D 122527 **(Pages : 2)** **Name.....**

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2025

(CBCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer all **eight** *questions.*
Each question carries 1 *weightage.*

1. Prove that $\sin(x+iy) = \sin x \cosh y + i \cos x \sinh y$.
2. Explain the concept of residues.
3. For a group $\left(A, A^2, A^3 = E\right)$, find the elements conjugate to A and A^2.
4. Draw the group multiplication table for 3 element permutation group.
5. Give with proof any *two* properties of representations of groups.
6. Apply Euler equation to find the shortest distance between two points in space.
7. Define Fredholm and Volterra types of integral equations with one example for each.
8. Define one dimensional Green's function.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions.*
Each question carries 5 *weightage.*

9. a) Explain Cauchy's integral theorem.
   b) Derive Cauchy's integral formula.

Turn over

10. Distinguish between discrete and continuous groups. Give characteristics of special orthogonal groups $SO(2)$ and $SO(3)$.

11. Explain the method of Lagrange's undetermined multipliers in problems of minimization under constraints. Illustrate your answer with an example.

12. Explain the procedure of converting a 2nd order first degree non-homogeneous Differential Equation into an integral equation. Illustrate with the example of linear oscillator.

(2 × 5 = 10 weightage)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

13. Evaluate the following integral by Cauchy's residue theorem : $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2}$ where p is a real number fixed between 0 and 1, $(0<p<1)$.

14. List with proper proof, the sub groups of the symmetry elements of an equilateral triangle .Find the normal sub group for this group.

15. Explain "Eight fold way" for the classification of particles.

16. Derive a Volterra integral equation corresponding to $y''(x)-y(x)=0$. Given $y(0)=0$ and $y'(0)=1$.

17. Find the Neumann series solution for the integral equation

$$\phi(x)=1-2\int_0^x t\,\phi(t)\,dt.$$

18. Find the Green's function solution for the equation $\nabla^2\psi(\mathbf{r})=f(\mathbf{r})$.

19. Apply calculus of variations to find the path of a point mass falling under gravity.

(4 × 3 = 12 weightage)

D 102184 **(Pages : 2)** **Name.....**

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE REGULAR/SUPPLEMENTARY EXAMINATION, APRIL 2024

(CBCSS)

Physics

PHY2C06—MATHEMATICAL PHYSICS – II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all eight** *questions.*
Each question carries 1 *weightage.*

1. Distinguish between simply connected and multiple connected regions. Explain their relevance in complex analysis
2. Show that z* is not analytic
3. State and explain Cauchy's integral theorem
4. State the conditions for the formation of a group
5. Distinguish between reducible and irreducible representations
6. Draw the fundamental SU(3) representation weight diagrams for the u d s quarks and anti-quarks
7. Give any two different forms of Euler equation and show their equivalence
8. Distinguish between integral equations of I and II kinds.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions.*
Each question carries 5 *weightage.*

9. Explain the term residue . State and prove Cauchy's residue theorem
10. a) Explain with examples: Cyclic groups, Cosets, Normal sub group and factor group.
    b) Distinguish between homomorphism and isomorphism with at least two properties. Give an example.

Turn over

11. Explain the features of calculus of variation . Derive Euler equation for a stationary path

12. Give a brief account of quantum mechanical theory of scattering using Green's function

(2 × 5 =10 weightage)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

13. Evaluate the following integral by Cauchy's Integral formula $\oint \frac{dz}{z^2-1}$ b) $\oint \tan z \, dz$ where the contour of integration is a circle centered at origin and having radius = 2.

14. Find the classes and subgroups of the group of symmetry elements of a square (C4v group).

15. Using the calculus of variations, find the optical path near event horizon of a black hole

16. Prove that any non-unitary representation of a finite group is equivalent to a representation by unitary matrices.

17. Find the Green's function to solve the following boundary value problem $y'' + k^2 y = f(x)$ with the initial condition $y(0) = 0, y(\mathrm{L}) = 0$ where k and L are constants.

18. Derive the Volterra integral equation corresponding to :

    a) $y''(x) - y(x) = 0$ with $y(0) = 0$ and $y'(0) = 1$.

    b) $y''(x) - y(x) = 0$ with $y(0) = 1$ and $y'(0) = -1$.

19. Find the Neumann series solution for the integral equation :

    $\phi(x) = 1 - 2\int_0^x t\phi(t)dt$.

(4 × 3 = 12 weightage)

C 42802 **(Pages : 2)** **Name.....**

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2023

(CBCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 Short questions answerable within 7.5 minutes.
Answer **all** *questions, each question carries 1 weightage.*

1. Write the Cauchy-Reimann differential equations and explain their significance.
2. What conditions should be satisfied for a group to be abelian ?
3. Mention any two problems solved using the variation principle.
4. Find the Neumann series solution for the Fredholm integral equation of the second kind.
5. Enlist different types of integral transforms. Represent the mathematical form of any one of the integral transform.
6. Why is homomorphism also called multiple-isomorphism?
7. Describe Fredholm integral equation of first kind.
8. Briefly summarize the properties of Green's function.

(8 × 1 = 8 weightage)

Section B

4 essay questions answerable within 30 minutes.
Answer any **two** *questions, each question carries 5 weightage.*

9. Obtain the expansion of the Green's function for a boundary value problem in terms of the eigen functions of the corresponding eigen value problem.
10. Explain the operations associates with point groups that lead to representation of SO (2) and SO (3) groups.

Turn over

11. Deduce the Cauchy-Reimann condition for a function to be analytic.

12. Explain the Rayleigh-Ritz variation technique for the computation of approximate solutions to partial differentiation equations.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions, each question carries 3 weightage.*

13. A complex variable $z = x + iy$. Check if z^{-1} is analytic ?

14. Find the residue of $\frac{z^3 - z^2 + 1}{z^3}$ at infinity.

15. Prove that group of order 3 is always cyclic.

16. Find Laurent series of function $f(z) = \frac{1}{(1 - z^2)}$ with centre at $z = 1$.

17. Solve the integral equation $s = \int_0^s e^{s-t}\, g(t)\, dt$.

18. Prove that the inverse of the product of two elements of a group is the product of the inverse in reverse order.

19. Maximize $I(y) = \int_{x1}^{x2} 1 + y'^2\, dx$ where $y(x1) = y(x2) = 0$.

(4 × 3 = 12 Weightage)

C 23368 **(Pages : 2)** **Name.....**

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE [REGULAR/SUPPLEMENTARY] EXAMINATION, APRIL 2022

(CBCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

(8 short questions answerable within 7.5 minutes)
(Answer all questions, each question carries weightage 1.

1. How can a function $f(z)$ be expanded where $f(z)$ is singular ? Briefly explain.
2. Show that three cube roots of unity form an abelian group under multiplication.
3. Discuss about the generators of the SU (2) group.
4. Using the variation principle discuss the problem on curve of shortest length connecting two points in a plane.
5. Explain the role of Lagrange Multipliers.
6. Define an integral equation and explain its significance.
7. Explain the symmetry property of Dirac-delta function.
8. State and provide proof of Cauchy's integral formula.

(8 × 1 = 8 weightage)

Turn over

Section B

(4 essay questions answerable within 30 minutes)
Answer any **two** *questions, each question carries weightage* 5.

9. Obtain the solution to the Poisson's equation using Green's function.

10. Show that a twofold homomorphism exists between the group of 2 × 2 unitary matrices and the SO (3) group.

11. Explain the Rayleigh-Ritz variation technique for the computation of approximate solutions to partial differentiation equations.

12. Deduce the Cauchy-Reimann condition for a function to be analytic.

(2 × 5 = 10 weightage)

Section C

(7 problems answerable within 15 *minutes)*
(Answer any **four** *questions, each carry Weightage* 3*)*

13. Find Laurent series of function $f(z) = \frac{1}{(1 - z^2)}$ with centre at $z = 1$.

14. Construct the group multiplication table for the Vierrer group.

15. Find the residues of $f(z) = \frac{ze^z}{(z = a)^3}$ at $z = a$.

16. Obtain the eigen functions for Green's function.

17. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3}\, dx$.

18. Prove that the inverse of the product of two elements of a group is the product of the inverse in reverse order.

19. Solve the integral equation $s = \int_0^s e^{s-t}\, g(t)\, dt$.

(4 × 3 = 12 weightage)

C 4758 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2021

(CBCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS-II

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

8 *Short questions answerable within* 7.5 *minutes.*
Answer **all** *questions.*
Each question carries weightage 1.

1. State and provide proof of Cauchy's integral formula.
2. Explain isomorphism.
3. Explain the method of Lagrange Multipliers briefly.
4. Describe a Fredholm integral equation of the second kind.
5. Explain the symmetry property of Dirac-delta function.
6. Discuss about the generators of the SU (2) group.
7. Mention any two problems solved using the variation principle.
8. Enlist different types of integral transforms. Represent the mathematical form of any one of the integral transform.

(8 × 1 = 8 weightage)

Section B

4 *essay questions answerable within* 30 *minutes.*
Answer any **two** *questions.*
Each question carries weightage 5.

9. Discuss the representation of the two dimensional unitary group SU (2).

Turn over

10. Obtain the Green's function for a one-dimension operator.

11. Explain the Rayleigh-Ritz variation technique for the computation of approximate solutions to partial differentiation equations.

12. Deduce the Cauchy-Reimann condition for a function to be analytic.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions.*
Each question carries weightage 3.

13. Evaluate the integral $\oint_c \frac{dz}{z^2+z}$.

14. Prove that a group of order 4 may or may not be a cyclic group. Give example in both cases.

15. Find the residue of $f(z)=\frac{e^z}{z^2+a^2}$ at its singularities.

16. Maximize $I(y)=\int_{x1}^{x2} 1+y'^2 dx$ where $y(x1)=y(x2)=0$.

17. Obtain the eigen functions for Green's function.

18. Solve the integral equation $S=\int_0^s e^{s-t} g(t)\,dt$

19. Find Laurent series of function $f(z)=\frac{1}{(1-z^2)}$ with centre at $z = 1$.

(4 × 3 = 12 weightage)

C 83072 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION, JUNE 2020

(CBCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS–II

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Attempt **all** *questions.*
1 *weightage per question.*

1. Given that $f(z)=u+iv$ is analytic, show that both u and v satisfy the two dimensional Laplace equation.
2. How many distinct groups are there of order four ? Give the multiplication table for a non-cyclic group of order four.
3. What is the isotopic spin formalism for nucleons ?
4. Show that the shortest distance path in the two dimensional Euclidean plane is the straight line.
5. Write down the integral transform relations between a function and its Laplace transform.
6. Give the general form for the Green's function for the three dimensional Poisson equation.
7. Write down the form of F[(x)] that forms the basis of the Rayleigh-Ritz method for the computation of eigen functions and eigenvalues under a given normalizing condition.
8. Define conjugacy classes in the case of groups.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions.*
5 *weightage per question.*

9. Determine the definite integral $I(\sigma)=\int_{-\infty}^{+\infty}\frac{x\sin x}{x^2-\sigma^2}\,dx$, where σ is real and positive, in two different ways such that a) $I(\sigma)$ represents a standing wave and b) $I(\sigma)$ represents an outgoing wave.

Turn over

10. Set up the variational problem for getting a stationery value for $\int f\left(y_i, \frac{\partial y_i}{\partial x_j}, x_j\right) dx_j$ under the constraints $\varphi_k(y_i, x_j) = 0$ and obtain the corresponding Euler -Lagrange equations.

11. For a Fredholm equation of the second kind with a separable kernel outline a method of solution.

12. Given that the operator $\mathcal{L}$ is self-adjoint, obtain the eigen function expansion for the Green's function for the operator $\mathcal{L} - \lambda$.

(2 × 5 = 10 weightage)

Part C

Answer any **four** *questions.*
3 *weightage per question.*

13. Prove the identity $\left(\frac{ia-1}{ia+1}\right)^{ib} = \exp\left(-2b\cot^{-1}(a)\right)$.

14. Two coaxial wire circles are connected by a surface of minimum area generated by revolving a curve $y(x)$ about the x-axis. The curve is required to pass through fixed end points (x_1, y_1) and (x_2, y_2). Determine the equation for the curve $y(x)$. How are the integration constants of the general solution fixed?

15. Solve the integral equation $f(x) = \int_{-1}^{+1} \frac{\varphi(t)}{(1-2xt+x^2)^{1/2}} dt, -1 \le x \le +1$ for $\varphi(t)$ if $f(x) = x^{2s+1}$.

16. Determine the Green's function for the operator $\frac{d^2}{dx^2}$ given $y(0) = 0$ and $y'(1) = 0$.

17. Derive the differential equation equivalent to the integral equation $y(x) = x + a^2 \int_0^x (t-x)\, y(t) dt$.

18. If a non-trivial subgroup H of a group G consists of complete classes of G show that H is a normal subgroup of G.

19. Show that the operator for the z component of angular momentum is the generator for rotations about the z-axis.

(4 × 3 = 12 weightage)

C 82889 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION JUNE 2020

(CUCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS–II

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Write Cauchy-Riemann conditions.
2. Show that z^2 is analytic.
3. What is homomorphism ?
4. What are generators of a group ?
5. What is a representation of a group ?
6. What are discrete and continuous groups ?
7. Define Lagrange multipliers.
8. What is Rayleigh Ritz variational technique ?
9. Give 2 uses of calculus of variations.
10. Explain integral transform.
11. What is a non-homogenous equation ? Give example.
12. What is Green's function technique ?

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*
Each question carries 6 *weightage.*

13. Obtain Cauchy-Riemann conditions. Showthat z^* is not analytic.
14. Construct the group of a square C_{4v}. Write down its multiplication table and subgroups.

Turn over

15. Describe the Neumann series method of solving a linear integral equation of the second kind having a separable kernel.

16. a) Define Green's function and show it can solve a non-homogenous differential equation.

    b) Find expressions for 1 dimensional Green's function.

(2 × 6 = 12 marks)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

17. Using Cauchy-Reimann condition show that $d\left(\frac{\ln z}{dz}\right)=\frac{1}{z}$.

18. Obtain the Laurent series for $\frac{1}{z(z-1)}$.

19. Show that if a group H of order h is a subgroup of a group G of order g, then g is an integral multiple of h.

20. Show that every subgroup of index 2 is a normal subgroup.

21. Obtain the Neumann series for $\phi(x)=x+\frac{1}{2}\int_{-1}^{1}(t-x)\phi(t)\,dt$.

22. Deduce Green's function of the operator $\left(\frac{d^2y}{dx}+k^2\right)$ with boundary condition $y(0) = 0$, $y(L) = 0$

(4 × 3 = 12 weightage)

C 63084

(Pages : 2)

Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS – II

(2012 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries a weightage of 1.

1. Express Cauchy-Riemann conditions in polar form.
2. Differentiate between essential singularities and poles.
3. State Cauchy's residue theorem.
4. Give an example for a cyclic group.
5. What is meant by irreducible representation ?
6. Show that the identity element of a group is a class by itself.
7. Explain the concept of variation.
8. Prove the symmetry of Greens function.
9. Define Volterra equations of the first and second kind.
10. What are separable kernals ?
11. Explain the Eigen function expansion of Green's function.
12. Explain how Lagrangian multipliers are used in calculus of variations.

(12 × 1 = 12 weightage)

Part B

Answer any **two** *questions.*

Each question carries a weightage of 6.

13. Derive Cauchy's integral formula and its first derivative.
14. Explain homomorphism of groups Establish the homomorphism of SU(2) and SO(3).
15. Discuss the Neumann series method for the solution of linear integral equations with an example.
16. Explain the Rayleigh-Ritz variation technique for the computation of Eigen values.

(2 × 6 = 12 weightage)

Turn over

Part C

Answer any **four** *questions.*

Each question carries a weightage of 3.

17. Real part of an analytic function is $u = \frac{1}{2} \ln (x^2 + y^2)$. Find the imaginary part and the complex function.

18. Form the multiplication table for the group of symmetry operations of an equilateral triangle.

19. Find the residues at the poles of $1/(z^2 + 1)$.

20. Find the equation to a line connecting two parallel coaxial wire circles such that the wire revolving about the x-axis produces the minimum surface area.

21. Convert the equation $y'' + w^2 y = 0$ to an integral equation.

22. Ground state energy of a particle in a box is given by $E = \frac{h2}{8m} (1/a^2 + 1/b^2 + 1/c^2)$. Find the shape of the box that will minimize the energy subject to the condition that the volume is a constant.

(4 × 3 = 12 weightage)

C 63088 (Pages : 2)

Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II

(2017 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Find the residue of $g(z)=\frac{1}{z^2+1}$ at $=-i$.
2. Check whether the function $f(z) = z^2$ is analytic.
3. Evaluate $\int_C \frac{dz}{z^2-2z}$ where C is the circle $|z| = 1$.
4. Define group, subgroup and class.
5. What is meant by irreducible representation of a group ?
6. What is Lie group ?
7. Show that shortest distance between two points is a straight line.
8. Explain Hamilton's principle.
9. What are the advantages of integral equations ?
10. Show that the function $y(x)=\frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$ is a solution of the Volterra integral equation

$$y(x)=\frac{1}{\left(1+x^2\right)}-\int_0^x \frac{t}{\left(1+x^2\right)} y(t)\cdot dt.$$

11. Explain how Green's function can be used to solve a non-homogeneous differential equation.
12. What are the properties of one-dimensional Green's function ?

(12 × 1 = 12 weightage)

Turn over

Section B

Answer any **two** *questions.*
Each question carries 6 *weightage.*

13. State and explain the theorem of residues. Using this theorem, show that $\int_0^{2\pi} \frac{d\theta}{1-2t\cos\theta+t^2} = \frac{2\pi}{1-t^2}$ for $|t|<1$.

14. Explain the homomorphism between SU(2) and SO(3) groups.

15. Explain Lagrangian multipliers. Find the ratios of the sides of the rectangular parallelpiped subjected to constraint that the volume is constant.

16. What are integral equations ? Explain different types of integral equations. Describe how will you convert a linear second order differential equation into an integral equation.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

17. Using Cauchy's integral formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} \cdot dz$, where C is the circle $|z| = \frac{3}{2}$.

18. Show that three cube roots of unity form an abelian group under multiplication.

19. Find out the equation of motion of a simple pendulum using Lagrangian equations.

20. Using Neumann series, solve the integral equation $\varphi(x) = x + \frac{1}{2}\int_{-1}^{1} (t-x)\,\varphi(t) \cdot dt$.

21. Obtain Green's function for the non-homogeneous differential equation $\frac{d^2y}{dx^2} - k^2y = f(x)$ subject to $y(\pm\infty) = 0$.

22. Using Green's function solve the boundary value problem $y'' = f(x)$ with $y(0) = 0, y'(1) = 0$.

(4 × 3 = 12 weightage)

D 43532 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2018

(CUCSS–PG)

Physics

PHY 2C 06 – MATHEMATICAL PHYSICS – II

(2017 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*

Each question carries a weightage of 1.

1. Write a non-analytic function over the entire z plane.
2. What is the condition for the existence of the derivative of a complex function $f(z)$?
3. Explain reducible and irreducible representations.
4. Write any *two* examples of a group.
5. Explain the significance of Brachistochrone problem in the variational technique.
6. What is Hamilton's principle?
7. Explain the significance of scattering problem in quantum mechanics related to integral equations.
8. Write two examples to invert integral equations using standard integral transforms.
9. Explain how Green's function acts as a propagator function.
10. Explain the relation between Green's function and the Dirac Delta function.
11. Check whether the limit of the function z^* exist or not (* denotes complex conjugation).
12. Write any *two* conservation laws with their corresponding symmetries.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*

Each question carries a weightage of 6.

13. Prove Cauchy's integral theorem for multiply connected regions.
14. Explain the homomorphism between SU(2) and SO(3) groups.
15. Write and solve the Euler equations to make the integral from s_1 to s_2 of (ds/ s) stationary.
16. Convert the linear oscillator problem to integral equations with boundary conditions.

(2 × 6 = 12 weightage)

Turn over

Section C

Answer any **four** *questions.*

Each question carries a weightage of 3.

17. **Evaluate the closed contour integral from 0 to 2π of $[d\theta/(5 + 4\cos\theta)]$.**
18. **Explain the special unitary group SU(3).**
19. **Explain the Rayleigh-Ritz variational technique.**
20. **Find the Fourier transform solution with the kernel type $k(x, t)$.**
21. **Prove the symmetry property of the Green's function.**
22. **Give the eigen function expansion of Green's function.**

(4 × 3 = 12 weightage)

D 43528 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2018

(CUCSS–PG)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II (4C)

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. State the necessary and sufficient conditions for a function to be analytic.
2. Differentiate between poles and essential singularities of an analytic function.
3. What do you mean by residue of a function ? How is it evaluated ?
4. Explain the concept of conjugate elements and classes in a group.
5. Differentiate isomorphism and homomorphism.
6. Using variational Calculus derive Lagrange's equation.
7. What is the kernel of an integral equation ?
8. Write a short note on Neumann series. What is its significance ?
9. Write a note on SU(2) – SO(3) homomorphism.
10. Develop the Hilbert-Schmidt theory of integral equations.
11. If a function $f(z)$ is analytic at z_0, prove that it is continuous there.
12. What are the advantages of using Green's function technique in solving boundary value problems ?

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*
Each question carries 6 *weightage.*

13. Find the n^{th} derivative of a function $f(z)$ of the complex variable z in the domain of its analyticity and using the obtained formula, show that

$$\left(\frac{d^n e^{-z}}{dz^n}\right)_{z=1} = (-1)^n e^{-1}.$$

Turn over

14. Give an account of SU(2) group. Is it abelian ? Why ? Discuss the different irreducible representations of the SU(2) group.

15. From the concept of variation, derive Euler's equation. Also explain any two applications of this equation.

16. Obtain the Green's function for the Laplace's equation. Hence find out an expression for the electrostatic potential at a point due to a given charge distribution.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

17. Determine the poles of the function $f(z)=\frac{e^{1/z}}{z-3}$. Also find the residues at these poles.

18. Apply Calculus of residues to show that $\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)}=\frac{\pi}{4}$.

19. Show that the three cube roots of unity form an Abelian group under multiplication.

20. Estimate the ground state energy of a quantum harmonic oscillator by the method of Calculus of variation.

21. Using Neumann series solve :

$$\phi(x)=x-\int_0^x (t-x)\phi(t)\,dt.$$

22. Find the Green's function for the operator

$$Ly(x)=\frac{d^2y(x)}{dx^2}+y(x)$$

with the boundary conditions $y(0)=0=y'(1)$.

(4 × 3 = 12 weightage)

C 4674 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Physics

PHY2C06—MATHEMATICAL PHYSICS—II

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. What are Cauchy-Riemann conditions for analyticity ?
2. Define poles and zeros of a function.
3. Indicate how a simply connected region is converted into a multiply connected region.
4. Give an example for a cyclic group.
5. What is meant by irreducible representation ?
6. Show that the identity element of a group is a class by itself.
7. Explain the concept of variation.
8. Prove the symmetry of Greens function.
9. What is the equation to a plane curve along which a particle acted upon by gravity alone would descent down ?
10. What are Lagrange multipliers ?
11. Explain separable kernals.
12. Define Volterra equations of the first and second kind.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*
Each question carries a weightage of 6.

13. Obtain the Laurent series expansion of a complex function
14. Explain homomorphism of groups. Establish the homomorphism OF SU(2) and SO(3).
15. Discuss the Neumann series method for the solution of linear integral equations with an example.
16. Obtain the Green's function solution of Poisson's equation.

(2 × 6 = 12 weightage)

Turn over

Section C

Answer any **four** *questions.*
Each question carries a weightage of 3.

17. Find the sum of the residues of the function $f(z) = \frac{\tan z}{z}$.

18. Show that the order of each sub group of a group is a divisor of the order of the group.

19. Integrating over a suitable contour evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$.

20. Find the equation to a line connecting two parallel coaxial wire circles such that the wire revolving about the x-axis produces the minimum surface area.

21. Derive Fredholm equation, corresponding to $y''(x) - y(x) = 0$; $y(1) = 1, y(-1) = 1$ by integrating twice.

22. Convert the equation $y'' + \omega^2 y = 0$ to an integral equation.

(4 × 3 = 12 weightage)

C 83629 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CUCSS)

Physics

PHY 2C 06 – MATHEMATICAL PHYSICS – II (4C)

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*

Each question carries a weightage of 1.

1. Define an analytic function. Give an example.
2. Find the nature of singularity of the function $f(z) = \exp(1/z)$.
3. Indicate how a simply connected region is converted into a multiply connected region.
4. Give an example for an abelian group.
5. What is meant by irreducible representation?
6. Show that the identity element of a group is a class by itself.
7. Explain the concept of variation.
8. Prove the symmetry of Greens function.
9. What is the equation to a plane curve along which a particle acted upon by gravity alone would descent down?
10. Explain the conversion of a differential equation into an integral equation.
11. What are separable kernels?
12. How is Green's function related to spherical Bessel function?

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*

Each question carries a weightage of 6.

13. State and prove Cauchy's residue theorem.
14. Explain homomorphism of groups. Establish the homomorphism of SU (2) and SO (3).

Turn over

15. Solve the equation $\phi(x) = f(x) + \lambda \int_a^b k(x, t)\, \phi(t)\, dt$ by the Neumann series method.

16. Derive Euler's equation for one independent and one dependent variable.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*

Each question carries a weightage of 3.

17. Evaluate $\int_C e^z/(z^2 + 1)^2$ where C is the circle $|z - 1| = 3$.

18. Show that every group of prime order is cyclic.

19. Expand $f(z) = \ln(1 + z)$ as a taylor series about $z = 0$.

20. Find the equation to a line connecting two parallel coaxial wire circles such that the wire revolving about the x-axis produces the minimum surface area.

21. Convert the equation $y'' + \omega^2 y = 0$ to an integral equation.

22. From the lens equation $1/u + 1/v = 1/f$, find the minimum object image distance $(u + v)$ for the formation of real image, applying Lagrangian multipliers.

(4 × 3 = 12 weightage)

C 63095 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Physics

PHY 2C 06—MATHEMATICAL PHYSICS—II

(2012 Admissions)

Time : Three Hours Maximum : 36 Weightage

Section A

Answer **all** *questions.*
Each question carries weightage of 1.

1. What are the Cauchy — Riemann Conditions for analyticity ?
2. What is meant by continuity of a function ?
3. Find the poles of the function $f(z) = \tan z$.
4. What are Co sets of a group ?
5. Prove Lagrange's theorem regarding the order of a group and that of its sub group.
6. What are the properties of Lie group ?
7. State Hamilton's principle.
8. What is meant by transversality condition ?
9. Explain the concept of variance.
10. Prove the symmetric property of Green's function.
11. What is the advantages of using Green's function technique in solving boundary value problems.
12. Define integral equation.

(12 × 1 = 12 weightage)

Section B

Answer any **two** *questions.*
Each question carries a weightage of 6.

13. State and prove Cauchy residue theorem. Find the residue at the poles of

$$f(z) = \frac{e^z}{z^2 + a^2}.$$

14. Explain homomorphism of the groups SU(2) and SU(3).

Turn over

15. Formulate the Green's function for Sturm —Liouvelle differential operator in one dimension.

16. Apply the Rayleigh–Ritz variation Principle for the computation of Eigenvalues and Eigenfunctions of the ground state of Helium atom.

(2 × 6 = 12 weightage)

Section C

Answer any **four** *questions.*
Each question carries 3 *weightage.*

17. Find the analytic function whose imaginary part is V = – Cos X Sin hy

18. Integrating over a suitable contour evaluate $\int_0^{2\pi} \frac{d\theta}{a + b \sin\theta}$.

19. Show that the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ form a group.

20. Apply Euler equation to find the shortest distance between two points in Euclidean space.

21. Find the integral equation corresponding to the boundary value problem

$Y''(x) + \lambda Y(x) = 0, \; Y(0) = Y(1) = 0.$

22. Determine the Green's function in terms of the Eigenvalues and Eigenfunctions of operator L for the differential equation $\frac{d^2\psi}{dx^2} = f(x)$ $0 \le x \le \lambda$ with the boundary condition $\psi(0) = 0 = \psi(l)$.

(4 × 3 = 12 weightage)