D 50198 **(Pages : 5)** **Name..**

Reg. No......................................

FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Solve the initial value problem $dy/dt = -y + 5, y(0) = y_0$.
2. Verify that $y = e^t$ is a solution of the differential equation $y'' - y = 0$.
3. Find the order of the differential equation $y'' + 7y' + 5y = 0$.
4. Show that $(2x + 3) + (2y - 2)y' = 0$ is an exact equation.
5. Find the Wronskian of the functions $y_1(t) = \cos t, y_2(t) = \sin t$.
6. Write the characteristic equation of the differential equation $y'' + y' + y = 0$.
7. Transform the equation $u'' + 0.125u' + u = 0$ in to a system of first order equations.
8. Find $\mathcal{L}\left(t^2 + 1\right)$.
9. Define the Heaviside function.
10. Find $\mathcal{L}^{-1}\left(\frac{s}{s^2 + 4}\right)$.
11. Find the fundamental period of the function $f(x) = \sin 3x$.
12. Show that $f(x) = x^2 \cos 4x$ is an even function.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20° throughout and whose ends are maintained at 0° for all time $t \geq 0$.

14. Show that the wave equation $a^2 u_{xx} = u_{tt}$ can be reduced to the form $u_{\zeta\eta} = 0$ by the change of variables $\zeta = x - at, \eta = x + at$.

15. Determine the co-efficients in the Fourier Series of $f(x)$ where $f(x) = \begin{cases} -x, & -2 \leq x < 0 \\ x, & 0 \leq x < 2 \end{cases}$ and $f(x+4) = f(x)$.

16. Using Convolution Integral, find the inverse transform of $H(s) = \dfrac{s}{s^2\left(s^2 + a^2\right)}$.

17. Define the unit impulse function δ and show that $\mathcal{L}(\delta(t)) = 1$.

18. Consider the function $f(t) = \begin{cases} 2, & 0 \leq t < 4, \\ 5, & 4 \leq t < 7, \\ -1, & 7 \leq t < 9, \\ 1, & t \geq 9. \end{cases}$

    Express $f(t)$ in terms of $u_c(t)$.

19. Prove the following : If $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots\ldots, \mathbf{x}^{(n)}$, are solutions of $\mathbf{x}' = \mathbf{P}(t)\mathbf{x}$ on the interval $\alpha < t < \beta$, then in this interval $W\left[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots\ldots, \mathbf{x}^{(n)}\right]$ either is identically zero or else never vanishes.

20. Given that $y_1(t) = t^{-1}$ is a solution of $2t^2 y'' + 3ty' - y = 0, t > 0$, find a fundamental set of solutions.

21. Find a general solution of the initial value problem $y'' + y' + 9.25y = 0, y(0) = 2, y'(0) = 8$.

22. State The Existence and Uniqueness Theorem.

23. Solve the differential equation $\left(y \cos x + 2xe^{y}\right) + \left(\sin x + x^{2}e^{y} - 1\right)y' = 0.$

24. Solve the initial value problem $y' = y^2, y(0) = 1$ and determine the interval in which the solution exists.

25. Show that the equation $y'' = \dfrac{x^2}{1 - y^2}$ is separable, and then find an equation for its integral curves.

26. Solve $\dfrac{dy}{dx} = \sin(x + y)$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks each.*

27. (a) Let $y = y_1(t)$ is a solution of $y' + p(t)\,y = 0$ and let $y = y_2(t)$ be a solution of $y' + p(t)\,y = g(t)$. Show that $y(t) = y_1(t) + y_2(t)$ is also a solution of $y' + p(t)\,y = g(t)$.

    (b) Find the value of b for which he given equation $\left(xy^2 + bx^2\, y\right)dx + (x + y)\,x^2\,dy = 0$ is exact.

28. Find an integrating factor for the equation $\left(3xy + y^2\right) + \left(x^2 + xy\right)y' = 0$ and hence solve it.

29. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t} + 2\sin t$.

30. Suppose that a mass of weighing 10 lb stretched in a spring 2 in. If the mass is displaced an additional 2 in. and is then set in motion with an initial upward velocity of 1 ft/s, determine the position of the mass at an later time. Also determine the period , amplitude and phase of the potion.

31. If the function f defined by $f(t) = \begin{cases} \sin t, 0 \le t < \pi/4, \\ \sin t + \cos(t - \pi/4), t \ge \pi/4 \end{cases}$.

    Find $\mathcal{L}(t)$.

32. (a) Show that if c is a positive constant, then $\mathcal{L}(ct) = \frac{1}{c} \mathrm{F}(s/a), s > ca$.

(b) Show that if a and b are constants with $a > 0$, then $\mathcal{L}^{-1} \mathrm{F}\left[(as+b)\right] = \frac{1}{a} e^{-bt/a} f\left(\frac{t}{a}\right)$.

33. State and prove The Convolution Integral theorem for Laplace Transform.

34. Find a Fourier sine series for $f(x) = \begin{cases} x, 0 < x < \pi/2 \\ \pi - x, \pi/2 < x < \pi \end{cases}$.

35. Show that $\int_{-\mathrm{L}}^{\mathrm{L}} \sin\left(\frac{m\pi x}{\mathrm{L}}\right) \sin\left(\frac{n\pi x}{\mathrm{L}}\right) dx = \begin{cases} 0, m \neq n \\ \mathrm{L}, m = n \end{cases}$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 *marks each.*

36. (a) Consider a vibrating string of length L = 30 that satisfy the wave equation $4u_{xx} = u_{tt}, 0 < x < 30, t > 0$. Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position

$$u(x, 0) = f(x) = \begin{cases} x/10, 0 \leq x \leq 10, \\ (30 - x)/20, 10 < x \leq 30. \end{cases}$$

Find the displacement $u(x, t)$ of the string and describe its motion through one period.

(b) Find the Fourier series for the function $f(x) = \begin{cases} x, 0 < x < \pi \\ 2\pi - x, \pi < x < 2\pi \end{cases}$ and hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

37. (a) Using Laplace transform, find the solution of the initial value problem $y'' + y = \sin 2t,\ y(0) = 2,\ y'(0) = 1$.

(b) Find the inverse Laplace Transform of $\dfrac{1}{s\left(2s^2 + s + 2\right)}$.

38. (a) State and Prove Abel's Theorem.

(b) Using the Method of Variation of Parameters, solve $y'' + 4y = 3\csc t$.

(2 × 13 = 26 marks)

255712

D 30180 **(Pages : 4)** Name.......................................

Reg. No...................................

FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : The function $f(t)$ for which $\mathcal{L}\{f(t)\}=1$ is ________________.

2. What can you say about the product of two odd functions ? Prove your assertion.

3. Write down the Euler formulas for computing the coefficients of a periodic function $f(t)$ of period 2L.

4. Write down the differential equation whose solution is $y = c_1e^{2t} + c_2e^{-2t}$.

5. Define the Wronskian of the functions $y_1(t)$ and $y_2(t)$.

6. Find inverse Laplace transform of $\dfrac{2(s-2)}{s+2}$.

7. What is the integrating factor of $x(x-2)\dfrac{dy}{dx} + y = \sin x$?

8. Find the order of the p.d.e. $\left(\dfrac{\partial u}{\partial x}\right)^7 - x^{-1/2}\, y^7\, \dfrac{\partial^2 u}{\partial y^2} + x^2y^3\, \dfrac{\partial^2 u}{\partial x^2} = 7$.

9. Find the fundamental solutions of $y'' - 2y = 7t$.

Turn over

255712

10. Write one dimensional wave equation ?

11. What is inverse Laplace transform of $t^{-1/2}$?

12. Solve the system : $\frac{dy}{dt} = x, \frac{dx}{dt} = 2.$

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Applying method of separation of variables, solve the BVP : $\frac{\partial y}{\partial t} = 2\frac{\partial y}{\partial x}$ with boundary conditions $y(x, 1) = y(x, 0) = 0.$

14. Write down the fundamental solutions of $y'' - 1 = y$ and find their Wronskians.

15. Convert $y'' + 2y' + 3y = 0$ into a system of first order equations.

16. Find the inverse Laplace transform of $\log\left((s-1)/(s-2)\right)$.

17. Calculate the Laplace transform of Dirac's impulse function.

18. Solve the Cauchy's differential equation $t^2x'' - tx' - x = 0.$

19. State the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.

20. Calculate a_n for $f(x) = 2x^2, x \in [-1, 1]$ in its Fourier series expansion.

21. Show that the Laplace transform is linear.

22. Solve : $\frac{dy}{dx} = \tan(x + y + 1).$

23. Solve : $y' - y = 0$ using Laplace transform.

24. Solve the system : $\frac{dy}{dt} = 2x - y, \frac{dx}{dt} = x - 2y.$

25. State Abel's theorem.

26. Find the second order p.d.e. for which $y = f(x + ct) + g(x - ct)$ is a solution.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Express the function $f(t) = \begin{cases} t \sin t, & \text{if } 0 \le t < \pi/2 \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0, & \text{elsewhere} \end{cases}$ in terms of combination of unit step functions and hence find its Laplace transform.

28. Evaluate the Laplace inverse transforms of $4 - \cot^{-1}(s/a)$ and $\frac{1}{(s^2 - 5s + 6)^2}$.

29. Find the solution by the checking the exactness of $(3y^2 - 2xy + 2)\,dx + (6xy - x^2 + y^2)\,dy = 0$.

30. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

31. A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high. Neglect air resistance, (a) Find the maximum height above the ground that the ball reaches, (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

32. Find the Fourier cosine series for the function $f(t) = \pi|t - \pi|, t \in [0, \pi]$.

33. Find the solution of the heat conduction problem :

$$25u_{xx} = u_t, 0 < x < 1, t > 0 \;;\; u(0, t) = 0, u(1, t) = 0, \; t > 0 \;;\; u(x, 0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

Turn over

34. State and prove convolution theorem for Laplace transforms.

35. Evaluate (i) $\mathcal{L}^{-1}\left(\frac{1-e^{-2s}}{2s}\right)$; and (ii) $\mathcal{L}\left(t^2 \cos 2t - te^t \sin t\right)$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Apply method of variation of parameters to solve : $y'' + y = \sec t$.

(b) Solve $(3x + y + 1)\,dx + (x + 3y + 1)\,dy = 0$.

37. (a) Find the two half range Fourier series of the function: $f(t) = |\sin t|, t \in [0, \pi]$ and draw the graphs of the corresponding periodic extensions.

(b) Find an expression for $\mathcal{L}\left(\frac{f(t)}{t}\right)$ in terms of $\mathcal{L}(f(t))$ and prove the same.

38. (a) Derive the d'Alemberts solution of one dimensional wave equation.

(b) Find the solution of the p.d.e. $\frac{\partial^2 u}{\partial x^2} = 2$.

(2 × 13 = 26 marks)

D 10233 **(Pages : 3)** **Name....................................**

Reg. No..................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. Prove that the product of two odd functions is an even function.
2. Prove $L^{-1}\{1\}$.
3. Write down the differential equation whose solution is $y = c_1 e^{5t} + c_2 e^{-2t}$.
4. Evaluate $W\left[e^{\mu\cos\lambda t}, e^{\mu\sin\lambda t}\right]$.
5. Compute $L\left\{t^2 e^{\lambda t}\right\}$.
6. Find the integrating factor of $(x-2)(x+1)\dfrac{dy}{dx} + 3y = x$.
7. Solve the system $\dfrac{dy}{dt} - x = 0, \dfrac{dx}{dt} - y = 0$.
8. Find the fundamental solutions of $y'' + 25y = t^{-1/2}$.
9. Find the value of b_n in the Fourier sine series expansion of 2π-periodic function $f(x) = -x, x \in [-\pi, \pi]$.
10. Write one dimensional heat equation with all the assumptions involved.
11. What do you mean by an exact differential equation ? Give an example.
12. Find the complementary function corresponding to $y'' - 2y' + 2y = t$.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Convert $y'' + 2y' = 0$ into a system of first order equations.

Turn over

14. Find the Fourier cosine series for the 2π-periodic function $f(x) = -x, x \in [-\pi, \pi]$.

15. Find the integrating factor for $(2x + 3y)\,dx + (2x - 3y)\,dy = 0$.

16. Find the inverse Laplace transform of $\log\left((s-a)/(s-b)\right)$.

17. Define unit step function and find its Laplace transform.

18. Solve : $t^2x'' - 2tx' - 3x = 0$.

19. Write the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.

20. Show that the inverse Laplace transform is linear.

21. State Abel's theorem.

22. Solve : $\dfrac{dy}{dx} = (3x + 2y + 1)^2$.

23. Evaluate $L\left\{te^t \cos 2t\right\}$.

24. Find the second order p.d.e. for which $y = \phi(x + at) + \psi(x - at)$ is a solution.

25. Solve the system : $\dfrac{dy}{dt} = x - y, \dfrac{dx}{dt} = x + y$.

26. Solve : $y' - 2y = 0$ using Laplace transform.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Express the function $f(t) = \begin{cases} t\sin t, & \text{if } 0 \le t < \pi/2 \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0, & \text{elsewhere} \end{cases}$ in terms of combination of unit step functions and hence find its Laplace transform.

28. Evaluate the Laplace inverse transforms of $4 - \cot^{-1}(s/a)$ and $\dfrac{1}{\left(s^2 - 5s + 6\right)^2}$.

29. Find the solution by the checking the exactness of $\left(3y^2 - 2xy + 2\right)dx + \left(6xy - x^2 + y^2\right)dy = 0$.

30. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

31. A ball with mass 0.15 kg. is thrown upward with initial velocity 20 m/s from the roof of a building 30 m. high. Neglect air resistance (a) Find the maximum height above the ground that the ball reaches ; (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

32. Find the Fourier cosine series for the function $f(t)=\pi|t-\pi|, t\in[0,\pi]$.

33. Find the solution of the heat conduction problem :

$$25u_{xx}=u_t, 0<x<1, t>0; u(0,t)=0, u(1,t)=0, t>0; u(x,0)=\sin(2\pi x)-\sin(5\pi x), 0\le x\le 1.$$

34. State and prove Convolution theorem for Laplace transforms.

35. Evaluate (i) $L^{-1}\left(\frac{1-e^{-s}}{s}\right)$ and (ii) get a formula for $L(f(t))$ where $f(t)$ is a periodic function of period T.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) Use method of separation of variables and solve the one-dimensional heat equation completely. State the assumptions involved therein explicity.

(b) Find the solution of the p.d.e. $\frac{\partial^2 u}{\partial y\,\partial x}=2x$.

37. (a) Solve the following differential equation in two ways, one of them must be using Laplace transform. $4y''-y=t, y(0)=1, y(1)=0$.

(b) Find the Fourier series of $f(x)=x^2, x\in[-2,2]$ treating it as a periodic function of period 4.

38. (a) Apply method of variation of parameters to solve : $y''-y=\sec t$.

(b) Solve $(2x+y+3)\,dx+(x-3y+2)\,dy=0$.

(2 × 13 = 26 marks)

D 90233 (Pages : 4) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS–UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : The solution of $y' = 2y, y(0) = 1$ is $y(t) =$ ———.

2. Write the wave equation and the conditions assumed for its D'Alemberts solution.

3. Find the general solution of $y'' - 2y = 0$.

4. Find the Laplace transform of cosh (2 *at*).

5. Write the Euler formulas for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period 2L.

6. Define an exact differential equation. Is $(2x + y)dy - (x - y)dx = 0$ exact ? Why ?

7. Solve the system : $\dfrac{dx}{dt} = 2y, \dfrac{dy}{dt} = -2x$.

8. Define Dirac's delta function and write its Laplace transform.

9. Give an example of a non-linear differential equation in the dependent variable y and the independent variable t of second order.

10. Show that $u(x,y) = f(x - ay) + g(x + ay)$ is a solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}.$$

11. Show that product of two odd functions is an even function.

12. Compute the Wronskian of the functions e^{3t} and e^{-3t}.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. $\left(x+e^{-y/3}\right)\frac{dy}{dx}=3, y(0)=0.$

14. Use Laplace transform to find the solution of $\frac{dy}{dt}=t, y(0)=1.$

15. Using convolution find the inverse Laplace transform of $\frac{1}{(s-2)(s-1)}.$

16. Show that any separable equation $M(x)+N(y)y'=0$ is also exact.

17. Solve : $t^2y''+ty'+y=0.$

18. Use method of variation of parameters to solve : $y''+4y=3\csc t.$

19. Compute the Wronskian of vectors $x^{(1)}(t)=\begin{pmatrix} t \\ 1 \end{pmatrix}$ and $x^{(2)}(t)=\begin{pmatrix} t^2 \\ 2t \end{pmatrix}.$

20. Find an integrating factor and solve it : $y'=e^{2x}+y-1.$

21. Find the formula for computing $\mathcal{L}\left(t^n\right)$ when n is an integer in terms of the gamma function.

22. Find the value of a_n, the Fourier coefficient of cos nx in the Fourier expansion of $f(x)=k$ for $x\in[-\pi,\pi].$

23. State Abels theorem.

24. Solve : $\left(3xy+y^2\right)+\left(x^2+xy\right)y'=0.$

25. Briefly describe about mathematical modelling by giving a physical example.

26. Find the longest interval in which the solution of the initial value problem :

$$\left(t^2-3t\right)y''+ty'-(t+3)y=0, y(1)=2, y'(1)=1$$ is certain to exist.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 marks.
All questions can be attended.
Overall Ceiling 45.

27. Determine whether the equation is exact or not and hence find its solution :

$$\left(3x^2 - 2xy + 2\right)dx + \left(6y^2x^2 + 3\right)dy = 0.$$

28. Find an integrating factor for the equation $\left(3xy + y^2\right) + \left(x^2 + xy\right)y' = 0$ and solve it.

29. Find the general solution of $y'' - 2y' + y = 2\cos(2t) - t^2$.

30. Find the Fourier cosine series expansion of $f(t) = |\,t\,| + t \in [0, \pi]$.

31. Find (a) $\mathcal{L}(2te^{-2t} - t^2\cos(2t))$ and (b) $\mathcal{L}^{-1}\left[\frac{\left(1-e^{-s}\right)}{s}\right]$.

32. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove that it is so.

33. Find the solution of the heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1,\ t > 0; u(0,t) = 0,\ u(1,t) = 0, t > 0; u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

34. (a) Show that $W\left(e^{\beta t}\cos\alpha t, e^{\beta t}\sin\alpha t\right) = \alpha e^{2\beta t}$ and (b) write the differential equation of second order for which $e^{\beta t}\cos\alpha t$ and $e^{\beta t}\sin\alpha t$ form a fundamental set of solutions.

35. A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m. high. Neglect air resistance :

    (a) Find the maximum height above the ground that the ball reaches.

    (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

(5 × 9 = 45 marks)

Section D

*Answer any **one** question.*
Each question carries 15 marks.

36. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

37. Find the solution of the initial value problem $y'' - 2y - 1 = 0, y(0) = 0, y'(0) = 1$ in two ways ; one of them must be using Laplace transforms.

38. Find the two half range expansions of the function :

$$f(x) = \begin{cases} \dfrac{2kx}{L} & , \text{if } 0 < x < L/2 \\ \dfrac{2k(L-x)}{L} & , \text{if } L/2 < x < L. \end{cases}$$

(1 × 15 = 15 marks)

D 70323 (Pages : 4)

Name..............................

Reg. No..............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. The inverse Laplace transform of the function $f(t) = 1$ is ————.

2. The integrating factor fot the linear differential equation $y' - \frac{1}{t} y = 0$ is ————.

3. Write the order of the differential equation :

$$\frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + 2y^2 = 0.$$

4. Show that the differential equation :

$(2xy + y - \tan y) + \left(x^2 - x\tan^2 y + \sec^2 y + 2\right)y' = 0$ is exact.

5. Solve $y'' + y = 0$.

6. Are the functions $e^{\pi t}$ and $\frac{1}{\pi}e^{\pi t}$ linearly independent ?

7. Find the Laplace Transform of $e^{at}\cos bt$.

8. Define step function.

9. If $f(x)$ is an even function, the co-efficient of sines in the Fourier series expansion of $f(x)$ is evaluated by the integral ————.

Turn over

10. What is the fundamental period of $\cos\left(\frac{\pi x}{3}\right)$.

11. If $f(x) = x + k$ is an odd function, the value of k must be ————.

12. What is the heat conduction equation ?

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Show that $u(x, y) = \cos x \cosh y$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$.

14. Find the solution of the initial value problem $y' = (1 - 2x)y^2, y(0) = \frac{1}{6}$.

15. Find the value of b for which the following equation is exact :

$$\left(xy^2 + b x^2 y\right) dx + (x + y) x^2 \, dy = 0.$$

16. Write the conditions for the existence of the Laplace transform of a function.

17. Find the Wronskian of the functions x and xe^x.

18. Find the general solution of $y'' + 2y' + 5y = 0$.

19. Find the Laplace transform of $f(t) = e^{\omega t}, t \geq 0$.

20. Find the Laplace transform of $f(t) = 5e^{-2t} - 3 \sin 4t, t \geq 0$.

21. Find the inverse Laplace transform of the function $\frac{1}{s^2 - 4s + 5}$.

22. Show that sum of two even functions is even.

23. Assuming the required equations, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

24. Find $L\left(e^{5t}\cos 3ht\right)$.

25. Find the inverse Laplace transform of the function $\ln\frac{s+a}{s+b}$.

26. Find a_0 for the periodic function :

$$f(x)=\begin{Bmatrix} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{Bmatrix}.$$

(10 × 4 = 40 marks)

Section C

*Answer any **six** out of nine questions.*
Each question carries 7 marks.

27. Show that the differential equation $\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$ homogeneous and hence solve.

28. Find an integrating factor for the equation $\left(3xy+y^2\right)+\left(x^2+xy\right)y'=0$ and then solve the equation.

29. Transform the equation $u''+2u'+2u=0$ into a system of first order equation.

30. State and prove Abel's Theorem.

31. Using the method of Laplace transform, solve :

$$y''-3y'+2y=4e^{2t},\ y(0)=-3,\ y'(0)=5.$$

32. State and prove the convolution theorem for Laplace transform.

33. Find the inverse transform of $F(s)=\frac{1-e^{-2s}}{s^2}$.

34. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5) ; y(0) = 0, y'(0) = 0.$$

35. Solve using the method of separation of variables :

$$\frac{\partial u}{\partial x} = a^2 \frac{\partial u}{\partial y}, u(x, 0) = 1, u(0, y) = -1.$$

(6 × 7 = 42 marks)

Section D

*Answer any **two** out of three questions.*
Each question carries 13 marks.

36. (a) Solve by method of variation of parameters :

$$y'' + y = \tan t, 0 < t < \frac{\pi}{2}.$$

(b) Find the general solution of $t^2 y'' - 4ty' + 6y = 0, t > 0$.

37. Let $f(x) = 1 - x^2$ if $-1 \leq x \leq 1$ and $f(x+2) = f(x)$. Then :

(a) Sketch the graph of the function f and state whether the function is even or odd.

(b) Find the Fourier series of f.

(c) Deduce that : $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

38. Derive the wave equation by stating the assumptions involved and find it's D'Alembert's solution.

(2 × 13 = 26 marks)

D 50602 **(Pages : 3)** Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCBCSS–UG)

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all the twelve** *questions.*
Each question carries 1 *mark.*

1. $L(t) =$ ———————.
2. Show that $y = e^{-t}$ is a solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$.
3. Write the 2-dimensional wave equation.
4. Show that the equation $\frac{ydx - xdy}{y^2}$ is exact.
5. Find the order of the differential equation : $\frac{t\,d^2y}{dx^2} + 2y = \cos 3t$.
6. Find $W\left(t, t^{-1}\right)$.
7. Write the characteristic equation of the differential equation $\frac{d^2y}{dt^2} - \frac{4\,dy}{dt} + 5y = 0$.
8. Find $L^{-1}\left[\frac{1}{s^2 + a^2}\right]$.
9. Find the general solution of the differential equation $\frac{d^2x}{dt^2} + 16x = 0$.
10. When we say that a function is odd ?
11. Solve : $\frac{dy}{dx} + y^2 = 0$.
12. Is $\begin{bmatrix} 1 \\ \alpha \end{bmatrix}$ is an eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Justify your answer.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Solve : $\frac{d^2y}{dt^2} - \frac{6\,dy}{dt} + 9y = 0.$

14. Find $L^{-1}\left[\frac{1}{s^2\left(s^2+4\right)}\right].$

15. Find the Fourier coefficients corresponding to the function $f(t) = 1, 0 \le t \le 2\pi.$

16. Find the interval in which the initial value problem $ty^1 + 2y = 4t^2, y(1) = 2$ has a unique solution.

17. Solve : $\left(y\cos x + 2x\,e^y\right) + \left(\sin x + x^2 e^y - 1\right)y^1 = 0.$

18. Given that Y_1 and Y_2 are solutions of the homogeneous equation $y'' + p(t)y' + q(t)y = 0.$ Show that $2y_1 + 3y_2$ is also a solution of $y'' + p(t)y' + q(t)y = 0.$

19. Show that $W\left(e^{\lambda t}\cos\mu t, e^{\lambda t}\sin\mu t\right) = \mu e^{2\lambda t}.$

20. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t}.$

21. Let n be a +ve integer ; show that $L\left(t^n\right) = \frac{n!}{s^{n+1}}, s > 0.$

22. Find the Fourier sine series for the function $f(t) = 1, 0 \le t \le \pi.$

23. What is "Linearization" ? Give an example.

24. Find the value of b for which the equation $\left(xy^2 + bx^2y\right)dx + (x+y)x^2dy = 0$ is exact and then solve using that value of b.

25. Find a differential equation whose general solution is $y = c_1e^{2t} + c_2e^{-2t}.$

26. Find the inverse transform of $H(s) = \frac{a}{s^2\left(s^2+a^2\right)}$ by convolution integral.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Find the general solution of $y'''' + y''' - 7y'' - y' + 6y = 0$. Also find the solution that satisfies the initial conditions $y(0) = 1, y'(0) = 0, y''(0) = -2$, and $y'''(0) = -1$.

28. Given that $y_1 = e^t, y_2 = te^t, y_3 = e^{-t}$ are solutions of the homogeneous equation corresponding to $y''' - y'' - y' + y = g(t)$, determine a particular solution in terms of an integral.

29. Using the method of undetermined coefficients, solve : $y'' + 4y = 3\sin 2t$.

30. Find the fundamental set of solutions for the differential equation $y'' - y = 0$ using the initial point $t_0 = 0$.

31. State and prove "Abel's theorem".

32. State and prove "The Convolution Integral Theorem.

33. Using Laplace transforms, solve the initial value problem $\frac{d^2y}{dt^2} + \frac{2\,dy}{dt} - 3y = \sin t$ given that $y(0) = y'(0) = 0$.

34. Evaluate : $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t}\, dt$.

35. Suppose that a mass weighing 10 *lb* stretches a spring 2 in. If the mass is displaced an additional 2 in. and is then set in motion with an initial upward velocity of 1 ft/sec. Determine the position of the mass at any later time. Also, determine the period, amplitude of the motion.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Solve : $y'' + 4y = 3\csc t$.

37. The motion of a spring-mass system is governed by the differential equation $u'' + 0.125\,u' + u = 0$, where u is measured in feet and t in seconds. If $u(0) = 2$ and $u'(0) = 0$, determine the position of the mass at any time. Also find the quasi frequency and the quasi period time at which the mass first passes through the equilibrium position.

38. Find the Fourier series to represent $f(x)$ in $[-\pi, \pi]$, given $f(x) = x + x^2$ in $-\pi < x < \pi$; and $f(x) = \pi^2$ when $x = \pm\pi$. Also deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$.

(2 × 13 = 26 marks)

C 30308 (Pages : 4) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. $L\left(e^{-t}\right) =$

2. Show that $y = \sin t$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

3. Find $w(\sin t, \cos t)$.

4. Write the 2-dimensional wave equation.

5. Find the order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y = 0$.

6. Write the characteristic equation of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 0$.

7. Show that the equation $ydx + xdy = 0$ is exact.

8. When we say that a function is periodic ?

9. Find the general solution of the differential equation $\frac{d^2x}{dt^2} + x = 0$.

10. Find $L^{-1}\left[\frac{1}{s+a} + \frac{1}{s-a}\right]$.

Turn over

11. Solve : $\frac{dy}{dx} + xy = 0$.

12. Show that the function $f(t) = t \sin t$ is even.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Solve : $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0$.

14. Find $L^{-1}\left[\frac{1}{s^2\left(s^2 + a^2\right)}\right]$.

15. Find the Fourier coefficients corresponding to the friction $f(t) = 2, 0 \le t \le 2\pi$.

16. Find the interval in which the initial value problem $ty^1 + 2y = 4t^2,\ y(1) = 2$ has a unique solution.

17. Show that $W\left(e^{2t}\cos 3t, e^{2t}\sin 3t\right) = 2e^{4t}$.

18. Find a particular solution of $y'' - 4y' - 4y = 2e^{2t}$.

19. Find the Laplace transform of $f(t) = \begin{cases} 1, 0 \le t < 1 \\ 0, 1 \le t < 2 \\ 1, 2 \le t < 3 \\ 0, t \ge 3 \end{cases}$.

20. Find the integrating factor of the equation $\left(3xy + y^2\right) + \left(x^2 + 2y\right)y' = 0$.

21. Given that Y_1 and Y_2 are solutions of the non-homogeneous equation $y'' + p(t)y' + q(t)y = g(t)$. Show that their difference $Y_1 - Y_2$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

22. Find the differential equation whose general solution is $y = c_1e^t + c_2e^{-2t} + c_3e^{-t}$.

23. Find the inverse transform of $\mu(s) = \frac{2}{s^2(s^2+4)}$ by convolution integral.

24. Solve the differential equation $\frac{dy}{dx} + 4y = 3t$ and find the particular solution whose graph contains the point (0, 1).

25. What is "Linearization" ? Give an example.

26. Find the value of b for which the equation $\frac{y^2\,dx + bxy\,dy}{y^4} = 0$ is exact and then solve using that value of b.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Solve the initial value problem : $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$ and determine the interval in which solution emits.

28. Using the method of indetermined coefficients, solve : $y'' + 9y = 2\sin 3t$.

29. Evaluate $\int_0^\infty \frac{e^{-t}\sin t}{t}\,dt$.

30. Using Laplace transforms, solve the initial value problem :

$$\frac{d^2y}{dt^2} + \frac{2dy}{dt} - 3y = \sin t \quad \text{given that } y(0) = y'(0) = 0.$$

31. State and prove "Abel's Theorem".

32. Find the solution of the initial value problem :

$$2y'' + y' + 2y = s(t-5),\ y(0) = 0,\ y'(0) = 0.$$

33. Given that $y(t) = \frac{1}{t}$ is a solution of $2t\,y'' + 3t\,y' - y = 0,\ t > 0$. Find a second linearly independent solution.

34. Consider a vibrating string of length L = 30 that satisfies the wave equation $4u_{xn} = u_{tt},\ 0 < x < 30$, $t > 0$. Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position :

$$u(x, 0) = f(x) = \begin{cases} \frac{x}{10}, & 0 \le x \le 10 \\ \frac{30 - x}{20}, & 10 \le x \le 30 \end{cases}$$

Find the displacement $u(x, t)$ of the string, and describe its motion through one period.

35. Find the Fourier series for $f(x) = |x|$ in $[-\pi, \pi]$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Solve : $y'' + 4y = 3\csc t$.

37. Solve : $y'' + 2y' + 5y = t^2 \sin 2t + (6t + 7)\cos 2t$.

38. When x lies between $\mp\pi$ and m is not an integer, prove that :

(i) $$\sin mx = \frac{2}{\pi}\sin m\pi\left[\frac{\sin x}{1 - m^2} - \frac{2\sin 2x}{2^2 - m^2} + \frac{3\sin 3x}{3^2 - m^2} \ldots\ldots\right].$$

(ii) $$\cos mx = \frac{2}{\pi}\sin m\pi\left[\frac{1}{2m} + \frac{m\cos x}{1 - m^2} - \frac{m\cos 2x}{2^2 - m^2} + \ldots\ldots\right].$$

(2 × 13 = 26 marks)

D 11548 (Pages : 3) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Section A

Each question carries a weightage of ¼.

1. What is the order of $y''^3 + y'^5 + 5y = \tan t$.
2. Is $y' = y^3 \cos x$ separable ?
3. Check for exactness $\left(2xy^2 + 2y\right) dx + \left(2x^2y + 2x\right) dy = 0$.
4. Solve $y'' - 4y = 0$.
5. State Abel's theorem.
6. Find the Wronskian of $y_1 = e^t, y_2 = e^{-t}$.
7. Find $L\left\{e^{-2t}\right\}$.
8. State True *or* False :

   The Laplace transform is a linear operator.
9. Find $L\{\sin 5t\}$.
10. What is the fundamental period of sin $5x$?
11. What is the one-dimensional wave equation ?
12. Is the following function even, odd or neither sec x.

(12 × ¼ = 3 weightage)

Turn over

Section B

Each question carries a weightage of 1.

13. Verify that $y = t/3$ is a solution of $y'''' + 4y''' + 3y = t$.

14. State the existence and uniqueness theorem for First order initial value problem.

15. Verify whether 'y^3' is an integrating factor of $\frac{dx}{y} + \frac{2x}{y^2}dy = 0$.

16. Solve $y'' - 5y' + 6y = e^{5t}$.

17. Solve $y'''' - 16y = 0$.

18. Find $L\{\cosh at\}$.

19. Show that convolution is commutative.

20. Show that the sum of an even and odd function is odd.

21. Graph the full function $f(x) = 0, -3 < x < -1, f(x+6) = f(x)$
$1, -1 < x < 1$
$0, 1 < x < 3$

(9 × 1 = 9 weightage)

Section C

Each question carries a weightage of 2.

22. Find an integrating factor and solve : $dx + \left(\frac{x}{y} - \sin y\right)dy = 0$.

23. Solve the initial value problem : $(x+4)\left(y^2+1\right)dx + y\left(x^2+3x+2\right)dy = 0, y(0) = 1$.

24. Solve the initial value problem : $y'' + 2y' + 5y = 4^{e-t}\cos 2t, y(0) = 1, y'(0) = 0$.

25. Solve $y'' - y = \cosh t + \cos t$.

26. Find $L^{-1}\left\{\frac{2s-3}{s^2-4}\right\}$.

27. Find $L\{t \cos at\}$.

28. Find the Fourier cosine series of $f(x) = L - x, 0 \le x \le L, f$ being of period 2 L.

(5 × 2 = 10 weightage)

Section D

Each question carries a weightage of 4.

29. Solve by the method of variation of parameters :

$$4y'' + y = 2\sec\left(t/2\right), -\pi < t < \pi.$$

30. (i) Using convolution, find $L^{-1}\left\{\frac{s}{(s+a)^2}\right\}$.

(ii) Use Laplace transforms to solve $y'' + 3y' + 2y = 0, y(0) = 1, y'(0) = 0$

31. Find the Fourier series expansion of :

$$f(x) = \begin{cases} 0, -L < x < 0 \\ L, 0 < x < L \end{cases}, \ f \text{ of period } 2\text{ L}.$$

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

(2 × 4 = 8 weightage)

D 11164 (Pages : 4)

Name..

Reg. No..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours | Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

Fill in the blanks :

1. The brachistochrone problem was first solved by ————.
2. Write the heat equation for a rod of finite length completely as a boundary value problem.
3. Find the general solution of $y'' - y = 0$.
4. Find the Laplace transform of $\cosh(2\,at)$.
5. Write the formulas for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period 2L.
6. Define an exact differential equation. Is $(x + y)dy - (x - y)dx = 0$ exact ? Why ?
7. Solve the system : $\dfrac{dx}{dt} = y,\ \dfrac{dy}{dt} = x.$
8. Define unit step function and write its Laplace transform.
9. Give an example of a non-linear differential equation in the dependent variable y and the independent variable x of second order.
10. Show that $u(x, y) = f(x - ay) + g(x + ay)$ is a solution of the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} = a^2 \dfrac{\partial^2 u}{dy^2}$.

Turn over

11. Show that sum and product of two even functions are even functions.

12. Compute the Wronskian of the functions e^{t} and e^{-t}.

(12 × 1 = 12 marks)

Section B

*Answer any **ten** out of fourteen questions.*

Each question carries 4 marks..

13. $\left(x+e^{-y/2}\right)\dfrac{dy}{dx}=2,\ y(0)=0.$

14. Use Laplace transform to find the solution of $\dfrac{dy}{dt}=t,\ y(0)=1.$

15. Using convolution find the inverse Laplace transform of $\dfrac{1}{(s-2)(s-1)}$.

16. Show that any separable equation $M(x)+N(y)y'=0$ is also exact.

17. Solve : $t^2y''+ty'+y=0.$

18. Use method of variation of parameters to solve : $y''+4y=3\operatorname{cosec} t$.

19. Given that $y_1(t)=t^{-1}$ is a solution of $2t^2y''=3ty-y=0, t>0$. Find a fundamental set of solutions.

20. If $f(x)=x, -\pi\le x\le\pi$ is a 2π-periodic function, find a_n, the coefficient of $\cos(nx)$ in its Fourier series expansion.

21. Find the values of a and b such that the equation $(ax+by)\dfrac{dy}{dx}=bx+ay$ is exact and hence solve it.

22. Find the Laplace transform of the function :

$$f(t)=\begin{cases} 2, & \text{if } 0<x<\pi \\ 0, & \text{if } \pi<x<2\pi \\ \sin t, & \text{if } x>2\pi \end{cases}$$

23. State the conditions for the convergence of a Fourier series of a 2π periodic function.

24. Transform the equation $u'' + 0.5\,u' + u = 0$ into a system of first order differential equations.

25. Show that Wronskian of the fundamental solutions of $y'' + y = 0$ is actually non-zero.

26. Write the conditions for the existence of the Laplace transform of a function.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*

Each question carries 7 marks.

27. Solve:

(a) $(3x + 4y)\dfrac{dy}{dx} = 2x + y,\ y(0) = 0.$

(b) $y - y' = 2xy,\ y(0) = 1.$

28. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)y' = 0$ and solve it.

29. Find the general solution of $y'' - 2y' + y = 2\cos(2t) - t^2$.

30. Find the Fourier cosine series expansion of $f(x) = \sin\left(\pi x/L\right)$ when $0 < x < L$.

31. Find :

(a) $\mathcal{L}(\cosh(at)\cos(at))$.

(b) $\mathcal{L}^{-1}\left(\dfrac{1}{(s^2 + \omega^2)^2}\right)$.

32. Solve the boundary value problem using Laplace transform : $y'' - y = 1$, where $y(0) = 0,\ y\left(\pi/2\right) = 1$.

33. State and prove Abel's theorem.

34. Prove the convolution theorem for Laplace transform.

35. (a) Solve using the method of separation of variables : $\frac{\partial u}{\partial x} = a^2 \frac{\partial u}{\partial y}$, $u(x,0)=1$, $u(0,y)=-1$

(b) Solve : $y'' + y' + y = 2t$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Find the Fourier series of :

$$f(x) = \begin{cases} k, \text{ if } -\pi/2 < x < \pi/2 \\ 0, \text{ if } \pi/2 < x < 3\pi/2 \end{cases},$$

assuming it is period 2π and deduce that $\pi/4 = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$

37. Find the solution of the initial value problem $y'' - 2y - 1 = 0$, $y(0) = 0$, $y'(0) = 1$ in two ways ; one of them must be using Laplace transforms.

38. Derive the wave equation by stating the assumptions involved and find its D'Alembert's solution.

(2 × 13 = 26 marks)

D 90908 (Pages : 3) Name.....................................

Reg. No.....................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Is $\frac{dy}{dx} + x^2 y = \frac{1}{y}$ linear ?
2. Find an integrating factor of $xdy - ydx = 0$?
3. Does $\frac{dy}{dt} = e^t$ basic a solution passing through $(0, 1)$?
4. Find the solution of $y'' - 4y' + 4y = 0$.
5. Find the differential equation whose solution is $y = c_1 e^{kx} + c_2 e^{-kx}$.
6. Are x and x^2 linearly independent.
7. What is the Laplace Transform of sinh at ?
8. State the shifting properly of Laplace Transforms.
9. Define step function.
10. Give the wave equation.
11. Is the function $f(x) = x^2 \cos nx$ even.
12. If $f(x)$ is an odd function, the coefficient of cosines in the Fourier series expansion of $f(x)$ is ————.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *questions.*

13. Solve $x(1 + y^2)dx + y(1 + x^2)dy = 0$.
14. Show that $\mu(x) = x$ is an integrating factor of $(x^2 - 2x + 2y^2)dx + 2xy\,dy = 0$.

Turn over

15. State the existence and uniqueness theorem for solution of a first order differential equation.

16. $y'''' + 8y'' + 16y = 0$ – Solve this.

17. Solve $y'' - y' - 6y = 20e^{-2x}$.

18. Find $L\{e^{-7t}\cos 3t\}$.

19. Find $L^{-1}\left\{\frac{1}{(s+9)^3}\right\}$.

20. Find a_o for the periodic function (of period 2π) : $f(x)\begin{cases}-k, & -\pi < x < 0 \\ k & 0 < x < \pi.\end{cases}$

21. Define the convolution integral and show that it is commutative.

(9 × 1 = 9 weightage)

Part C

Answer any **five** *questions.*

22. Solve $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$.

23. (i) Define linear and non-linear first order differential equations with examples.

(ii) When is a differential equation said to be exact ? Derive a necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact.

24. Show that $y = c_1 x + c_2 x^2$ is the general solution of $x^2 y'' - 2xy' + 2y = 0$ on any internal not containing zero and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

25. Find the general solution by the method of variation of parameters : $y'' - 2y' + y = 2x$.

26. Using the method of Laplace Transforms, solve $y'' - 3y' + 2y = 4e^{2t}$, $y(0) = -3$, $y'(0) = 5$.

27. Using convolution properly, show that $L^{-1}\left\{\frac{1}{(s-2)(s-3)}\right\} = e^{3t} - e^{2t}$.

28. Show that $u = e^{nx + iny}$ and $u = e^{nx - iny}$ are both solutions of $u_{xx} + u_{yy} = 0$.

(5 × 2 = 10 weightage)

Part D

Answer any **two** *questions.*

29. Solve $\left(D^2 + 2D + 5\right)y = x + \sin 2x$.

30. Find the Fourier series of $f(x) = |x|$ in $[-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$.

31. Solve by the method of separation of variables : $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

(2 × 4 = 8 weightage

D 70943 (Pages : 3) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.–CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. What is the order of $(y''')^2 + (y'')^7 + y = \sin t$?
2. Give the general form of a separable equation.
3. Test for exactness : $(2x+4y)\,dx+(2x-2y)\,dy=0$.
4. Solve $y''-y=0$.
5. State Abel's theorem.
6. Find the Wronskian of $y_1 = e^{2t}, y_2 = e^{-3t}$.
7. What is $\mathrm{L}\left\{e^{-at}\right\}$?
8. State true *or* false : The Laplace transform is a linear operator.
9. Find $\mathrm{L}\{\sin 3t\}$.
10. What is the fundamental period of sin 7 t ?
11. What is the heat conduction equation ?
12. Is the function $f(x) = x|x|$ even, odd or neither ?

(12 × ¼ = 3 weightage)

Turn over

Section B

Answer **all** *questions.*
Each question carries 1 *weightage.*

13. Verify that $y = 3t + t^2$ is a solution of $ty^1 - y = t^2$.
14. State the existence and uniqueness theorem for first order initial value problems.
15. Verify whether 'y' is an integrating factor of $ydx + 2xdy = 0$.
16. Solve $2y'' - 5y' + 3y = 2e^{4t}$.
17. Solve $y'' + a^2y = 0$.
18. Find $L\{\sinh 7t\}$.
19. Show that convolution is commutative.
20. Show that the sum of two even functions is even.
21. Graph the square wave function.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Find an integrating factor and solve :

$$\left(2x^2 + y\right) dx + \left(x^2 y - x\right) dy = 0.$$

23. Solve the initial value problem :

$$(y+2)\, dx + y(x+4)\, dy = 0 ;\ y(-3) = -1.$$

24. Solve the initial value problem :

$$y'' - 2y' + y = te^{t+4},\ y(0) = 1, y'(0) = 1.$$

25. Solve $y'' + y = \sin t \sin 2t$.
26. Find $L^{-1}\left\{2/\left(s^2 + 3s - 4\right)\right\}$.
27. Find $L\left\{t^2 e^{at}\right\}$.
28. Find the Fourier sine series of $f(x) = \begin{cases} x, 0 \le x < 1, & f \text{ is of} \\ 1, 1 \le x < 2, & \text{period } 4 \end{cases}$.

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 weightage.

29. Solve by the method of variation of parameters :

$$y'' + y = \tan t, 0 < t < \frac{\pi}{2}.$$

30. (i) Using convolution, find $L^{-1}\left\{\frac{1}{s^2(s+2)}\right\}$.

(ii) Using Laplace transforms, solve $y'' - y' - 6y = 0$
$y(0) = 1, y'(0) = -1.$

31. Find the Fourier series expansion of :

$$f(x) = \begin{cases} -x, -2 \le x < 0, f(x+4) = f(x) \\ x, 0 \le x < 2 \end{cases}.$$

Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

(2 × 4 = 8 weightage)

D 50725 **(Pages : 3)** Name.......................................

Reg. No.......................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2013

(UG–CCSS)

Mathematics (Core Course)

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer all **twelve** *questions.*

1. State $f(x) = x \cos x$ is even or odd.
2. Solve $\dfrac{dy}{dx} + \dfrac{x}{y} = 0$.
3. Write the order of the differential equation $\dfrac{d^3y}{dx^3} + 2\left(\dfrac{d^2y}{dx^2}\right)^2 - \dfrac{dy}{dx} + y = 0$.
4. Write the necessary condition for the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact.
5. Show that $(Ax, By)\,dx + (Cx, Dy)\,dy = 0$ is exact iff $B = C$.
6. Verify that $\sin x$ is a solution of $\dfrac{d^2y}{dx^2} + y = 0$.
7. Write the homogeneous equation of $\dfrac{d^2y}{dx^2} + y = x$.
8. Laplace transform of t is ———.
9. If $L\{F(t)\} = f(s)$, then $L\{e^{-at}F(t)\} =$ ———.
10. Find $(F * G)\,t$ if $F(t) = 1$, $G(t) = 1$.
11. $L\{e^{-at}\sin bt\} =$ ———.
12. Show that $(x^2 + y)\,dx + (y^2 + x)\,dy = 0$ is exact.

(12 × ¼ = 3 weightage)

Turn over

Answer **all** *questions.*

13. Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.

14. Define a homogeneous differential equation.

15. Find the integrating factor of $(1+xy)\,ydx + (1-xy)\,xdy = 0$.

16. Determine N (x, y) such that the equation $(x^3 + xy^2)\,dx + N(x,y)\,dy = 0$ is exact.

17. Find the Laplace transform of $\cos at$.

18. Find $(F * G)\,t$ if $F(t) = t, G(t) = e^t$.

19. Determine whether $\sin 7x$ is periodic. If so find its fundamental period.

20. Find the Laplace transform of $2e^{4t} + 3x^{-2t}$.

21. Find the Wronskian of $\sin x$ and $\cos x$.

(9 × 1 = 9 weightage)

Answer any **five** *questions from seven.*

22. Solve $(x^2 + y^2)\frac{dy}{dx} = xy$.

23. Solve the initial value problem $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0,\ y(0) = -3, y'(0) = -1$.

24. Transform the equation $u'' + 2u' + 2u = 0$ into a system of first order equation.

25. If $\{F(t)\} = f(s)$ then prove that $L\{e^{at}F(t)\} = f(s-a)$.

26. Find the inverse transform of $\frac{3s+7}{s^2 - 2s - 3}$.

27. Using Convolution property, find $L^{-1}\left[\frac{1}{s(s^2+a^2)}\right]$.

28. Solve the boundary value problem $y'' + 2y = 0,\ y(0) = 1, y(\pi) = 0$.

(5 × 2 = 10 weightage)

Answer any **two** *questions.*

29. Find the integrating factor and hence solve

$$x^2 y\,dx - \left(x^3 + y^3\right)dy = 0.$$

30. Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x}.$$

31. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = \tan x.$$

(2 × 4 = 8 weightage)

C 30802 (Pages : 2) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, NOVEMBER 2017

(UG-CCSS)

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each question carries ¼ weightage.

1. What is the order of $y''' + 7.y''^2 + y' = \cos t$?
2. Is $y' = x^3\left(y - y^2\right)$ separable ?
3. Check for exactness : $3x^2 - 2xy + 2 + y'\left(6y^2 - x^2 + 3\right) = 0$.
4. Solve : $y'' - 9y = 0$.
5. State Abd's theorem.
6. Find the Wronskian of $y_1 = \cos t, y_2 = \sin t$.
7. Find $L\left\{e^{-7t}\right\}$.
8. State True *or* False. The Laplace transform is a linear operator.
9. What is L {sin 4*t*} ?
10. What is the fundamental period of cos 5*t* ?
11. What is the heat conduction equation ?
12. Is the following function even, odd or neither : $x^3 - 4x$.

(12 × ¼ = 3 weightage)

Section B

Answer **all** *questions.*
Each question carries 1 *weightage.*

13. Verify that $y = 3t + t^2$ is a solution of $ty' - y = t^2$.
14. State the existence and uniqueness theorem for first order initial value problems.
15. Verify whether y^2 is an integrating factor of $\frac{dx}{y} + \frac{2x}{y^2} dy = 0$.
16. Solve $y'' + 4y' - 12y = e^{3t}$
17. Solve $y''' - y = 0$.

Turn over

18. Find L {sinh at}.

19. Show that convolution is commutative.

20. Show that the sum of two odd functions is odd.

21. Graph the sawtooth wave function.

(9 × 1 = 9 weightage)

Section C

Answer any **five** *questions.*
Each question carries 2 *weightage.*

22. Find an integrating factor and solve : $y' = e^{2x} + y - 1$.

23. Solve the initial value problem : $(3x+8)\left(y^2+4\right)dx - 4y\left(x^2+5x+6\right)dy = 0,\ y(1) = 2$.

24. Solve the initial value problem : $y'' - 2y' - 3y = 3te^{2t},\ y(0) = 1,\ y'(0) = 0$.

25. Solve $y'' + 4y = \sin^2 t$.

26. Find $L^{-1}\left\{\dfrac{3s}{s^2 - s - 6}\right\}$.

27. Find L {$t \sin at$}.

28. Find the Fourier cosine series of :

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases},\ f \text{ of period } 4.$$

(5 × 2 = 10 weightage)

Section D

Answer any **two** *questions.*
Each question carries 4 *weightage.*

29. Solve by the method of variation of parameters $y'' + 4y = 3 \operatorname{cosec} 2t,\ 0 < t < \pi/2$.

30. (a) Using convolution, evaluate $L^{-1}\left\{\dfrac{1}{s(s+2)^3}\right\}$.

(b) Use Laplace transforms to solve $y'' - 2y' + y = 0,\ y(0) = 0,\ y'(0) = 1$.

31. Find the Fourier series expansion of

$$f(x) = \begin{cases} 0, & -L < x < 0 \\ L, & 0 < x < L \end{cases},\ f \text{ of period } 2L.$$

Hence deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \ldots$

(2 × 4 = 8 weightage)