C 40187 **(Pages : 5)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2023

Mathematics

MAT 6B 11—NUMERICAL METHODS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. State Ramanujan's method to find a real root of the equation.

2. Form the forward difference table of the function $f(x) = \frac{1}{x}$ for the values $x = 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$.

3. State Newton's general interpolation formula with divided differences.

4. Evaluate $\Delta^2 (\cos 2x)$, interval of differencing being h.

5. State Gauss' backward central difference formula.

6. What do you mean by inverse interpolation ?

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_0}$.

8. Give Simpson's $1/3$-rule of integration.

9. What is Partial pivoting.

10. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$.

Turn over

11. In solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ write down Taylor's series for $y(x_1)$.

12. State Predictor formula.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find by iteration method, a real root of the equation $x^3 = 1 - x^2$ on the interval $[0, 1]$ with an accuracy of 10^{-4}.

14. Prove that (i) $\mu = \cosh\left(\frac{hD}{2}\right)$; and (ii) $\delta = \Delta E^{-1/2}$.

15. Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

16. Find the missing terms in the following table :

x	:	0	5	10	15	20	25	30
y	:	1	3	–	73	225	–	1153

17. Certain corresponding values of x and $\log_{10}x$ are (300, 2.4771), (304, 2.4829), (305,2.4843) and (307, 2.4871). Find $\log_{10} 301$.

18. Evaluate the integral $\int_{-2}^{2} \frac{x}{5 + 2x} dx$ using the trapezoidal rule with five ordinates.

19. Use Gauss elimination method to solve the system

$2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

20. Decompose the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ in the form LU where L is a lower triangular matrix and U is unit upper triangular matrix.

21. If D stands for the differential operator $\frac{d}{dx}$, prove that $D = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - ......\right]$.

22. Find the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

23. Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$, obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places.

24. The following table gives the population of a town during the last six censuses. Estimate, using Newton's interpolation formula, the increase in the population during the period 1946 to 1948 :

Year	:	1911	1921	1931	1941	1951	1961
Population (in thousands)	:	12	15	20	27	39	52

25. Determine the real root of the equation $x = e^{-x}$, using the secant method.

26. The distances (x cm) traversed by a particle at different times (t seconds) are given below :

t	:	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x	:	3.01	3.16	3.29	3.36	3.40	3.38	3.32

Find the velocity of the particle at $t = 0.3$ seconds.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Using Newton-Raphson's method, find a real root, correct to 3 decimal places, of the equation $\sin x = \frac{x}{2}$ given that the root lies between $\frac{\pi}{2}$ and π.

28. Find a root of the equation $4e^{-x} \sin x - 1 = 0$ by regula-falsi method given that the root lies between 0 an 0.5.

Turn over

29. Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}.$$

30. Values of x (in degrees) and $\sin x$ are given in the following table :

x :	15	20	25	30	35	40
$\sin x$:	0.2588190	0.3420201	0.4226183	0.5	0.5735766	0.6427876

Determine the value of sin 38.

31. Using Gauss's forward formula, find the value of $f(32)$ given that $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3386$ and $f(40) = 0.3794$.

32. Given the table of values :

x :	1.4	1.5	1.6	1.7
e^x :	4.0552	4.4817	4.9530	5.4739

Use the method of successive approximations to find x and $e^x = 4.7115$.

33. From the following values of x and y, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$:

x :	0	1	2	3	4	5	6
y :	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

34. Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ using LU decomposition Method.

35. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25, 0.5$ and 1.0 correct to three decimal places.

($6 \times 7 = 42$ marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. a) From the following table, find the number of students who obtained marks between 60 and 70 :

Marks obtained	:	0-40	40-60	60-80	80-100	100-120
No. of Students	:	250	120	100	70	50

b) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following table of values. Hence determine the value of ln 2.7.

x	:	2	2.5	3.0
$y = \ln x$	:	0.69315	0.91629	1.09861

37. Solve the system $8x - 3y + 2z = 20\,;\, 6x + 3y + 12z = -35\,;\, 4x + 11y - z = 33$ using both Jacobi and Gauss-Seidel method

38. a) Given the differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, compute $y(0.02)$ using Euler's modified method.

b) Using Milne's method to obtain the value of $y(0.3)$ given that

$$\frac{dy}{dx} = x^2 + y^2 - 2,\ y(-0.1) = 1.0900,\ y(0) = 1.0000,\ y(0.1) = 0.8900 \text{ and } y(0.2) = 0.7605.$$

(2 × 13 = 26 marks)

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C 20209 (Pages : 4) Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 11—NUMERICAL METHODS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 mark.

1. What is the minimum number of iterations required in bisection method to achieve an accuracy $\in$?
2. State the condition for convergence of Newton-Raphson method.
3. Define the central difference operator.
4. Evaluate $\Delta\left(x^2 + \sin x\right)$, interval of differencing being h.
5. State Newton's backward difference interpolation formula.
6. Show that the Lagrange interpolating polynomial is unique.
7. Given $f(x) = \frac{1}{x^2}$, find the divided differences $[a, b]$ and $[a, b, c]$.
8. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_n}$.
9. State general formula for numerical integration.
10. What is complete pivoting ?
11. Write Runge-Kutta formula to fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y\left(x_0\right) = y_0$.
12. Write Adams-Moulton corrector formula.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the methods of Regula-Falsi to determine it.

Turn over

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14. Prove that (i) $\delta \equiv \Delta E^{-1/2}$; (ii) $E \equiv e^{hD}$ where E is the shift operator and D is the differential operator.

15. Given $\log_{10} 100 = 2$, $\log_{10} 101 = 2.0043$, $\log_{10} 103 = 2.0128$, $\log_{10} 104 = 2.0170$, find $\log_{10} 102$.

16. The function $y = \sin x$ is tabulated below :

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \sin x$	0	0.70711	1.0

Using Lagrange's interpolation formula, find the value of $\sin\left(\frac{\pi}{6}\right)$.

17. Prove that the nth divided difference of a polynomial of nth degree are constant.

18. Given the set of tabulated points (0, 2), (1, 3), (2, 12) and (15, 3587) satisfying the function $y = f(x)$, compute $f(4)$ using Newton's divided difference formula.

19. Using Simpson's $\frac{3}{8}$-rule with $h = \frac{\pi}{6}$, evaluate the integral $\int_0^{\pi/2} \sin x \, dx$.

20. Solve the system $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$ by the Gauss-Jordan method.

21. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU where L is a unit lower triangular matrix and U is a upper triangular matrix.

22. Find the smallest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

23. Use Picard's method to obtain $y(0.1)$ of the problem defined by $\frac{dy}{dx} = x + yx^4, y(0) = 3$.

24. Explain briefly the method of iteration to compute a real root of the equation $f(x) = 0$, stating the condition of convergence of the sequence of approximations.

25. A rod is rotating in a plane about one of its ends. The angle θ (in radians) at different times t (seconds) are given below :

t	0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.15	0.50	1.15	2.0	3.20

Find its angular acceleration when $t = 0.6$ seconds.

26. Solve the tridiagonal system of equations $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Using the secant method, find a real root of the equation $f(x) = xe^x - 1 = 0$.

28. Using bisection method find the positive root, between 0 and 1, of the equation $x = e^{-x}$ to a tolerance of 0.05 %.

29. Using Newton's forward interpolation formula, find y at $x = 8$ from the following table :

x	0	5	10	15	20	25
y	7	11	14	18	24	32

30. From the following table, find the value of $e^{1.17}$ using Gauss' forward formula :

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

31. Given the table of values

x	2	3	4	5
x^3	8	27	64	125

Use the method of successive approximations to find x when $x^3 = 10$.

32. Find the first and second derivatives of the function tabulated below at the point $x = 2.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

33. Use Gauss elimination to find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$.

34. If $\dfrac{dy}{dx} = \dfrac{1}{x^2 + y}$ with $y(4) = 4$ compute the values of $y(4.1)$ and $y(4.2)$ by Taylor's series method.

Turn over

35. A curve is given by the points of the table given below :

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	23	19	14	11	12.5	16	19	20	20

Apply Simpson's rule to find the area bounded by the curve, the x-axis and the extreme ordinates.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^1 \frac{dx}{1+x}$ using :

(a) Trapezoidal rule taking $h = 0.25$.

(b) Simpson's $\frac{1}{3}$-rule taking $h = 0.125$.

37. Solve the system $10x+2y+z=9; 2x+20y-2z=-44; -2x+3y+10z=22$ using both Jacobi and Gauss-Seidel method.

38. (a) Use Runge-Kutta fourth order formula to find $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}, y(0)=1.$

(b) Solve the initial value problem $\frac{dy}{dx}=1+y^2, y(0)=0$ with $h = 0.2$ on the interval [0, 0.6] using Milne's method.

(2 × 13 = 26 marks)

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Name.......................................

Reg. No...................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours | Maximum : 120 Marks

Section A

Answer **all** *questions. Each question carries* 1 *mark.*

1. Set up a Newton's iteration for computing $\sqrt{5}$.
2. What do you mean by interpolation ?
3. Find the second divided difference of $f(x) = \frac{1}{x}$ for the values $x = 1, 2, 3$.
4. Evaluate $\Delta\left(\frac{2x}{(x+1)!}\right)$, interval of differencing being unity.
5. State Gauss' forward central difference formula.
6. Give the Lagrange's interpolation formula.
7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{dy}{dx}\right]_{x_n}$
8. State Trapezoidal rule of integration.
9. In numerical integration, what should be the number of intervals to apply Simpson's $1/3$-rule and by Simpson's $\frac{3}{8}$-rule.
10. Determine the eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 5 & 3 \end{bmatrix}$.
11. State Picard's method of successive approximations.
12. Write Milne's corrector formula.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions. Each question carries* 6 *marks.*
All questions can be attended. Overall Ceiling 48.

13. Find a real root of the equation $x^3 - 2x - 5 = 0$, correct to 3 decimal places using bisection method.
14. Prove that $\mu = \sqrt{1 + \frac{1}{4}\delta^2}$

Turn over

15. Find the missing term in the following table :

x	:	0	1	2	3	4
y	:	1	3	9	–	81

Explain why the result differs from $3^3 = 27$.

16. Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x	:	0	1	3	4
y	:	– 12	0	12	24

17. Prove that the divided differences are symmetrical in all their arguments.

18. Using the divided differences, show that the data :

x	:	– 1	0	3	6	7
y	:	3	– 6	39	822	1611

represents polynomial of degree 4.

19. Evaluate the integral $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$-rule with $h = \frac{1}{4}$.

20. Use Gauss elimination with partial pivoting to solve the system

$2x + y - z = -1; x - 2y + 3z = 9; 3x - y + 5z = 14$.

21. Decompose the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ into the form LU where L is unit lower triangular and U an upper triangular matrix.

22. Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

23. Find the solution of the initial value problem $\frac{dy}{dx} = 2y - x, y(0) = 1$, by performing three iterations of the Picard's method.

24. The following table gives angular displacements θ (in radians) at different times t (seconds) : (0, 0.052), (0.02, 0.105), (0.04, 0.168), (0.06, 0.242), (0.08, 0.327), (0.10, 0.408), (0.12, 0.489). Calculate the angular velocity at $t = 0.06$.

25. Derive Simpson's $\frac{3}{8}$-rule $\int_{x_0}^{x_3} y dx = \frac{3}{8} h (y_0 + 3y_1 + 3y_2 + y_3)$.

26. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Trapezoidal rule find the velocity of the rocket at $t = 80$.

t sec	:	0	10	20	30	40	50	60	70	80
f (cm/sec^2)	:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*

Each question carries 9 *marks.*

All questions can be attended.

Overall Ceiling 45.

27. Find the smallest root, correct to 4 decimal places of the equation $3x - \cos x - 1 = 0$.

28. Use the method of iteration to find a real root, correct to three decimal places, of the equation $2x - 3 = \cos x$ lying in the interval $\left[\frac{3}{2}, \frac{\pi}{2}\right]$.

29. Find the cubic polynomial which takes the following values : $y(1) = 24, y(3) = 120, y(5) = 336$ and $y(7) = 720$. Hence or otherwise obtain the value of $y(8)$.

30. State Gauss's backward formula and use it to find the value of $\sqrt{12525}$, given that

$\sqrt{12500} = 111.8034, \sqrt{12510} = 111.8481, \sqrt{12520} = 111.8928, \sqrt{12530} = 111.9375$ and $\sqrt{12540} = 111.9822$.

31. By means of Newton's divided difference formula, find the values of $f(8)$ and $f(15)$ from the following table :

x	:	4	5	7	10	11	13
$f(x)$	:	48	100	294	900	1210	2028

32. Given the table of values :

x	:	51	55	57
$\sqrt[3]{x}$	:	3.708	3.803	3.848

Use Lagrange's formula to find x when $\sqrt[3]{x} = 3.780$.

33. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$.

x	:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

34. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss-Jordan Method.

35. Compute the values of $y(1.1)$ and $y(1.2)$ using Taylor's series method for the solution of the problem $y'' + y^2 y' = x^3$, $y(1) = 1$ and $y'(1) = 1$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) Using Newton's forward difference formula, find the sum

$$S_n = 1^3 + 2^3 + 3^3 + \ldots + n^3.$$

(b) From the following table, find the number of students who obtained less than 45 marks :

Marks obtained	:	30–40	40–50	50–60	60–70	70–80
No. of Students	:	31	42	51	35	31

37. Solve the system $6x + y + z = 20$; $x + 4y - z = 6$; $x - y + 5z = 7$ using both Jacobi and Gauss-Seidel method.

38. (a) Solve, by Euler's modified method, the problem $\frac{dy}{dx} = x + y$ with $y(0) = 0$. Choose $h = 0.2$ and compute $y(0.2)$ and $y(0.4)$.

(b) Using Milne's formula, find $y(0.8)$ given that

$$\frac{dy}{dx} = x - y^2,\ y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795 \text{ and } y(0.6) = 0.1762.$$

(1 × 15 = 15 marks)

C 80271 (Pages : 4) Name.......................................

Reg. No..................................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*

Each question carries 1 *mark.*

1. State the sufficient condition for the convergence of sequence of approximations $x_{n+1} = \phi(x_n)$ in iteration method.

2. Construct a forward difference table from the following data :

x	:	0	1	2	3	4
$y = f(x)$	:	1	1.5	2.2	3.1	4.6

3. State Newton's backward interpolation formula.

4. What do you mean by central differences ?

5. Evaluate $\Delta^2 \left(ab^x\right)$, interval of differencing being unity.

6. Write the relation between divided differences and forward differences.

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_n}$.

8. State general formula for numerical integration.

9. State Adams-Bashforth formula.

10. What is the order of the error in Trapezoidal rule ?

11. In solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$, write down Taylor's series for $y(x_1)$.

Turn over

12. Write Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find a real root of the equation $x^3 - x - 1 = 0$, that lies between 1 and 2, using bisection method.

14. Prove that (i) $\Delta = E\nabla = \nabla E$; (ii) $E = e^{hD}$ where E is the shift operator and D is the differential operator.

15. Find the missing term in the following table :

x	:	1	2	3	4	5	6	7
y	:	2	4	8	—	32	64	128

16. Show that $e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + ...\right) = u_0 + u_1 x + u_2\frac{x^2}{2!} + ...$

17. A solid of revolution is formed by rotating about the x-axis, the lines $x = 0$ and $x = 1$, and a curve through the points with the following co-ordinates :

x	:	0.00	0.25	0.50	0.75	1.00
y	:	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed, giving the answer to three decimal places.

18. Derive Simpson's (3/8)-rule $\int_{x_0}^{x_3} y\,dx = \frac{3}{8}h\,(y_0 + 3y_1 + 3y_2 + y_3)$.

19. Explain Trapezoidal rule.

20. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU.

21. From the following table, estimate the number of men getting wages between 100 and 150 :

Wages in Rupees	:	0–100	100–200	200–300	300–400
No. of Men	:	9	30	35	42

22. Find $\sqrt{12516}$ using Gauss' backward interpolation formula given that

$\sqrt{12500} = 111.8033, \sqrt{12510} = 111.8481, \sqrt{12520} = 111.8928$ and $\sqrt{12530} = 111.9374$.

23. Solve the system of equations $4x + 11y - z = 33$; $8x - 3y + 2z = 20$; $6x + 3y + 12z = 35$ by Gauss-Seidel iteration method .

24. Use Picard's method to approximate the value of y when $x = 0.1$, given that $y = 1$ at $x = 0$ and $\frac{dy}{dx} = 1 + xy$.

25. Using Adams-Moulton method , find :

$y(1.4)$ given $\frac{dy}{dx} = x^2(1+y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548$ and $y(1.3) = 1.979$.

26. Find the largest eigen value of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

28. Find by Newton's method, the real root of the equation $3x = \cos x + 1$.

29. Using Newton's forward difference formula , find the sum $S_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

30. Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \log x$ defined by the following table of values. Hence determine the value of log 2.7.

x	:	2	2.5	3
$y = \log x$	:	0.69315	0.91629	1.09861

31. Prove that n^{th} divided differences of a polynomial of n^{th} degree are constants.

32. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 2.2$.

x	:	1.00	1.20	1.40	1.60	1.80	2.00	2.20
y	:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

33. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss-Jordan method.

34. Apply Lagrange's formula inversely to obtain a root of the equation $f(x) = 0$, given that $f(30) = -30, f(34) = -13, f(38) = 3$ and $f(42) = 18$.

35. Using Euler's method, find an approximate value of y corresponding to $x = 0.2$, given that $\frac{dy}{dx} = 3x + \frac{1}{2}y, y(0) = 1 \ (h = 0.05)$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ using :

(a) Trapezoidal rule taking $h = 1$.

(b) Simpson's $\frac{1}{3}$ rule taking $h = 1$.

(c) Simpson's $\frac{3}{8}$ rule taking $h = 1$.

37. Solve the system of equations

$x_1 + x_2 + x_3 + x_4 = 2$; $x_1 + x_2 + 3x_3 - 2x_4 = -6$; $2x_1 + 3x_2 - x_3 + 2x_4 = 7$; $x_1 + 2x_2 + x_3 - x_4 = -2$ by Gauss elimination method.

38. Using Runge-Kutta method of fourth order, find y for $x = 0.1, 0.2, 0.3$ given that $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$. Continue the solution at $x = 0.4$ using Milne's method.

(2 × 13 = 26 marks)

C 60050 **(Pages : 3)** Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Give an example of a transcendental function.
2. What do you mean by complete pivoting ?
3. What is the advantage of Gauss Jordan method over Gauss Elimination method ?
4. Write Newton's forward difference interpolation formula.
5. Write Lagrange's interpolation formula of degree n.
6. State Trapezoidal rule.
7. What is meant by ill-conditioned system of equations ?
8. What is a differential equation ?
9. Find the degree and order of differential equation $y^1 + ay^2 = 0$.
10. What is interpolation ?
11. What is homogeneous equation ?
12. Give an example of linear function.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Use the method of fixed point iteration to find a positive root, between 0 and 1, of the equation $xe^x = 1$.
14. Find a real root of the equation $f(x) = x^3 - x - 1 = 0$.
15. Evaluate $e^{1.24}$, given that $e^{1.1} = 3.0042$ and $e^{1.4} = 4.0552$.
16. State Simpson's $1/3^{rd}$ rule.

Turn over

17. Explain the limitations of using Newton-Raphson's method.

18. Construct a divided difference table for 4 data points.

19. State the formula of Picard's method to solve the differential equation of type $dy/dx = f(xy)$.

20. Comment the accuracy of Euler's method.

21. What is triangularisation of equations ?

22. State the second order Newton's divided difference interpolation polynomial.

23. What are the limitations of Taylor's series method.

24. Write the fourth order Runge-Kutta formula.

25. By the matrix inversion method solve :

$$2x + y = 1$$
$$x + 3y = 2.$$

26. What is the difference between Gauss elimination and Gauss-Jordan method ?

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Solve the system by using Gauss-Jordan method.

$$2x_1 + 4x_2 - 6x_3 = -8$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12.$$

28. Using linear interpolation formula estimate the square root of 2.5 :

X	:	1	2	3	4	5
$\sqrt{x}$	:	1	1.4142	1.7321	2	2.2361

29. The table below gives the values of distance travelled by a car at various time intervals during the initial running :

Time (s)	:	5	6	7	8	9
Distance (km)	:	10	14.5	19.5	25.5	32

Estimate velocity at time $t = 5$, $t - 7$ and $t = 9$.

30. Evaluate the $\int_a^b x^3 + 1$ by using Trapezoidal rule for the (1, 2) and (1, 1.5).

31. Use Taylor's method to solve the equation $y' = x^2 + y^2$ for $x = 0.25$ and $x = 0.5$ given $y(0) = 1$.

32. Obtain a polynomial using Lagrange formula :

x	:	0	1	2	3
$e^x - 1$	:	0	1.7183	6.3891	19. 0855

Use the polynomial to estimate the value of $e^{1.5}$.

33. Find the root of the equation $x^2 - 3x + 2$ in the vicinity of $x = 0$ using Newton's Raphson method.

34. Explain the principle of false position method.

35. Given the equation $dy/dx = 3x^2 + 1$ with $y(1) = 2$. Find $y(2)$ by Euler's method using $h = 0.5$ and $h = 0.25$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Use bisection method to find the root of the equation $x^2 - 4x - 10 = 0$.

(b) Use false position method to determine the root of the equation $f(x) = x^2 - x - 2 = 0$ in the range $1 < x < 3$.

37. (a) Solve the system by using Gauss-Seidel method :

$$2x_1 + x_2 + x_3 = 5$$
$$3x_1 + 5x_2 + 2x_3 = 15$$
$$2x_1 + x_2 + 4x_3 = 8.$$

(b) Solve the system by using Triangular factorization method :

$$3x_1 + 2x_2 + x_3 = 10$$
$$2x_1 + 3x_2 + 2x_3 = 14$$
$$x_1 + 2x_2 + 3x_3 = 14.$$

38. Use Runge-Kutta method to estimate $y(0.4)$ when $y'(x) = x^2 + y^2$ with $y(0) = 0$ and assume $h = 0.2$.

(2 × 13 = 26 marks)

D 40901 **(Pages : 4)** Name.........................

Reg. No.........................

SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION MARCH 2018

(CCSS)

MM 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 30 Weightage

Part I

Answer all **twelve** *questions.*
Each question carries weightage ¼.

1. Write Newton-Raphson formula.

2. A real root of the equation $x^3 - 3x + 1 = 0$ lies in the interval ————.

3. Define Simpson's $3/8$ rule.

4. Write the relation connecting backward difference operator Δ and shift operator E.

5. The forward difference $\Delta y_6 =$ ————.

6. Write Newton's backward difference interpolation formula.

7. The process of finding the value of x for a certain value of y is called ————.

8. Define the characteristic polynomial of a square matrix A.

9. If the eigen values of the matrix A are 3,3,6. Then the degree of the characteristic polynomial is ————.

10. Write Taylor's series for $y(x)$ around $x = x_0$.

11. Let $x_i = 1, 0, 3, 6$ and $y_i = 3, -6, 39, 45, i = 0, 1, 2, 3$. Then the divided difference $[x_1, x_2]$ is ————.

12. If the eigen values of the matrix A are $2, 2, 6, -4$. Then the spectral radius of A is ————.

(12 × ¼ = 3 weightage)

Turn over

Part II

*Answer all **nine** questions.*
Each question carries weightage 1.

13. Evaluate $\Delta^n \left(e^{ax+b}\right)$.

14. Write Newton's forward difference formula.

15. Define spectrum of a square matrix A.

16. Write the fourth order formula for Runge-Kutta method.

17. The function $y = \sin x$ is tabulated below :

x	...	0	$\pi/4$	$\pi/2$
$y = \sin x$	...	0	0.707	1

Using Lagrange's interpolation formula find the value of $\sin(\pi/6)$.

18. Define central difference operator δ.

19. Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

20. If the sum and product of eigen values of a 2×2 matrix are 2 and -8 respectively. Find the eigen values.

21. What is y (0.01) for the equation $\frac{dy}{dx} + y = 0$ with $y(0) = 1$ using Euler's method.

$(9 \times 1 = 9$ weightage$)$

Part III

Answer any **five** *questions.*
Each question carries weightage 2.

22. Find the positive root of $x^3 - x - 1 = 0$ by bisection method.

23. Find the value of y at $x = 21$ and $x = 28$ from the following data :

x	...	20	23	26	29
y	...	0.342	0.3907	0.4384	0.4848

24. Find the eigen values of :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

Also find the eigen vector corresponding to largest eigen value.

25. Evaluate $\int_0^6 \frac{1}{1+x}\,dx$ using Simpson's 1/3 rule.

26. Compute y at $x = 0.25$ by modified Euler's method given $\frac{dy}{dx} = 2xy,\ y(0) = 1$.

27. Fit a Lagrangian polynomial of degree 3 :

x	...	5	6	9	11
$f(x)$	...	12	13	14	16

and find $f(10)$.

28. Use Newton-Raphson method to find a root of the equation $4x - e^x = 0$ lies between 2 and 3.

(5 × 2 = 10 weightage)

Part IV

Answer **two** *questions.*
Each question carries weightage 4.

29. Solve $\frac{dy}{dx} = y - x^2, y(0) = 1,$ by Picard's method upto third approximation. Hence find the value of $y(0.1)$.

30. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 2.2$:

x	...	1	1.2	1.4	1.6	1.8	2	2.2
y	...	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

31. Solve the equations $2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8$ by LU decomposition.

(2 × 4 = 8 weightage)

D 40043 (Pages : 4) Name.................................

Reg. No.................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS–UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 2x - 5 = 0$.

2. State Newton's forward interpolation formula.

3. Evaluate $\Delta^2 ab^x$, interval of differencing being unity.

4. Write the relation between divided differences and forward differences.

5. Form the table of backward differences of the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$.

6. Give the names of two interpolation formula applicable for unequally spaced values of the argument.

7. What do you mean by inverse interpolation ?

8. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{dy}{dx}\right]_{x_0}$?

9. Give Simpson's 1/3 -rule of integration.

10. What is complete pivoting ?

11. In numerical integration, what should be the number of intervals to apply Simpson's 1/3- rule and by Simpson's 3/8-rule.

12. Write Milne's corrector formula.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Find the iterative method based on the Newton-Raphson method for finding $\sqrt{N}$ where N is a positive real number. Apply the method to N = 18 to obtain the results correct to two decimal places.

14. Prove that (i) $\nabla = I - E^{-1}$; (ii) $\delta = \Delta E^{1/2}$, where E is the shift operator and δ is the central differential operator.

15. Certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Find $\log_{10}$ 301.

16. Find the missing values in the following table :

x	45	50	55	60	65
y	3.0	–	2.0	–	–2.4

17. Construct the divided difference table for the following data :

x	–1	0	3
$f(x)$	– 4	– 5	16

Determine the approximate value of $f(1)$ using divided difference interpolation.

18. Find a polynomial satisfied by (– 4, 1245), (–1,33), (0,5), (2,9) and (5,1335).

19. Obtain the first derivative of $\sqrt{x}$ at $x = 15$ from the table :

x	15	17	19	21	23	25
$\sqrt{x}$	3.873	4.123	4.359	4.583	4.796	5

20. Find, from the following table, the area bounded by the curve and the x-axis from x = 7. 47 to x = 7.52 :

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

21. Evaluate the integral $\int_0^{\pi/2} \sin x \, dx$, using Simpson's (3/8)-rule.

22. Solve the system of equations $3x + y - z = 3 ; 2x - 8y + z = -5 ; x - 2y + 9z = 8$, using Gauss-elimination method.

23. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU.

24. Solve by Jacobi's iteration method, the equations $20x + y - 7z = 17 ; 3x + 20y - z = -18 ;$ $2x - 3y + 20z = 25$.

25. Find by Taylor's series method the value of y at $x = 0.1$ correct to five places of decimals from $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$.

26. Given $\frac{dy}{dx} = x^2 (1 + y)$ and $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$ evaluate y (1.4) by Milne's predictor-corrector method.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Using Ramanujan's method find a real root of the equation :

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \ldots = 0.$$

28. Find the 7^{th} term and the general term of the series 3, 9, 20, 38, 65, ————.

29. The equation $f(x) = \log_e x - x + 3 = 0$ has a root in the interval (4, 5). Obtain the root correct to three decimal places using regula-falsi method.

30. The following table of the function $f(x) = e^{-x}$ is given :

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

(i) Using Gauss forward central difference formula compute $f(0.55)$.

(ii) Using Gauss backward central difference formula compute $f(0.45)$.

31. From the following table, find x, correct to two decimal places, forwhich y is maximum and find this value of y.

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

32. Derive Simpson's 1/3- rule using the method of undetermined coefficients.

33. Tabulate $y = x^3$ for $x = 2, 3, 4$ and 5, and calculate the cube root of 10 correct to three decimal places.

34. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ using Gaussian elimination and use the result to solve the system of equations : $3x + 2y + 4z = 7, 2x + y + z = 7, x + 3y + 5z = 2$.

35. Using Euler's method, find an approximate value of y corresponding to $x = 0.1$, given that $\frac{dy}{dx} = x^2 + y, y(0) = 1$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) A rod is rotating in a plane about one of its ends. If the following table gives the angle θ radians which the rod has turned for different values of time t seconds, find its angular velocity and angular acceleration when $t = 0.7$ seconds.

t seconds	0.0	0.2	0.4	0.6	0.8	1.0
θ radians	0.0	0.12	0.48	1.10	2.0	3.20

(b) Find the value of $\int_1^2 \frac{dx}{x}$ by Trapezoidal rule. Hence find approximate value of $\log_e 2$.

37. Solve the system of equations $x - y + 5z = 5$; $2x - 3y + z = 0$; $x + 3y + 7z = 11$ by LU decomposition.

38. (a) Solve the following by Euler's modified method : $\frac{dy}{dx} = \log_{10}(x + y), y(0) = 2$ at $x = 1.2$ and 1.4 with $h = 0.2$.

(b) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$.

(2 × 13 = 26 marks)

C 21072 (Pages : 4)

Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Set up a Newton's iteration for computing $\sqrt{2}$.

2. Construct the difference table of $f(x) = x^3 - 3x^2 + 5x + 7$ for the values $x = 0, 2, 4, 6, 8$.

3. Give the Gauss's forward interpolation formula.

4. What do you mean by central differences ?

5. Evaluate $\Delta^n \left(e^x\right)$, interval of differencing being unity.

6. Write the relation between divided differences and forward differences.

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_0}$.

8. State general formula for numerical integration.

9. State Adams-Bashforth formula.

10. What is the order of the error in Simpson's 1/3 - rule.

11. In solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$, write down Taylor's series for $y(x_1)$.

12. Write Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method.

14. Prove that (i) $\Delta = E - I$; (ii) $E = e^{hD}$ where E is the shift operator and D is the differential operator.

15. Find the missing term in the following table :

x	...	0	1	2	3	4
y	...	1	3	9	-	81

16. Find the divided differences of $f(x) = x^2 + x + 2$ for the arguments 1, 3, 6, 11.

17. Prove that n^{th} divided differences of a polynomial of n^{th} degree are constants.

18. Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x	...	0	1	3	4
y	...	−12	0	12	24

19. The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table :

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Find the distance travelled by the car in 2-minutes.

20. Derive Simpson's (3/8)-rule $\int_{x_0}^{x_3} y\,dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3)$.

21. Explain Simpson's 1/3-rule.

22. Decompose the matrix $\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$ in the form LU.

23. From the following table, estimate the number of men getting wages between 100 and 150 :

Wages in Rupees	0–100	100–200	200–300	300–400
No. of Men	9	30	35	42

24. Solve the system of equations $28x + 4y - z = 32$; $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$ by Gauss-Seidel iteration method .

25. Using Picard's method obtain a solution upto the fifth approximation to the equation $\frac{dy}{dx} = y + x$, such that $y(0) = 1$.

26. Using Adams-Moulton method , find $y(1.4)$ given

$$\frac{dy}{dx} = x^2(1+y),\ y(1) = 1,\ y(1.1) = 1.233,\ y(1.2) = 1.548 \text{ and } y(1.3) = 1.979.$$

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$ correct to five decimal places.

28. Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \ldots + (-1)^n u_{x-n}.$$

29. Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

30. From the following table, find the value of sin $38°$:

x (in degrees)	15	20	25	30	35	40
sin x	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

31. Given $\sum_{1}^{10} f(x) = 500426$, $\sum_{4}^{10} f(x) = 329240$, $\sum_{7}^{10} f(x) = 175212$ and $f(10) = 40365$ find $f(1)$.

32. Find the first and second derivatives of the function tabulated below at the point $x = 1.5$:

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.0	13.625	24.0	38.875	59.0

33. Apply Lagrange's formula inversely to obtain a root of the equation $f(x)=0$, given that $f(30)=-30, f(34)=-13, f(38)=3$ and $f(42)=18$.

34. Apply Runge-Kutta method, to find an approximate value of y when x = 0.2 given that $\frac{dy}{dx}=x+y, y(0)=1$.

35. Find the inverse of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ using Gauss Elimination Method.

($6 \times 7 = 42$ marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using

(a) Trapezoidal rule taking $h=1$.

(b) Simpson's $\frac{1}{3}$ rule taking $h=1$.

(c) Simpson's $\frac{3}{8}$ rule taking $h=1$.

37. Solve the system of equations :

$x_1+2x_2+x_3-x_4=-2; x_1+x_2+3x_3-2x_4=-6; 2x_1+3x_2-x_3+2x_4=7; x_1+x_2+x_3+x_4=2$

by Gauss Jordan method.

38. (a) Apply Milne's method, to find a solution of the differential equation $\frac{dy}{dx}=x-y^2$ in the range $0 \le x \le 1$ for the boundary condition $y=0$ at $x=0$.

(b) Determine the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

($2 \times 13 = 26$ marks)

C 21564 (Pages : 3) Name.................................

Reg. No.................................

SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MARCH 2017

(UG–CCSS)

Mathematics

MM 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 30 Weightage

Part I

Answer **all twelve** *questions.*
Each question carries ¼ weightage.

1. The bookward difference ∇y_n = ———.
2. A real root of the equation $x^3 - x - 1 = 0$ lies in the interval ———.
3. Write Newton Raphson formula.
4. Write the relation connecting forward difference operator Δ and shift operator E.
5. $\frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) =$ ———.
6. Write Newton's forward difference interpolation formula.
7. Define Trapezoidal rule.
8. Define the characteristic polynomial of a square matrix A.
9. If the eigen values of the matrix A are – 2, 4, 6. Then the order A is ———.
10. Write Taylor's series for $y(x)$ around $x = x_0$.
11. Let x_i = – 1, 0, 3, 6 and y_i = 3, – 6, 39, 45, i = 0, 1, 2, 3. Then the divided difference $|x_0, x_1|$ is ———.
12. The process of finding the value of x for a certain value of y is called ———.

(12 × ¼ = 3 weightage)

Part II

Answer **all** *nine questions.*
Each question carries 1 weightage.

13. Find the forward difference table for the following values : $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$.
14. Show that $e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2}\Delta^2 u_0 + \ldots\right) = u_0 + u_1 x + u_2\frac{x^2}{2} + \ldots$.
15. Write Gauss forward formula.

Turn over

16. If $y_1 = 4, y_3 = 12, y_4 = 19$ and $y_x = 7$, find x.

17. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

18. If $A = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$ then the spectral radius is ———.

19. What is $y(0.01)$ for the equation $\frac{dy}{dx} + y = 0$ with $y(0) = 1$ using Euler's method.

20. Define central difference operator δ.

21. Find the sum of eigen values of a matrix whose characteristic equation $\lambda^2 - 2\lambda - 3 = 0$.

(9 × 1 = 9 weightage)

Part III

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ using the method of False Position.

23. Find the missing term in the following table :—

x	:	0	1	2	3	4
y	:	1	3	9	—	81

24. Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ using Simpson's 3/8 rule.

25. Determine the largest eigen value and the corresponding eigen vector of the matrix :

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

26. Given $\frac{dy}{dx} = 1 + y^2$, where $y(0) = 0$. Find $y(0.2)$ by Runge-Kutta method.

27. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$ with the initial condition $y = 0$ when $x = 0$. Use Picard's method to obtain y for $x = 0.25$.

28. Certain values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843), (307, 2.4871). Find $\log_{10} 301$ by Newton's divided difference method.

(5 × 2 = 10 weightage)

Part IV

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Solve the equations $2x+3y+z=9, x+2y+3z=6, 3x+y+2z=8$ by LU decomposition. From the following table of values of x and y, obtain $\frac{dy}{dx}$ for $x = 1.2$.

30. The population of a certain town is given below. Find the rate of growth of the population in 1931 :

Year	1931	1941	1951	1961	1971
Population (in lakhs)	40.62	60.80	79.95	103.56	132.65

31. Determine the value of y when $x = 0.1$ and $x = 0.2$ modified Euler's method. Given that $y(0) = 0$ and $\frac{dy}{dx} = 1 - y$.

(2 × 4 = 8 weightage)

C 60110 (Pages : 4)

Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2014

(U.G.—CCSS)

Mathematics

MM 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 30 Weightage

I. Answer all *twelve* questions :—

1 Forward difference $\Delta y_n =$ ————.

2 $1 - E^{-1} =$ ————.

(a) ∇. (b) δ.

(c) μ. (d) Δ.

3 Define averaging operator μ.

4 $\Delta =$ ———— $= \delta E^{1/2}$.

(a) ∇E. (b) ΔE.

(c) μE. (d) δE.

5 Show that $e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \ldots\ldots \right)$

$$= \left(1 + xE + \frac{x^2 E^2}{2!} + \ldots\ldots \right) u_0 .$$

6 Write Simpson's $\frac{3}{8}$-Rule.

7 Define the characteristic polynomial of a square matrix A.

8 The shift operator E is defined as $E_{y_r} =$ ————.

9 $\frac{1}{2}\left(E^{1/2} + E^{-1/2}\right) =$ ————.

Turn over

10 Write Newton's forward difference interpolation formula.

11 $1+\frac{1}{4}\delta^2 =$ ________.

(a) Δ^2. (b) ∇^2.

(c) μ^2. (d) δ^2.

12 Write Gauss backward formula.

(12 × ¼ = 3 weightage)

II. Answer all *nine* questions :—

13 Define central difference opertor δ ?

14 Prove that $e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + ...\right)$.

$$= u_0 + xu_1 + \frac{x^2}{2!}u_2 + ...$$

15 Write Gauss forward formula.

16 The function $y = \sin x$ is tabulated below :

x	0	$\pi/4$	$\pi/2$
$y = \sin x$	0	0.70711	1

Using Lagrange's interpolation formula find the value of sin $(\pi/6)$.

17 Write the Trapezoidal rule.

18 Define the eigen value of a square matrix.

19 Find the integers between which the real root of $xe^x - 1 = 0$ lies.

20 Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$. Find $y(0.1)$ correct to four decimal places by Runge-Kutta second order formula.

21 Define the spectral radius of a square matrix.

(9 × 1 = 9 weightage)

III. Answer any *five* questions from seven :

22 Using Newton's general interpolation formula with divided differences find $f(x)$ as a polynomial in x. Given

x	:	-1	0	3	6	7
$f(x)$	:	3	-6	39	822	1611

23 Using Trapezoidal rule evaluate $I = \int_0^1 \frac{1}{1+x} dx$ correct to three decimal places. Take $h = 0.5$.

24 Determine the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

25 Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y = 0$ when $x = 0$. Use Picard's method to obtain y for $x = 0.25$.

26 Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x	0	1	3	4
y	-12	0	12	24

27 Find the following table find the value of $e^{1.17}$ using Gauss' forward formula.

x	1	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

28 Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

(5 × 2 = 10 weightage)

IV. Answer *two* questions from *three* :

29 By using Newton's forward difference interpolation formula find the cubic polynomial which takes the following values :

$$y(1) = 24,\ y(3) = 120,\ y(5) = 336, y(7) = 720.$$

30 Solve the equation $2x + 3y + z = 9,\ \ x + 2y + 3z = 6,\ \ 3x + y + 2z + 8$ by LV decomposition.

31 Using Modified Euler's method determine the value of y when $x = 0.1$ given that $y(0) = 1,\ y' = x^2 + y$.

(2 × 4 = 8 weightage)

C 30027 (Pages : 3) Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions from this part.*

1. If $f(x)$ is continuous in $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs then which of the following is true :
   - (a) There exists exactly one root of $f(x) = 0$ between a and b.
   - (b) There exist at least one root of $f(x) = 0$ between a and b.
   - (c) There exist at most one root between a and b.
   - (d) There is no root between a and b.
2. Find the second approximation of a real root of $x^2 - 2x - 5 = 0$ using bisection method.
3. Write the Newton-Raphson formula.
4. Define the central difference operators.
5. Write the Newton's backward difference formula.
6. Write the Lagrange polynomial of degree 2.
7. Write the general form of the unit lower triangular matrix.
8. Find the characteristic equation of the matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

9. $y' = x + y^2$ with $y(0) = 1$. Find the second approximation $y^{(2)}$ using Picard's method.
10. Write Simpson's $\frac{1}{3}$ rule.
11. In Adams-Moulton method ———— formula is used to derive Predictor-corrector formula.
12. Write the Milne's corrector formula.

(12 × ¼ = 3 weightage)

Turn over

Part B

Answer **all** *questions from this part.*

13. Find the second approximation of a real root of the equation $x^3 - 4x - 9 = 0$ using bisection method.

14. Find an iteration formula used to find a root of the equation $x \sin x + \cos x = 0$ using Newton-Raphson formula.

15. Using Ramanujan's method obtain the first two convergents of the equation $x + x^3 = 1$.

16. Prove that $E = e^{hD}$ where D is the differential operator.

17. Write Bessel's interpolation formula.

18. Fine the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

19. Evaluate $\int_0^1 \frac{1}{1+x} dx$ correct to three decimal places Simpson's $\frac{1}{3}$ rule taking $h = 0{\cdot}5$.

20. Find the unit lower triangular matrix L in the LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix}.$$

21. $\frac{dy}{dx} = 1 + xy$ and $y(0) = 1$, obtain the Taylor series for $y(x)$.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions from this part.*

22. Find a real root of the equation $x^3 - x^2 - 2 = 0$ by Regula-Falsi method.

23. Using method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_{x-n} u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \ldots + (-1)^n u_{x-n}.$$

24. The Population of a town in decennial census was given below. Estimate the population for the year 1925 :

Year (x) :	1891	1901	1911	1921	1931
Population (y) (in thousands) :	46	66	81	93	101

25. Using Lagrange interpolation formula, express the function $\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as sums of partial fractions.

26. By Gauss elimination method solve the system of equations

$5x - y - 2z = 142, \quad x - 3y - z = -30, \quad 2x - y - 3z = -50.$

27. Determine the largest eigen value and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

28. Apply Runge-Kutta method to find an approximate value of y for $x = 0{\cdot}1$ taking $h = 0{\cdot}1$, if

$\frac{dy}{dx} = x + y^2, \; y(0) = 1.$

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions from this part.*

29. From the following table, find the value of $e^{1\cdot17}$ using Gauss's forward formula.

x :	1·00	1·05	1·10	1·15	1·20	1·25	1·30
e^x :	2·7813	2·8577	3·0042	3·1582	3·3201	3·4903	3·6693

30. Solve the system of equations using factorization method :

$x + 2y + 3z = 14, \quad 2x + 5y + 2z = 18, \quad 3x + y + 5z = 20.$

31. Solve the Initial value problem $\frac{dy}{dx} = 1 + xy^2, \; y(0) = 1$ for $x = 0{\cdot}4$ by using Milne's method. Given that

x :	0·1	0·2	0·3
y :	1·105	1·223	1·354.

(2 × 4 = 8 weightage)