D 100186 **(Pages : 4)** **Name.....**

Reg. No.....

SIXTH SEMESTER UG (CUCBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MAT 6B 01—COMPLEX ANALYSIS

(2018 Admissions only)

Time : Three Hours Maximum Marks : 120

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. For the complex numbers $z_0 = a + ib$ and $z_1 = c + id$, $\lim_{z \to z_0} z_1 =$ _____.
2. State True or False : The function

$$f(z) = \begin{cases} \dfrac{\operatorname{Re} z}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is continuous at $z = 0$.

3. Find the derivative of $f(z) = z^2$.
4. Compute the principal value of $\log_e z$ when $z = 1 + i$.
5. Find all the roots of the equation $\tan z = 1$.
6. Evaluate $\int_i^{1+4i} z^2 dz$.
7. If C is the simple closed contour given by the circle $|z| = 2$, then $\int_C dz =$ _____.
8. Integrate $\dfrac{z^2+1}{z^2-1}$ in the contour clock wise sense around a circle of radius 1 with centre at the point $z = \dfrac{1}{2}$.
9. State Maximum modulus principle.
10. If the sequence $\sqrt[n]{|a_n|}, n = 1, 2, \ldots$ converges with the limit L > 0, then the radius of convergence of the power series

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n = a_0 + a_1 (z - z_0) + a_2 (z - z_0)^2 + \ldots$$

is _____.

Turn over

11. Define removable singular point.

12. State Cauchy's Residue Theorem.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Prove that if the limit of the function $f(z)$ exists at a point z_0, then it is unique.

14. Verify Cauchy-Riemann equations for the function f given by $f(z) = \frac{x - iy}{x^2 + y^2}$.

15. Examine the differentiability at the origin of the function f given by $f(z) = |z|^2$.

16. Examine the analyticity of $f(z) = \cosh x \cos y + i \sinh x \sin y$.

17. Find the real part of e^{-3z}.

18. Prove that $\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_1$.

19. Evaluate $\int_0^1 (1 + it)^2 dt$.

20. If $f'(z) = 0$ everywhere in a domain D, then prove that $f(z)$ must be constant throughout D.

21. If M be any non-negative cosntant such that $|f(z)| \leq M$ everywhere on a contour C and L is the length of C, then prove that $\left| \int_C f(z) dz \right| \leq ML$.

22. Evalaute $\int_C \frac{dz}{z - a}$, where C is the circle $|z - a| = r$ oriented in the positive direction.

23. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ on its circle of convergence.

24. Find the expansion for $f(z) = z^2 e^z$.

25. What kind of singularity the function $\frac{\cot \pi z}{(z - a)^2}$ has at the point $z = a$.

26. Find the residue of the function $f(z) = \frac{z}{z^4 + 4}$ at the isolated singular point $1 + i$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 *marks.*

27. If $w = f(z) = \bar{z}$, show that $\frac{dw}{dz}$ does not exist at the origin.

28. Evaluate $\oint_C \frac{dz}{z^2+9} dz$, where C is the unit circle.

30. Using principle of deformation of paths, evaluate $\int_C \frac{1}{z} dz$ where C is any positively oriented closed contour surrounding the origin.

31. Find an analytic function whose real part $e^x(x\cos y - y\sin y)$ and which takes the value e at $z = 1$.

32. If R_1 and R_2 are the radius of convergences of the power series $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ respectively, show that the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n b_n z^n$ is R_1R_2.

33. Find series representation of $f(z) = \frac{-1}{(z-1)(z-2)}$.

34. If C is a simple closed contour containing the origin, show that $\frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}} dz = \frac{a^n}{n!}$.

35. Show that if $s > 0, a > 0$, then prove that $\int_{-\infty}^{\infty} \frac{e^{1sx}}{x^2+a^2} dx = \frac{\pi}{a} e^{-as}$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) Find the Laurent series of $f(z) = \frac{1}{1-z^2}$ with centre at $z = 1$.

(b) If R be the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ what is the radius of convergence of $\sum_{n=0}^{\infty} a_n^2 z^n$.

Turn over

37. (a) Evaluate $\oint_C \sec z\,dz$, where C is the unit circle.

(b) If $w = f(z) = \bar{z}$, show that $\frac{dw}{dz}$ doesn't exist at any point.

38. (a) Prove by contour integration that $\int_0^{2\pi} e^{\cos\theta}\cos(n\theta - \sin\theta)\,d\theta = \frac{2\pi}{n!}$ (n positive integer).

(b) Show that $\int_{-\infty}^{\infty} \frac{\cos 3x}{\left(x^2+1\right)^2}\,dx = \frac{2\pi}{e^3}$.

(2 × 13 = 26 marks)

C 20208 **(Pages : 3)** **Name.....**

Reg. No.....

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. A complex function $f(z)$ is analytic at a point $z = z_0$ if ————.
2. An analytic function with constant argument is ————.
3. Give an example of a complex function which is Differentiable at a point but not analytic at that point.
4. Find the simple poles, if any for the function $f(z) = \frac{(z+2)^2}{z^5(x^4-1)}$.
5. Write the polar form of Cauchy-Riemann equations.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh $(2z)$ is ————.
8. Fill in the blanks : $f(z) = e^z$ is periodic with period = ————.
9. A point $z = z_0$ is a singular point of a complex function $w = f(z)$ if ————.
10. Fill in the blanks : $\text{Res}_{z=\pi/2} \tan z =$ ————.
11. The solution of the equation $e^z = -3$ is ————.
12. The principal value of i^i is ————.

$(12 \times 1 = 12$ marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that $f(z) = \sin z$ is analytic for all z.
14. Find the principal value of $(1-i)^{1+i}$.
15. Show that $\tanh^{-1}(z) = \frac{1}{2}\log\frac{1+z}{1-z}$.
16. Show that the zeros of an analytic function are isolated.

Turn over

17. Determine and classify the singular points of $f(z)=\dfrac{(z+2)^2}{z^5\left(z^4-1\right)}$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty}\dfrac{n!z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Discuss the nature of singularities if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solution of $e^z = 2$.

22. Find the residue of $f(z) = \cot(z)$ at its poles.

23. Evaluate $\oint_C \dfrac{\sin\pi z}{\left(z^6\right)}dz$ around C = $|z| = 1$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Evaluate $\oint_{|z|=2}\overline{z}dz$.

26. Illustrate entire function by an example.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \dfrac{1}{(z-1)(z-2)}$ around the simple closed curve C = $|z| = 4$.

28. Determine the nature of the singularities of the function $f(z) = \sec(1/z)$.

29. Expand $f(z)=\dfrac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0<|z+1|<2$.

30. If $f(z)=u(x,y+iv(x,y))$ is analytic in a domain D, then prove that its component functions are harmonic in D.

31. Find the analytic function $f(z)$ is terms of z, if $u(x,y)=\mathrm{Re}(f(z))=e^x(x\cos y-y\sin y)$.

32. Show that the function $f(z)=\sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. State and prove Morera's theorem.

34. Show that the derived series has the same radius of convergence as the original series.

35. Evaluate $\oint_{|z-2|=2}\dfrac{z^3}{(z-1)^4(z-2)(z-3)}dz$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) State and prove Cauchy's integral formula.

    (b) Prove or disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

    (b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{a + \cos\theta} d\theta$.

    (b) Evaluate $\int_0^{\infty} \dfrac{1}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)

C 1247 (Pages : 4) Name.....

Reg. No.....

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. An analytic function with constant modulus is _____.
2. Fill in the blanks : The real part of $f(z) = \ln(z)$ is _____.
3. Fill in the blanks : $f(z)$ is singular at infinity if _____.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^2(z^2+1)}$.
5. Define harmonic function.
6. Give an example of a complex function which is nowhere analytic.
7. Fill in the blanks : $\text{Res}_{z=0} \cot z =$ _____.
8. State Morera's theorem.
9. Solution of $\sinh(z) = 0$ is _____.
10. The radius of convergence of the series $\sum_{n=0}^{\infty} \dfrac{(z-i)^n}{n!}$ is _____.
11. Fill in the blanks : For $f(z) = \dfrac{\tan z}{z}$; $z = 0$ is _____.
12. Find the value of $\text{Log}(-10i)$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Prove or disprove : $|\sin(z)| \leq 1$ for all complex numbers z. Justify your claim.

14. Verify Cauchy-Riemann equations for the function $f(z) = \ln z$.

15. If $f(z) = u + iv$ is analytic then derive the condition under which $v + iu$ is analytic .

16. Show that the poles of an analytic function are isolated.

17. Evaluate $\oint_{|z|=1} \bar{z}dz$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!(z-i)^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^2$ when the contour of integration is the circle with centre at origin and radius 5 units.

20. Locate the singular points if any, of $f(z) = \frac{1}{\sin(\pi/z)}$ in the complex plane.

21. Find all the solutions of $e^z = -10$.

22. Evaluate the integral of $f(z)$ around the circle $|z| = 2$, where $f(z) = \frac{\cos z}{z^2}$.

23. Find the Residue of $\tan z$ at $z = \pi/2$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \cos(z)$.

26. Find the principal value of $(1-i)^{1+i}$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Evaluate $\oint_C \frac{z^2+1}{(z^2-1)}$, where $C = |z-1| = 1$.

28. Show that $\tan^{-1}(z) = \frac{i}{2}\log\frac{i+z}{i-z}$.

29. Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor series about $z = 1$.

30. State and prove Liouvillie's theorem.

31. Find the harmonic conjugate of $u(x, y) = \text{Re}(f(z)) = \frac{x}{x^2+y^2}$.

32. Derive the polar form of Cauchy-Riemann Equations.

33. State and prove the Cauchy's Integral formula.

34. Find an analytic function in terms of z, whose real part is $e^x(x\cos y - y\sin y)$.

35. Find the residues of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) State and prove Laurents theorem.

(b) Expand $f(z) = \frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0 < |z+1| < 2$.

37. (a) State and prove Cauchy's Residue theorem.

(b) Evaluate $\oint_{|z|=1} \frac{\exp z}{\cos \pi z} dz$.

38. (a) Evaluate using the method of residues $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta$.

(b) Evaluate $\int_0^{\infty} \frac{x^2}{\left(x^2 + a^2\right)^2} dx, a > 0$.

(1 × 15 = 15 marks)

C 80270 (Pages : 4) Name.....

Reg. No.....

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define an analytic function and give an example of a function which is not analytic at the origin.
2. Fill in the blanks : The locus of the points z satisfying $|z+2i|^2 = 2|i+1|$ is a/an ———.
3. Verify whether $f(z) = \bar{z}/z$ is analytic or not at $z = 0$?
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^3(z^2+9)}$.
5. Is $u(x, y) = x^2 - y^2 + xy$ a harmonic function ? Justify your claim.
6. Define essential singularity of a complex valued function.
7. Fill in the blanks : The real part of log $(2z)$ is ———.
8. Write the formula for the evaluation of n^{th} derivative of an analytic function with full assumptions involved.
9. Solve for z : $3z - 1 = 2\bar{z}$.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum a_n z^{3n}$.
11. What do you mean by a Jordan curve ?
12. Find the value of i^i + Log $(2i)$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Evaluate the line integral of $f(z) = z^2$ over the line joining $2i$ to $i - 1$.

14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

15. Show that $\tan^{-1}(z) = \frac{i}{2} \log \frac{i+z}{i-z}$.

16. Show that the poles of an analytic function are isolated.

17. Which one is bigger : $\| z_1| - |z_2 \|$ *or* $|z_1 - z_2|$. Prove your claim.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!\, z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 2$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the circle containing zero inside it.

23. Using the definition of continuity show that $\sin z$ is continuous through out the plane.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \sin(z)$.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $|z - \pi/2| \leq 3$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

28. Determine the nature of the singularities of the function $f(z) = \cos(1/z)$. Does this function have zeros ? Find them if any.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(2z-3)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z)) = e^x(x \cos y - y \sin x)$. You should express $f(z)$ finally only in terms of z.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. Prove the formulas for conversion Cauchy-Riemann equation into the corresponding polar form in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-3|/[z-2] = 2$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Derive the formula involving integral to compute the first derivative of an analytic function by stating all the assumptions involved.

(b) Prove or disprove : $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$ for all complex numbers z_1 and z_2.

37. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \dfrac{\sin z}{(z-1)^2 (z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{5 + 2\cos\theta} d\theta$.

(b) Evaluate $\int_0^{\infty} \dfrac{1}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)

C 60049 (Pages : 4) Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Solve for $z : 5z = 2i\bar{z}$.
2. State Cauchy-Goursat theorem with full assumptions involved.
3. Verify whether $f(z) = \bar{z}$ is analytic or not at $z = 0$.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-2)^2}{z^3(z^2+1)}$.
5. Is $u(x, y) = x^2 + y^2 - xy$ a harmonic function ? Justify your claim.
6. Define a simply connected domain.
7. Fill in the blanks : The real part of $\cosh(2z)$ is ————.
8. Fill in the blanks : The locus of the points z satisfying $|z - 2i| = 2|i - 1|$ is a/an ————.
9. If an infinite series of complex numbers converges, then show that its n^{th} term converges to zero.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum n^2 a_n z^n$.
11. What do you mean by a contour ?
12. Find i^i.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Find the real and imaginary parts of the function $f(z) = \log(z)$.

14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

15. Show that $\tan^{-1}(z) = \frac{i}{2} \log \frac{i+z}{i-z}$.

16. Show that the zeros of an analytic function are isolated.

17. Evaluate the line integral of $f(z) = z^2$ over the line joining i to $2i - 1$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!\, z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^2$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \cos(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 3$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the ellipse containing zero inside it.

23. Using the definition of continuity show that the composite of two continuous functions is continuous.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Which one is bigger : $\| z_1| - |z_2 \|$ or $|z_1 - z_2|$. Prove your claim.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $|z - \pi/2| \leq 2$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Determine the nature of the singularities of the function $f(z) = \sin(1/z)$. Does this function have zeros ? Find them if any.

28. Evaluate $\oint_C \frac{z}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(z-1)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the harmonic conjugate of $u(x, y) = e^x (x \cos y - y \sin x)$ and find the corresponding analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z))$. Express the result for $f(z)$ in terms of z only.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. How do we convert the Cauchy-Riemann equation into the corresponding polar form ? Prove the formulas for conversion in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-1| + |z-2| = 3$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ at its poles.

37. (a) State and prove Liouvillies theorem.

(b) Prove or disprove : $|\sin(z)| \leq 1$ for all complex numbers z. Justify your claim.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta, a > b > 0.$

(b) Evaluate $\int_0^{\infty} \frac{1}{x^4 + a^4} dx, a > 0.$

(2 × 13 = 26 marks)

D 40900 (Pages : 3) Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION MARCH 2018

(CCSS)

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

Each question carries weightage ¼.

1. What is the value $\lim_{z\to\infty} \frac{3z+4i}{2z+4}$.

2. The imaginary part of an analytic function is harmonic. Say True or False.

3. What is the real part of $\frac{1}{z}$.

4. $|e^{i\pi}|$ is ————————.

   (a) 1. (b) -1.

   (c) π. (d) e.

5. Express the cosine function in terms of exponential function.

6. Given an example of an complex function which is entire and bounded.

7. What is the parametric form of the circle $|z| = r$.

8. Define removable singularity of $f(z)$.

9. For the function $f(z) = \frac{\sin z}{z^2}$, $z = 0$ is :

   (a) Simple pole. (b) Removable singularity.

   (c) Double pole. (d) Essential singularity.

Turn over

10. Identify the poles of $\frac{z+2}{z^2+1}$.

11. For an analytic function $f(z)$ if $u_x = 2x$ and $u_y = -2y$ then find $f^1(z)$.

12. Write the formula for residue of $f(z)$ at a pole $z = a$ of order 3.

(12 × ¼ = 3 weightage)

Part B

Answer all **nine** *questions.*
Each question carries weightage 1.

13. Show that $u = \log\sqrt{x^2+y^2}$ is harmonic.

14. Show that the function $f(z) = xy + iy$ is nowhere differentiable.

15. If a function $f(z)$ is analytic in a domain D then show that its real part is harmonic in D.

16. Find the principle value of $(-i)^i$.

17. Test whether $f(z) = y$ is analytic or not.

18. State Laurent's theorem.

19. Evaluate $\int_i^{i/2} e^{\pi z}\, dz$.

20. Find the order of the pole $f(z) = \frac{1-\sin z}{z^5}$ at $z = 0$.

21. Find residue of the function $\frac{1}{z^2+a^2}$ at $z = ai$.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions from* **seven.**
Each question carries weightage 2.

22. Show that $u(x, y) = x^3 - 3xy^2$ is a harmonic function and find a harmonic conjugate $v(x, y)$ of u.

23. Prove Cauchy-Riemann equations in polar form for the function $f(z)$.

24. Find constants a and b so that the function $f(z) = a(x^2 - y^2) + ibxy + c$ is differentiable at every point.

25. Find all roots of the equation $\cosh z = \frac{1}{2}$.

26. Using Cauchy's residue theorem evaluate $\int \frac{3\cos z}{2i - 3z}\, dz$ around the unit circle.

27. Expand $\frac{1}{z}$ into a Taylor's series about the point $z = 1$ and determine the region of convergence.

28. If $f(z)$ and $f^-(z)$ are analytic in a region D then show that $f(z)$ is constant in that region.

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions.*
Each question carries weightage 4.

29. Expand $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in a Laurent's series valid for : (i) $0 < |z-1| < 4$; (ii) $|z-1| > 4$.

30. State and prove Cauchy's integral formula.

31. Find $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

(2 × 4 = 8 weightage)

C 21071 (Pages : 3) Name..... Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries **1** *mark.*

1. Define the limit of a complex valued function.
2. Write Cauchy's integral formula with full assumptions involved.
3. Verify whether $f(z) = 2i\bar{z}$ is analytic or not at $z = 0$?
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^3(z^2+2)}$.
5. Is $u(x, y) = x^2 + y^2 + xy$ a harmonic function ? Justify your claim.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh $(2z)$ is ———.
8. Fill in the blanks : The locus of the points z satisfying $|z + 2i|^2 = 2|i + 1|$ is a/an ———.
9. Solve for z : $5iz = 2\bar{z}$.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum a_n z^{2n}$.
11. What do you mean by a simply connected domain ?
12. Find the value of $i^i + \log(i)$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries **4** *marks.*

13. Which one is bigger : $\big||z_1| - |z_2|\big|$ or $|z_1 - z_2|$. Prove your claim.
14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

Turn over

15. Show that $\tan^{-1}(z) = \frac{i}{2}\log\frac{i+z}{i-z}$.

16. Show that the poles of an analytic function are isolated.

17. Evaluate the line integral of $f(z) = z^2$ over the line joining $2i$ to $i - 1$.

18. Find the radius of convergence of the power series :

$$\sum_{n=0}^{\infty} \frac{n!z^n}{n^n}.$$

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 2$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the circle containing zero inside it.

23. Using the definition of continuity show that $\sin z$ is continuous through out the plane.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \sin(z)$.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $|z - \pi/2| \leq 3$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

28. Determine the nature of the singularities of the function $f(z) = \cos(1/z)$. Does this function have zeros ? Find them if any.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(2z-3)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z)) = e^x(x \cos y - y \sin x)$. You should express $f(z)$ finally only in terms of z.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy-Riemann equations are satisfied at that point.

33. Prove the formulas for conversion Cauchy-Riemann equation into the corresponding polar form in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-2|-|z-1|=2$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) State and prove Liouvillies theorem.

(b) Prove *or* disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{3+2\cos\theta}\, d\theta$.

(b) Evaluate $\int_0^{\infty} \dfrac{x^2}{x^4+a^4}\, dx, a>0$.

(2 × 13 = 26 marks)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, MARCH 2017

(UG–CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. The real part of an analytic function is harmonic. Say True *or* False.
2. What is the value $\lim_{z\to\infty}\frac{2z^3-1}{z^2+1}$?
3. What is the imaginary part of z^3?
4. $e^{2+3\pi i}$ is _____.

   (a) e^2 ; (b) e ; (c) 1 ; (d) $-e^2$.
5. Express the sin hx in terms of exponential function.
6. Give an example of a complex function which is entire and bounded.
7. What is the parametric form of the circle $|z| = 4$?
8. Find pole of the function $f(z)=\frac{z-1}{z^2-1}$.
9. Determine the singular points of $\frac{z+1}{z^2-2x}$.
10. Write the formula for residue of $f(z)$ at a pole $z = a$ of order m.
11. For an analytic function $f(z)$ if $v_x = -2y$ *and* $v_y = 2x$ then find $f^1(z)$.
12. Find the value of $\oint_{|z|=1}\frac{dz}{z+2}$.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *nine questions.*
Each question carries 1 *weightage.*

13. Show that the function $f(z) = e^z$ is differentiable everywhere.
14. Show that $u = e^x \cos y$ is harmonic.

Turn over

15. Prove that an analytic function whose imaginary part is constant is itself a constant.

16. Test whether $f(z)=\frac{1}{z}$ is analytic or not.

17. Show that $\cosh(z+\pi i)=\cosh z$.

18. State Taylor's theorem.

19. Evaluate $\int_C \frac{\sin z}{z-\pi/2}\,dz$ where C is the unit circle.

20. Find the residue of $f(z)=\frac{1}{2z+3}$ at $z=-\frac{3}{2}$.

21. Find the singularities of the function $ze^{1/z}$.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Show that $u(x, y)=2x-x^3+3xy^2$ is a harmonic function and find a harmonic conjugate $v(x, y)$ of u.

23. Prove that the families of level curves $u(x, y)=c_1$ and $v(x, y)=c_2$ are orthogonal for an analytic function $f(z)=u(x, y)+iv(x, y)$.

24. Prove that $f(z)=z\mathrm{Im}z$ is differentiable only $z=0$ and find $f^1(0)$.

25. Using Cauchy's residue theorem evaluate $\int \frac{e^{2z}}{(z+1)^3}\,dz$ around the circle $|z|=\frac{3}{2}$.

26. Find all roots of the equation $\sinh z = i$.

27. Expand $\sin z$ into a Taylor's series about the pont $z=\frac{\pi}{4}$ and determine the region of convergence.

28. Evaluate $\int_C \frac{3z-4}{z(z-1)}\,dz$ where C is the circle $|z| = 2$ using Cauchy's integral formula.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. Expand $f(z)=\frac{z}{(z-1)(2-z)}$ in a Laurent's series valie for (i) $|z| < 1$; (ii) $0 < |z-2| < 1$; (iii) $|z-1| > 1$.

30. Using contour integration find the value of $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

31. Evaluate using (i) Cauchy's integral formula ; (ii) residue theorem $\int_C \frac{z+1}{z^2+2z+4}$ where C is the circle $|z+1+i| = 2$.

(2 × 4 = 8 weightage)

C 1743

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2016

(UG–CCSS)

Core Course—Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions :

1 State Fundamental theorem of Algebra.

2 Verify Cauchy–Riemann equations for the function :

$f(z) = \sin x \cosh y + i \cos x \sinh y$.

3 Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.

4 Define Pole.

5 Find the poles of $f(z) = \dfrac{z+2}{(z^2+1)^3 (z-2)}$ and write their orders.

6 Define Removable singularity.

7 State Cauchy's Integral Formula.

8 Prove that an analytic function $f(z) = u + iV$ is constant if its real part is constant.

9 $\cos(iy) =$ _____.

10 $\int_C \dfrac{1}{z-1}\, dz =$ _____ where C is the circle $|z| = \dfrac{1}{2}$.

11 If $e^z = e^{x+iy}$ then $|e^z| =$ _____.

12 Define the Residue at a pole.

(12 × ¼ = 3 weightage)

Turn over

II. Answer all *nine* questions :

13 Prove that $\left| e^{-2z} \right| < 1$ if and only if real part of $z > 0$.

14 Find the harmonic conjugate of $u = x^2 - y^2$.

15 Show that $f'(z)$ does not exist at any point of z when $f(z) = \text{Re}\,(z)$.

16 Find the principal values of $(-i)^i$.

17 Evaluate $\int_C \frac{1}{z-3} dz$ where C is $|z| = 4$.

18 Evaluate $\int_C \frac{1}{z-b} dz$ where C is the circle $|z-b| = r$.

19 Prove that $\exp(4 \pm 3\pi i) = -e^4$.

20 Prove that $\sinh(-z) = -\sinh z$.

21 Show that $f'(z)$ does not exist at any point of z when $f(z) = \bar{z}$.

(9 × 1 = 9 weightage)

III. Answer any *five* questions :

22 Using Cauchy's Residue Theorem evaluate $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$.

23 Prove that differentiable functions are continuous.

24 Obtain the Taylor series expansion $\cos z$ at $z = \frac{\pi}{2}$.

25 Obtain Laurent series expansion of $\frac{1}{z(z-1)^2}$ about $z = 1$.

26 State and prove Liouville's theorem.

27 Prove that $f(x) = \sqrt{|xy|}$ is not analytic at the origin even if the Cauchy-Riemann equations are satisfied at that point.

28 State and prove Cauchy's Residue Theorem.

(5 × 2 = 10 weightage)

IV. Answer any *two* questions :

29 Evaluate $\int_{-\infty}^{\infty} \frac{\cos 2x}{(x^2+1)(x^2+4)}\,dx$.

30 Obtain Laurent series expansion of $\frac{3z+7}{(z+2)(z+3)}$ in $2 < |z| < 3$.

31 State and prove Maximum Modulus Principle.

(2 × 4 = 8 weightage)

C 80026

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 30 Weightage

Answer **all** *questions.*

1. Define a harmonic function.
2. Sin $(i\,y)$ = _____.
3. Given $f(z) = \dfrac{z^3\,(z-1)^4\,(z+1)^5}{(z+2)^2\,(z+6)}$. Write the order of the zero $z = 1$.
4. State Cauchy's residue theorem.
5. Cos $h^2\,z$ – sin $h^2\,z$ = _____.
6. Define isolated singularity.
7. If $e^z = e^{\,x+iy}$ then arg (e^z) = _____.
8. $\int\limits_c \dfrac{z^2}{z-3}\,dz$ = _____ where c is the circle $|z| = 2$.
9. Prove that $u = x^2 - y^2$ is harmonic.
10. State Liouville's theorem.
11. Verify Cauchy-Riemann equation for the function $f(z) = (3x+y) = i\,(3y-x)$.
12. Define Pole.

(12 × ¼ = 3 weightage)

Answer all **nine** *questions.*

13. Find the harmonic conjugate of $u = x^4 - 6x^2y^2 + y^4$.
14. Prove that $f'(z)$ does not exist at any point if $f(z) = z - \bar{z}$.

Turn over

15. Prove that $f(z) = \frac{\bar{z}}{z}$ does not have a limit when $z \to 0$.

16. Evaluate $\int_c \frac{e^z}{z^5}\, dz$ where c is $|z| = 1$.

17. Give an example of removable singularity.

18. Evaluate the residue at the pole $z = 1$ of $f(z) = \frac{z+1}{z^2(z-1)}$.

19. Determine the order of zero of the function $z\left(e^z - 1\right)$ at $z = 0$.

20. Find the principal value of $(-i)^i$.

21. Prove that $\exp\left(\frac{2+\pi i}{4}\right) = \sqrt{e}\left(\frac{1+i}{\sqrt{2}}\right)$.

(9 × 1 = 9 weightage)

Answer any **five** *questions.*

22. Find an analytic function $f(z) = u + iv$ given $u = \sin x \cos hy + 2 \cos x \sin hy + x^2 - y^2 + 4xy$.

23. Prove that $\text{Log}(1-i) = \frac{1}{2}\text{Log}\, 2 - i\frac{\pi}{4}$.

24. State and prove Cauchy's Integral formula.

25. Using Taylor's series prove that

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}.$$

26. Using Cauchy's residue theorem evaluate :

$$\int_c \frac{z+1}{z^2}\, dz \text{ where c is } |z| = 1.$$

27. Prove that differentiable functions are continuous.

28. Let $f(z)$ be an analytic function such that $|f(z)| \leq A|z|$ for every z, where A is constant. Prove that $f(z) = a, z$ where a, is a complex constant.

(5 × 2 = 10 weightage)

Answer any **two** *questions.*

29. Obtain Laurent series expansion of $\frac{1}{(z-1)(z-2)}$ in $|<|z|<2$.

30. Prove that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & : z \neq 0 \\ 0 & : z = 0 \end{cases}$ is not analytic at $z = 0$. But Cauchy-Riemann equations are satisfied at that point.

31. Evaluate $\int_0^\infty \frac{1}{(x^2+1)^2}\,dx$.

(2 × 4 = 8 weightage)

C 40395 (Pages : 3) Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2013

(CCSS)

Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *the questions.*

1. What is the value of :

$$\lim_{z\to\infty}\left(\frac{2z+i}{z+1}\right).$$

2. Find the imaginary part of $z + i$.

3. $f(z)=u(x,y)+iV(x,y)$ is analytic in a domain D if and only, V is _____ of u.

4. What is the real part of e^z ?

5. What is the period of $\sin z$?

6. Express $\cos x$ in terms of e^{ix}.

7. The value of $e^{i\pi}$ is :

   (a) 1. (b) e.

   (c) -1. (d) 0.

8. The value of $\int_{|z|=1}\frac{dz}{z-4}$ is _____.

   (a) $2\pi i$. (b) 0.

   (c) 2π. (d) 4.

9. The region of convergence of the series :

$$1+\frac{z}{1!}+\frac{z^2}{2!}+\ldots+\frac{z^n}{n!}+\ldots \text{ is } \_\_\_\_\_.$$

10. What is the sum function of the series $1 + z + z^2 + \ldots + z^{n-1} + \ldots$

11. For $f(z)=\frac{z^2-4}{z-2}$, $z=2$ is a _____.

   (a) Removable singular point. (b) Pole of order 1.

   (c) Pole of order 2. (d) Essential singular point.

Turn over

12. Identify the poles of :

$$\frac{2z}{z^2-1}.$$

(12 × ¼ = 3 weightage)

Section B

Answer all **nine** *questions.*

13. Show that $f'(z)$ does not exist at any point for $f(z)=2x+ixy^2$.

14. If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, prove that $f(z)$ is a constant throughout D.

15. Show that :

$$\text{Log}\,(1-i)=\frac{1}{2}\ln 2-\frac{\pi}{4}i.$$

16. Show that :

$$|\sinh z|^2=\sinh^2 x+\sin^2 y.$$

17. State Cauchy's integral formula.

18. Evaluate :

$$\int_C \frac{dz}{z-1}, \text{ where C is } |z-1|=2.$$

19. Show that when $z\neq 0$

$$\frac{e^z}{z^2}=\frac{1}{z^2}+\frac{1}{z}+\frac{1}{2!}+\frac{z}{3!}+\ldots$$

20. State Cauchy's residue theorem.

21. For the function $f(z)=\dfrac{1-\cosh z}{z^3}$ determine the order of the pole at $z=0$ and the corresponding residue.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*

22. Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic and find a harmonic conjugate $v(x,y)$ of u.

23. If $f(z)=u(x,y)+iV(x,y)$ is analytic in a domain D, prove that u and V are harmonic in D.

24. Find the general solution of the equation :

$$\cosh z = \frac{1}{2}.$$

25. Evaluate $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is the circle $|z| = 2$.

26. State and prove Liouville's theorem.

27. State and prove Taylor's theorem.

28. Evaluate $\int_0^\infty \frac{dx}{x^2+1}$.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*

29. State and prpove maximum modulus principle.

30. Give two Laurent series expansions in powers of z for the function :

$$f(z) = \frac{1}{z(1+z^2)},$$

the regions of validity of expansions.

31. Using residues, evaluate :

$$\int_{-\infty}^{+\infty} \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)} \quad (a > b > 0)$$

(2 × 4 = 8 weightage)

D 40042 (Pages : 3) Name.....

Reg. No.....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2018

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. What is the period of $f(z) = e^{2iz}$?
2. Give an example of an entire function.
3. What is the complex, form of Cauchy-Riemann equations ?
4. Define entire function.
5. What are the singularities of $f(z) = |z|^2$.
6. State Cauchy's integral formula.
7. State Morera's theorem.
8. State Gausss mean value theorem.
9. Define Residue of a complex function.
10. Give an example of an essential singularity.
11. Define removable singularity of a complex function.
12. What, is the residue at a removable singularity ?

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of* ***fourteen*** *questions.*
Each question carries 4 *marks.*

13. Define Analytic functions. Give an example.
14. Show that $(z) = \sin x \cos h\, y + i \cos x \sin h\, y$ is an entire function.

Turn over

15. If $f = u + iv$ is analytic, then show that u and v are harmonic.

16. Prove of disprove : $\text{Log}(a^b) = b\text{Log}(a)$. where Log is the principal branch of logarithm and $a, b \in \mathbb{C}$.

17. State Cauchy's integral formula and its extension.

18. Find all the values of $\sin^{-1}(-i)$.

19. Is Cauchy-Goursat theorem valid for arbitrary connected domains? Prove your claim.

20. Using Liouville's theorem prove the fundamental theorem of algebra.

21. Evaluate $\int_C \frac{e^{-z}\,dz}{z - i\,\pi/2}$, where C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$.

22. Suppose $z_n = x_n + iy_n$, $n = 1, 2, 3, ...$, and $S = X + iY$. If $\sum_{n=1}^{\infty} z_n = S$, then show that $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

23. State Laurent theorem.

24. Give an example of a non isolated singularity.

25. Using Cauchy's integral theorem, evaluate $\int_C \frac{z+1}{z^2 - 2z}\,dz$, where C is the circle $|z| = 3$ in the positive sense.

26. Define pole and its order of a complex function.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Derive Cauchy-Riemann equations.

28. If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then show that u and v are harmonic in D.

29. Find the harmonic conjugate of $u(x, y) - \sin h\, x \sin y$.

30. Using contour integration, evaluate $\int_C z^{1/2}\, dz$, where C is the path given by $z = 3e^{i\theta}$, $0 \le \theta \le \pi$.

31. State and prove the principle of domination of paths.

32. Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.

33. If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 (z_1 \neq z_0)$ then show that it is absolutely convergent at each point z in the open disk $|z - z_0| < R_1$, where $R_1 = |z_1 - z_0|$.

34. State- and prove Cauchy's residue theorem.

35. Using residue evaluate $\int_0^{\infty} \frac{dx}{x^4 + 1}$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 *marks.*

36. State and prove reflection principle.

37. (a) State and prove Liouville's Theorem.

(b) Using Liouville's theorem, prove fundamental theorem of algebra.

38. (a) Show that the power series $S(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ is analytic each point, z interior to the circle of convergence of that series.

(b) Find the residue of $\frac{1}{z + z^2}$ at $z = 0$.

(2 × 13 = 26 marks)