

C 42953

(Pages : 4)

Name.....

Reg. No.....

**SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2023**

Mathematics

MAT 2C 02—Mathematics

(2017—2018 Admissions)

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)

Answer all questions.

Each question carries 1 mark.

1. Write $\sinh x$ in terms of exponential function.
2. If f is smooth on $[a, b]$, what is the length of the curve $y = f(x)$ from a to b .
3. Write the formula for n^{th} term of the sequence $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$
4. Define geometric series.
5. Evaluate $\lim_{n \rightarrow \infty} (\ln n) / n$.
6. What is the sum of the infinite series $1 + x + x^2 + x^3 + \dots + x^n + \dots, |x| < 1$.
7. Define power series about $x = a$.
8. Replace the Cartesian equation $x^2 + y^2 = 4$ with equivalent polar equation.
9. Find the Cartesian co-ordinate of the point $\left(\sqrt{2}, \frac{\pi}{4}\right)$ given in polar co-ordinates.
10. Find the domain of the function $f(x, y) = \sqrt{y - x - 2}$.

Turn over

11. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$.

12. Find $\partial f / \partial x$ and $\partial f / \partial y$, given $f(x, y) = x^2 - xy + y^2$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** questions.

Each question carries 2 marks

13. Evaluate $\int_0^1 \sinh^2 x dx$.

14. Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

15. Find the limit of the sequence $\{a_n\}$, where $a_n = \frac{n + (-1)^n}{n}$.

16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{7}{4^n}$.

17. Show that the series $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$ diverges.

18. For what values of x , the series $\sum_{n=0}^{\infty} x^n$ converges absolutely.

19. Replace the polar equations $r = \frac{4}{2 \cos \theta - \sin \theta}$ by equivalent Cartesian equations, and identify the graph.

20. Find a polar equation for the conic section with eccentricity $e = 1$ and with one focus at the origin and the directrix $x = 2$.
21. Find the area of the regions bounded by the circle $r = 2 \sin \theta$ for $\pi/4 \leq \theta \leq \pi/2$.
22. Find a spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$.
23. Find $\partial^2 w / \partial x \partial y$ if $w = xy + \frac{e^y}{y^2 + 1}$.
24. Find dw/dt if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any six questions.
Each question carries 5 marks.*

25. Derive the formula $\sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right)$ for all real x .
26. Find the volume of the solid generated by revolving the region between the y -axis and the curve $x = 2/y$, $1 \leq y \leq 4$, about the y -axis.
27. The line segment $x = 1 - y$, $0 \leq y \leq 1$, is revolved about the y -axis to generate the cone. Find its lateral surface area.
28. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n! n!}$.
29. Find the sum of series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$.

Turn over

30. For the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

- Find the radius and interval of convergence.
- For what values of x does the series converge absolutely?
- For what values of x does the series converge conditionally?

31. Find the Taylor series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.

32. Find the area inside the smaller loop of the limaçon $r = 2 \cos \theta + 1$.

33. Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ is not continuous at the origin.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** questions.

Each question carries 10 marks.

34. (a) Evaluate $\int_{-8}^1 \frac{dx}{x^{1/3}}$. (5 marks)

(b) Find the volume of the solid generated by revolving about y -axis, the infinite region in the first quadrant between the curve $y = e^{-x}$ and the x -axis. (5 marks)

35. (a) Test for convergence the series $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdots (2n-1)}{4^n 2^n n!}$. (5 marks)

(b) Prove the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2} \right)^n$ converges. (5 marks)

36. Find the linearization $L(x, y)$ of

$$f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3.$$

at the point $(3, 2)$. Find an upper bound for the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R: |x-3| \leq 0.1, |y-2| \leq 0.1$. Express the upper bound as a percentage of $f(3, 2)$, the value of f at the center of the rectangle. (10 marks)

[2 × 10 = 20 marks]

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Name.....

Reg. No.....

**SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2022**

Mathematics

MAT 2C 02—MATHEMATICS

(2016—2018 Admissions)

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)*Answer all twelve questions.**Each question carries 1 mark.*

1. Find the n^{th} term of the sequence 2, 5, 10, _____.
2. $\frac{d}{dx}(\tanh x) =$ _____.
3. If f is continuous on $[a, \infty)$, then $\lim_{b \rightarrow \infty} \int_a^b f(x) dx =$ _____.
4. The n^{th} term of a converging infinite series has the limit _____.
5. Find the domain of the function $w = \sin xy$.
6. Define level curve of a function f .
7. State Euler's mixed derivative theorem.
8. If $f(x, y) = \cosh xy$, find $\frac{\partial}{\partial x} f(x, y)$.
9. Which point is the foot of the perpendicular from the origin to the line $r \cos(\theta - \theta_0) = r_0$?
10. The name of the curve with the polar equation $r = a(1 + \cos \theta)$ is _____.

Turn over

11. Define partial derivative of $u = f(x, y, z)$ with respect to y .
12. State the chain rule for partial differentiation in the case of functions with two independent variables and three intermediate variables.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** questions.

Each question carries 2 marks.

13. Investigate the convergence of $\int_0^1 \frac{1}{1-x} dx$.
14. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x -axis is revolved about the x -axis to generate a solid. Find its volume.
15. Determine the sequence $a_n = \frac{3n-1}{n+1}$ is nondecreasing and if it is bounded from above.
16. Find the cartesian form of the curve $r = 2a \sin \theta$.
17. Find $\frac{\partial}{\partial z}$ if the equation $yz - \ln z = xy$ defines z as a function of the two independent variables x and y and the partial derivatives exists.
18. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.
19. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = x^2 + y^2 - 2z^2$.
20. Investigate the convergence of $\int_1^\infty \frac{\sin^2 x}{x^2}$.

21. Find the sum of the series : $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$
22. Find the Maclaurin series expansion of $f(x) = (1+x)^n$.
23. Find $f_x(1, 2)$ if $f(x, y) = \cosh^2(xy) - \sinh^2(xy)$.
24. Test the function for continuity at origin :

$$f(x, y) = \begin{cases} \frac{2xy}{x+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{otherwise} \end{cases}.$$

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any six questions
Each question carries 5 marks.*

25. Show that $\tanh(2x) = \frac{2 \tanh x}{1 + \tanh^2 x}$.
26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$ and find the actual value in case of convergence
27. Find the sum of the series $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^n}$.
28. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).
29. Find the length of the cardioid $r = 2(1 - \cos \theta)$.
30. Use partial differentiation to find $\frac{dx}{dy}$ if $x^3 + y^3 - 3xy = 0$.

Turn over

31. Find the cartesian form of the surface $\rho = 2 \cos \theta$.
32. Find the radius and centre of the circle $r = 4 \cos \theta$ in cartesian plane.
33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (2x-1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any two questions

Each question carries 10 marks.

34. If the resistors $R_1, R_2,$ and R_3 hms are connected in parallel to make an R ohm resistor satisfying the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ find $\frac{\partial R}{\partial R_2}$ when $R_1 = 30, R_2 = 45$ and $R_3 = 90$ ohms.
35. (a) Find a curve through the point $(1, 1)$ whose length integral is $L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$.
- (b) How many such curves are there? Give reasons for your answer.
36. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq \frac{1}{2}$ about the x -axis.

(2 × 10 = 20 marks)

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Name.....

Reg. No.....

**SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2021**

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)*Answer all twelve questions.**Each question carries 1 mark.*

1. Find $\int a^x dx$.
2. Define the partial derivative of $f(x, y)$ with respect to x at (x_0, y_0) .
3. If f is continuous on $[a, \infty)$, then $\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \dots\dots\dots$
4. If a series $\sum a_n$ converges, then $\lim a_n = \dots\dots\dots$
5. Find the n^{th} term of the sequence $-2, 2, -2, 2, \dots\dots$
6. Find the domain of the function $z = \sin xy$.
7. The polar form of the line $y = 2$ is $\dots\dots\dots$
8. $\frac{d}{dx} \cosh x = \dots\dots\dots$
9. If $f(x, y) = 2x^2y$ then find $\frac{\partial^2 f}{\partial x \partial y}$.
10. Give an example of conditionally converging series.

Turn over

11. State Sandwich theorem for sequences.
12. Write the transformation equations for Cartesian co-ordinates to spherical polar co-ordinates.
(12 × 1 = 12 marks)

Part B (Short Answer Type)

*Answer any **nine** questions.
Each question carries 2 marks.*

13. Evaluate $\int_0^{\log 2} 4e^x \sinh x \, dx$.
14. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x -axis is revolved about the x -axis to generate a solid. Find its volume.
15. Determine whether the sequence $a_n = \frac{2n+1}{3n+1}$ is non-decreasing and if it is bounded from above.
16. Describe the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 1}$.
17. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$.
18. Find $\frac{\partial z}{\partial y}$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivatives exists.
19. Is the area under the curve $y = \ln x/x^2$ from $x = 1$ to $x = \infty$ finite? If so, what is it?
20. Evaluate $\int_0^1 \frac{dx}{\sqrt{3+4x^2}}$.
21. Write the polar form of the circle $x^2 + (y-3)^2 = 9$.
22. Draw the polar curve $r = 2 \cos \theta$.

23. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

24. Find the volume of $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** questions.

Each question carries 5 marks.

25. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

27. Does the sequence whose n th term is $\left(\frac{n+1}{n-1}\right)^n$ converge? If so, find its limit.

28. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{n-1} - 1}{3^n}$.

29. Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$.

30. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = 2x + 2y - z^2, x = r/s, y = r^2 + \ln s, z = 2r$.

31. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

Turn over

32. Show that $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges.

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any two questions.

Each question carries 10 marks.

34. Find the length of the cardioid $r = 1 - \cos \theta$.

35. a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

b) Show that $\tanh^2 x = 1 - \operatorname{sech}^2 x$

36. a) Using partial differentiation find $w'(0)$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

b) If $f(x - y, y - z, z - x) = 0$, show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$.

(2 × 10 = 20 marks)

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

*Answer all twelve questions.**Each question carries 1 mark.*

1. Find a formula for n th term of the sequence 1, -1, 1, -1,.....
2. Define a non-decreasing sequence.
3. Write $\tanh x$ in terms of exponential function.
4. State Sandwich theorem for sequences.
5. Find the domain of the function $w = \sqrt{y - x^2}$.
6. Define contour line of a function $f(x, y)$.
7. Find $\lim_{(x,y) \rightarrow (0,1)} \frac{2 - xy + 3}{x^2 y + 5xy - y^3}$.
8. Define absolute convergence of a series.
9. $\frac{d}{dx} \sinh 2x =$ _____.
10. Define level curve of a function.
11. Find $\lim_{n \rightarrow \infty} n^{1/n}$.
12. $\int_0^1 \sinh x dx =$ _____.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

*Answer any nine questions.**Each question carries 2 marks.*

13. Is the area under the curve $y = 1/\sqrt{x}$ from $x = 0$ to $x = 1$ finite? If so, what is it?
14. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

Turn over

15. Show that $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.
16. Determine whether the sequence $a_n = \frac{2^n 3^n}{n!}$ is non-decreasing and bounded from above.
17. Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0); \\ 0, & (x, y) = (0, 0); \end{cases}$ is continuous at every point except the origin.
18. Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.
19. Find the volume of the solid generated by revolving the region between y -axis and the curve $x = 2/y, 1 \leq y \leq 4$ about the y -axis.
20. Find $\int_0^{\ln 2} 4e^x \sin x dx$.
21. Graph the sets of points whose polar co-ordinates satisfies the conditions $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$.
22. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ equivalent cartesian equation.
23. Find the spherical co-ordinates equation for the cone $z = \sqrt{x^2 + y^2}$.
24. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any six questions.
Each question carries 5 marks.*

25. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.
26. Show that $(-1)^n \frac{n-1}{n}$ diverges.
27. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.
28. Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

29. Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (x_0, y_0, z_0) .
30. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?
31. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.
32. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$.
33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

*Answer any two questions.
Each question carries 10 marks.*

34. Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

35. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

36. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \leq x \leq 4$ about the x -axis.

(2 × 10 = 20 marks)

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(Pages : 3)

Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2019

(CUCBCSS-UG)

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Types)

Answer all twelve questions.

1. Define a sequence.
2. Fill in the blanks : $\frac{d}{dx} \cosh^3(3x) = \underline{\hspace{2cm}}$.
3. For what values of real numbers x , does the series $\sum_{n=1}^{\infty} \sin^n x$ converge ?
4. Fill in the blanks : The polar equation of the circle with centre origin and radius a is $\underline{\hspace{2cm}}$.
5. Find the n^{th} term of the sequence $2, -2, 2, -2 \dots$.
6. Fill in the blanks : If $f(x, y) = 1 - \sinh(1 - xy)$, then $f_x(1, 1) = \underline{\hspace{2cm}}$.
7. Fill in the blanks : If f is continuous on $[a, b]$, then $\lim_{c \rightarrow b} \int_a^c f(t) dt = \underline{\hspace{2cm}}$.
8. Write explicitly the ratio test for the convergence of the series $\sum_{n=0}^{\infty} a_n$.
9. State alternating series test of Leibniz.
10. Define $\frac{\partial}{\partial x} f(x, y)$ using limit.
11. The power series $\sum_{n=0}^{\infty} a_n (x - a)^n$ always converges to a_0 when $x = \underline{\hspace{2cm}}$.
12. What do you mean by linearization of a function in two variables at a point.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Types)*Answer any nine questions.*

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.
14. Test the convergence of the integral $\int_0^{\frac{1}{2}} \frac{1}{1-2x} \, dx$.
15. State the non-decreasing sequence theorem.
16. Describe the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2} - 1$.
17. Graph the sets of points whose polar co-ordinates satisfy the condition $0 \leq r \leq 2$.
18. Evaluate $\int_0^1 \frac{3dx}{\sqrt{4+9x^2}}$.
19. Find $\tanh x$, if $\cosh x = \frac{17}{15}$, $x > 0$.
20. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ if $f(x, y) = \log \sqrt{x^2 + y^2}$.
21. Find a cylindrical co-ordinate equation for the surface $x^2 + (y - 3)^2 = 9$.
22. Find $\frac{\partial z}{\partial r}$ if $z = x + 2y$, $x = \frac{r}{s}$ and $y = 2rs$.
23. Find $\lim_{n \rightarrow \infty} \frac{n}{2n+1}$.
24. Write the Maclaurin series for $\sin x$.

 $(9 \times 2 = 18 \text{ marks})$ **Part C (Short Essay Types)***Answer any six questions.*

25. Find the length of the curve $y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$.
26. Find the limit of the function $f(x, y) = \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ as (x, y) tends to $(0, 0)$.

27. Replace the polar equation $r = \frac{4}{2\cos\theta + \sin\theta}$ by equivalent Cartesian equation and draw the graph in Cartesian form.
28. Find a power series for $\log(1+x)$ and find the radius of convergence of that series.
29. Show that $\tanh^{-1}x = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$.
30. Find the volume of the solid of revolution when the region between the parabola $x = y^2 + 1$ and the line $x = 3$ is revolved about the line $x = 3$.
31. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n - 1}{4^n}$.
32. Find the radius and interval of convergence of the series: $\sum_{n=0}^{\infty} (-1)^n (2x-1)^n$.
33. Evaluate: $\int \frac{\cosh^4 \sqrt{x}}{\sqrt{x}} dx$.

(6 × 5 = 30 marks)

Part D (Essay Types)*Answer any two questions.*

34. Show that the function $f(x,y) = \frac{2xy}{x^2 + y^2}$ when $(x,y) \neq (0,0)$ and 0, otherwise is continuous everywhere except at the origin.
35. (a) Find the linearization of the function $f(x,y) = x^2 - xy + y^2/2 + 3$ at $(3, 2)$.
 (b) Find the area of the region enclosed by the cardioid: $r = 2(1 + \cos\theta)$.
36. Find the area of the surface generated by revolving the curve $y = x^3/9, 0 \leq x \leq 2$ about the x -axis.

(2 × 10 = 20 marks)

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2018

(CUCBCSS-UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)*Answer all twelve questions.**Each question carries 1 mark.*

1. Write an example for a sequence which has no upper bound.
2. Find the domain of the function $w = xy \ln z$.
3. Define the level surface of a function f .
4. State two path test for non-existence of limit.
5. If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$.
6. $\frac{d}{dx} \sinh x =$ _____.
7. Write $\tanh x$ in terms of exponential function.
8. Find $\lim_{n \rightarrow \infty} \sqrt[n]{n}$.
9. $\int \cosh 2x =$ _____.
10. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + 2y}{3x - 2}$.
11. Find $\frac{\partial}{\partial x} \sin 2xy$.
12. Define conditional convergence of a series.

(12 × 1 = 12 marks)

Part B (Short Answer Type)*Answer any nine questions.**Each question carries 2 marks.*

13. Investigate the convergence of $\int_0^{\infty} e^{-x^2} dx$.
14. Show that $\lim_{n \rightarrow \infty} k = k$, where k is a constant.

Turn over

15. Find $\lim_{n \rightarrow \infty} \frac{\cos n}{n}$.
16. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.
17. Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.
18. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.
19. Use chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. What is the derivative's value at $t = \pi/2$?
20. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.
21. Show that if u is a differentiable function of x whose values are greater than 1, then
- $$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}.$$
22. Graph the sets of points whose co-ordinates satisfies the condition $2\pi/3 \leq \theta \leq 5\pi/6$ (no restriction on r).
23. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.
24. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

*Answer any six questions.
Each question carries 5 marks.*

25. Compare $\int_1^{\infty} \frac{dx}{x^2}$ and $\int_1^{\infty} \frac{dx}{1+x^2}$ with limit comparison test.
26. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent or divergent.
27. Find the linearization of the function $f(x, y) = x^2 + y^2 + 1$ at $(0, 0)$.
28. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s$ and $y = r + s$.
29. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
30. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = e^{3x+4y} \cos 5z$.

31. Find the Maclaurin series for the function $f(x) = xe^x$.
32. Does series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ convergent.
33. Find the surface area generated by revolving the curves $x = t + \sqrt{2}$, $y = \frac{t^2}{2} + \sqrt{2}t$, $-\sqrt{2} \leq t \leq \sqrt{2}$ about y -axis.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any two questions.

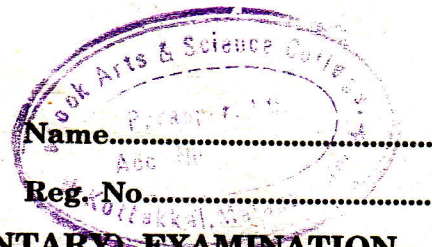
Each question carries 10 marks.

34. Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.
35. Find the points of intersection of $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.
36. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the functions's local and absolute extrema values.

(2 × 10 = 20 marks)

C 25917

(Pages : 3)



SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION
APRIL 2017

(UG—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Answer all questions.

1. Integrate $\operatorname{sech}^2\left(x - \frac{1}{2}\right)$.
2. $\tanh^2 x = 1 - \underline{\hspace{2cm}}$.
3. Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
4. $\lim_{n \rightarrow \infty} \frac{\ln(n)}{n} = \underline{\hspace{2cm}}$.
5. Give an example of a constant sequence.
6. State n^{th} Root Test.
7. Find the Taylor polynomial of order zero generated by $f(x) = \sin x$ at $x = \frac{\pi}{4}$.
8. If $f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$. Find the series for $f^4(x)$.
9. Define Absolute convergence.
10. If $f(x, y) = x \cos y + y e^x$ find $\frac{\partial^2 f}{\partial x^2}$.
11. Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.
12. Differentiate $\tanh \sqrt{1+t^2}$.

(12 × ¼ = 3 weightage)

Turn over

Answer all the questions.

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.

14. Evaluate $\int_{-\pi/4}^{\pi/4} \cosh(\tan \theta) \sec^2 \theta \, d\theta$.

15. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

16. Evaluate $\int_{-\infty}^{\infty} \frac{2x \, dx}{(x^2 + 1)^2}$.

17. Given $a_1 = 2$, $a_{n+1} = (-1)^{n+1} \frac{a_n}{2}$. Write the first four terms of the sequence.

18. For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely.

19. Graph the set of points whose polar co-ordinates satisfy the conditions $r \leq 0$ and $\theta = \frac{\pi}{4}$.

20. Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$.

21. Find an equation for the hyperbola with eccentricity $\frac{1}{2}$ and directrix $x = 2$.

(9 × 1 = 9 weightage)

Answer any five questions.

22. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.
23. Find the derivative of $f(x, y) = x e^y + \cos(x, y)$ at the point $(2, 0)$ in the direction of $A = 3i - 4j$.
24. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point $(3, 2)$.
25. Find :

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

26. Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.
27. Find the Taylor series generated by $f(x) = x^4 + x^2 + 1$ at $a = 2$.
28. Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$.

(5 × 2 = 10 weightage)

Answer any two questions.

29. Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \dots + \frac{1}{n^p} + \dots$ (p in a real number) converges if $p > 1$ and diverges if $p \leq 1$.
30. Find an upper bound $|E|$ of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R: |x - 2| \leq 0.1, |y - 1| \leq 0.1$ for $f(x, y) = x^2 - 3xy + 5$.
31. Find the area inside the smaller loop of $r = 2 \cos \theta + 1$.

(2 × 4 = 8 weightage)

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS—UG)

Complementary Course

MAT 2C 02—MATHEMATICS



Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)*Answer all questions.*

1. Find the derivative of y with respect x , where $y = \ln (\sinh x)$.
2. Evaluate $\int_5^2 \frac{d x}{1-x^2}$.
3. Find the value of $\int \frac{d u}{\sqrt{a^2+u^2}}$ when $a > 0$.
4. Write the formula for the length of the curve $x = g(y)$, $c \leq y \leq d$.
5. Write the limit comparison test for improper integrals.
6. Show that $\sum_{n=1}^{\infty} \frac{n+1}{n}$ diverges.
7. Find the Maclaurin series for the function e^{-x} .
8. Replace the following Cartesian equation by equivalent polar equation.
 $xy = 2$.
9. Find an equation for the hyperbola with $\frac{3}{2}$ eccentricity and directrix $x = 4$.
10. Evaluate $\lim_{(x,y) \rightarrow (0,1)} \frac{x-xy+3}{x^2 y+5xy-y^3}$.
11. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, where $f(x, y) = x^2 - xy + y^2$.
12. If $w = f(x, y, z)$, $x = g(r, s)$, $y = h(r, s)$ and $z = k(r, s)$ write $\frac{\partial w}{\partial s}$.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)*Answer any nine questions.*

13. Find the volume of the solid generated by revolving the region bounded by the lines

$$y = 2, x = 0 \text{ and the curve } y = 2\sqrt{x}.$$

14. Find the length of the curve $y = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

15. Find the area of the surface generated by revolving the curve $x = \frac{y^3}{3}, 0 \leq y \leq 1$ about the y -axis.

16. Evaluate $\int \tanh \frac{x}{7} dx$.

17. Investigate the convergence of $\int_0^{\frac{\pi}{2}} \tan \theta d\theta$.

18. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{4n}$.

19. Find the Maclaurin series for the function $\frac{1}{1-x}$.

20. Find the polar equation for the circle $x^2 + (y - 3)^2 = 19$.

21. Find the directrix of the parabola $r = \frac{25}{10^{-5} \cos \theta}$.

22. What point satisfies the equations $r = 2, \theta = \frac{\pi}{4}$?

23. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ where $f(x, y) = \frac{1}{x+y}$.

24. State chain rule for two independent variables and three intermediate variables.

(9 × 2 = 18 marks)

Part C (Short Essays)*Answer any six questions.*

25. Find the volume of the solid generated by revolving the region bounded by the Curve $x = \frac{\sqrt{2y}}{y^2 + 1}$ and the lines $x = 0$ and $y = 1$.
26. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1, 0 \leq x \leq 1$.
27. Evaluate $\int_0^{\ln 2} 4e^{-\theta} \sinh \theta d\theta$.
28. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.
29. Find the sum of the series $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$.
30. (a) Graph the curve $r = 1 - \cos \theta$.
- (b) Show that the point $\left(2, \frac{\pi}{2}\right)$ lie on the curve $r = 2 \cos 2\theta$.
31. Find the points of the intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.
32. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$.
33. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $x^2 + y^2 = r - s, y = r + s$.

(6 × 5 = 30 marks)**Turn over**

Part D (Essay Type)*Answer any two questions.*

34. (a) Show that if u is a differentiable function of x whose values are greater than 1, then :

$$\frac{d}{dx} (\cosh^{-1} u) = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}$$

(b) Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

35. (a) Find all the second order partial derivatives of $f(x, y) = x^2 y + \cos y + y \sin x$.

- (b) Draw the tree diagrams and chain rules for the derivatives $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for

$$z = f(x, y), x = g(t, s), y = h(t, s).$$

36. (a) Find a polar equation of the conic with $e = \frac{1}{5}$, one focus at origin and directrix $y = -10$ corresponding to that focus.

- (b) Sketch the circle $r = 2a \sin \theta$. Give polar co-ordinates for the centers and identify the radius.

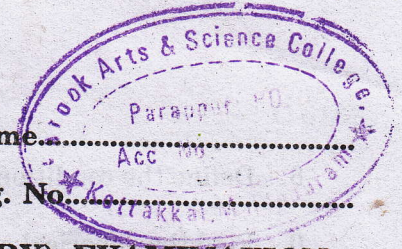
(2 × 10 = 20 marks)

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(Pages : 4)

Name.....

Reg. No.....



**SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION
MAY 2016**

(UG—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

2a

Time : Three Hours

Maximum : 30 Weightage

Unit I

Answer all questions.

1. $\frac{d}{dx}(\operatorname{csch}^{-1} x)$ is equal to :

(a) $\frac{1}{|x|\sqrt{x^2+1}}$

(b) $\frac{-1}{|x|\sqrt{x^2+1}}$

(c) $\frac{1}{x\sqrt{1-x^2}}$

(d) $\frac{-1}{x\sqrt{x^2-1}}$

2. Write $\sinh^{-1} x$ as a logarithmic function.

3. $\int \coth 5x \, dx$ is equal to :

(a) $\log \sinh 5x + c$

(b) $\log \cosh 5x + c$

(c) $\frac{1}{5} \log \sinh 5x + c$

(d) $\frac{-1}{5} \log \cosh 5x + c$

4. Write the n^{th} term of the sequence $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

5. Find $\lim_{x \rightarrow \infty} \left(\frac{\ln x}{x} \right)$.

6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{5(-1)^n}{4^n}$.

7. Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+5} \right)$.

Turn over

8. Define the conditional convergence of the series Σa_n .
9. Find the spherical co-ordinate equation of the sphere $x^2 + y^2 + (z - 1)^2 = 1$.
10. Find the domain and range of the function $w = \sin xy$.

11. Find $\lim_{(x,y) \rightarrow (0,1)} \left(\frac{x - xy + 3}{x^2 y + 5xy - y^3} \right)$.

20

12. Find $\frac{\partial^2 w}{\partial x \partial y}$ if $w = xy + \frac{e^4}{y^2 + 1}$.

(12 × ¼ = 3 weightage)

Unit II

Answer any nine questions.

13. Differentiate $t^2 \tan h \frac{1}{t}$ w.r. to t .
14. Find $\int \operatorname{csch}^2 (5 - x) dx$.
15. Find $\int_{-\infty}^{\infty} \frac{2x}{(x^2 + 1)^2} dx$.
16. Does $\sum_{n=1}^{\infty} \frac{\log n}{n^{3/2}}$ converge? Why?
17. For what value of x the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$ converges?
18. Find the Taylor series expansion of $f(x) = x^2 - 2x + 4$ at $x = 2$.
19. Find the polar equation of the circle $x^2 + (y - 3)^2 = 9$.
20. Find the centre and radius of the circle $r = 4 \sin \theta$.

21. Find $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} \right)$.

2C

22. Find f_x, f_y, f_z if $f(x, y, z) = 1 + xy^2 - 2z^2$.

23. Define the linearization of $f(x, y)$ at (x_0, y_0) .

24. Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

(9 × 1 = 9 weightage)

Unit III

Answer any five questions.

25. Evaluate $\int_0^1 \frac{2 dx}{\sqrt{3+4x^2}}$.

26. Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

27. Using geometric series prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

28. Find the Maclaurin's series for $\cos x$.

29. Find the points of intersection of the pair of curves $r = 4 \cos \theta, r = 1 - \cos \theta$.

30. Find the tangent plane to the surface $z = x \cos y - y e^x$ at $(0, 0, 0)$.

31. Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at $(1, 1)$.

32. Find the area of the region enclosed by $r = 2(1 + \cos \theta)$.

(5 × 2 = 10 weightage)

Turn over

Unit IV

Answer any two questions.

33. (a) State and prove the Leibnitz theorem for the alternating series $u_1 - u_2 + u_3 - u_4 + \dots$

(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.

2d

34. Find the points of intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$.

35. Find the region enclosed by the cardioid $r = 2 \cos \theta + 1$.

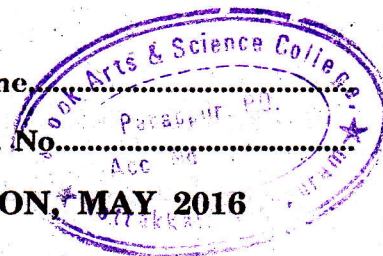
(2 × 4 = 8 weightage)

C 5622

(Pages : 3)

Name.....

Reg. No.....



SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2016

(CUCBCSS-UG)

Complementary Course

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all questions.

1. Define a smooth curve.
2. Find $\frac{d}{dt} (\tanh \sqrt{1+t^2})$.
3. Find the first four terms of the sequence $a_n = (-1)^{n+1} \frac{1}{n}, n \geq 1$.
4. Give an example of a sequence which has no upper bound.
5. State Leibnitz's theorem for the convergence of sequence.
6. Show that $(2, 3\pi/4)$ lies on the curve $r = 2\sin 2\theta$.
7. Write the formula for finding the length of the curve in polar co-ordinates.
8. Find the Cartesian equation of the surface $z = r^2$.
9. Define level surface of f .
10. When we say that a function f is continuous?
11. Which order of differentiation will calculate f_{xy} faster, x first or y first for $f(x, y) = x \sin y + e^y$.
12. Define $\cosh x$.

(12 × 1 = 12 marks)

Part B

Answer all questions.

13. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

14. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$.

Turn over

15. Find the volume of the solid generated by revolving the region between the curve :

$$y = \sqrt{x}, 0 \leq x \leq 4 \text{ and the } x\text{-axis.}$$

16. Find the n^{th} term of the sequence 1, -4, 9, -16, 25, ...

16

17. Replace $r \cos \theta = -4$ to the equivalent Cartesian equation.

18. Graph the set of points whose polar co-ordinates satisfy $1 \leq r \leq 2, \theta = \pi/4$.

19. Find all polar co-ordinates pairs which label the same as $(2, -\pi/3)$.

20. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - y^2}{x - y}$.

21. Find f_x if $f(x, y) = 2y/(y + \cos x)$.

22. Find the length of the curve $r = 1 - \cos \theta$.

23. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

24. Show that $\lim_{(x,y) \rightarrow (3,-4)} \sqrt{x^2 + y^2} = 5$.

(9 × 2 = 18 marks)

Part C

Answer any six questions.

25. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the x -axis to generate a solid. Find the volume of the solid by Washer Method by explaining steps in detail.

26. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

27. Show that $\frac{d}{dx} (\sinh^{-1}(u/a)) = \frac{1}{\sqrt{a^2 + u^2}} \frac{du}{dx}$.

28. Express 5.232323... as the ratio of two integers.

29. Check the convergence of $\sum a_n$ where $a_n = \begin{cases} n/2^n, & n \text{ odd} \\ 1/2^n, & n \text{ even} \end{cases}$

30. Graph the curve $r = 1 - \cos\theta$.

31. Find the equivalent Cartesian equation of $r = \frac{4}{2\cos\theta - \sin\theta}$.

32. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

33. Find $\partial z / \partial x$ if $yz - \ln z = x + y$ defines z as a function of two independent variables x and y and the partial derivative exists.

(6 × 5 = 30 marks)

Part D

Answer any two questions.

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq 1/2$ about the x -axis.

35. Find the radius and interval of convergence of $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \sum_{n=1}^{\infty} n! x^n$.

36. (a) Write the chain rule and draw the tree diagram for finding $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ if $w = x^2 + y^2, x = r - s,$
 $y = r + s.$

(b) Using Implicit differentiation, find dy/dx if $x + \sin y - 2y = 0.$

(2 × 10 = 20 marks)

C 83001

(Pages : 3)

Name.....

Reg. No.....



SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2015

(CUCBCSS—UG)

Complementary Course

Mathematics

MAT 2C 02—MATHEMATICS

4a

Time : Three Hours

Maximum : 80 Marks

Part A

Answer all questions.

1. Define a smooth curve.
2. Write down the relation connecting $\sin x$ and $\sinh x$.
3. Evaluate $\int_0^1 \frac{dx}{\sqrt{x}}$.
4. Give an example of a non-decreasing sequence.
5. State Sandwich theorem for the sequence.
6. Define absolute convergent sequence.
7. Find the equation for a hyperbola with eccentricity $= 3/2$ and directrix $x = 2$.
8. What is the formula in polar co-ordinates for the area of the surface generated by revolving the curve about the x -axis.
9. Find the equation of the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.
10. Define level surface of f .
11. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x - xy + 3}{x^2 + 5xy + y^3}$.
12. Write down the chain rule for finding dw/dt if $w = f(x, y, z)$ is differentiable and all x, y, z are differentiable functions of t .

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** questions.

13. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.
14. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \leq x \leq 1$.
15. Find the area under the curve $y = 1/\sqrt{x}$ from $x = 0$ to $x = 1$.
16. Show that $\lim_{n \rightarrow \infty} k = k$.
17. Find $\lim_{n \rightarrow \infty} 1/2^n$.
18. Graph the set of points whose polar coordinates satisfy $1 \leq r \leq 2, 0 \leq \theta \leq \pi/2$.
19. Find all Cartesian equation of $r \cos \theta = -4$.
20. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y - 2x + 2}{x - 1}$.
21. Find f_x if $f(x, y) = x^2 + 3xy + y - 1$.
22. Find the length of the curve $r = 1 - \cos \theta$.
23. Find the length of curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.
24. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

(9 × 2 = 18 marks)

Part C

Answer any **six** questions.

25. Compare $\int_1^{\infty} \frac{dx}{x^2}$ and $\int_1^{\infty} \frac{dx}{1+x^2}$.

26. Find the lateral surface area of the cone generated by revolving the line segment $x = 1 - y$, $0 \leq y \leq 1$ about y -axis.
27. Find the length of $y = x^{3/2}$ from $x = 0$ to $x = 4$.
28. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.
29. Find the Taylor series generated by $f(x) = x^3 - 2x + 4$ about $a = 2$.
30. Graph the curve $r^2 = 4 \cos \theta$.
31. Find the area of the region lie inside $r = 1$ and outside $r = 1 - \cos \theta$.
32. Show that $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches to $(0, 0)$.
33. Find dw/dt at $t = 0$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

(6 × 5 = 30 marks)

Part D*Answer any two questions.*

34. Write down the shell formula. Using this find the volume of the solid generated for the following problems.
- (a) The region bounded by $y = \sqrt{x}$, the x -axis and the line $x = 4$ revolved about x -axis.
- (b) The region in the first quadrant bounded by $y = x^2$, y -axis and the line $y = 1$ revolved about $x = 2$.
35. Define radius and interval of convergence. Investigate the convergence of $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$, $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.
36. (a) Write the chain rule and draw the tree diagram for finding $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$ if $w = x^2 + y^2$
 $x = r - s$, $y = r + s$.
- (b) Using Implicit differentiation, find dy/dx if $x^2 + \sin y - 2y = 0$.

(2 × 10 = 20 marks)

C 82136

(Pages : 3)

Name.....

Reg. No.....

**SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT)
EXAMINATION, APRIL/MAY 2015**

(UG—CCSS)

Complementary Course—Mathematics

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Objective Type Questions : (Answer *all* questions.)

1 Differentiate $\coth 5x$.

2 Show that $\sinh 2x = 2\sinh x \cosh x$.

3 Integrate $\operatorname{sech}^2\left(x - \frac{1}{2}\right)$.

4 Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

5 Give an example of a constant sequence.

6 $\lim_{n \rightarrow \infty} \sqrt[n]{n^2} = \underline{\hspace{2cm}}$.

7 State Alternating Series Test (Leibniz's Theorem).

8 Write the series for $\ln(1+x)$.

9 Find the Taylor polynomial of order 0 generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.

10 $|r| = 1$ is the equation for a $\underline{\hspace{2cm}}$.

11 Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$.

12 Define gradient vector.

(12 × ¼ = 3 weightage)

Turn over

II. Short Answer Type Questions : (Answer all *nine* questions).

13 Find f_x if $f(x, y) = \frac{2y}{y + \cos x}$.

14 Find an equation for the hyperbola with eccentricity $\frac{1}{2}$ and directrix $x = 2$.

15 Graph the set of points whose polar co-ordinates satisfy the conditions $-3 \leq r \leq 2$ and $\theta = \frac{\pi}{4}$.

16 Find the Maclaurin's series for $f(x) = \frac{1}{1+x}$.

17 For what values of x do the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely.

18 Given $a_1 = a_2 = 1$, $a_{n+2} = a_{n+1} + a_n$. Write the first 4 terms of the sequence.

19 Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

20 Evaluate $\int_1^2 \frac{\cosh(\ln t)}{t} dt$.

21 Use the definition of $\cosh x$ and $\sinh x$ to show that $\cosh^2 x - \sinh^2 x = 1$.

(9 × 1 = 9 weightage)

III. Short Essay Questions. (Answer any *five* questions).

22 Is the area under the curve $y = \frac{\ln x}{x^2}$ from $x = 1$ to $x = \infty$ finite. If so, what is it?

23 Investigate the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n! n!}$$

24 Find the Taylor series generated by :

$$f(x) = x^4 + x^2 + 1 \text{ at } a = -2.$$

25 Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

26 Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is continuous at every point except the origin.

27 Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

28 Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$:

- (a) Increases most rapidly ; and
 (b) Decreases most rapidly at the point (1, 1).

(5 × 2 = 10 weightage)

IV. Essay Questions. (Answer any two questions) :—

29 Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + z \sin z$ at the point (2, 1, 0). Find an upper bound for the error occurred in replacing f by L over the rectangle $R : |x - 2| \leq 0.01$, $|y - 1| \leq 0.02$, $|z| \leq 0.01$.

30 Find the length of the cardioid $r = 1 - \cos \theta$.

31 Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

(2 × 4 = 8 weightage)

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Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

(U.G.—CCSS)

Complementary Course

MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

Unit I

Answer all twelve questions.
Each question carries $\frac{1}{4}$ weightage.

1. Tanh x is equal to :

(a) $\frac{e^x - e^{-x}}{2}$.

(b) $\frac{e^x + e^{-x}}{2}$.

(c) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$.

(d) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$.

2. Write $\operatorname{csch}^{-1} x$ as a logarithmic function :

3. Find $\frac{d}{dx} \left(6 \sinh \frac{x}{3} \right)$.

4. Find $\int_0^1 \frac{1}{\sqrt{x}} dx$.

5. The n^{th} term of the sequence
0, 3, 8, 15, 24

6. Find $\lim_{n \rightarrow \infty} \sqrt[n]{3n}$.

7. Find the sum of the series $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$.

8. Test the convergence of the series

$$1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$$

Turn over

Unit IV

7d

Answer any two questions.
Each question carries 4 weightage.

33. (a) Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.

(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{4^n (n!)^2}{(2n)!}$.

34. Find the length of the cardioid $r = 1 - \cos \theta$.

35. Find the linearization of $f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$ at the point (3, 2).

(2 × 4 = 8 weightage)

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(Pages : 3) ⁺

Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL/MAY 2013

(CCSS)

MM 2C 02—MATHEMATICS

3a

Time : Three Hours

Maximum : 30 Weightage

I. Objective type questions. Answer *all* questions, weightage $\frac{1}{4}$ each :

- 1 Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
- 2 Investigate the convergence of $\int_0^1 \frac{1}{1-x} dx$.
- 3 Find $\lim_{n \rightarrow \infty} \frac{\ln(n^2)}{n}$.
- 4 Define the convergence of a sequence.
- 5 Find a formula for the n^{th} term of the sequence 1, -4, 9, -16, 25, ...
- 6 Define the alternating series test.
- 7 The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$ is _____.
- 8 Graph the set of points whose polar co-ordinates satisfy the conditions $r \leq 0$ and $\theta = \frac{\pi}{4}$.
- 9 Show that the point $\left(2, \frac{\pi}{2}\right)$ lies on the curve $r = 2 \cos 2\theta$.
- 10 Find $\frac{\partial t}{\partial x}$ at (4, -5) if $f(x, y) = x^2 + 3xy + y - 1$.
- 11 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.
- 12 If x, y and z are independent variables and $f(x, y, z) = x \sin(y + 3z)$ find $\frac{\partial f}{\partial z}$.

(12 \times $\frac{1}{4}$ = 3 weightage)

II. Short answer type questions. Answer all *nine* questions, weightage 1 each :

- 13 Find the derivative of $y = 2\sqrt{t} \tanh \sqrt{t}$ with respect to t .

Turn over

- 14 Show that the series $\sum_{n=1}^{\infty} n^2$ diverges.
- 15 Test the convergence of $\sum_{n=1}^{\infty} \frac{n+1}{n}$. 3b
- 16 Examine the convergence of $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$.
- 17 For what values of x does the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ converges.
- 18 Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical co-ordinates.
- 19 Define the gradient of $f(x, y)$.
- 20 Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at $(-2, 1)$.
- 21 Find $\frac{\partial z}{\partial x}$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivatives exists.

(9 × 1 = 9 weightage)

III. Short essay questions. Answer any *five* questions, weightage 2 each :

- 22 Compare $\int_1^{\infty} \frac{dx}{x^2}$ and $\int_1^{\infty} \frac{dx}{1+x^2}$ with the limit comparison test.
- 23 Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge ?
- 24 Prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
- 25 Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
- 26 Verify that $W_{xy} = W_{yx}$ if $W = e^x + x \ln y + y \ln x$.
- 27 Find $\frac{\partial w}{\partial r}$ when $r = 1, s = 1$ if $W = (x + y + z)^2, x = r - s, y = \cos(r + s), z = \sin(r + s)$.
- 28 Find the length of the asteroid $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq 2\pi$

(5 × 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions, weightage 4 each :

29 Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$ (p is a real number) converges if $p > 1$ and diverges if $p \leq 1$.

30 Find the Taylor series and the Taylor polynomial generated by $f(x) = e^x$ at $x = 0$.

31 Find the linearization $L(x, y)$ of $f(x, y) = e^x \cos y$ at $P_0(0, 0)$ and find an upper bound for $|E|$ of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle $R: |x| \leq 0.1, |y| \leq 0.1$.

(2 × 4 = 8 weightage)