D 93823 (Pages : 3)

Name .....

Reg No .....

FIRST SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions (1 - 12).*
Each question carries 1 *mark.*

1. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = ...$

2. $\lim_{x\to 0} \frac{\sin(2+x)-\sin 2}{x} = ...$

3. Define removable discontinuity.

4. State the condition(s) for local maximum of the function $y = f(x)$.

5. What is (are) the vertical asymptote(s) of the curve $xy^3 - 2xy^2 - 2y^3 - 4 = 0$.

6. State Rolle's theorem.

7 Find $\frac{d}{dx}(\cosh(3x-2))$.

8. State the second derivative test for concavity of a function $y = f(x)$.

9. State the mean value theorem for definite integral.

10. $\sum_{k=1}^{4}\left(k^2 - 3k\right) = ...$

11. Let f be a continuous function on $[a,b]$. Then what is the average value of f on $[a,b]$.

12. Area bounded by the curves $y = f(x), y = g(x)$ and the ordinates $x = a$ and $x = b$ is given by _____.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions (13 - 24).*
Each question carries 2 *marks.*

13. If $\sqrt{3-2x} \le f(x) \le \sqrt{3-x}$, find $\lim_{x \to 0} f(x)$.

14. Find $\lim_{x \to 2} \frac{x-3}{x^2-4}$.

15. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

16. Find the absolute extrema of $f(x) = x^{2/3}$ on $[-2, 3]$.

17. Find the points of inflection of the function $y = 2 + \cos x, x \ge 0$.

18. Find $\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

19. Find the horizontal asymptotes of the graph of the function $f(x) = \frac{-8}{x^2 - 4}$.

20. Find the linearization of $f(x) = x^3 - 2x + 3$ at $x = 2$.

21. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

22. Find $\lim_{x \to 1} \frac{1-x}{\log x}$.

23. Find $\lim_{x \to \infty} x^{1/x}$.

24. Verify Rolle's theorem for the function $f(x) = \tan x$ in $[0, \pi]$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions (25 - 33).*
Each question carries 5 *marks.*

25. State and prove the rule for the limit of a sum.

26. Show that if a function f has a derivative at $x = c$, then show that f is continuous at $x = c$.

27. State and prove Rolle's theorem.

28. Verify mean value theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in $\left[0, \frac{1}{2}\right]$.

29. Find the intervals on which $f(x) = -x^3 + 12x + 5, x \in [-3,3]$ is increasing and decreasing.

30. Find all the asymptotes of $f(x) = \frac{x^2 - 3}{2x - 4}$.

31. Give an example of a function which is not Riemann integrable. Prove your claim.

32. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

33. Verify the mean value theorem for integrals for the function $f(x) = \frac{x}{\sqrt{x^2 + 16}}$ in [0, 3].

(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any **two** *questions (34 - 36).*
Each question carries 10 *marks.*

34. A dynamite blast blows a heavy rock straight up with a velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t seconds.

    a) How high does the rock go ?

    b) What is the velocity and speed of the rock when it is at 256 ft above the ground on the way up? on the way down ?

    c) What is the acceleration of the rock at any time t during its flight ?

35. Sketch the graph of the function $y = x^4 - 4x^3 + 10$, by inspecting increasing, decreasing, concavity, points of inflection, local extrema etc.

36. a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge by slicing method.

    b) Find the area of the region bounded by the curves $y = x^2$ and $y = x^4 - 4x^2 + 4$.

(2 × 10 = 20 marks)

D 73167 (Pages : 4) Name.....

Reg. No.....

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. Find $\lim_{x \to 2} \frac{-x}{(x-3)^2}$.

2. Differentiate $\cos\left(x^2+2\right)$ with respect to x.

3. Suppose $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$. Find $\lim_{x \to c} \frac{f(x)}{g(x)}$.

4. Find second derivative of $y = \sin x + \cos x$.

5. Define a decreasing function.

6. Evaluate $\sum_{k=1}^{10} k^2$.

7. Find $\lim_{x \to \infty} \frac{\pi\sqrt{3}}{x^2}$.

8. Absolute maximum of the function $y = x^2$ on $[0, 2]$ is _____.

9. Find dy if $y = x^5 + 37x$.

10. Define critical point of a function.

Turn over

11. Define norm of a partition.

12. Suppose that $\int_1^4 f(x)\,dx = -2$. Evaluate $\int_4^1 f(x)\,dx$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

14. Find absolute extremes of $h(x) = x^{2/3}$ on $[-2, 3]$.

15. Fmd $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t dt$.

16. Given $1 - x^2/4 \leq u(x) \leq 1 + x^2/2$, for all $x \neq 0$. Find $\lim_{x \to 0} u(x)$.

17. If $\lim_{x \to -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \to -2} \frac{f(x)}{x^3}$.

18. Show that $\lim_{x \to 1} 5x - 3 = 2$.

19. Evaluate $\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

20. Find the derivative of $y = \frac{x^2 + 1}{x^2 - 1}$.

21. Find the linearization of $x^3 - x$ at $x = 1$.

22. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(c_k^2 - 3c_k\right) \Delta x_k$ as definite integral where P is the partition of $[-7, 5]$.

23. Give an example of a function with no Riemann integral. Explain.

24. Show that the value of $\int_0^1 \sqrt{1+\cos x}\, dx$ cannot possibly be 2.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

26. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

27. Find the interval on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

28. Show that the functions with zero derivatives are constant.

29. Show that $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ has acontinuous extension to $x = 2$, and find that extension.

30. For what values of a is $f(x) = \begin{cases} x, & x < -2; \\ ax^2, & x \ge -2 \end{cases}$ continuous at every x ?

31. Find the slope of the parabola $y = x^2$ at the point P $(2, 4)$. Write an equation for the tangent to the parabola at this point.

32. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so where ?

33. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$. about the x-axis.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. Find the volume of the solid generated by revolving the regions bounded by the curve $x=\sqrt{5}y^2, x=0, y=-1\ y=1$ about x-axis.

35. Let $f(x)=\begin{cases}3-x, & x<2;\\ \frac{x}{2}+1, & x>2.\end{cases}$

a) Find $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$.

b) Does $\lim_{x\to 2} f(x)$ exist ? If so, what is it ? If not, why not ?

c) Find $\lim_{x\to 4^+} f(x)$ and $\lim_{x\to 4^-} f(x)$.

d) Does $\lim_{x\to 4} f(x)$ exist ? If so, what is it ? If not, why not ?

36. Find the center of mass of a thin plate of constant density δ covering the region bounded above by parabola $y=4-x^2$ and below by x-axis.

(2 × 10 = 20 marks)

D 52761

(Pages : 4)

Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS—UG)

Complementary Course—Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Suppose $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$, find $\lim_{x \to c} 2f(x)\, g(x)$.

2. Find dy if $y = \cos 3x + x^4$.

3. Write the sum without sigma notation : $\sum_{k=1}^{1} 0k^2 + 3k$.

4. Find the interval in which the function $y = x^3$ is concave up.

5. Find absolute extrema of $y = x^2$ on (0, 2).

6. Find $\lim_{x \to -1} \frac{-1}{(1-x)^3}$.

7. Define vertical asymptote.

8. $\frac{d}{dx} \int_{-\pi}^{x} \cos t dt =$ .....

9. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \frac{1}{1-c_k} \Delta x_k$ where P is the partition of [2, 3] as an integral.

10. State Mean Value Theorem.

Turn over

11. Find all possible functions with derivative $y' = x^3$.

12. Shortest interval length of a partition is called ———.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Show that if $\lim_{x \to c} |f(x)| = 0$, then $\lim_{x \to c} f(x) = 0$.

14. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$.

15. If $\lim_{x \to -2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \to -2} \frac{f(x)}{x}$.

16. Find the slope of the curve $y = 1/x$ at $x = a$.

17. Differentiate $f(x) = \frac{x}{x-1}$. Where does the curve $y = f(x)$ have slope -1 ?

18. Show that $y = —x —$ is not differentiable at $x = 0$.

19. Find the equation for the tangent to the curve $y = x + \frac{2}{x}$ at $(1, 3)$.

20. Find absolute extrema of $y = x^{2/3}$ on $[-2, 3]$.

21. Find the function whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.

22. Show that $\lim_{x \to \infty} \frac{1}{x} = 0$.

23. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

24. Express the solution of the initial value problem $\frac{dy}{dx} = \tan x$, $y(1) = 5$ as an integral.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

26. When does a function not have a derivative at a point ? Explain.

27. Show that if f has a derivative at $x = c$, then f is continuous at $x = c$.

28. Show that functions with same derivatives differ by a constant.

29. Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

30. Find $\lim\limits_{x \to \infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$.

31. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

32. Find $\lim\limits_{x \to 0^+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.

33. Define $f(3)$ in a way that extends $f(x) = \dfrac{x^2 - 9}{x - 3}$ to be continuous at $x = 3$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}\, y^2, x = 0, y = -1, y = 1$ about x-axis.

36. Let $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \le x < 1; \\ 1, & 1 \le x < 2; \\ 2, & x = 2. \end{cases}$

(a) What are the domain and range of f?

(b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist?

(c) At what points does only the left-hand limit exists?

(d) At what points does only the right-hand limit exists?

(2 × 10 = 20 marks)

C 33326 (Pages : 4) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. At what points are function $f(x) = \frac{1}{(x+2)^2} + 4$ continuous ?
2. Define critical point of a function.
3. Suppose $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$. Find $\lim_{x \to c} f(x)\, g(x)$.
4. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.
5. Find absolute minima of $y = x^2$ on $(0, 2]$.
6. Find the interval in which $y = x^3$ is concave up.
7. $\frac{d}{dx} \int_a^x f(t)\, dt =$ _____.
8. Find dy if $y = x^5 + 37x$.
9. Define average value of a function f on $[a, b]$.
10. Find $\lim_{x \to -\infty} \frac{\pi\sqrt{3}}{x^2}$.
11. Define horizontal asymptote of the graph of a function.
12. Find $\lim_{x \to 2} \frac{3 - x}{3 + x}$.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. If $2 - x^2 \le g(x) \le 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

14. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.

15. Find the derivative of $y = \sqrt{x}$ for $x > 0$. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

16. Area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter when the diameter is 10 m ?

17. Find absolute extreme values of $g(t) = 8t - t^4$ on $[-2, 1]$.

18. Show that $\lim_{x \to -\infty} \frac{1}{x} = 0$.

19. The radius r of a circle increases from $r_0 = 10\ m$ to 10.1 m. Estimate the increase in the circle's area A by calculating dA. Compare this with true change ΔA.

20. Find a lower bound for the value of $\int_0^1 \cos x\, dx$ using the inequality $\cos x \ge 1 - x^2/2$.

21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1 + x^2}\, dx$.

22. Find the area of the region between $y = 4 - x^2, 0 \le x \le 3$ and the x-axis.

23. Find the function with derivative $f'(x) = 2x - 1$ passing through the point $P(0, 0)$.

24. Find $\frac{d}{dx} \int_0^{t^4} \sqrt{u}\, du$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find the slope of the curve $y = 1/x$ at $x = a$. Where does the slope equal $-1/4$? What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes ?

26. Show that functions with zero derivatives are constant.

27. Find the asymptotes of the graph of $f(x) = \dfrac{-8}{x^2 - 4}$.

28. Find $\lim\limits_{x \to 0+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.

29. Show that functions with the same derivative differ by a constant.

30. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

31. Express the solution of the initial value problem $\dfrac{ds}{dt} = f(t), s(t_0) = s_0$ in terms of integral.

32. Show that if f is continuous on $[a, b], a \ne b$ and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

33. Show that if f has a derivative at $x = a$ then f is continuous at a.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ is increasing and decreasing. What are the critical points ? When does the function assume extreme values and what are these values ?

35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^3, x = 0, y = -1, y = 1$ about x-axis.

36. Let $f(x) = \begin{cases} 3-x, & x<2; \\ 2, & x=2;. \\ \frac{x}{2}+1, & x>2 \end{cases}$

(a) Find $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$. and $f(2)$.

(b) Does $\lim_{x\to 2} f(x)$ exist ? If so, what is it ? If not, why not?

(c) Find $\lim_{x\to -2^+} f(x)$ and $\lim_{x\to -2^-} f(x)$.

(d) Does $\lim_{x\to -2} f(x)$ exist ? If so, what is it ? If not, why not ?

(2 × 10 = 20 marks)

D 92865

(Pages : 4)

Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. Find $\lim_{x \to 1} \frac{-1}{(3x-1)^2}$.

2. If f is the identity function , then find $\lim_{x \to x_0} f(x)$ where x_0 is a point in the domain of $f(x)$.

3. Find $\lim_{x \to \infty} \left(5 + \frac{1}{x}\right)$.

4. Differentiate $\sin(2x+3)$ with respect to x.

5. Find the second derivative of $y = -x^2 + 3$.

6. Define absolute maximum of a function.

7. Define an increasing function.

8. State the Mean Value theorem.

9. Find the critical points of $y = x^{2/3}(x+2)$.

10. Evaluate $\sum_{k=1}^{7} -2k$.

11. Give an example of a function which is not Riemann integrable.

12. The volume of a solid of cross-section area A (x) from $x = a$ to $x = b$ is .....

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. Show that if $\lim_{x \to c} |f(x)| = 0$, then $\lim_{x \to c} f(x) = 0$.

14. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

15. For the function $f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $\lim_{x \to 0} f(x)$ or explain why they do not exist.

16. Show that the derivative of a constant function is zero.

17. Find the derivative of $y = \frac{x^2 - 1}{x^2 + 1}$.

18. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.

19. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.

20. Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

21. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(3 c_k^2 - 2 c_k + 5\right) \Delta x_k$ as definite integrals, where P is a partition of $[-3, 5]$.

22. Find $\frac{dy}{dx}$ if $y = \int_0^{x^2} \cos t \, dt$.

23. Using the inequality $\cos x > \left(1 - x^2/2\right)$, find a lower bound for the value of $\int_0^1 \cos x \, dx$.

24. State and prove the Mean Value theorem for definite integrals.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find the continuous extension to $x = 2$ of the function $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$.

26. Let $f(x) = \begin{cases} 0 & x \leq 0 \\ \sin 1/x & x > 0 \end{cases}$.

(a) Does $\lim_{x \to 0^+} f(x)$ exist ? If so what is it ? If not, why not ?

(b) Does $\lim_{x \to 0^-} f(x)$ exist ? If so what is it ? If not, why not ?

(c) Does $\lim_{x \to 0} f(x)$ exist ? If so what is it ? If not, why not ?

27. Find the equation of the tangent to the curve $f(x) = (x+1)^2$ at the point (1, 4).

28. If $s = t^2 - 3t + 2, 0 \leq t \leq 2$ gives the positions of a body moving on a coordinate line, with s in metres and t in seconds.

(a) Find the body's displacement and average velocity for the given time interval.

(b) Find the body's speed and acceleration at the endpoints of the interval.

29. Show that $\lim_{x \to 0+} (1+x)^{1/x} = e$.

30. Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

31. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \leq x \leq 3$ is increasing and decreasing. Where does the function assume extreme values ?

32. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

33. Find the average value of $f(x) = 4 - x^2$ on [0, 3]. At what point in the interval does this function assume its average value ?

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*

34. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, prove that $\lim_{x \to c} (f(x) + g(x)) = L + M$.

35. The first derivative of the continuous function $y = f(x)$ is $y' = 2 + x - x^2$. Find y'' and sketch the general shape of the graph of f.

    (a) What are the critical points of f?

    (b) On what intervals is f increasing or decreasing ?

    (c) At what point if any, does f assumes local maximum and minimum values ?

36. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(2 × 10 = 20 marks)

D 53525 (Pages : 4) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2014

(U.G.-CCSS)

Complementary Course

MM IC 01——MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$.

2. At what points is $y = \frac{x+1}{x^2-4x+3}$ continuous ?

3. Evaluate $g'(2) : f\, g(t) = \frac{1}{t^2}$.

4. What is the parametric form of $x^2 + y^2 = 4$?

5. Find the absolute extrema of $g(\theta) = \theta^{3/5}, -32 \le \theta \le 1$.

6. State Rolle's Theorem.

7. Find the value of 'c' satisfying $f'(c) = \frac{f(b)-f(a)}{b-a}$ in the mean value theorem for

$f(x) = x + \frac{1}{x}$ in $\left[\frac{1}{2}, 2\right]$.

8. If $f(x) = -9$ on $[-1, 6]$, $\int_{-1}^{6} f(x)\, dx = ?$

9. State the Fundamental Theorem of calculus to evaluate definite integrals.

10. Define a smooth curve.

Turn over

11. $\frac{0}{0}$ is an ——— form.

12. $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-1}{x} = ?$

$(12 \times \frac{1}{4} = 3$ weightage)

Part B

Answer any **nine** *questions.*

13. State Sandwich Theorem and use it to find $\lim\limits_{x \to 0} g(x)$ if $3 - x^3 \leq g(x) \leq 3 \sec x \; \forall \; x$.

14. Find a δ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$, $f(x) = \frac{1}{x}, L = \frac{1}{4}, x_0 = 4, \epsilon = 0.05$.

15. Evaluate $\lim\limits_{x \to 0} \frac{\tan 3x}{\sin 8x}$.

16. Show that $\lim\limits_{x \to -\infty} \frac{2}{x} = 0$.

17. Find a horizontal asymptote of $y = \frac{2}{x} - 3$.

18. Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

19. Evaluate $\sum\limits_{k=1}^{4} \frac{6k}{k+1}$.

20. If $\int\limits_0^3 f(z)\,dz = 3$, $\int\limits_0^4 f(z)\,dz = 7$ and f is continuous, find $\int\limits_3^4 f(z)\,dz$ and $\int\limits_4^3 f(z)\,dz$.

21. Graph the function $f(x) = -x^2$ in $[0, 1]$.

22. Give the formula for volume of a solid by revolving about the y-axis.

23. Define an astroid and sketch it.

24. Find $\lim_{x \to 0+} (1+x)^{1/x}$.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*

25. Show that $y = \sqrt{x}$ is not differentiable at $x = 0$.

26. Find an equation of the tangent line to $y = 1 + \sqrt{4 - x}$ at (3, 2).

27. Find the critical points, intervals in which increasing or decreasing and local extrema if they exist if $f'(x) = (x-1)(x+2)(x-3)$.

28. For what values of a, m, b does

$$f(x) = \begin{cases} 3, x = 0 \\ -x^2 + 3x + a, 0 < x < 1 \\ mx + b, 1 \le x \le 2 \end{cases}$$

Satisfy the hypothesis of the Mean Value Theorem.

29. Define average value of a function f. Find the average value of $f(x) = x^2 - 1$ on $\left[0, \sqrt{3}\right]$.

30. Find the area of the region between the x-axis and $y = x^3 - 3x^2 + 2x, 0 \le x \le 2$.

31. Find the area between $y = x^2$ and $y = -x^2 + 4x$.

32. Find the lateral surface area of the cone generated by revolving $y = x/2, 0 \le x \le 4$ about the x-axis.

(5 × 2 = 10 weightage)

Part D

Answer **both** *questions.*

33. (i) Find the equation(s) of the asymptotes and graph them : $y = \dfrac{x^2}{x-1}$.

(ii) Let $f(x)=\begin{cases} x^3, x \neq 1 \\ 0, \ x=1 \end{cases}$. Do the following exist ? Find them if they do :

(a) $\underset{x \to 1-}{\text{Lt}} f(x)$ (b) $\underset{x \to 1+}{\text{Lt}} f(x)$.

(c) $\underset{x \to 1}{\text{Lt}} f(x)$

(iii) Find an oblique asymptote of $f(x)=\dfrac{2x^2-3}{7x+4}$.

34. (i) Find the volume of the solid generated be revolving the region bounded by revolving $y=\sqrt{9-x^2}$, $y=0$ about the x-axis.

(ii) Find the length of $x=\dfrac{y^3}{3}+\dfrac{1}{4y}$ from $y=1$ to $y=3$.

(2 × 4 = 8 weightage)

D 32494 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JANUARY 2013

(CCSS)

Mathematics (Complementary Course)

MA 1C 01—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

Part A (Objective Type Questions)

Answer all **twelve** *questions.*

Each bunch of four questions carries 1 *weightage.*

1. Find $\lim_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5}$.

2. Find a point of discontinuity of the function $y = \frac{x+2}{\cos x}$.

3. Find $\frac{d\gamma}{d\theta}$ if $\gamma = 4 - Q^2 \sin\theta$.

4. The curve $y = x^2 - 2x + 1$ has a horizontal tangent at $x =$ _____.

(4 × ¼ = 1 weightage)

5. Define Rolle's theorem.

6. The formula for finding the sum of squares of first 'n' natural numbers is _____.

7. Express $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(3C_k^2 - 2C_k + 5\right) Dx_k$ as an integral if p denotes a partition of the interval $[-1, 3]$.

8. Evaluate $\int_{-4}^{-1} \frac{\pi}{2} d\theta$.

(4 × ¼ = 1 weightage)

9. Suppose $\int_1^9 f(x)\,dx = -1$ and $\int_7^9 f(x)\,dx = 5$ then $\int_1^7 f(x)\,dx =$ _____.

10. If f is integrable on $[a, b]$ then the average value of f on $[a, b]$ is av $(f) =$ _____.

11. Where does the function $y = \sec x$ have vertical asymptotes ?

12. Use L' Hopital's rule find $\lim_{t \to 0} \frac{\sin t^2}{t}$.

(4 × ¼ = 1 weightage)

Turn over

Part B (Short Answer Type Questions)

Answer all **nine** *questions.*
Each question carries 1 *weightage.*

13. Find $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+3}-2}$.

14. If $1-\dfrac{x^2}{4}\le u(x)\le 1+\dfrac{x^2}{4}$ for all $x\neq 0$ then find $\underset{x\to 0}{\text{Lt}}\, u(x)$.

15. Suppose $\lim_{x\to 0} f(x)=1$ and $\lim_{x\to 0} g(x)=-5$ find $\lim_{x\to 0}\dfrac{2f(x)-g(x)}{[f(x)+7]^{2/3}}$.

16. Find the slope and equation of the tangent at the point $(3,3)$ to the curve $g(x)=\dfrac{x}{x-2}$.

17. Find the function $f(x)$ whose derivative is sin x-and whose graph passes through the point $(0,2)$.

18. Find the linearization of $f(x)=\sqrt{1+x}$ at $x=3$.

19. Find the area between the curve $y=\dfrac{x}{2}+1$ and the x-axis on the interval $[0,b]$.

20. Evaluate $\dfrac{d}{d\theta}\int_0^{\tan\theta}\sec^2 y\,dy$.

21. Find the length of the curve $x=\dfrac{y^3}{3}+\dfrac{1}{4y}$ from $y=1$ to $y=3$.

(9 × 1 = 9 weightage)

Part C (Short Essay Questions)

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Find the first and second derivatives of the function $w=\left(\dfrac{1+3z}{3z}\right)(3-z)$.

23. If $f(x)=x+1, \mathrm{L}=5, x_0=4, \in=\cdot 01$, find an open interval containing x_0 and a value of $\delta>0$ such that $0<|x-x_0|<\delta$ implies $|f(x)-\mathrm{L}|<\in$.

24. The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and the line $y = x$ is a tangent to the curve at the origin. Find a, b, c.

25. Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

26. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

27. Use max–min inequality find upper and lower bounds for the value of $\int_0^1 \dfrac{1}{1+x^2}\,dx$.

28. For what values of a, m and b does the function $f(x) = \begin{cases} 3 & , x = 0 \\ -x^2 + 3x + a & , 0 < x < 1 \\ mx + b & , 1 \le x \le 2 \end{cases}$

Satisfy the hypotheses of the mean value theorem on the interval $[0, 2]$.

(5 × 2 = 10 weightage)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. Find y' and y'' and graph the function $y = x^4 - 4x^3 + 10$. Include the co-ordinates of any local extreme points and inflection points.

30. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$ about the x-axis.

31. Find the volume of the solid generated by revolving the region between the curve $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

(2 × 4 = 8 weightage)

C 43533 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, JULY 2013

(CCSS)

Mathematics

MM 1C 01—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

I. Answer *all* twelve questions :

1 Evaluate $\lim_{x\to\infty} \frac{5x^2+8x-3}{3x^2+2}$.

2 Find dy if $y = \sin 3x$.

3 Write the sums without sigma notation and then evaluate the sum $\sum_{k=1}^{3} (-1)^{k+1} \sin\frac{\pi}{k}$.

4 Suppose that $\int_1^2 f(x)\,dx = 5$. Find $\int_1^2 -f(x)\,dx$.

5 Evaluate $\int_0^4 \left(3x - \frac{x^3}{4}\right) dx$.

6 Evaluate $\lim_{x\to -5} \frac{x^2+3x-10}{x+5}$.

7 Define the continuity of a function f at a right end point $x = b$ of its domain.

8 Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$.

9 Find the slope of the curve $f(x) = x^2 + 1$ at (2, 5).

10 At what points do the graph of the function $f(x) = x^2 + 4x - 1$ has horizontal tangents.

11 State the mean value theorem.

12 The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circles area A by calculating d A.

(12 × ¼ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions :

13 Find $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

Turn over

14 Find the absolute maximum and minimum value of $g(t) = 8t - t^4$ on [–2, 1].

15 Find the asymptotes of the curve $y = \frac{x+3}{x+2}$.

16 Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

17 Evaluate $\sum_{k=1}^{4} (k^2 - 3k)$.

18 Find the average value of $f(x) = -3x^2 - 1$ on [0, 1].

19 Evaluate $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$.

20 Find the volume of the solid generated by revolving the region bounded by :

$y = x^2, y = 0, x = 2$.

21 Where does the slop of the curve $y = \frac{1}{x}$ equal $-\frac{1}{4}$?

(9 × 1 = 9 weightage)

III. Short essay. Answer any *five* questions from seven :

22 Show that $\lim_{x \to 0^+} (1+x)^{1/x} = e$.

23 Prove that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and $(0, \infty)$ but has no derivative at $x = 0$.

24 Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$.

25 Express the solution of the following initial value problem as an integral

Differential equation : $\frac{dy}{dx} = \tan x$.

Initial condition : $y(1) = 5$.

26 Find the total area between the region $y = -x^2 - 2x, -3 \le x \le 2$ and the x-axis.

27 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

28 Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}, 0 \le x \le 4$ about the x-axis.

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions from three :

29 The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

30 Find the area of the region in the first quadrant that is bounded and above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

31 Evaluate $\lim_{\theta \to 0} \frac{\cos\theta - 1}{e^{\theta} - \theta - 1}$.

(2 × 4 = 8 weightage)

D 13817 **(Pages : 4)** **Name**.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours **Maximum : 80 Marks**

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{t \to 1} \frac{t^2 + t - 2}{t^2 - 1}$.

2. Show that the sum of two continuous functions is continuous.

3. If $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$, find $\lim_{x \to c} 2f(x)\, g(x)$.

4. Find $K'(-1)$ if $k(z) = \frac{1 - z}{2z}$.

5. If $p = \frac{1}{\sqrt{q+1}}$, find $\frac{dp}{dq}$.

6. What is the derivative product rule ?

7. Does differentiability of a function imply continuity ? Is the converse true ?

8. Find $\lim_{x \to -\infty} \frac{\sqrt{7e}}{x^3}$.

9. Find $\lim_{x \to +\infty} \frac{3x + 7}{x^2 - 2}$.

10. Is the x-axis an aymptote of $f(x) = \frac{1}{x}$?

Turn over

11. Give an example of a non-integrable function on [0, 1].

12. Find the area of the region between $y = 3x^2$ and the x-axis on $[0, b]$.

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find $\underset{x \to 9}{\text{Lt}} = \dfrac{\sqrt{x} - 3}{x - 9}$.

14. Prove that $\underset{x \to x_0}{\text{Lt}}\ x = x_0$.

15. Prove that the derivative of a constant function is zero.

16. Find the first derivative of $r = \dfrac{(\theta - 1)(\theta^2 + \theta + 1)}{\theta^3}$.

17. Find the first four derivatives of $y = x^3 - 7x^2 + 8x - 9$.

18. Find $\underset{x \to \infty}{\text{Lt}} \dfrac{2x^3 + 7}{x^3 - x^2 + x + 7}$.

19. Find a linearization of $f(x) = \sqrt{2 + x}$ at $x = 1$.

20. Find dy if $y = x^3 - 3\sqrt{x}$.

21. If $\int_1^2 f(x)dx = -4$, $\int_1^5 f(x)dx = 6$ and $\int_1^5 g(x)dx = 8$, find $\int_2^5 f(x)dx$ and $\int_1^5 [4f(x) - g(x)]dx$.

22. Define a continuous function.

23. Find the tangent to $y^2 = x$ at (0, 0).

24. State the first derivative test.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Applying the definition of limit, show that $\lim_{x \to 1}\left(\frac{3x-1}{2}\right)$ is $\frac{1}{2}$.

26. Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

27. Graph the curve $y = 1/x$. Find its slope at $x = a \neq 0$. When is the slope equal to $-1/4$?

28. Find $\lim_{x \to \infty} x \sin\left(1/x\right)$.

29. Find the total area between the region $y = -x^2 - 2x, -3 \leq x \leq 2$ and the x-axis.

30. Show that $y = |x|$ is not differentiable at the origin.

31. Show that if n is a positive integer, $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$.

32. State the Mean Value Theorem for definite integrals. Also show that if 'f' is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ atleast once in $[a, b]$.

33. State and prove the fundamental theorem of calculus for the evaluation of definite integrals using anti-derivatives.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (i) Find an equation for the tangent to the curve $y = x + \frac{2}{x}$ at (3, 4).

(ii) Find the second derivative of $p = \frac{q^3 + 3}{12q}\left(\frac{q^4 - 1}{q^3}\right)$.

(iii) At time 't', the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ metres. Find the body's acceleration each time the velocity is O.

35. (i) Find the critical points of 'f', the intervals in which f is increasing or decreasing and local extrema, if they exist, given $f'(x) = (x-1)^2 (x+2)$.

(ii) Evaluate :

(a) $\underset{x \to \pi/4}{\text{Lt}} \dfrac{\sin x - \cos x}{x - \pi/4}$.

(b) $\underset{x \to \infty}{\text{Lt}} \dfrac{5x^2 - 3x}{7x^2 + 1}$.

36. (i) Find the area between $y = x^4$ and $y = 8x$.

(ii) The circle $x^2 + y^2 = 9$ is rotated about the x-axis to generate a sphere. Find its volume.

(2 × 10 = 20 marks)

D 74380 (Pages : 4) Name.....

Reg. No.....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(CUCBCSS-U.G.)

Complementary Course—Mathematics

MAT IC 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer all **twelve** *questions.*

1. Evaluate $\lim_{x \to -5} \frac{x^2+3x-10}{x+5}$.

2. At what points are the function $y = \frac{\cos x}{x}$ continuous ?

3. Find the slope of $f(x) = x^2 + 1$ at (3, 7).

4. Find the derivative of $y = x^2$ using the definition of derivative.

5. Find the second derivative of $y = \frac{1}{3x^2} - \frac{5}{2x}$.

6. How fast does the area of a circle change with respect to the diameter when the diameter is 8 m ?

7. Find the critical points of $f(x) = \frac{2x^3 - 3x^2}{6}$.

8. Graph the parabola $y = x^2$.

9. Find $\lim_{x \to \infty} \frac{2x+3}{5x+7}$.

10. Evaluate the sum of the first 20 cubes.

11. State the mean value theorem for definite integrals.

12. Find the intersection points of $f(x) = 2 - x^2$ and $g(x) = -x$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer all **nine** *questions.*

13. If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}, -1 \le x \le 1$, find $\underset{x \to 0}{\text{Lt}} f(x)$.

14. Prove that $\underset{x \to 1}{\text{Lt}} f(x) = 1$ if $f(x) = \begin{cases} x^2, x \neq 1 \\ 2, \ x = 1 \end{cases}$.

15. Suppose $\underset{x \to c}{\text{Lt}} f(x) = 5$ and $\underset{x \to c}{\text{Lt}} g(x) = -2$. Find :

(i) $\underset{x \to c}{\text{Lt}} [f(x) + 3g(x)]$; and (ii) $\underset{x \to c}{\text{Lt}} \dfrac{f(x)}{f(x) - g(x)}$.

16. Find the derivative of $y = \dfrac{1}{(x^2-1)(x^2+x+1)}$.

17. Find the equation of the tangent to the curve $y = x^3 - 4x + 1$ at (2, 1).

18. Find $\underset{x \to 0}{\text{Lt}} \dfrac{8x^2}{\cos x - 1}$.

19. Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

20. Graph the function $y = \dfrac{1}{2x+4}$.

21. Find the area of the region enclosed by $y = x^2 - 2$ and $y = 2$.

22. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through (0, 2).

23. Find the derivatives of all orders of $y = x^5/120$.

24. State both parts of the fundamental theorem of calculus.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*

25. Show that $y = \sin(1/x)$ has no limit point as x approaches zero from either side. Also sketch the graph of this function.

26. Evaluate $\underset{x\to 1}{\text{Lt}} \frac{x-1}{\sqrt{x+3}-2}$.

27. The curve $y = ax^2 + bx + c$ passes through (1, 2) and is tangent to $y = x$ at the origin. Find a, b, c.

28. State and prove the product rule for derivatives. Use it to find the derivative of $y=\left(x^2+1\right)\left(x^3+3\right)$.

29. Find the intervals on which $f(x)=\frac{x^2-3}{x-2}, x \neq 2$ is increasing and decreasing. Identify local extrema if they exist.

30. Define average value of an integrable function over a closed interval. Find the average value of $f(x) = -3x^2 - 1$ on [0,1]. where in the given interval does $f(x)$ assume its average value.

31. Show that $\underset{x\to 0+}{\text{Lt}} (1+x)^{1/x} = e$.

32. An object is dropped from the top of a 100 m high tower. Its height above ground after 't' seconds is $(100 - 4.9\, t^2)\, m$. How fast is it falling 2 seconds after it is dropped ?

33. Find the derivative $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\, dt$ by (i) evaluating the integral and differentiating the result ; and (ii) by differentiating the integral directly.

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*

34. (i) Find the area of the region enclosed by the curves $x+4y^2=4$ and $x+y^4=1$ for $x \geq 0$.

(ii) Find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 0, x = 2$ about the x-axis.

35. (i) Graph the function $y = x^4 - 4x^3 + 10$ by finding the first and second derivative.

(ii) Evaluate $\underset{x\to 0}{\text{Lt}} \frac{x^2}{\ln(\sec x)}$.

(iii) Evaluate $\sum_{k=1}^{4} \cos k\pi$.

36. (i) Let $f(x)=\begin{cases}3-x, x<2\\ \frac{x}{2}+1, x>2\end{cases}$. Find :

(a) $\underset{x\to 2+}{\text{Lt}} f(x)$ and $\underset{x\to 2-}{\text{Lt}} f(x)$.

(b) Does $\underset{x\to 2}{\text{Lt}} f(x)$ exist ? why or why not ?

(c) $\underset{x\to 4-}{\text{Lt}} f(x)$ and $\underset{x\to 4+}{\text{Lt}} f(x)$.

(d) Does $\underset{x\to 4}{\text{Lt}} f(x)$ exist ? Why or why not ?

(ii) Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

$(2 \times 10 = 20$ marks)

36. (i) Let $f(x)=\begin{cases}3-x, x<2\\ \frac{x}{2}+1, x>2\end{cases}$. Find :

(a) $\underset{x\to 2+}{\text{Lt}} f(x)$ and $\underset{x\to 2-}{\text{Lt}} f(x)$.

(b) Does $\underset{x\to 2}{\text{Lt}} f(x)$ exist ? why or why not ?

(c) $\underset{x\to 4-}{\text{Lt}} f(x)$ and $\underset{x\to 4+}{\text{Lt}} f(x)$.

(d) Does $\underset{x\to 4}{\text{Lt}} f(x)$ exist ? Why or why not ?

(ii) Show that the line $y = mx + b$ is its own tangent at any point $(x_0, mx_0 + b)$.

$(2 \times 10 = 20$ marks)