

D 120574

(Pages : 4)

Name.....

Reg. No.....

**FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2025**

Mathematics

MTS4B04—LINEAR ALGEBRA

(2019—2023 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer type Question)

Each question carries 2 marks.

All questions can be attended.

Overall ceiling 25.

1. Give an example of a system of linear equation with the following properties.
 - (i) Infinite number of solutions.
 - (ii) No solution.
2. Write all elementary row operations.
3. Let $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find the matrix polynomial $P(A)$ for

$$p(x) = x^2 - 2x - 3.$$
4. Solve the following system by inverting the co-efficient matrix :

$$x_1 + x_2 = 2$$

$$5x_1 + 6x_2 = 9.$$
5. Let $W = \{(x, y) : x \geq 0, y \in \mathbb{R}\}$. Show that W is not a subspace of \mathbb{R}^2 over \mathbb{R} .
6. Write a basis for \mathbb{R}^2 over \mathbb{R} . Express $(1, 2)$ as the linear combination of the basis.
7. Show that $\{1, x, x^2, \dots, x^n\}$ spans P_n , where P_n is the vector space of polynomials of dimension $n + 1$.

Turn over

8. Define linearly independent set. Show that the set $\{(1, 0), (0, 1)\}$ is linearly independent.
9. Find the number of parameters in the general solution of $Ax = 0$, if A is a 5×7 matrix of rank 3.
10. Find the reflection of the vector $(1, 1)$ about the line through origin that makes an angle of $\frac{\pi}{6}$ with the x -axis.
11. Write all fundamental spaces of a matrix A .
12. Find the image of the unit square under the multiplication by the invertible matrix $\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$.
13. Define eigen values and eigen vectors of a matrix.
14. Find the eigen values of $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$.
15. Define an orthonormal set. Give an example.

(Ceiling 25 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.

All questions can be attended.

Overall ceiling 35.

16. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ define by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3)$. Find the standard matrix of the operator T .
17. Let $A = \begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$ be the augmented matrix of a linear system. Find the values of a and b such that the system has a unique solution.
18. Show that the vectors $v_1 = \{1, 2, 1\}$, $v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$ form a basis of \mathbb{R}^3 .

19. Let $S = \{v_1, v_2, \dots, v_n\}$ is a basis for a vector space V . Then show that every element $v \in V$ can be expressed in uniquely in the form of :

$$v = c_1v_1 + c_2v_2 + \dots + c_nv_n, c_i \text{ are scalars for every } i.$$

20. Find the rank and nullity of the matrix :

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

21. Let $T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, be the reflection about y-axis and $T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the reflection about the x-axis. Verify that $T_1 \circ T_2 = T_2 \circ T_1$.

22. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be vectors in \mathbb{R}^2 . Verify that

$$\langle u, v \rangle = 3u_1v_1 + 2u_2v_2,$$

satisfies all axioms of inner product.

23. Show that the basis $\left\{ (0, 1, 0), \left(-\frac{4}{5}, 0, \frac{3}{5}\right), \left(\frac{3}{5}, 0, \frac{4}{5}\right) \right\}$ form an orthonormal basis for \mathbb{R}^3 .

(Ceiling 35 marks)

Section C (Essay Type Question)

Answer any **two** questions.

Each question carries 10 marks.

24. (a) Solve $x_1 + 3x_2 + 4x_4 + 2x_5 = 0$

$$x_3 + 2x_4 = 0$$

$$x_6 = \frac{1}{3}.$$

- (b) Find the row reduced echelon form of $\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$.

Turn over

- (b) Determine whether the polynomials

$$1 - x, 5 + 3x - 2x^2, 1 + 3x - x^2,$$

Are linearly independent or not.

- (b) Find the Wronskian of :

$$\{1, e^x, e^{2x}\}.$$

26. (a) Define rank of a matrix. Find the rank of :

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}.$$

- (b) Define nullity of a matrix. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, defined by the formula

$$T(x_1, x_2) = (x_1 + 3x_2, x_1 - x_2, x_1).$$

Find the nullity of T.

27. (a) Define Gram-Schmidt process.
(b) Use Gram-Schmidt process transform the basis.

$$\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$$

into orthonormal basis.

(2 × 10 = 20 marks)

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(Pages : 4)

Name.....

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FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION**APRIL 2024**

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Type Question)*All questions can be attended.**Each question carries 2 marks.**Overall ceiling 25.*

1. Give an example of a system of linear equation with the following properties :
 - (i) Unique solution
 - (ii) Infinite number of solutions
2. Solve the system $x + y = 2, x - y = 0$ by using any method.
3. Give an example to show that the matrix multiplication is need not be commutative.
4. Find the row reduced echelon form of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}.$$

5. Let $W = \{(x, y, z) : x + y + z = 0\}$. Show that W is subspace of \mathbb{R}^3 .
6. Show that $\{(1, 0), (0, 1)\}$ spans \mathbb{R}^2 .
7. Define Wronskian. Find the Wronskian of $\sin 5x$ and $\cos 5x$.
8. Define linearly independent set. Give an example.
9. Define row space and column space of a matrix.

Turn over

10. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.
11. Show that the operator $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, that projects onto the x -axis in the xy -plane is not one-one.
12. Find the eigen values of $\begin{bmatrix} 3 & 0 \\ 8 & 1 \end{bmatrix}$.
13. Define similar matrices. Show that if A and B are similar the determinant is equal.
14. Let $\mathbf{u} = \mathbf{U} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{v} = \mathbf{V} = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$. Evaluate $\langle \mathbf{u}, \mathbf{v} \rangle$, where, $\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{U}, \mathbf{V} \rangle = \text{trace}(\mathbf{U}^T \mathbf{V})$.
15. Define orthogonal matrix. Show that $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ is orthogonal.

(Ceiling 25 Marks)

Section B (Paragraph/Problem Type Questions)*All questions can be attended.**Each question carries 5 marks.**Overall Ceiling 35.*

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, find, $\det(A)$, A^{-1} , A^{-2} , A^{-3} and A^{-5} .

17. Using row reduction, evaluate the determinant of:

$$\begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

18. Determine the set $\{6, 3 \sin^2 x, 2 \cos^2 x\}$ is independent or not.

19. Show that the matrices

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Form a basis for the vector space M_{22} of 2×2 matrices.

20. Find a basis for row space of the matrix

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}.$$

21. Describe the null space of the matrix $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$.

22. Define inner product. Consider P_2 with the inner product $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$. Verify that x and x^2 are orthogonal with respect to above inner product.

23. Let $f = f(x)$ and $g = g(x)$ be two functions on $C[a, b]$. Show that $\langle f, g \rangle = \int_a^b f(x)g(x)dx$ defines an inner product on $C[a, b]$.

(Ceiling 35 Marks)

Turn over

Section C (Essay Type Question)

Answer any two questions.

Each question carries 10 marks.

24. (a) For what values of b_1 , b_2 and b_3 the following system of equations are consistent ?

$$x_1 + x_2 + 2x_3 = b_1$$

$$x_1 + 0x_2 + x_3 = b_2$$

$$2x_1 + x_2 + 3x_3 = b_3.$$

- (b) Let A and B are symmetric matrices of same size. Then show that the followings.

(i) A^T is symmetric

(ii) $A + B$ and $A - B$ are symmetric

(iii) kA is symmetric, where k is any scalar.

25. Let $u = \{1, 2, -1\}$, $v = \{6, 4, 2\}$ in \mathbb{R}^3 .

(a) Show that $w = \{9, 2, 7\}$ is in the linear combination of u and v .

(b) Show that $w = \{4, -1, 8\}$ is not in the linear combination of u and v .

26. Consider the matrix,

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

(a) Verify that $\text{rank}(A) = \text{rank}(A^T)$.

(b) Verify dimension theorem for the matrix A.

27. Find an orthogonal matrix P that diagonalizes :

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}.$$

(2 × 10 = 20 marks)

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(Pages : 4)

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**FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023**

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer type Question)*Each question carries 2 marks.**All questions can be attended.**Overall ceiling 25.*

1. Give an example of a system of linear equation with the following properties :
 - (i) Unique solution ; and
 - (ii) No solution.
2. For any 2×2 matrices, A and B, prove that
 $\text{trace}(A + B) = \text{trace}(A) + \text{trace}(B)$.
3. Define all subspaces of the vector space \mathbb{R}^3 over \mathbb{R} .
4. Define linear combination of vectors in a vector space. Write $(2, 3)$ as the linear combination of $(1, 0)$ and $(0, 1)$.
5. Define basis of a vector space. Write a basis of P_n , where P_n is the polynomials of degree less than or equal to n .
6. Consider the basis $B = \{u_1, u_2\}$ and $B' = \{u'_1, u'_2\}$ of \mathbb{R}^2 , where $u_1 = (1, 0)$, $u_2 = (0, 1)$, $u'_1 = (1, 1)$ and $u'_2 = (2, 1)$. Find the transformation matrix from $B' \rightarrow B$.
7. Let $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$. Find the dimension of W.

Turn over

8. Give an example of an infinite dimensional vector space.
9. Define rank and nullity of a matrix.
10. Find the image of $x = (1, 1)$ under the rotation of $\frac{\pi}{6}$, about the origin.
11. Define eigen values and eigen vectors of a matrix.

12. Find the eigen values of $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$.

13. If λ is the eigen values of a matrix A, show that λ^n is the eigen values of A^n .
14. Show that $(1, 1)$ and $(1, -1)$ are orthogonal vectors with respect to the Euclidean inner product.
15. Let W be the subspace spanned by the orthonormal vector $v_1 = (0, 1, 0)$. Find the orthogonal projection of $u = (1, 1, 1)$ on W.

(Ceiling 25 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 35.

16. Solve the following linear system by Gauss-Elimination method,

$$\begin{aligned}x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10.\end{aligned}$$

17. Prove that, if A and B are invertible matrices of same size, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

18. Show that the set $\{(1, 1, 2), (1, 0, 1), (2, 1, 3)\}$ spans \mathbb{R}^3 .
19. Show that the operator $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by the equations

$$\begin{aligned}w_1 &= 2x_1 + x_2 \\w_2 &= 3x_1 + 4x_2\end{aligned}$$

is one-one, and find $T^{-1}(w_1, w_2)$.

20. Let T be the operator which is the reflection about the xz plane in \mathbb{R}^3 . Find the matrix of T with respect to the standard basis.
21. Find the rank and nullity of the matrix

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

22. Find the bases of the eigen spaces of the matrix

$$\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}.$$

23. Show that a square matrix A is invertible if and only if 0 is not an eigen value of A .

(Ceiling 35 marks)

Turn over

Section C (Essay Type Question)

Answer any two questions.

Each question carries 10 marks.

24. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Define the followings with examples :

- (i) Diagonal matrices ;
- (ii) Lower triangular matrices ;
- (iii) Upper triangular matrices ;
- (iv) Symmetric matrices ; and
- (v) Singular matrices.

25. Let $v_1 = \{1, 2, 1\}$, $v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$.

(a) Show that $\{v_1, v_2, v_3\}$ is a basis of \mathbb{R}^3 .

(b) Find the co-ordinate vector of $v = (5, -1, 9)$ relative to the basis $\{v_1, v_2, v_3\}$.

26. Consider the following linear system :

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}.$$

(a) Show that the above system is consistent.

(b) Solve the above system of linear equations.

27. (a) Define similar matrices.

(b) Show that the following matrix is not diagonalizable :

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}.$$

(2 × 10 = 20 marks)

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(Pages : 4)

Name.....

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FOURTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Type Questions)*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

1. Show that the linear system of equations $4x - 2y = 1$ has infinitely many solutions.
 $16x - 8y = 4$

2. Write any two facts about row echelon forms and reduced row echelon forms.

3. Express the linear system
- $$\begin{aligned} 4x_1 - 3x_3 + x_4 &= 1 \\ 5x_1 + x_2 - 8x_4 &= 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 &= 0 \\ 3x_2 - x_3 + 7x_4 &= 2 \end{aligned}$$

in the form $AX = B$.

4. Let $V = \mathbb{R}^2$ and define addition and scalar multiplication as follows. For $\bar{u} = (u_1, u_2), \bar{v} = (v_1, v_2)$,
 $\bar{u} + \bar{v} = (u_1 + v_1, u_2 + v_2)$ and for a real number $k, k\bar{u} = (ku_1, 0)$. For $\bar{u} = (1, 1)$ and $\bar{v} = (-3, 5)$ find
 $\bar{u} + \bar{v}$ and for $k = 5$, find $k\bar{u}$. Also show that one axiom for vector space is not satisfied.
5. Define basis for a vector space.
6. How will you relate the dimension of a finite dimensional vector space to the dimension of its subspace. Give two facts.
7. Give a solution to the change of basis problem.
8. When you can say that a system of linear equation $Ax = b$ is consistent. What is meant by a particular solution of the consistent system $Ax = b$.
9. Find the rank of a 5×7 matrix A for which $Ax = 0$ has a two-dimensional solution space.

Turn over

10. If $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a matrix transformation. Then define its kernel $\ker(T_A)$ and Range of (T_A) . What is $\ker(T_A)$ in terms of null-space of A.
11. Discuss the geometric effect on the unit square of multiplication by a diagonal matrix $A = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$.
12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen value, if $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
13. Let \mathbb{R}^2 have the weighted Euclidean inner product $\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$. For $u = (1, 1), v = (3, 2)$, compute $d(u, v)$.
14. If u and v are orthogonal vectors in a real inner product space, then show that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$.
15. State four properties of orthogonal matrices.

(10 × 3 = 30 marks)

Section B (Paragraph/ Problem Type Questions)*Answer at least five questions.**Each question carries 6 marks.**All questions can be attended.**Overall Ceiling 30.*

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form

$$\text{as } \begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix} \text{ solve the system.}$$

17. If A is an invertible matrix, then show that A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$.
18. Let V be a vector space and \bar{u} , a vector in V and k a scalar. Then show that (i) $0\bar{u} = 0$;
(ii) $(-1)\bar{u} = -\bar{u}$.

19. If $S = [v_1, v_2, \dots, v_n]$ is a basis for a vector space V , then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$ in exactly one way. What are the co-ordinates of v relative to the basis S ?
20. Consider the basis $B = [u_1, u_2]$ and $B' = [u'_1, u'_2]$ for \mathbb{R}^2 , where $u_1 = (2, 2)$ $u_2 = (4, -1)$
 $u'_1 = (1, 3)$ $u'_2 = (-1, -1)$.
- (a) Find the transition matrix from B' to B .
- (b) Find the transition matrix from B to B' .
21. If A is a matrix with n columns, then define rank A , nullity of A and establish a relationship between them.
22. Define eigen space corresponding to an eigen value λ of a square matrix A . Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.
23. Use the Gram–Schmidt process for an orthonormal basis corresponding to the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$.

(5 × 6 = 30 marks)

Section C (Essay Type Questions)

*Answer any two questions.
 Each question carries 10 marks.*

24. Show that the following statements are equivalent for an $n \times n$ matrix A :
- (a) A is invertible.
- (b) $Ax = 0$ has only the trivial solution.
- (c) The reduced row echelon form of A is I_n .
- (d) A is expressible as a product of elementary matrices.
25. (a) Define Wronskian of the functions $f_1 = f_1(x), f_2 = f_2(x) \dots f_n = f_n(x)$ which are $n - 1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.
- (b) Show that the vectors $v_1 = (1, 2, 1), v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ form a basis for \mathbb{R}^3 .

Turn over

26. (a) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting entirely

row vectors from A .

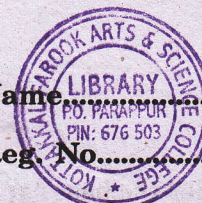
- (b) Find the standard matrix for the operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that first rotates a vector counter clockwise about z -axis through an angle θ , reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.

27. (a) On P_2 , polynomial in $[-1,1]$, define innerproduct as $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$. Find $\|p\|, \|q\|$

and $\langle p, q \rangle$ for $p = x$ and $q = x^2$.

- (b) If A is an $n \times n$ matrix with real entries, show that A is orthogonally diagonalizable if and only if A has an orthonormal set of n eigenvectors.

(2 × 10 = 20 marks)



FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION

APRIL 2021

Mathematics

MTS 4B 04—LINEAR ALGEBRA

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Type Questions)

Answer at least ten questions.

Each question carries 3 marks.

All questions can be attended.

Overall Ceiling 30.

1. Describe different possibilities for solution (x, y) of a system linear equations in the xy plane. What are consistent system ?

2. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form

$$\text{as } \begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \text{ solve the system.}$$

3. Define trace of a square matrix. Find the trace of the matrix $A =$

$$\begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$$

4. Show that the standard unit vectors

$$e_1 = (1, 0, \dots, 0), e_2 = (0, 1, 0, \dots, 0), e_3 = (0, 0, 1, 0, \dots, 0), \dots, e_n = (0, 0, \dots, 1) \text{ span } \mathbb{R}^n.$$

5. Find the co-ordinate vector of $w = (1, 0)$ relative to the basis

$$s = [\bar{u}_1, \bar{u}_2] \text{ of } \mathbb{R}^2, \text{ where } \bar{u}_1 = (1, -1) \text{ and } \bar{u}_2 = (1, 1).$$

6. Write two important facts about the vectors in a finite dimensional vector space V .

Turn over

7. Consider the bases $B = [\bar{u}_1, \bar{u}_2]$ and $B' = [\bar{u}'_1, \bar{u}'_2]$ where

$\bar{u}_1 = (1, 0)$, $\bar{u}_2 = (0, 1)$, $\bar{u}'_1 = (1, 1)$, $\bar{u}'_2 = (2, 1)$. Find the transition matrix $P_{B' \rightarrow B}$ from B' to B .

8. Define row spaces and null spaces an $m \times n$ matrix.

9. If $R = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the row reduced echelon form of a 3×3 matrix A , then verify the rank-

nullity formula.

10. Show that the operator $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that rotates vectors through an angle θ is one-one.

11. Find the image of the line $y = 4x$ under multiplication by the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen value if

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

13. Let A be an $n \times n$ matrix. Define inner product on \mathbb{R}^n generated by A . Also write the generating matrix of the weighted Euclidean inner product $\langle u, v \rangle = w_1 u_1 v_1 + w_2 u_2 v_2 + \dots + w_n u_n v_n$.

14. If u, v are vectors in a real inner product space V , then show that $\|u + v\| \leq \|u\| + \|v\|$.

15. If A is an $n \times n$ orthogonal matrix, then show that $\|Ax\| = \|x\|$ for all x in \mathbb{R}^n .

(10 × 3 = 30 marks)

Section B (Paragraph/Problem Type Questions)

Answer at least five questions.

Each question carries 6 marks.

All questions can be attended.

Overall Ceiling 30.

16. Describe Column Row Expansion method for finding the product AB for two matrices A and B . Use

this to find the product $AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \\ -3 & 5 & 1 \end{bmatrix}$.

17. If A is an invertible matrix, then show that A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$.
18. Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in \mathbb{R}^3 . Show that $w = (9, 2, 7)$ is a linear combination of u and v and that $w' = (4, -1, 8)$ is not a linear combination of u and v .
19. If $s = \{v_1, v_2, \dots, v_n\}$ is a basis for a vector space V , then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$ in exactly one way. What are the co-ordinates of v relative to the basis s .
20. If A is a matrix with n columns, then define rank of A and show that $\text{rank}(A) + \text{nullity}(A) = n$.
21. Find the standard matrix for the operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that first rotates a vector counter clockwise about z -axis through an angle θ , then reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.
22. Define eigen space corresponding to an eigen value λ of a square matrix A . Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.
23. If w is a sub-space of real inner product space v , then show that :

(a) w^\perp is subspace of v .

(b) $w \cap w^\perp = \{0\}$.

(5 × 6 = 30 marks)

Turn over