value of decision variable ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 *marks*.

13. Prove that the intersection of two convex sets is a convex set.
14. Write the standard form of LPP.
15. What is the basic principle of linear programming ?

Turn over

16. Does a feasible solution exist for the LPP : Maximize $z = 2x_1 + 10x_2$ subject to $x_1 - x_2 \geq 1, - x_1 + x_2 \geq 2, x_1 \geq 0, x_2 \geq 0$? Give reasons.
17. For linear inequalities, show that the solution set for a group of inequalities is a convex set.
18. Write the dual of the LPP :

    Maximize $f(x) = 3x_1 + 2x_2$ subject to $2x_1 + x_2 \leq 20, x_1 + 3x_2 \leq 20, x_1 \geq 0, x_2 \geq 0$.
19. Define a loop in transportation table.
20. How does a loop in a transportation table related to a basic feasible solution ?
21. Define a triangular basis. Write the role of triangular basis in transportation problem.
22. How do you solve an unbalanced transportation problem ?
23. Write the disadvantage of North-West corner rule.
24. "An assignment problem is a particular case of a transportation problem." Justify.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that the set $S = \{(x_1, x_2) : 5x_1 + 2x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex.
26. (a) Define convex linear combination of a finite set of vectors.

    (b) Show that the set of all convex linear combinations of a finite number of vectors $u_1, u_2, u_3, \ldots, u_k \in \mathbb{R}^n$ is a convex set.
27. Prove that the set of feasible solutions to an LPP is a convex set.
28. Show that the following system of linear equations has a degenerate solution :

    $2x_1 + x_2 - x_3 = 2, 3x_1 + 2x_2 + x_3 = 3.$
29. Let $x_1 = 2, x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations $2x_1 - x_2 + 2x_3 = 2,\ x_1 + 4x_2 = 18.$ Reduce this feasible solution to a basic feasible solution.
30. Use graphical method to solve the following LPP :

    Maximize $z = 6x_1 + x_2$ subject to $2x_1 + x_2 \geq 3, x_2 - x_1 \geq 0, x_1 \geq 0, x_2 \geq 0$.
31. Prove that the dual of the dual is the primal.
32. How do you resolve the problem of degeneracy in transportation problem ?

33. Obtain an initial basic feasible solution to the following transportation problem using North-West Corner rule :

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Use Simplex method to solve the LPP :

Maximize $Z = 4x_1 + 10x_2$ subject to

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90 \text{ and } x_1 \ge 0, x_2 \ge 0.$$

35. Use Vogel's approximation method to obtain an initial basic feasible solution to the transportation problem :

	D_1	D_2	D_3	D_4	Availability
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	

36. A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below :

Tasks	Men			
	T_1	T_2	T_3	T_4
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How the task should be allocated, one task to a man, so as to minimize the total man-hours ?

(2 × 10 = 20 marks)

C 1251 (Pages : 4) Name.................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions. Each question carries* 1 *mark.*

1. What is meant by a convex polyhedron ?
2. Explain the convex combination of set of vectors.
3. Write down the standard form of a general LPP.
4. Define feasible and optimal solution of a linear programming problem.
5. State fundamental theorem of linear programming.
6. In the two-phase simplex method when phase I terminates.
7. Write down the dual of the following LPP :

   Maximize $z = x_1 + 2x_2 + x_3$

   subject to the constraints :

   $$2x_1 + x_2 - x_3 \leq 2$$
   $$-2x_1 + x_2 - 5x_3 \geq -6$$
   $$4x_1 + x_2 + x_3 \leq 6$$
   $$x_1, x_2, x_3 \geq 0.$$

8. Give the mathematical formulation of the transportation problem.
9. Define loop in a transportation table.
10. State the necessary condition for the existence of feasible solution to the transportation problem.
11. What is an assignment problem ?
12. State Konig theorem.

($12 \times 1 = 12$ marks)

Section B

Answer at least **eight** *questions. Each question carries* 3 *marks.*
All questions can be attended. Overall Ceiling 24.

13. A manufacturer produces two types of models M_1 and M_2. Each model of the type M_1 requires 4 hours of grinding and 2 hours of polishing whereas each model of the type M_2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works 40 hours a week and each polisher works for 60 hours a week. Profit on M_1 model is Rs. 3.00 and on model M_2 is Rs. 4.00. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week.

Turn over

14. Show that the set $S = \{ (x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \le 4, x_1 + 2x_2 - x_3 \le 1 \}$ is a convex set.

15. Rewrite in standard form the following linear programming problem :

Minimize $Z = 12x_1 + 5x_2$

subject to the constraints

$$6x_1 + 3x_2 \ge 15$$
$$7x_1 + 2x_2 \le 14$$
$$x_1, x_2 \ge 0.$$

16. Verify the Minimax theorem for the function $f(x) = \{10, 8, 5, 2, 1\}$.

17. State the general rules for converting any primal LPP into its dual.

18. Verify that the dual of dual is primal for the following LPP :

Maximize $z = 8x_1 + 3x_2$

subject to the constraints :

$$x_1 - 6x_2 \le 2$$
$$5x_1 + 7x_2 = -4$$
$$x_1, x_2 \ge 0.$$

19. Prove that in a balanced transportation problem having m origins and n destinations $(m, n \ge 2)$, the exact number of basic variables is $m + n - 1$.

20. Prove that a set X of column vectors of the co-efficient matrix of a transportation problem is linearly dependent if their corresponding cells in the transportation table contains a loop.

21. Write all the steps for the North-West corner rule of solving a transportation problem.

22. How to solve the degeneracy in transportation problem ?

23. Write steps for solving assignment problem by Hungarian method.

24. State the difference between transportation problem and assignment problem.

(8 × 3 = 24 marks)

Section C

Answer at least **five** *questions.*

Each question carries 6 *marks.*

All questions can be attended.

Overall Ceiling 30.

25. Prove that the set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots x_k$ in R^n is a convex set.

26. Let S be a convex subset of the plane, bounded by lines in the plane. Prove that a linear function $z = c_1x_1 + c_2x_2$, where c_1 and c_2 are scalars, attains its extreme values at the vertices of S only.

27. Solve the following LPP by graphical method :

Maximize $Z = 5x_1 + 7x_2$

subject to the constraints :

$$x_1 + x_2 \le 4$$
$$3x_1 + 8x_2 \le 24$$
$$10x_1 + 7x_2 \le 35$$
$$x_1, x_2 \ge 0.$$

28. Find all the basic solution to the system of linear equations : $x_1 + 2x_2 + x_3 = 4$ and $2x_1 + x_2 + 5x_3 = 5$. Are the solutions degenerate ?

29. Use simplex method to solve the following LPP :

Maximize $z = 4x_1 + 10x_2$

subject to the constraints :

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$
$$x_1, x_2 \ge 0.$$

30. Use Charne's penalty method to :

Minimize $z = 2x_1 + x_2$

subject to the constraints :

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 3$$
$$x_1, x_2 \ge 0.$$

31. Prove that if the k^{th} constraint of the primal problem is an equality then the k^{th} dual variable will be unrestricted in sign.

32. A company has 5 jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs on different machines are given below. Assign the jobs for different machines so as to minimize the total cost :

	Machines				
Jobs	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

33. Determine an initial feasible solution to the following transportation problem using the Vogel's approximation method :

	A_1	B_1	C_1	D_1	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	950

(5 × 6 = 30 marks)

Section D

Answer any **one** *question.*
The question carries 14 *marks.*

34. Prove that there is a one-to-one correspondence between the optimum solutions to the General LPP and its reformulated LPP.

35. Solve the LPP by simplex method :

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$
subject to the constraints :

$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

36. Find the optimal solution of the following transportation problem whose cost matrix is given as under :

	D_1	D_2	D_3	D_4	Supply
O_1	1	5	3	3	34
O_2	3	3	1	2	15
O_3	0	2	2	3	12
O_4	2	7	2	4	19
Required	21	25	17	17	80

(1 × 14 = 14 marks)

C 80274 (Pages : 4) Name..................................

Reg. No..............................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *the twelve questions.*
Each question carries 1 *mark.*

1. Define a convex set.
2. Determine the convex hull of the set $A = \left\{(x_1, x_2) : x_1^2 + x_2^2 = 1\right\}$.
3. State graphical solution algorithm for an LPP involving two variables.
4. Define slack and surplus variables.
5. State the condition of optimality for a basic feasible solution to an LPP to be maximum.
6. Define artificial variable.
7. Write down the following LPP in standard form :

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$

   subject to the constraints : $2x_1 - 3x_2 \leq 3$

   $x_1 + 2x_2 + 3x_3 \geq 5$

   $3x_1 + 2x_3 \leq 2$

   $x_1, x_2 \geq 0$

   and $x_3 \geq 0$.
8. Write the dual of the following LPP :

   Maximize $Z = 3x_1 - x_2 + x_3$

   subject to the constraints : $4x_1 - x_2 \leq 8$

   $8x_1 + x_2 + 3x_3 \geq 12$

   $5x_1 - 6x_3 \leq 13$

   $x_1, x_2, x_3 \geq 0$.

Turn over

9. State fundamental theorem of linear programing.
10. Define triangular basis in a transportation problem.
11. What is an assignment problem ?
12. What is degeneracy in transportation problem ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 marks.

13. Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram and 12 units of vitamin B and costs 20 paise per gram ; The daily minimum requirements of vitamin A and vitamin B are 100 units and 120 units respectively. Formulate this as a Linear programming problem to find the minimum cost of product mix.

14. Show that the set $S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4\} \subset R^3$ is a convex set.

15. Plot the feasible region in x_1, x_2 plane for the LPP constraints :

$$3x_1 - 2x_2 \leq 12 ; x_1 - 6x_2 \leq 1 ; -x_1 + 2x_2 \leq 4 \text{ and } x_1, x_2 \geq 0.$$

16. State the characteristics of canonical form and write the canonical form of LPP in matrix form.

17. The column vector (1, 1, 1) is a feasible solution to the system of equations :

    $x_1 + x_2 + 2x_3 = 4$ and $2x_1 - x_2 + x_3 = 2$. Reduce the given feasible solution to a basic feasible solution.

18. Verify Minimax theorem for the function $f(x) = \{9, 7, 5, 3, 1\}$.

19. State the general rules for converting any primal LPP into its dual.

20. Explain the North- West corner rule for obtaining an initial basic feasible of a transportation problem.

21. Prove that every loop in a transportation table has an even number of cells.

22. What are the chief characteristics of a transshipment problem.

23. Write steps for solving assignment problem by Hungarian method.

24. In an assignment problem with cost (c_{ij}), if all $c_{ij} > 0$, then prove that feasible solution (x_{ij}) which satisfies $\sum\sum c_{ij} x_{ij} = 0$ is optimal.

(9 × 2 = 18 marks)

Section C

*Answer any **six** out of nine questions.*
Each question carries 5 marks.

25. Show that set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots x_k$ in R^n is a convex set.

26. Use graphical method to solve the LPP : Maximize $z = 5x_1 + 7x_2$ subject to the constraints,

$$x_1 + x_2 \le 4$$
$$3x_1 + 8x_2 \le 24$$
$$10x_1 + 7x_2 \le 35$$
$$x_1, x_2 \ge 0.$$

27. Obtain all the basic feasible solution to the system of linear equations :

$$x_1 + 2x_2 + x_3 = 4 \text{ and } 2x_1 + x_2 + 4x_3 = 5.$$

28. Write the algorithm to solve LPP using Simplex method.

29. Use penalty method to :

Minimize $z = 2x_1 + x_2$

subject to the constraints, $3x_1 + x_2 = 3$

$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 3 \text{ and } x_1, x_2, x_3 \ge 0.$$

30. Verify that dual of dual is primal for the following LPP :

Maximize $z = 2x_1 + 5x_2 + 6x_3$

subject to the constraints : $5x_1 + 6x_2 - x_3 \le 3$

$$-2x_1 + x_2 + 4x_3 \le 4$$
$$x_1 - 5x_2 + 3x_3 \le 1$$
$$-3x_1 - 3x_2 + 7x_3 \le 6$$
$$x_1, x_2, x_3 \ge 0.$$

31. Obtain an initial basic feasible solution to the following transportation problem using the matrix minima method.

	D_1	D_2	D_3	D_4	*Supply*
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Required	4	6	8	6	24

Turn over

32. Prove that there always exist an optimal solution to a balanced transportation problem.

33. A company has 5 jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs on different machines are given below. Assign the jobs for different machines so as to minimize the total cost.

	Machines				
Jobs	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks..*

34. Solve the following linear programming problem by simplex method :

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

subject to the constraints :

$$2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$$

$$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$$

$$7x_1 + x_4 \leq 70 \text{ and } x_1, x_2, x_3, x_4 \geq 0.$$

35. Let X be set of column vectors of the co-efficient matrix of a transportation problem. Prove that a necessary and sufficient condition for vectors in X to be linearly dependent is that the set of their corresponding cells in the transportation contains a loop.

36. Determine the optimum basic feasible solution to the following transportation problem with the initial solution obtained by Vogel's approximation method :

	D_1	D_2	D_3	*Availability*
O_1	50	30	220	1
O_2	90	45	170	4
O_3	250	200	50	4
Demand	4	2	3	9

(2 × 10 = 20 marks)

C 21075 (Pages : 4) Name........Anagha.M........

Reg. No........FPAOSMT023........

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 14 (E 02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Define convex hull of a set.
2. Examine whether the set $S = \{(x_1, x_2) : 5x_1 + 2x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex.
3. State graphical solution algorithm for an LPP involving two variables.
4. Define slack and surplus variables.
5. Reduce the following LPP to its standard form :

   Maximize $Z = x_1 - 3x_2$

   subject to the constraints :

   $-x_1 + 2x_2 \leq 15$

   $x_1 + 3x_2 = 10$

   x_1 and x_2 unrestricted in sign.
6. When does the simplex method indicate that the LPP has unbounded solution ?
7. Write the dual of the following LPP :

   Maximize $Z = 3x_1 - x_2 + x_3$

   subject to the constraints : $4x_1 - x_2 \leq 8$

   $8x_1 + x_2 + 3x_3 \geq 12$

   $5x_1 - 6x_3 \leq 13$

   $x_1, x_2, x_3 \geq 0.$
8. State Minimax theorem.

Turn over

9. What is transportation problem ?
10. State the necessary condition for the existence of feasible solution to the transportation problem.
11. Give the mathematical formulation of the assignment problem.
12. What is an unbalanced transportation problem ?

($12 \times 1 = 12$ marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 *marks.*

13. Formulate the following problem as a Linear Programming Problem : A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C respectively per carton. If the liquid product is sold for Rs. 3 per jar and the dry product is sold for Rs. 2 per carton, how many units of each product should be purchased, in order to minimize the cost and meet requirements.
14. Prove that a hyperplane in R^n is a convex set.
15. Obtain graphically the maximum value of $z = \{\min(3x_1 - 10), \min(-5x_1 + 5)\}$ such that $0 \le x_1 \le 5$.
16. Write the characteristics of standard form of Linear Programming Problem.
17. The column vector (1, 1, 1) is a feasible solution to the system of equations : $x_1 + x_2 + 2x_3 = 4$ and $2x_1 - x_2 + x_3 = 2$. Reduce the given feasible solution to a basic feasible solution.
18. Verify Minimax theorem for the function $f(x) = \{9, 7, 5, 3, 1\}$.
19. State the general rules for converting any primal LPP into its dual.
20. Write all the steps for Vogel's Approximation method of solving a transportation problem.
21. Prove that every loop in a transportation table has an even number of cells.
22. How to solve the degeneracy in transportation problems ?
23. Write steps for solving assignment problem by Hungarian method.
24. State the difference between transportation problem and assignment problem.

($9 \times 2 = 18$ marks)

Section C

Answer any **six** *out of nine questions.*

Each question carries 5 marks.

25. Show that set of all convex combinations of a finite number of vectors $x_1, x_2, \dots x_k$ in R^n is a convex set.

26. Use graphical method to solve the LPP :

    Maximize $Z = 6x_1 + 11x_2$

    subject to the constraints,

    $2x_1 + x_2 \leq 104$

    $x_1 + 2x_2 \leq 76$

    $x_1, x_2 \geq 0$

27. Show that the following system of linear equations has a degenerate solution : —

    $2x_1 + x_2 - x_3 = 2$ and $3x_1 + 2x_2 + x_3 = 3$.

28. Use simplex method to solve the LPP :

    Maximize $Z = 3x_1 + 2x_2$

    subject to the constraints

    $4x_1 + 3x_2 \leq 12$

    $4x_1 + x_2 \leq 8$

    $4x_1 - x_2 \leq 8$ and $x_1, x_2 \geq 0$.

29. Explain the Charne's Big-M method.

30. Prove that dual of the dual is primal.

31. Determine an initial basic feasible solution to the following transportation problem using the row minima method.

	D_1	D_2	D_3	*Supply*
O_1	50	30	220	1
O_2	90	45	170	4
O_3	250	200	50	4
Required	4	2	3	9

32. Prove that there always exist an optimal solution to a balanced transportation problem.

33. The owner of a small machine shop has four machinists available to do jobs for the day. Five jobs are offered with expected profit for each machinist on each job as follows :

Jobs	*Machinists*			
	1	2	3	4
A	32	41	57	18
B	48	54	62	34
C	20	31	81	57
D	71	43	41	47
E	52	29	51	50

Find, by using assignment method, the assignment of machinists to jobs that will result in a maximum profit.

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks.*

34. Let $A \subseteq R^n$ be any set. Prove that $< A >$, the convex hull of A, is the set of all finite convex combination of vectors in A.

35. Use Simplex method to solve the LPP :

Minimize $Z = x_2 - 3x_3 + 2x_5$

subject to the constraints

$$3x_2 - x_3 + 2x_5 \leq 7$$

$$-2x_2 + 4x_3 \leq 12$$

$$-4x_2 + 3x_3 + 8x_5 \leq 10 \text{ and } x_2, x_3, x_5 \geq 0.$$

36. Obtain an optimum basic feasible solution to the following degenerate transportation problem :

	D_1	D_2	D_3	*Availability*
O_1	7	3	4	2
O_2	2	1	3	3
O_3	3	4	6	5
Demand	4	1	5	10

(2 × 10 = 20 marks)

C 1749 (Pages : 4) Name.....................................

Reg. No.....................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2016

(UG–CCSS)

Elective Course—Mathematics

MM 6B 13 (E 02)—LINEAR PROGRAMMING

(2010 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. Give the Canonical form of a minimization linear programming problem.

2. Is Minimise $Z = 5x_1 - 6x_2$

   subject to $x_1 + x_2 = 5$

   $3x_1 + 2x_2 = 23$

   $x_1, x_2 \geq 0$ in the standard form ?

3. True *or* False : $\mathbb{R}^n$ is convex.

4. Define a non-degenerate Basic Feasible Solution of the system of linear equations AX = B.

5. State the fundamental theory of linear programming.

6. Define a surplus variable.

7. If the dual has no feasible solution, then the primal problem has an objective function that is ————.

8. Name the method used to solve a linear programming problem when surplus variables arise ?

9. What is the maximum number of basic variables in a balanced transportation problem with '*m*' rows and '*n*' columns ?

10. Consider a 5 × 7 transportation problem. Does the set of cells $\{(1,1),(1,3),(5,3),(5,7),(3,7),(3,1)\}$ form a loop ?

11. True *or* False : An assignment problem is a special type of transportation problem.

12. A non-degenerate basic feasible solution of a transportation Problem with '*m*' rows and '*n*' columns has ———— zeros.

(12 × ¼ = 3 weightage)

Turn over

Section B

Answer **all** *questions.*
Each question carries 1 *weightage.*

13. Standardize :

Maximize $Z = 3x_1 - 2x_2$

subject to
$$x_1 + x_2 \le 1$$
$$x_1 + 2x_2 \ge 7$$
$$x_1, x_2 \ge 0.$$

14. Show that $A = \{(2,0), (0,2)\} \subset \mathbb{R}^2$ is not convex.

15. Define a half space in $\mathbb{R}^n$.

16. State the optimality criterion for the Basic Feasible Solution of a Linear programming problem.

17. Find the dual of :

Minimize $Z = x_1 - 3x_2 - 2x_3$

subject to
$$2x_1 - 4x_2 \ge 12$$
$$3x_1 - x_2 + 2x_3 \le 7$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$
$$x_1 \le 0,\ x_2 \ge 0$$
x_3 unrestricted.

18. When is Charnes method used to solve a linear programming problem ? Define penalty.

19. Give the matrix notation of a transportation problem.

20. Find an initial basic feasible solution by NWCR :

	D_1	D_2	D_3	Supply
O_1	6	8	4	14
O_2	4	9	8	12
O_3	1	2	6	5
Demand	6	10	15	

21. Show that a balanced transportation problem always possesses a finite feasible solution and an optimal solution.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 weightage.

22. Solve graphically :

Minimize $Z = 2x_1 - x_2$

subject to $x_1 + x_2 \le 1$
$2x_1 + 3x_2 \ge 6$
$x_1, x_2 \ge 0.$

23. Show that the extreme points of the set of feasible solutions of $Ax = b$, are the basic feasible solutions.

24. Solve :

Maximize $Z = x_1 + x_2 + 3x_3$

subject to $3x_1 + 2x_2 + x_3 \le 3$
$2x_1 + x_2 + 2x_3 \le 2$
$x_1, x_2, x_3 \ge 0.$

25. Solve :

Maximize $Z = 3x_1 + 2x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \le 2$
$3x_1 + 4x_2 + 2x_3 \ge 8$
$x_1, x_2, x_3 \ge 0.$

26. Show that the dual of the dual is the primal itself.

27. Find an initial basic feasible solution by VAM :

	D_1	D_2	D_3	Supply
O_1	3	5	7	150
O_2	6	4	10	200
O_3	8	10	3	100
Demand	100	300	50	

28. Solve to find minimum cost (Assignment problem) :

	M_1	M_2	M_3	M_4	M_5
1	9	8	7	6	4
2	5	7	5	6	8
3	8	7	6	3	5
4	8	5	4	9	3
5	6	7	6	8	5

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Formulate as an L.P.P. and solve :

A firm produces two products – A and B, with profits per unit Rs. 100 and Rs. 40 respectively. Each product is processed on machines M1, M2 and M3 product A requires 10 minutes on M 1, 3 minutes on M2 and 6 minutes on M3 product B requires 4 minutes on M 1, 2 minutes on M2 and 12 minutes on M3. M1 is available for at most 33 hours. 20 minutes per week and M2 for at most 15 hours and M3 for at most 50 hours per week. Find the number of units of A and B to be made per week to maximize profit.

30. Use principle of Duality to solve :

Maximize $Z = x_1 + x_2 + x_3$

subject to $2x_1 + x_2 + 2x_3 \leq 2$
$4x_1 + 2x_2 + x_3 \leq 2$
$x_1, x_2, x_3 \geq 0.$

31. Obtain an optimal solution to minimize cost :

	D_1	D_2	D_3	D_4	Supply
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O3	40	8	70	20	18
Demand	5	8	7	14	

(2 × 4 = 8 weightage)

C 40400 (Pages : 4) Name..................................

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 30 Weightage

Part I

Answer **all** *questions.*

1. Define a slack variable.
2. Define a convex set in $\mathbb{R}^3$.
3. State True or False :

   The union of two convex sets is convex.
4. Define non-degenerate basic solution of the system AX = B.
5. What is the optimality criterion for the basic feasible solution of a maximization L.P.P. ?
6. Define artificial variable.
7. The primal has 5 decision variables and 3 constraints. Then its dual has ——— decision variables and ——— constraints.
8. Dual of the dual is the ———.
9. Define a loop in a Transportation Problem (TP).
10. State True *or* False :

    A balanced TP always possesses a finite feasible solution and an optimal solution.
11. The number of zeros in a non-degenerate basic feasible solution of a balanced Transportation Problem with 4 sources and 5 destinations is ———.
12. The decision variables in an Assignment problem are :
    (a) 1 only.
    (b) 0 only.
    (c) Either 1 or 0.
    (d) None of these.

($12 \times \frac{1}{4} = 3$ weightage)

Turn over

Part II

Answer any **nine** *questions.*

13. Write in standard form :

Maximize $z = 3x_1 - x_2$

subject to

$$x_1 - 2x_2 \le -3$$
$$4x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

14. Show that $k = \{(x, y) \in \mathbb{R}^2 ; x^2 + y^2 \le 9\}$ is convex.

15. Show that every hyperplane in $\mathbb{R}^n$ is convex.

16. Show that a set 'S' is convex in E^n implies that every convex combination of points in S lies in S.

17. Show that an optimal solution of

Minimize $z = cx$

subject to $Ax \le b,\ x \ge 0$ is also an optimal solution of

Maximize $z' = -cx$,

$Ax \le b\ \ x \ge 0$.

18. Find the dual of :

Minimize $z = 4x_1 - x_2$

subject to

$$x_1 + x_2 \le 4$$
$$2x_1 - x_2 \ge 3$$
$$x_1 \ge 0$$

x_2 unrestricted.

19. State the Fundamental Theorem of Linear Programming.

20. Find an IBFS by NWCR :

	A	B	C	D	
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
	6	10	12	15	

21. Find an IBFS to the above problem by the matrix minimum method.

22. Test optimality of the Basic Feasible Solution $x_{12} = 30, x_{21} = 10, x_{22} = 10, x_{23} = 30,$ $x_{31} = 10, x_{33} = 10$ for the Transportation Problem given below :

	A	B	C	D	
I	2	1	3	4	30
II	3	2	1	4	50
III	5	2	3	8	20
	20	40	30	10	

23. Give the Mathematical Form of the Assignment Problem.

24. What is a restrictive Assignment Problem and how is it tackled ?

(9 × 1 = 9 weightage)

Part III

Answer any **five** *questions.*

25. State and prove a necessary and sufficient condition for a set S in E^n to be convex.

26. Show that the basic feasible solutions of $Ax = b$ are the extreme points.

27. Explain in simple steps the computational procedure of the simplex method.

28. Solve : Maximize $z = x_1 + 5x_2$

subject to

$$x_1 + 10x_2 \le 20$$
$$x_1 \le 2$$
$$x_1, x_2 \ge 0.$$

29. Solve : Maximize $z = 2x_1 - 3x_2$

subject to

$$-x_1 + x_2 \ge -2$$
$$5x_1 + 4x_2 \le 46$$
$$7x_1 + 2x_2 \ge 32$$
$$x_1, x_2 \ge 0.$$

30. Find an IBFS by VAM and test for optimality :

	A	B	C	
I	10	9	8	8
II	10	7	10	7
III	11	9	7	9
IV	12	14	10	4
	10	10	8	

31. Solve the following minimization Assignment Problem :

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

32. Prove that in a balanced TP, there are at most $m+n-1$ basic variables ($m-no$: of sources, $n-no$: of destinations).

(5 × 2 = 10 weightage)

Part IV

Answer **both** *questions.*

33. Solve the following Transportation Problem to obtain the optimal solution :

	A	B	C	D	
I	6	1	9	3	70
II	11	5	2	8	55
III	10	12	4	7	90
	85	35	50	45	

34. Solve the following maximization Assignment Problem :

	1	2	3	4	5
A	9	3	4	2	10
B	12	10	8	11	9
C	11	2	9	0	8
D	8	0	10	3	7
E	7	5	6	2	9

(2 × 4 = 8 weightage)

D 40046 (Pages : 4) Name.................................

Reg. No...............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Define extreme point of a convex set.
2. A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$. Find in which half space does the point $(-6, 1, 7, 2)$ lie.
3. What do you mean by a degenerate basic solution ?
4. Define slack and surplus variable.
5. Reduce the following LPP to its standard form : Maximize $z = x_1 + 2x_2$ subject to the constraints,

$$2x_1 - 3x_2 \leq 3$$
$$4x_1 + x_2 \leq -4$$
$$x_1, x_2 \geq 0.$$

6. When does the simplex method indicate that the LPP has unbounded solution ?
7. Write the dual of the following LPP Min $4x_1 + 6x_2 + 18x_3$ subject to the constraints,

$$x_1 + 3x_3 \geq 3$$
$$x_2 + 2x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0.$$

8. When does the transportation problem is said to be unbalanced.
9. What is transportation problem ?

Turn over

10. Give mathematical formulation of the assignment problem.

11. State the necessary condition for the existence of feasible solution to transportation problem.

12. What do you mean by an unbalanced assignment problem ?

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Formulate the following problem as a LPP : A firm manufactures two type of products A and B and sells them at a profit of Rs. 2 on type A and Rs .3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires 1 minute of processing time on M_1 and 2 minutes on M_2 ; Type B requires 1 minute on M_1 and 1 minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes, while machine M_2 is available for 10 hours during any working day.

14. Prove that a hyperplane in $\mathbb{R}^n$ is a convex set.

15. Solve graphically, Maximize $z = 2x_1 + 3x_2$ subject to the constraints

$$x_1 + x_2 \leq 1;$$
$$3x_1 + x_2 \leq 4;$$
$$x_1, x_2 \geq 0.$$

16. Write the characteristics of standard form of LPP.

17. Find all basic solutions of the system,

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

18. Prove that the intersection of 2 convex sets is also a convex set.

19. Show that the dual of the dual of a given primal is the primal itself.

20. Write all the steps for North-West corner rule.

21. How to solve the degeneracy in transportation problem.

22. Give an algorithm to solve assignment problem.

23. Write the steps for solving assignment problem by Hungarian method.

24. Write the main differences between assignment problem and transportation problem.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Use graphical method to solve Maximize $z = 5x_1 + 7x_2$ subject to the constraints

$$x_1 + x_2 \leq 4$$
$$3x_1 + 8x_2 \leq 24$$
$$10x_1 + 7x_2 \leq 35$$
$$x_1, x_2 \geq 0.$$

26. Explain Charne's Big-M method.

27. Show that the set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots\ldots x_k$ in $\mathbb{R}^n$ is a convex set.

28. Using Simplex method to solve the LPP : Maximize $2x_1 + 4x_2$ subject to the constraints

$$2x_1 + 3x_2 \leq 48$$
$$x_1 + 3x_2 \leq 42$$
$$x_1 + x_2 \leq 21$$
$$x_1, x_2 \geq 0.$$

29. Let $x_1 = 2, x_2 = 4, x_3 = 1$ be a feasible solution to the system of equations $2x_1 - x_2 + 2x_3 = 2$, $x_1 + 4x_2 = 18$, reduce the feasible solution to a basic feasible solution.

30. Show that $S = (x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4$ in $\mathbb{R}^3$ is a convex set.

31. Prove that there always exist an optimal solution to a balanced transportation problem.

32. Solve :

	D_1	D_2	D_3	D_4	supply
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Demand	6	10	12	15	43

33. Prove that there always exist an optimal solution to a balanced transportation problem.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Use Simplex method to solve the LPP : Maximize $z = 2x_1 + x_2$ subject to the constraints

$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + x_2 \leq 8$$
$$4x_1 - x_2 \leq 8$$
$$x_1, x_2 \geq 0.$$

35. Let $A \subseteq \mathbb{R}^n$ be any set. Show that the convex hull of A, is the set of all finite convex combination of vectors in A.

36. Solve the transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	1	2	1	4	30
O_2	4	2	5	9	50
O_3	20	40	30	10	20
Demand	20	40	30	10	100

(2 × 10 = 20 marks)

C 60053 (Pages : 4)

Name...

Reg. No.....................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 13 (E 02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define a convex set.
2. What is a degenerate solution of an L.P.P. ?
3. What is a slack variable ?
4. Write the names of any two methods to a solve a transportation problem.
5. Write the following L.P.P. in standard form :

   Maximize $Z = 2x_1 - 8x_2$

   subject to $x_1 + x_2 \geq -1$

   $x_1 - x_2 \leq 3$

   $x_1, x_2 \geq 0.$
6. Show that $x_1 = 2, x_2 = 1$ is a feasible solution of the L.P.P. given below :

   Maximize $Z = 4x_1 + x_2$

   $x_1 + x_2 \leq 3$

   $x_1 - x_2 > 1$

   $x_1, x_2 \geq 0.$
7. Explain, why we not use 'Transportation Algorithm' to solve 'An assignment problem'.
8. Find the number of possible feasible solutions of the following L.P.P. :

   Maximize $Z = x_1 + x_2$

   subject to the constraints $x_1 + x_2 + x_3 \leq 5$

   $x_1, x_2, x_3 \geq 0.$
9. Write the necessary and sufficient condition for a basic feasible solution to a L.P.P. to be an optimum (maximum).
10. Write the following L.P.P. in matrix form :

    Minimize $Z = x_1 + x_2 - x_3$

    subject to $x_1 + x_3 \geq 2$

    $x_1 - x_2 \geq 4$

    $x_1, x_2 \geq 0.$

Turn over

11. Write the dual of the following L.P.P. :

Minimize $Z = x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 - x_2 \leq 1$

$x_1, x_2 \geq 0.$

12. When we say that a 'transportation problem' is unbalanced ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 marks.

13. Define a hyper sphere in $\mathbb{R}^n$.

14. Show that the following L.P.P. has no solution :

Maximize $Z = x_1 + x_2$

where $x_1 - x_2 \geq 0$

$3x_1 - x_2 \leq -3$

$x_1, x_2 \geq 0.$

15. Show that the intersection of two convex set is also a convex set.

16. Write a short note on 'The North-West Corner Rule'.

17. Write a short note on 'The Assignment Problem'.

18. Form the Mathematical formulation of the problem given below :

Prabha goes to the market to purchase buttons. She needs atleast 20 large buttons and 30 small buttons respectively. The shopkeeper sells buttons in two tons—(i) boxes ; and (ii) cards. A box contains 10 large buttons and 5 small buttons respectively ; whereas a card contains 2 large buttons and 5 small buttons respectively.

Determine the most economical way in which Prabha should purchase the buttons, if a box costs Rs. 25 and a card costs Rs. 10 only.

19. Write the dual problem of the following L.P.P. :

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3$

subject to the constraints. $5x_1 + 6x_2 - x_3 \leq 3$

$-2x_1 + x_2 + 4x_3 \leq 4$

$x_1 - 5x_2 + 3x_3 \leq 1$

$-3x_1 - 3x_2 + 7x_3 \leq 6$

$x_1, x_2, x_3 \geq 0.$

20. Find any basic feasible solution of the following transportation problem :

	m_1	m_2	m_3	m_4	
w_1	1	2	3	4	2
w_2	4	3	2	1	2
w_3	2	1	4	3	3
	3	2	1	1	

21. Given that $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ can be written in the form $a_1 = \lambda_1 b_1 + \lambda_2 c_1$. Find λ_1 and λ_2.

22. Write the Mathematical formulation of an assignment problem.

23. Check whether the set $A = \{(x_1, x_2) / x_1, x_2 \in \mathbb{R}\ s\ x_1^2 + x_2^2 = 1\}$ is a convex set.

24. Show that the vectors $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ are linearly dependent.

(9 × 2 = 18 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 5 *marks.*

25. Maximize $Z = 2x_1 + x_2$

where $x_1 + x_2 \le 4$

$x_1 + 2x_2 \le 6$

$x_1 \le 3$

$x_1, x_2 \ge 0.$

26. Prove that a hyperplane is a convex set.

27. Prove that the set of all feasible solutions to a L.P.P. constitutes a convex set.

28. Use simplex method to solve the following L.P.P. :

Maximize $Z = 2.5x_1 + x_2$

subject to the constraints $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0.$

29. Find a basic feasible solution of the following transportation problem by using Vogel's approximation method :

	I	II	III	IV	
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	–1	4	2	19
	21	25	17	17	80

30. Consider the problem of assigning five jobs to five persons. The assignment cost are given below :

		Job 1	2	3	4	5
	A	8	4	2	6	1
	B	0	9	5	5	4
Person	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Determine the optimum assignment schedule.

31. Show that the following L.P.P. has an unbounded solution :

Maximize $Z = 4x_1 + 5x_2$

subject to
$$x_1 + x_2 \geq 1$$
$$-2x_1 + x_2 \leq 1$$
$$4x_1 - 2x_2 \leq 1$$
$$x_1, x_2 \geq 0.$$

32. The column vector [1, 1, 1] is a feasible solution to the system of equations :

$$x_1 + x_2 + 2x_3 = 4$$
$$2x_1 - x_2 + x_3 = 2.$$

Reduce the given solution to a basic feasible solution.

33. State and prove 'Minimax Theorem'.

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks.*

34. S.T. any convex combination of K different optimum solutions to a L.P.P. is again an optimum solution to the problem.

35. Use simplex method to solve :

Maximize $Z = 107x_1 + x_2 + 2x_3$

subject to the constraints $14x_1 + x_2 - x_3 + 3x_4 = 7$
$$16x_1 + \frac{1}{2}x_2 - 2x_3 \leq 3$$
$$3x_1 \geq 0$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

36. Prove that a hyperplane is a closed set.

(2 × 10 = 20 marks)