643217

D 124490 **(Pages : 2)** **Name..............................**

Reg. No...............................

SECOND SEMESTER (CBCSS-UG) DEGREE SUPPLEMENTARY EXAMINATION, APRIL 2025

Statistics

STA 2C 02—PROBABILITY THEORY

(2019 Syllabus)

Time : Two Hours Maximum Marks : 60

Use of calculator and statistical tables are permitted.

Part A (Short Answer Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Mention two examples for discrete random variables.
2. Define pairwise independence of events.
3. State addition theorem of probability for three events.
4. Define sample space of a random experiment with an example.
5. Mention any two properties of distribution function of a discrete random variable.
6. Define continuous random variable.
7. Define characteristic function.
8. Write the expression for second central moment using raw moments.
9. Define moment measure of skewness.
10. Define joint probability mass function.
11. Define conditional distributions.
12. Define rth raw and central moments using expectation.

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. State and prove Baye's theorem.
14. What are the advantages of classical definition of probability.
15. If $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find the probability of (1) at least one of the events occur ; (2) exactly one of the events occur.

Turn over

643217

643217

16. A continuous random variable has pdf $f(x) = 3x^2 ; 0 < x < 1$ and zero elsewhere. Find two numbers a and b such that (1) $P(x \le a) = P(x \ge a)$ (2) $P(x \ge b) = 0.05$.

17. Show that $V(aX + bY) = a^2 V(X) + b^2 V(Y)$ where X and Y are independent random variables.

18. Explain any two properties of moment generating function.

19. Given the joint pdf $f(x, y) = 2 ; 0 < x < y < 1$ and zero elsewhere; obtain $E(X | Y)$ and Var $(X | Y)$.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. Given the joint pmf $f(x, y) = \frac{1}{72}(2x + 3y); x = 0, 1, 2$ and $y = 1, 2, 3;$ examine whether X and Y are independent.

21. (i) Prove that the expectatioin of the product of two independent random variables is equal to the product of their expectations, provided all the expectations exists.

(ii) Show by an example that the expectation of the product is equal to the product of the expectations does not imply that the variables are independent.

($1 \times 10 = 10$ marks)

643217

D 103789 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2024

Statistics

STA 2C 02—PROBABILITY THEORY

(2019–2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. When do you say a random variable is discrete or continuous ?
2. Give the axiomatic definition of probability.
3. State multiplication theorem and addition theorem of probability.
4. If $B \subset A$, show that $P(A \cap B') = P(A) - P(B)$.
5. A continuous random variable X has pdf given by $f(x) = 2x, 0 < x \leq 1$ and 0 elsewhere. Find (i) F (x) ; (ii) P (X $\leq$ 1/2).
6. What are the properties of distribution function ?
7. Examine whether the following is a density function :

$$f(x) = 2x \text{ if } 0 < x \leq 1$$
$$= 4 - 2x \text{ if } 1 < x < 2$$
$$= 0 \text{ elsewhere}$$

8. Define m.g.f. and give any two properties of it.
9. Define characteristic function.
10. Define raw moments and central moments. Give the relationship between them.

Turn over

520934

11. Determine c if $p(x, y) = c(2x + 3y)$ where x = 0, 1 and y = 1, 2 is a joint p.d.f. Also find the corresponding distribution function.

12. If $f(x, y) = 1/4$ when $(x, y) \in \{(0,0), (1,0), (0,1), (1,1)\}$ and 0 elsewhere. Examine whether the variables X and Y are independent.

(20 marks)

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. Distinguish between mutual independence and pairwise independence of a set events. Give an example to show that pairwise independence does not imply mutual independence.

14. If P_1 and P_2 are probability measures and $0 < \lambda < 1$, show that $\lambda P_1 + (1 - \lambda) P_2 = P$ is a probability measure.

15. Distinguish between probability density function and distribution function. How are the two functions related

16. Let a coin with probability $p, 0 < p < 1$ for turning up of head be tossed until a head appears. Let X denote the number of tails observed. Find P(X = r).

17. Define expectation of a random variable. If X and Y are independent random variables with means 10 and – 5 and variances 4 and 6 respectively. Find a and b such that Z = aX + bY will have mean 0 and variance 28.

18. The pdf of two random variables X and Y is $f(x, y) = 2, 0 \le x \le y \le 1$. Show that E (X) =1/3 and E (Y) = 2/3 and the correlation between X and Y is ½.

19. Define conditional mean and conditional variance in both discrete and continuous case.

(30 marks)

520934

520934

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. (a) State and prove Bayes' theorem.

    (b) The probabilities of X,Y and Z become managers are 4 : 2 : 3 .The probabilities that the Bonus scheme will be introduced if X,Y and Z becomes managers are 0.3, 0.5 and 0.8 respectively. If the Bonus scheme was introduced, What is the probability that X is appointed as the manager ?

21. $f(x,y)=\frac{1}{72}(2x+3y), x=0,1,2 \quad y=1,2,3$ is the joint density of (X,Y).

    (a) Find the distribution of X + Y.

    (b) Find the conditional distribution of X given X + Y = 3.

    (c) Examine whether X and Y are independent.

(1 × 10 = 10 marks)

520934

395024

C 43209 **(Pages : 2)** **Name..................................**

Reg. No..............................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2023

Statistics

STA 2C 02—PROBABILITY THEORY

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define field and sigma field.
2. State addition theorem of probability for two events.
3. Define the terms : (a) Mutually exclusive events ; and (b) Independent events.
4. If P (A) = 0.3, P (B) = 0.2 and $P(A \cap B) = 0.1$, find the probability that exactly one of the events will happen.
5. Define random variable.
6. Define pdf and give the properties of p.d.f.
7. A die is tossed until an odd number appears. Obtain the probability distribution of the number of tosses.
8. Three unbiased coins are tossed. Find the expectation of the number of heads.
9. What are the limitations of moment generating function.
10. What is meant by skewness ?
11. The joint p.d.f. of a two dimensional random variable(X, Y) is given by $f(x, y) = 2, 0 < x < 1$, $0 < y < x$ and 0 elsewhere. Find the marginal density functions of X and Y.
12. How do you interpret correlation co-efficient ?

Turn over

395024

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 *marks.*

Maximum marks that can be scored from this part is 30.

13. Define conditional probability of two events. If A and B are independent, show that (a) A and B^c are independent ; and (b) A^c and B^c are independent.

14. Define independence of two random variables. Give one example to show that pairwise independence does not imply mutual independence.

15. A continuous random variable X has the p.d.f. $f(x) = 3x^2, 0 \le x \le 1$. Find two numbers a and b such that : (i) $P(x \le a) = P(x \ge a)$; and (ii) $P(x \ge b) = 0.05$.

16. $f(x) = \frac{x}{15}$, when $x = 1, 2, 3, 4, 5$ and 0 elsewhere is the density function of the random variable X. Find its distribution function.

17. Two unbiased dice are thrown. Find the expectation of the sum of the number of points on them.

18. Find the m.g.f of $f(x) = \frac{1}{2} e^{-|x|}, \; -\infty < x < \infty$.

19. Joint distribution of X and Y is given by : $f(x, y) = 4xy\, e^{-\left(x^2 + y^2\right)}, x \ge 0, y \ge 0$. Test whether X and Y are independent. Also find the conditional density of X given $Y = y$.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. State Bayes' theorem. There are two identical boxes containing respectively 4 white and 3 red balls; 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. Find the probability that it is from the first box ?

21. Let $f(x, y) = 8xy, 0 < x < y < 1$; $f(x, y) = 0$ elsewhere. Find (i) $E(Y/X = x)$; (ii) $E(XY/X = x)$; and (iii) $Var(Y/X = x)$.

C 23897 **(Pages : 2)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Statistics

STA 2C 02—PROBABILITY THEORY

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 *marks.*

Maximum marks that can be scored from this part is 20.

1. Show that the probability of an impossible event is zero.
2. If A,B and C are 3 mutually independent events, then show that $A \cup B$ and C are also independent.
3. State multiplication theorem of probability.
4. Explain the terms : (i) Sample space ; (ii) Mutually exclusive events ; (iii) Exhaustive events ; and (iv) Equally likely events.
5. Define random variable. Distinguish between discrete and continuous random variable using examples.
6. Let X be a random variable with p.d.f. $f(x) = (1/2)^x, x = 1,2,...$ and zero elsewhere. Find the p.d.f of $Y = X^2$.
7. For the density function $f(x) = ke^{-\theta x}, x \geq 0, \theta > 0$ and 0 elsewhere, find the value of k.
8. If the first three moments about origin are respectively 1,7 and 38. Obtain the co-efficient of skewness.
9. If $\mu'_r = r!$ for a random variable X, find its m.g.f.
10. For two dimensional discrete random variables X and Y show that E (X + Y) = E (X) + E (Y).
11. For two random variables X and Y show that V (Y) = E [V(Y/X)] + V [E(Y/X)].
12. Define conditional mean and conditional variance.

Turn over

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. Two fair dice are thrown once. Find the probability that sum of the upper faces is 10 or above provided first die shows an even number.

14. Two urns I and II contain respectively 3 white and 2 black balls, 2 white and 4 black balls. One ball is transferred from urn 1 to the urn II and then a ball is drawn from the latter. It happens to be white. What is the probability that the transferred ball was white ?

15. What are the properties of p.d.f and distribution function?

16. A coin is known to come up heads three times as often as tails. This coin is tossed three times. Let X be the number of heads that appear. Write down the possibility density function of X.

17. Define skewness and kurtosis. Give any one measure for examining skewness and kurtosis.

18. State and prove Cauchy-Schwartz inequality for two random variables X and Y.

19. For the joint function, $f(x, y) = \frac{2}{3}(1+x)e^{-y}, 0 < x < 1, y > 0.$ Obtain the conditional distribution of x given $y = 1$ and that of y given $x = 1/3$.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. (i) State and prove addition theorem of probability.

    (ii) The probability of male birth is 0.6.Find the probability that in four births : (a) All are boys ; (b) All are girls ; (c) Atleast one is a girl ; and (d) There are two boys and two girls.

21. Two random variables X ang Y have the following p.d.f :

    $f(x,y) = 2 - x - y, 0 \le x \le 1, 0 \le y \le 1$ and zero elsewhere. Find :

    (i) Marginal probability density functions of X and Y.

    (ii) Conditional density functions.

    (iii) Var(x) and Var(Y).

C 22110 **(Pages : 2)** **Name.....................................**

Reg. No.................................

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Statistics

STA 2C 02—PROBABILITY THEORY

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of calculator and statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Distinguish between discrete and continuous sample space.
2. For two events A and B express the events (i) at least one of A and B ; (ii) exactly one of A and B ; (iii) none of A and B ; (iv) all of A and B.
3. If B is the subevent of A, show that $P(A \cap B^c) = P(A) - P(B)$.
4. Given P(A/B) = 0.4, find P(A) if P $(A \cup B)$ and P(B) are 0.8 and 0.5 respectively.
5. Verify whether $f(x) = 3x^2$, for $0 < x < 1; f(x) = 0$, elsewhere, is a p.d.f. If so find P $(0.5 < X < 15)$.
6. An unbiased die is tossed until the face with number 6 appears to start a game. If X is the number of tosses before 6 appears, obtain the probability mass function of X.
7. Define distribution function of a random variable X. Mention any two properties of distribution function.
8. Find the expected number of heads in 3 tosses of an unbiased coin.
9. For a random variable X, show that $V(aX - b) = a^2 V(X)$, when they exist.
10. If $M_X(t)$ is the m.g.f. of X, show that $M_{aX+b}(t) = e^{bt} M_X(at)$.
11. The joint p.m.f. of X and Y is $f(x, y) = \frac{x + y}{12}$, for $x = 1, 2 ; y = 1, 2$. Identify the probability distribution of $\mathbb{Z}$, where $\mathbb{Z}$ = X + Y.
12. Show that Cov(X, Y) = – Cov (X, Y), where X and Y are two random variables.

(8 × 3 = 24 marks)

Turn over

204501

Section B

*Answer at least **five** questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Define partition of the sample space. Prove that the events A and $(A^c \cap B)$ partition the sample space in to two, where A and B are exhaustive events.
14. State and prove Bayes' theorem.
15. Obtain the probability distribution of X, if the distribution function of X is,

$$F(x) = \begin{cases} 0, & \text{if } x < -2 \\ \frac{1}{8}, & \text{if } -2 \le x < 0 \\ \frac{1}{2}, & \text{if } 0 \le x < 3 \\ \frac{7}{8}, & \text{if } 3 \le x < 5 \\ 1, & \text{if } x \ge 5. \end{cases}$$

    Also find P(X > – 2/X < 4).

16. Given the p.d.f. of X as $f(x) = 2e^{-2x}$, for $x > 0$. Find the p.d.f. of Y = 2X – 5.
17. The p.d.f. of X is $f(x) = 1$, for $0 < x < 1$. Find the coefficient of skewness of X based on moments.
18. Define characteristic function of a random variable X. If $\phi_X(t)$ is the characteristic function of X, prove that $|\phi_X(t)| \le 1$.
19. State and prove Cauchy-Schwartz inequality for two random variables X and Y.

(5 × 5 = 25 marks)

Section C (Essay Type Question)

*Answer any **one** question.*
The question carries 11 marks.

20. (a) State and prove the addition theorem on probability for any two events A and B. Hence establish the addition theorem on probability for three events A, B and C.

    (b) Given P(A) = 0.2, P(B) = 0.5 and P(C) = 0.4. Find the probability of the happening of at least one of these events if A, B and C are mutually independent events.

21. (a) If X and Y are two random variables, prove that $E[X(X/Y = Y)] = E(X)$.

    (b) The joint p.d.f. of (X, Y) is $f(x, y) = 2 - x - y$; for $0 < x < 1, 0 < y < 1$, find Cov (X, Y).

(1 × 11 = 11 marks)

204501

C 4402 **(Pages : 3)** **Name.....................................**

Reg. No.................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Statistics

STA 2C 02—PROBABILITY THEORY

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define (a) Random experiment ; (b) Event.
2. If the events A = { 1, 2, 3, 4, 5} and B = {4, 6, 7} are exhaustive events, identify the events : (i) $A \cap B^c$; (ii) $(A \cup B)^c$.
3. If P (A) = 0.4, P (B) = 0.5 and $P(A \cup B) = 0.7$. Find $P(A/B^c)$
4. State multiplication theorem on probability for two events A and B. If P (A/B) = P (A) = P (B) = 0.4. Find $P(A \cup B)$.
5. Define probability density function and state any *two* of its properties.
6. Obtain the distribution function of X, with p.d.f. $f(x) = 3x^2$, for $0 < x < 1$.
7. Find the value of k, if $f(x) = \left(\frac{k}{2}\right)^x$, for $x = 1, 2, 3...$ is the probability mass function of X.
8. If E (X) = 2, E (X^2) = 8, find V (3X – 2).
9. Obtain the mean and variance of a random variable X with m.g.f. $M_X(t) = (1-t)^{-1}$, $t < 1$.
10. Define characteristic function of a random variable and state its advantage over m.g.f.

Turn over

111105

11. Find c, if $f(x, y) = c(x + 2y)$, for $x = 1,2$; $-y = 0, 1$ is the joint p.m.f. of (X, Y).

12. Define independence of two random variables X and Y.

(8 × 3 = 24 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. Mentioning the underlying assumptions clearly, state axiomatic definition of probability.

    Using this definition establish $0 \le P(A) \le 1$ for an event A.

14. A box contains 3 blue and 2 red balls. Another box contains 2 blue and 3 green balls. One of the identical boxes is selected and two balls were drawn without replacement. It is found that the two balls are blue. What is the probability that only green balls to remain in the selected box ?

15. The p.m.f. of X, $f(x) = \frac{2x^2 - 1}{k}$, for $x = 1, 2, 3, 4$ and $f(x) = 0$ *elsewhere* (i) Find k ; (ii) Write the distribution function $F(x)$.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -2 \log_e X$.

17. In a game three balls are drawn from a box containing 5 white and 7 black balls. 10 points are given for each white ball drawn and 5 points are given for each black ball drawn. Calculated the expected points per game for a long run of the game.

18. For X with p.d.f. $f(x) = kx(2 - x)$, for $0 < x < 1$; $f(x) = 0$, elsewhere. Obtain (a) k ; (b) Mean and variance of X.

19. For two random variables X and Y, prove that (i) $V(X - Y) = V(X) + V(Y) - 2\,\mathrm{Cov}(X, Y)$; (ii) $\mathrm{Cov}(X - a, Y - b) = \mathrm{Cov}(X, Y)$, where a and b are two constants.

(5 × 5 = 25 marks)

11105

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) If A and B are two independent events prove that A^c and B^c are also independent.

    (b) Define the mutual independence of three events A, B and C. Also illustrate that the pairwise independence of A, B and C need not imply their mutual independence.

21. (a) Cauchy-Schwartz Inequality for two random variables X and Y.

    (b) Using this inequality prove $-1 \leq r_{XY} \leq +1$, where r_{XY} is the coefficient of correlation between X and Y.

(1 × 11 = 11 marks)

11105 11105 11105

C 82459 (Pages : 2) Name..................................

Reg. No..............................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2C 02—PROBABILITY THEORY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define sample space, event of a random experiment.
2. Explain mutually exclusive and exhaustive events.
3. If P(A) = 0.6, P $(A \cup B)$ = 0.8, find P(B) when A and B are independent.
4. Define P(A/B), where A and B are two events. Also state the multiplication theorem on probability.
5. Differentiate discrete and continuous random variables.
6. Find k, if $f(x) = kx^2$, for $0 < x < 1$ is a probability density function of X.
7. For a random variable X with possible values 1, 2 and 3, identify with reason, the values F(0.5) and F(3.2) where F is the distribution function of X.
8. Define Mathematical expectation of a discrete random variable X. Also show that, for a random variable X, $[E(X)]^2 \leq E(X^2)$ if the expectations exist.
9. If $M_X(t)$ is the m.g.f. of X, identify the m.g.f. of 2X – 5.
10. Find the characteristic function of X, where $P(X = x) = 0.5$; for $x = 0.1$.
11. Express coefficient of correlation between two random variables X and Y in terms of expectations.
12. If the joint p.d.f. of X and Y is $f(x, y) = 1$, for $0 < x < 1; 0 < y < 1$, find P(X > 0.2/Y > 0.6).

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. For two events A and B, P(A) = 0.4, P(B) = 0.3, $P(A \cap B) = 0.2$. Find (i) P(At least one of A and B to happen) ; (ii) P(Exactly one of A and B to happen).
14. For two events A and B, prove that $P(A \cup B)/C = P(A/C) + P(B/C) - P(A \cap B/C)$.

Turn over

15. Identify the distribution function of X and sketch its graph when the possible values of X are $-1, 0, 1$ and 2 with respective probabilities 0.2, 0.35, 0.4 and 0.05.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -\log_e X$.

17. Given the p.d.f. of X as $f(x) = e^{-x}$, for $0 < x < \infty$. Find the m.g.f. of X and hence the variance of X using m.g.f.

18. The first three raw moments of X are $\lambda, \lambda^2 + \lambda$ and $\lambda^3 + 3\lambda^2 + \lambda$. Obtain the coefficient of skewness of X and identify the condition for symmetry.

19. State and prove Cauchy-Schwartz inequality for two random variables X and Y.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. State and prove Bayes' theorem. Result of a survey from a college consists of 40 % boys and 60% girls on a recently released film, reveals that 35 % of the boys like the film but 30 % of the girls not like the film. A randomly selected student from this college likes the film. What is the probability that the student is a girl ?

21. (a) State and prove the multiplication theorem on expectation for the two random variables X and Y.

    (b) If the joint p.d.f. of (X, Y) is $f(x, y) = cxy$, for $0 < x < y < 1$.

    (i) Find the value of c ; (ii) Verify whether X and Y are independent.