D 114598 **(Pages : 2)** **Name.....**

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 *Short questions answerable within* 7.5 *minutes*
Answer **all** *questions, each question carries weightage* 1.

1. Write down the divergence of a vector V in orthogonal curvilinear co-ordinates.
2. Write down the orthogonality relation of a Hermite polynomials.
3. Define beta function.
4. What do you mean by pseudo tensors ? Give an example.
5. Define an orthogonal matrix with an example.
6. Write down the Rodrigues formula of Legendre polynomial and obtain $P_2(x)$ from Rodrigues formula.
7. Explain Schmidt orthogonalization.
8. Explain the convolution theorem with an example.

(8 × 1 = 8 weightage)

Section B

4 *essay questions answerable within* 30 *minutes.*
Answer any **two** *questions, each question carries weightage* 5.

9. Define a Fourier transform. Explain the properties of Fourier transforms.
10. Prove the orthogonality relation for the Bessel functions.

Turn over

11. Find the eigen values and eigen vectors of

$$H = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

12. Explain the Frobenius' method of finding solution to homogenous differential equation of second order by taking the example of linear oscillator.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions, each question carries weightage 3.*

13. Prove that every (second rank) tensor can be resolved into symmetric and antisymmetric parts.

14. Find the Fourier transform of a derivative.

15. Express the spherical polar unit vectors in terms of Cartesian unit vectors

16. Define the singular points of an ordinary differential equation, obtain the singular points of Bessel's differential equation

17. Prove that $\nabla \cdot r^n \hat{r} = (n+2) r^{n-1}$.

18. Two matrices A and B are each Hermitian. Find a necessary and sufficient condition for their product AB to be Hermitian.

19. Find the value of $\int_0^\infty e^{-x^4}\, dx$ in terms of gamma function.

(4 × 3 = 12 weightage)

D 52838 **(Pages : 2)** **Name.....**

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 short questions answerable within 7.5 minutes.
Answer **all** *questions, each question carries 1 weightage.*

1. Write down the relation between cartesian coordinate system and spherical polar co-ordinate system.
2. What are Tensors ? Define the rank of the tensor.
3. Write the expression for Fourier co-efficients.
4. What do you mean by a self-adjoint differential equation ?
5. Write down the Rodrigues formula of Laguerre function and obtain $L_1(x)$ from Rodrigues formula.
6. Define a unitaiy matrix with an example.
7. Obtain the recurrence formula, $H_n'(x) = 2nH_{n-1}(x)$ from generating function.
8. Explain the convolution property of Fourier transform with an example.

(8 × 1 = 8 weightage)

Section B

4 essay questions answerable within 30 minutes.
Answer any **two** *questions, each question carries weightage 5.*

9. Discuss the orthogonality property of Legendre polynomials.
10. Explainthe Frobenius' method of finding solution to homogenous differential equation of second order.

Turn over

11. Prove that $\nabla \cdot r^n \hat{r} = (n+2) r^{n-1}$.

12. State and prove the Quotient rule in tensors.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions, each question carries weightage 3.*

13. Express the spherical polar unit vectors in terms of cartesian unit vectors.

14. Show that $\Gamma(p+1) = p\,\Gamma(p)$.

15. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

16. Find Laplace transform of the function, $f(t) = t^n$.

17. Define spherical Bessel function. Obtain the expression for $j_1(x)$.

18. Find the Fourier transform of the normalized Gaussian distribution

$f(t) = \frac{1}{\tau\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\tau^2}\right), -\infty < t < \infty$, where $\tau = \Delta t$ (root mean square deviation).

19. A and B are two non-commuting Hermitian matrices : AB – BA = iC. Prove that C is Hermitian.

(4 × 3 = 12 weightage)

D 32731 **(Pages : 2)** **Name.....**

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2022

(CBCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 *Short questions answerable within* 7.5 *minutes)*
Answer **all** *questions, each question carries weightage* 1.

1. If V represents a vector derive the divergence of V in orthogonal curvilinear co-ordinates.
2. Is the given matrix Hermitian ?
3. Explain quotient law for tensors.
4. Explain the general form of a second order differential equation and classify them based on being elliptic, parabolic or hyperbolic.
5. Explain the significance of the Dirac-Delta function.
6. Explain Gram-Schmidt orthogonalization.
7. Evaluate the Fourier co-efficients a_0, a_n and b_n for a piecewise continuous function where L is half of the period of the function.
8. Explain the convolution property of Fourier transform with an example.

(8 × 1 = 8 weightage)

Section B

4 *essay questions answerable within* 30 *minutes*
Answer any **two** *questions, each question carries weightage* 5

9. Explain the Frobenius' method of finding solution to homogenous differential equation of second order.
10. Using appropriate differential equation obtain the general form for Hermite polynomials of degree n.

Turn over

11. Define a Fourier transform. Explain any *five* properties of Fourier transforms.

12. Explain the algebraic operations of Tensors.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes

Answer any **four** *questions, each question carries weightage* 3.

13. A string of length π is stretched until the wave speed is 40 m/sec. It is given an initial velocity of $4 \sin(x)$ from its initial position. What is maximum displacement ?

14. Evaluate $\Gamma\left(-\frac{3}{2}\right)$.

15. If A_{ij} is antisymmetric tensor, find the component A_{11}.

16. Find Laplace transform of the function $F(t) = \frac{e^{at} - 1}{a}$.

17. For the Legendre polynomial prove that $P_n(x) = 1$.

18. Find the Fourier series of the function e^x in the interval $-\pi < x < \pi$.

19. Prove that the given matrix is unitary :

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

(4 × 3 = 12 weightage)

D 13158 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—1

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

8 *Short questions answerable within* 7½ *minutes.*
Answer **all** *questions, each carry weightage* 1.

1. If V represents a vector derive the curl of V in orthogonal curvilinear coordinates.
2. Is the given matrix Hermitian $\begin{bmatrix} 1 & -i & -3i \\ i & 5 & 0 \\ 3i & 0 & 2 \end{bmatrix}$.
3. Explain concept of outer product in tensors.
4. With an example explain features of a hyperbolic partial differential equation.
5. Show that $\int_{-1}^{+1} xP_n(x)\, p_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$.

Turn over

6. Explain the convolution theorem of Fourier transform.

7. Explain when can a second-order linear homogeneous differential equation can be called self-adjoint.

8. Distinguish between Fourier integral and Fourier transform.

(8 × 1 = 8 weightage)

Section B

4 essay questions answerable within 30 *minutes.*
Answer any **two** *questions, each carry weightage* 5.

9. What are orthogonal curvilinear co-ordinate systems ? Obtain the mathematical expression for divergence in terms of curvilinear coordinates.

10. Using appropriate differential equation explain Laguerre polynomials and associated Laguerre polynomials. Obtain their representation in series form.

11. Explain the following properties of Fourier series: (1) Convergence (2) Integration ; and (3) Differentiation. Obtain the sine and cosine series in the interval $(0, \pi)$ for a function $f(x)$.

12. Explain the Frobenius' method of finding solution to homogenous differential equation of second order.

(2 × 5 = 10 weightage)

Section C

7 *problems answerable within* 15 *minutes.*
Answer any **four** *questions, each carry weightage* 3.

13. Is the given matrix orthogonal $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

14. Prove that $P_{2m+1}(0) = 0$.

15. A string of length n is stretched until the wave speed is 40 m/sec. It is given an initial velocity of 4 sin (x) from its initial position. What is location of maximum displacement ?

16. Evaluate $\Gamma\left(-\frac{1}{2}\right)$.

17. Evaluate Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$.

18. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

19. Expand the function $f(x) = \sin x$ as a cosine series in the interval $(0, \pi)$

(4 × 3 = 12 weightage)

D 72979 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION DECEMBER 2019

(CBCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS–I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*
Each carry weightage 1.

1. Show that the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix.
2. Write down the expression for gradient and divergence in spherical polar coordinates.
3. What are Hermitian and unitary matrices?
4. What is meant by similarity transformation?
5. Evaluate $\int_{-\infty}^{\infty} e^{-x^2}\, x dx$.
6. Evaluate $\int_{-\infty}^{+\infty} e^{-(x-4)}\delta(x-4)dx$.
7. Find the Laplace transform of $f(t)=t$.
8. Find the Laplace transform of $f(t)=e^{-at}$.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions.*
Each carry weightage 5.

9. Explain the idea of Schmidt orthogonalisation.
10. Define beta and gamma functions and derive the relation between the two.
11. Define Fourier transform and inverse transform. Show that the Fourier transform of a Gaussian function is Gaussian.

Turn over

12. Prove the recurrence relation :

$$(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x).$$

(2 × 5 = 10 weightage)

Section C

Answer any **four** *questions.*
Each carry weightage 3.

13. Show that divergence of curl of a vector is always zero.

14. Show that direct product of two vectors A^{μ} and B^{ν} given by $C^{\mu\nu}$ is a second rank tensor.

15. What are the eigen functions and eigen values of the operator $-\frac{d^2}{dx^2}$ with the eigen functions satisfying the boundary conditions $f(x)=0$ at $x=0$ and $x=L$?

16. Argue that the Gaussian function given by $f(x)=\sqrt{\frac{a}{\pi}}\,e^{-ax^2}$ goes over to the Dirac delta function in the appropriate limit.

17. State and prove the convolution theorem in the context of Fourier transform.

18. Give a general solution of the Laplace equation.

19. Find the inverse Laplace transform of $\frac{1}{s(s+2)}$.

(4 × 3 = 12 weightage)

D 51453 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(CUCSS–PG)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS–I

[2017 Syllabus Year]

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. What is Laplace's equation ? How does it explain the motion of incompressible fluids ?
2. Find the volume element dV in cylindrical and curvilinear co-ordinates.
3. Prove that an arbitrary matrix can be decomposed into the sum of a Hermitian and anti-hermitian matrices.
4. Define outer product of tensors.
5. What are the difficulties involved when a differential equation is solved by applying Frobenius method ?
6. State and explain Fuch's theorem in differential equations.
7. Define Dirac delta function. State one situation where it finds application.
8. Explain what is meant by a Neumann function. Write down the expression for the function.
9. Write the Rodrigue's formula for Legendre polynomial and deduce the value of $P_0(x)$.
10. State the convolution theorem for Fourier transforms.
11. Using Fourier series prove that a square wave contains many high frequency components.
12. Discuss the important properties of Fourier transforms.

(12 × 1 = 12 weightage)

Turn over

Section B

Answer any **two** *questions.*
Each question carries a weightage of 6.

13. Explain about cylindrical and spherical polar co-ordinates and deduce expression for unit vectors in spherical co-ordinates and show that they are orthogonal.

14. Explain the method of diagonalisation and its importance. Diagonalise the matrix

$$\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}.$$

15. Obtain the series solution of Bessel differential equation using Frobenius method.

16. Obtain the orthogonality relation for Laguerre polynomials.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries a weightage of 3.

17. Prove that velocity and acceleration are contravariant vectors and the gradient of a scalar field is a covariant vector.

18. Prove that single contraction of a tensor A^{ijk}_{lm} is a tensor of rank 3.

19. Explain Gram- Schmitz orthogonalisation process.

20. Derive the relation between beta and gamma functions. Show that $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.

21. From the generating function of Hermite polynomials, obtain the following recurrence relations.

    (i) $2nH_{n-1}(x) = H'_n(x)$.

    (ii) $H'_n(x) = 2xH_n(x) - H_{n+1}(x)$.

22. Solve the Simple harmonic oscillator problem by applying Laplace transforms.

(4 × 3 = 12 weightage)

C 32345 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS—I

(2017 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*

Each question has weightage of 1.

1. Explain briefly how Laplace equation can be applied in describing electrostatic fields.
2. If A is irrotational, show that A × R is solenoidal.
3. Explain the physical significance of Unitary transformations.
4. Define outer product of tensors with an example.
5. What are Hermitian operators ? Mention any two properties.
6. What are even and odd functions ? Write corresponding Fourier series.
7. Write the Rodrigues Formula for Legendre Polynomial and deduce the value of $P_0(x)$.
8. Show that $\beta(m,n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1} (\cos\theta)^{2n-1}\, d\theta$.
9. What is Dirac delta function? Mention two situations where the function can be applied.
10. Show that $J_n(-x) = J_n(x)$ for even n and $J_n(-x) = -J_n(x)$ for odd n.
11. Discuss the advantages of using the transforms of derivatives.
12. Show that the convolution of $f(x)$ and $g(x)$ is commutative.

(12 × 1 = 12 weightage)

Turn over

Section B

Answer any **two** *questions.*
Each question has weightage of 6.

13. Describe cylindrical and spherical polar co-ordinates and show that these co-ordinate systems are orthogonal.

14. Discuss Hermitian, Unitary, and Orthogonal matrix with example. Show that eigen vectors of Hermitian matrix are orthogonal and eigen values are real.

15. (a) Explain Gram- Schmitz orthogonalisation process.

    (b) Find the distance from the point $y = (0, 0, 0, 1)$ to the subspace $V \subset R\,4$ spanned by vectors $x_1 = (1, -1, 1, -1)$, $x_2 = (1, 1, 3, -1)$, and $x_3 = (-3, 7, 1, 3)$.

16. Show that $(1 - 2tx + t^2)^{-1/2}$ is a generating function of $P_n(x)$. Hence prove the following recurrence relations.

    (i) $nP_n(x) = (2n-1)x\,P_{n-1}(x) - (n-1)\,P_{n-2}(x)$.

    (ii) $nP_n(x) = xP_n'(x) - P_{n-1}(x)$.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question has weightage of 3.

17. Consider a co-ordinate system (u, v, w) which is related to Cartesian co-ordinate system by $x = wv : y = uw : z = uv$. Obtain the metric tensor in the form of u, v, w.

18. State Stoke's theorem and verify it for the vector $A = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\,\hat{k}$ over the upper half surface of a sphere $x^2 + y^2 + z^2 = 1$.

19. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

20. Define beta and gamma functions. Evaluate the integral $\int_{-1}^{+1}\left(\frac{1+x}{1-x}\right)^{1/2} dx$ using the properties of beta and gamma functions.

21. Show with the help of an example how Laplace transforms can be used to evaluate a definite integral.

22. Expand $f_1(x) = x^2$ for $-\pi \leq x \leq \pi$ in a Fourier series.

(4 × 3 = 12 weightage)

D 13194 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS—I

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 *weightage.*

1. If $\vec{r} = \hat{x}\, r \cos(\omega t) + \hat{y}\, r \sin(\omega t)$, evaluate $\ddot{r} + \omega^2 r$ where the dot indicates differentiation with respect to time.

2. If a force $\vec{F}$ is given by $\vec{F} = \left(x^2 + y^2 + z^2\right)^n (\hat{x}\, x + \hat{y}\, y + \hat{z}\, z)$ find $\vec{\nabla} \cdot \vec{F}$.

3. Show that the operation of contraction reduces the rank of a tensor by 2.

4. What are "orthogonal matrices" ?

5. If A and B are Hermitian matrices, show that AB + BA is also Hermitian.

6. Show that $\int_{-\infty}^{+\infty} f(x)\, \delta(a\, x)\, dx = \frac{1}{a} f(0)$, where δ is the Dirac delta function.

7. What is meant by a "Wronskian" ?

8. Show that $B(m, n) = \frac{\Gamma(m)\, \Gamma(n)}{\Gamma(m + n)}$.

9. From the product of the generating functions, show that :

$$1 = \left[J_0(x)\right]^2 + 2\left[J_1(x)\right]^2 + 2\left[J_2(x)\right]^2 + \ldots\ldots$$

Turn over

10. What is the generating function of Legendre polynomials ?

11. What are the properties of Fourier series ?

12. Define Laplace transform of a function. Do all functions have a Laplace transform. Comment.

(12 × 1 = 12 weightage)

Part B

Answer any **two** *questions.*
Each question carries 6 *weightage.*

1. Derive the general expression for the divergence of a vector in a general, orthogonal curvilinear co-ordinate system. Find the expression for $\nabla \cdot \vec{V}$ in cylindrical co-ordinate system.

2. Define Trace and Transpose of a matrix giving one example each. If A is a 2×2 matrix, show that its eigenvalues λ satisfy the secular equation $\lambda^2 - \lambda\,(\text{trace A}) + \det(A) = 0$.

3. Explain the Gram-Schmidt orthogonalization process with an example.

4. Show that $P'_{n-1}(x) = -n P_n(x) + x P'_n(x)$. Express $\frac{3}{2}x^2 + x + \frac{1}{2}$ in terms of $P_0(X)$, $P_1(X)$ and $P_2(X)$.

(2 × 6 = 12 weightage)

Part C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

1. A potential is given by $V(r) = V\left(\sqrt{x^2 + y^2 + z^2}\right)$. Find gradient of $V(r)$.

2. Prove that $\vec{\nabla} \times \left(\varphi \vec{\nabla} \varphi\right) = 0$.

3. Find the eigen values of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and the corresponding eigenvectors.

4. Show that $B(a, b) = \frac{a+b}{b} B(a, b+1)$.

5. Show that $P_n'(1) = \frac{n(n+1)}{2}$.

6. Analyze by Fourier series of a square wave given by

$f(x) = 0$ for $-\pi < x < 0$ and $f(x) = h$ for $0 < x < \pi$.

(4 × 3 = 12 weightage)

D 92956

(Pages : 2)

Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS—I

(2012 Admission onwards)

Time : Three Hours

Maximum : 36 Weightage

Section A

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Define a vector in terms of its transformation under rotation of co-ordinates.
2. Which are the co-ordinate surfaces in spherical polar co-ordinates ?
3. Give the Laplacian operator in general currilinear co-ordinates.
4. State the quotient rule of tensors.
5. What is meant by Wronskian ?
6. Prove that the momentum operator is Hermitian.
7. Give an example for a self adjoint differential equation.
8. Define Dirac delta function.
9. What is meant by a unitary transformation ?
10. If λ is an eigenvalue of a matrix A, show that λ^2 is an eigenvalue of A^2.
11. What are the general properties of Fourier series ?
12. Show that $L\left\{e^{at}\right\} = \frac{1}{S-a}$ for $S > a$.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*
Each question carries a weightage of 6.

13. Derive the transformation relations from rectangular to spherical co-ordinates. Show that the spherical co-ordinate system is orthogonal.

Turn over

14. Diagonalize the matrix A by a similarity transformation.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

15. Explain the Gram-Schmidt orthogonalization procedure with a suitable example.

16. Establish the Orthogonality of Legendre Polynomials.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries a weightage of 3.

17. Transform the unit vectors i, j and k in to their components in a spherical polar co-ordinate system.

18. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

19. Find the regular singularities of Legendre equation.

20. Show that $\overline{|y_2|} = \sqrt{\pi}$.

21. Show that $J_0(x)^2 + 2\left[J_1^2(x) + J_2^2(x) + \ldots\ldots\right] = 1$.

22. Find the Fourier series expansion of the function $f(x) = e^x$ in the interval $0 < x < 2\pi$.

(4 × 3 = 12 weightage)

D 52981 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS – I

(2012 admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*

Each question carries a weightage of 1.

1. Define a vector in terms of its transformation under a rotation of co-ordinate system.
2. Give the expression for volume element in cylindrical co-ordinates. What are the scale factors ?
3. Resolve the spherical polar unit vectors into their Cartesian components.
4. If A is an orthogonal matrix prove that $\det A = \pm 1$.
5. Define Levi-Civita three index symbol.
6. State the quotient rule of tensors.
7. Explain how the roots of indicial equation. Provide an idea about the number of distinct solutions of an ODE.
8. What are the properties of Hermitian operators ?
9. What is meant by a singular point of a differential equation ?
10. Explain Fuchs' theorem.
11. State the symmetry of β-function.
12. Show that the term by term integration results in rapid convergence of Fourier series.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*

Each question carries a weightage of 6.

13. Find the general expression for Laplacian in general curvilinear co-ordinates. Deduce it for a cylindrical co-ordinate system.

Turn over

14. Define Orthogonal Hermitian and unitary matrices Diagonalise the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ by a similarity transformation.

15. Establish the Orthogonality of Bessel functions.

16. Define Fourier transform of a function. Find the Fourier sine transform of the function e^{-x}.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries a weightage of 3.

17. The Electrostatic field of a point charge f is given by $E = \frac{f}{AE\varepsilon_0} \frac{\hat{r}}{r^2}$. Calculate the divergence of E. What happens at the origin.

18. Find the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

19. Show that :

$$L_{n+1}(x) = 2L_n(x) - L_{n-1}(x).$$

20. Show that $P_{n(x)}$ is the co-efficient of t^n in the expansion of $(1 - 2xt + t^2)^{-1/2}$.

21. Derive the recurrence relation $\Gamma z + 1 = z\Gamma z$.

22. Find the Laplace transforms of e^{-at}.

(4 × 3 = 12 weightage)

D 72895 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Physics

PHY 1C 02—MATHEMATICAL PHYSICS – I

(2012 admission onwards)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*

Each question has a weightage of 1.

1. Write down the rotation matrix for the rotation of co-ordinates through an angle θ about the z-axis.
2. Give a physical meaning for the divergence of a vector.
3. If A is an orthogonal matrix, prove that det $A = \pm 1$.
4. Define Levi-Civita three index symbol.
5. Define a covariant tensor.
6. Explain how the roots of indicial equation provide an idea about the number of distinct solution of an ODE.
7. What are the properties of a Hermitian operator ?
8. What is meant by a singular point of a differential equation ?
9. Graphically represent $P_0(x)$, $P_1(x)$ and $P_2(x)$ in terms of x.
10. Explain Fuch's theorem.
11. Show that the term by term integration results in rapid convergence of Fourier series.
12. State the first shifting theorem of Laplace transform.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*

Each question has a weightage of 6.

13. Derive the expression for curl in general curvilinear co-ordinates. Deduce the curl in spherical co-ordinates.

Turn over

14. Define orthogonal, Hermitian and unitary matrices. Diagonalise the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ by a similarity transformation.

15. Establish the orthogonality of Bessel's function.

16. Explain Gram-Schmidt orthogonalisation procedure with a suitable example.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question has a weightage of 3.

17. Transform the unit vectors i, j, k into their components in a cylindrical co-ordinate system.

18. If λ is an eigen value of a matrix A, show that λ^2 is an eigen value of A^2.

19. Show that $\sqrt{1/2} = \sqrt{\pi}$.

20. Show that $J_0^2(x) + 2\left[J_1^2(x) + J_2^2(x) + \ldots\right] = 1$.

21. Find the Laplace transform of the function $e^{at} \sin t$.

22. Find the Fourier series of the function :

$f(x) = x^2, -\pi \le x \le \pi$.

(4 × 3 = 12 weightage)

C 32341 (Pages : 2) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2017

(CUCSS)

Physics

PHY 1C 03—MATHEMATICAL PHYSICS

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Evaluate $\nabla \cdot (\vec{r} f(r))$.

2. Express $\frac{\partial}{\partial y}$ in spherical polar co-ordinates.

3. Show that $\int_{-\infty}^{+\infty} \delta'(x) f(x)\, dx = -f'(0)$.

4. Show that the trace of the product of a symmetric and an anti-symmetric matrix is zero.

5. State the quotient rule for tensors.

6. Show that $(2n+1)!! = \frac{(2n+1)!}{2^n n!}$

7. What are partial differential equations ? How are they classified ?

8. From the generating function of the Legendre polynomials show that $P_n(1) = 1$.

9. State Laguerre's differential equation. What is the expression for the corresponding generating function ?

10. What is a "piecewise regular" function ?

11. Define Fourier transform of a function. What are sine and cosine transforms ?

12. Determine the Laplace transform of the Dirac delta function.

(12 × 1 = 12 weightage)

Turn over

Part B

Answer any **two** *questions.*
Each question carries 6 *weightage.*

1. Derive the general expression for the divergence of a vector in a general, orthogonal curvilinear co-ordinate system. Find the expression for $\nabla \bullet \vec{V}$ in spherical co-ordinate system.

2. A matrix A has eigenvalues of +1 and -1. The corresponding eigen vectors are $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. Construct A.

3. Prove, for Bessel functions, that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$ and $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

4. A wave is given by $F(x) = x$ for $0 < x < \pi$ and $F(x) = -x$ *for* $-\pi < x < 0$. Represent F (x) by a Fourier series.

(2 × 6 = 12 weightage)

Part C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

1. If a force is given by $\vec{F} = \left(x^2 + y^2 + z^2\right)^n (\hat{x}x + \hat{y}y + \hat{z}z)$, find the corresponding potential for $n \neq -1$ and $n = -1$.

2. Working in circular cylindrical co-ordinates, determine $\nabla \bullet \vec{r}$.

3. Find the eigen values of $A = \begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$ and the corresponding eigenvectors. Hence diagonalise A by means of a similarity transformation.

4. Show that $\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$ where $P_n(x)$ are the Legendre polynomials.

5. Show that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ where $H_n(x)$ are the Hermite polynomials.

6. Determine the inverse Laplace transform of $\frac{1}{s^2 + b^2}$.

(4 × 3 = 12 weightage)