

D 130227

(Pages : 3)

Name.....

Reg. No.....

**FIFTH SEMESTER CBCSS-UG DEGREE EXAMINATION
NOVEMBER 2025**

(2020 Syllabus)

Mathematics

MTS 5B 06—BASIC ANALYSIS

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Short Answer Type**All questions can be answered.**Each question carries 2 Marks. (Ceiling 25 Marks)*

1. Let $a \in \mathbb{R}$. Prove that $a \cdot 0 = 0$
2. Let $a \in \mathbb{R}$. with $a \neq 0$. Prove that $a^2 > 0$.
3. Let x, y belongs to the ϵ -neighbourhood of a, b . Prove that $x + y$ belongs to the 2ϵ -neighbourhood of $a + b$.
4. Prove that an upper bound of u of a non-empty set S in \mathbb{R} is the supremum of S if and only if for every $\epsilon > 0$ there exist an $s_\epsilon \in S$ such that $u - \epsilon < s_\epsilon$.
5. Let S be a non-empty subset of \mathbb{R} that is bounded above and let $a \in \mathbb{R}$. Prove that $\sup(a + s) = a + \sup S$.
6. Consider the sequences $X = (2, 4, 6, \dots, 2n, \dots)$ and $Y = \left(\frac{1}{1}, \frac{1}{2}, \frac{1}{8}, \dots, \frac{1}{n}, \dots\right)$. Find XY and X/Y .
7. Let (x_n) be a sequence of real numbers that converges to x . Prove that $(|x_n|)$ converges to $|x|$.
8. Let (x_n) be a sequence of real numbers that converges to x . Prove that any subsequence (x_{n_k}) of (x_n) also converges to x .

Turn over

9. Whether the sequence $(1 + (-1)^n)$ is a Cauchy sequence? Justify your answer.
10. Let $c > 1$, find the limit $\lim(c^n)$.
11. Find the modulus of the complex number $\frac{2i}{3 - 4i}$.
12. Find the polar form of the complex number $-\sqrt{3} - i$.
13. Express the number $3i^5 - i^4 + 7i^3 - 10i^2 - 9$ in the form of $a + ib$.
14. Sketch the graph of $\operatorname{Re}(z) = 5$ in the complex plane.
15. Find the real and imaginary parts u and v of the complex function $f(z) = z^3 - 2z + 6$.

(Ceiling 25 marks)

Section B

*Paragraph/Problem Type.
All questions can be answered.
Each question carry 5 marks.
(Ceiling 35 marks).*

16. Let A be any set. Prove that there is no surjection of A onto the set $\mathcal{P}(A)$ of all subsets of A .
17. Let $a, b \in \mathbb{R}$. Prove that (i) $|ab| = |a||b|$. (ii) $|a + b| \leq |a| + |b|$.
18. Let $x, y \in \mathbb{R}$ with $x < y$. Prove that there exist an irrational number z such that $x < z < y$.
19. Let $X = (x_n)$ be a sequence of real numbers that converges to x and $Z = (z_n)$ be a sequence of non-zero real numbers that converges to z with $z \neq 0$. Prove that X/Z converges to x/z .
20. Let (x_n) be a bounded increasing sequence. Prove that $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.

21. Find $\operatorname{Re}(z)$ and $\operatorname{Im}(z)$ where $z = \left(\frac{i}{3-i}\right)\left(\frac{1}{2+3i}\right)$.
22. Describe the set of points z in the complex plane satisfy $|z-i| = |z-1|$.
23. Prove that the union of an arbitrary collection of open subsets in \mathbb{R} is open.

(Ceiling 35 marks)

Section C*Essay type.**Answer any two of the following questions.**Each question carries 10 marks.*

24. Prove that the set \mathbb{R} of all real numbers is uncountable.
25. State and prove Nested Intervals Property.
26. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Prove that the sequence $X = (\sqrt{x_n})$ of positive square roots converges to \sqrt{x} .
27. Find the image of the vertical line $x = 1$ under the complex mapping $w = z^2$.

(2 × 10 = 20 marks)

D 110209

(Pages : 3)

Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2024**

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Maximum 25 marks.

1. Prove that there does not exist a rational number r such that $r^2 = 2$.
2. Determine the set A of $x \in \mathbb{R}$ such that $|2x + 3| \leq 7$.
3. If $a, b \in \mathbb{R}$, prove that $|a + b| \leq |a| + |b|$.
4. State the supremum property of \mathbb{R} .
5. If $S = \left\{ \frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N} \right\}$, find $\inf S$ and $\sup S$.
6. State and prove Archimedean property.
7. Let x and y be real numbers with $x < y$, prove there exists an irrational number z such that $x < z < y$.
8. State and prove squeeze theorem.
9. If a sequence (x_n) of real numbers converges to a real number x , prove that any subsequence (x_{n_k}) of (x_n) also converges to x .

Turn over

10. Prove that every Cauchy sequence of real numbers is bounded.
11. Let (x_n) and (y_n) be two sequence of real numbers and suppose that $x_n \leq y_n$ for all $n \in \mathbb{N}$. If $\lim x_n = +\infty$, prove that $\lim y_n = +\infty$.
12. Prove that the intersection of any finite collection of open sets in \mathbb{R} is open.
13. Compute $(1 + \sqrt{3}i)^9$.
14. Find the real and imaginary parts of $f(z) = z^2 - (2+i)z$ as a function of x and y .
15. Show that the complex function $f(z) = z + 3i$ is a one to one on the entire complex plane and find a formula for its inverse function.

Section B

Answer any number of questions.

Each question carries 5 marks.

Maximum 35 marks.

16. State and prove Cantor's theorem.
17. Let a and b be positive real numbers, prove that $\sqrt{ab} \leq \frac{a+b}{2}$ and the equality occurs if and only if $a = b$.
18. State and prove density theorem.
19. Prove that unit interval $[0, 1]$ is not countable.
20. State and prove monotone convergence theorem.
21. Let $F \subseteq \mathbb{R}$; prove that the following are equivalent :
 - (a) F is a closed subset of \mathbb{R} ;
 - (b) If $X = (x_n)$ is any convergent sequence of element in F , then $\lim X$ belongs to F .
22. Find an upper bound for $\left| \frac{-1}{z^4 - 5z + 1} \right|$ if $|z| = 2$.

23. For any *two* complex numbers, prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.

Section C

Answer any **two** questions.

Each question carries 10 marks.

24. a) If A_m is a countable set for each $m \in \mathbb{N}$, prove that $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

b) State and prove Bernoulli's inequality.

25. a) State and prove monotone convergence theorem.

b) Let $s_1 = 1$ and $s_{n+1} = \frac{1}{2} \left(s_n + \frac{a}{s_n} \right)$ for $n \in \mathbb{N}$. Prove that (s_n) converges to \sqrt{a} .

26. a) Prove that every contractive sequence is a Cauchy sequence.

b) Let $f_1 = 1, f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n+1}}$. Prove that $\lim x_n = \frac{-1 + \sqrt{5}}{2}$.

27. Find a complex linear function that maps the equilateral triangle with vertices

$1 + i, 2 + i$ and $\frac{3}{2} + \left(1 + \frac{1}{2}\sqrt{3}\right)i$ onto the equilateral triangle with the vertices $i, \sqrt{3} + 2i$ and $3i$.

(2 × 10 = 20 marks)

D 50666

(Pages : 3)

Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023**

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Maximum 25 marks.*

1. State Cantor's theorem.
2. Prove that there does not exist a rational number r such that $r^2 = 3$.
3. If $a, b \in \mathbb{R}$, prove that $\| |a| - |b| \| \leq |a - b|$.
4. Prove that an upper bound u of a nonempty set S in \mathbb{R} is the supremum of S if and only if for every $\epsilon > 0$ there exist an $s_\epsilon \in S$ such that $u - \epsilon < s_\epsilon$.
5. State and prove Archimedean property.
6. Prove that a sequence in \mathbb{R} can have at most one limit point.
7. Prove that $\lim \left(\frac{\sin n}{n} \right) = 0$.
8. Let $e_n = \left(1 + \frac{1}{n} \right)^n$ for $n \in \mathbb{N}$. Prove that $2 < e_n < 3$ for all $n \in \mathbb{N}$.
9. Give an example of an unbounded sequence that has a convergent subsequence.
10. If (x_n) and (y_n) are Cauchy sequences, prove that $(x_n + y_n)$ is a Cauchy sequence.

Turn over

11. Prove that the sequence $\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n}\right)$ diverges.
12. Define Cantor set.
13. Describe the set of points z in the complex plane that satisfy the equation $|z - 2| = \operatorname{Re}(z)$.
14. Find the image of the line segment from 1 to i under the complex mapping $w = \bar{iz}$.
15. Find the image of the rectangle with vertices $-1 + i$, $1 + i$, $1 + 2i$ and $-1 + 2i$ under the linear mapping $f(z) = 4iz + 2 + 3i$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Maximum 35 marks.

16. Prove that the following statements are equivalent :
 - a) S is a countable set ; and
 - b) There exists a surjection of \mathbb{N} onto S .
 - c) There exists an injection of S onto \mathbb{N} .
17. Determine the set $B = \{x \in \mathbb{R} : |x - 1| < |x|\}$.
18. Prove that \mathbb{R} of real numbers is not countable.
19. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Prove that $X \cdot Y$ converges to xy .
20. State and prove Cauchy convergence criterion for sequences.
21. Prove that
 - a) the union of an arbitrary collection of open subsets in \mathbb{R} is open.
 - b) the intersection of any finite collection of open sets in \mathbb{R} is open.

22. Determine whether the points $z_1 = -2 - 8i$, $z_2 = 3i$, $z_3 = -6 - 5i$ are the vertices of a right triangle.
23. Let $S = \{z \in \mathbb{C} : 1 \leq |z - 1 - i| < 2\}$. Determine whether the set S is :
- Open ;
 - Closed ;
 - Domain ;
 - Bounded ; and
 - Connected.

Section C

*Answer any two questions.
Each question carries 10 marks.*

24. (a) Let a and b be positive real numbers, prove that $\sqrt{ab} \leq \frac{1}{2}(a + b)$ and the equality holds if and only if $a = b$.
- (b) State and prove Bernoulli's inequality.
25. Prove that there exists a positive real number x such that $x^2 = 2$.
26. (a) Prove that every contractive sequence is a convergent sequence.
- (b) The polynomial equation $x^3 - 7x + 2 = 0$ has a root r with $0 < r < 1$. Use an appropriate contractive sequence to calculate r within 10^{-5} .
27. (a) Solve the simultaneous equations $|z| = 2$ and $|z - 2| = 2$.
- (b) Find the image of the triangle with vertices 0 , $1 + i$ and $1 - i$ under the mapping $w = z^2$.

(2 × 10 = 20 marks)

D 30569

(Pages : 3)

Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION
NOVEMBER 2022**

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 Marks.**Maximum 25 Marks.*

1. Define denumerable set. Give an example.
2. If $a \in \mathbb{R}$, then prove that $a \cdot 0 = 0$.
3. Let a, b, c be elements of \mathbb{R} and if $a > b$ and $b > c$, then prove that $a > c$.
4. Prove that $|-a| = |a|$ for all $a \in \mathbb{R}$.
5. Describe Fibonacci sequence.
6. State Monotone Convergence Theorem.
7. Define Cauchy sequence. Give an example.
8. Define properly divergent sequence.
9. Show that $\mathbb{R} = (-\infty, \infty)$ is open.
10. Describe any two properties of Cantor Set.
11. Whether the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \dots\right)$ is convergent? Justify your answer.
12. Find the principal cube root at the point $z = i$.
13. Define bounded subset of the complex plane.

Turn over

14. Find the reciprocal of $z = 2 - 3i$.
15. Express $-\sqrt{3} - i$ in polar form.

Section B

Answer any number of questions.

Each question carries 5 Marks.

Maximum 35 marks.

16. State and prove Cantor's Theorem.
17. Prove that there does not exist a rational number r such that $r^2 = 2$.
18. (a) Define supremum of a set of real numbers.
- (b) Prove that there can be only one supremum of a given subset S of \mathbb{R} , if it exists.
19. Prove that $\lim \left(\frac{1}{n^2} \right) = 0$.
20. If $0 < b < 1$, then prove that $\lim (b^n) = 0$.
21. Prove that the intersection of an arbitrary collection of closed sets in \mathbb{R} is closed.
22. Show that the complex function $f(z) = z + 3i$ is one-to-one on the entire complex plane and find a formula for its inverse function.
23. If $f(z) = \frac{z}{\bar{z}}$ then show by two path test that $\lim_{z \rightarrow 0} f(z)$ does not exist.

Section C

Answer any two questions.

Each question carries 10 marks.

24. State and prove Monotone Subsequence Theorem.
25. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded. If $X = (x_n)$ is a bounded increasing sequence, then prove that :

$$\lim (x_n) = \sup \{x_n : n \in \mathbb{N}\}.$$

26. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then

prove that the sequence $(\sqrt{x_n})$ converges and $\lim(\sqrt{x_n}) = \sqrt{x}$.

27. Show that the function f defined by :

$$f(z) = \sqrt{r}e^{i\theta/2}, -\pi < \theta < \pi$$

is a branch of the multiple-valued function $F(z) = z^{1/2}$.