

D 132760

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Name.....

Reg. No.....

**FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION
NOVEMBER 2025**

Mathematics

MAT1MN100/MAT1CJ101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours

Maximum : 70 Marks

Section A*All questions can be answered.**Each question carries 3 marks.**Ceiling 24 marks.*

1. Draw the graph of the equation $x^2 = 8y$.
2. Give an example of a piecewise-defined function.
3. Find $\lim_{x \rightarrow +\infty} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} \right)$.
4. If $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 4$, then find $\lim_{x \rightarrow 2} f(x)$.
5. Find $\frac{dy}{dx}$ if $y = \cos(x^3)$.
6. Find an equation for the tangent line for the function $y = x \cos 3x$ to the graph at $x = \pi$.
7. Find the absolute maximum and minimum values of $f(x) = 8x - x^2$ on the closed interval $[0, 6]$.
8. State Mean-Value Theorem.

Turn over

9. Locate the critical points of the function $f(x) = 3x^4 + 12x$ and identify which critical points are stationary points.
10. Give a graph of the function $f(x) = 4x^2 - 1$ and identify the locations of all critical points and inflection points.

Section B

All questions can be answered.

Each question carries 6 marks.

Ceiling 36 marks.

11. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 1$, then find $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} \frac{f(x)}{x}$.
12. Determine whether the statement “If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x)$ exists, then $\lim_{x \rightarrow a} \frac{g(x)}{f(x)}$ exists” is true or false. Explain your answer.
13. Define $g(4)$ in away that extends $g(x) = \frac{x^2 - 16}{x^2 - 3x - 4}$ to be continuous at $x = 4$.
14. Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.
15. Find the absolute extrema of $f(x) = x^{2/3}(3 - x)$ on $[-2, 2]$.
16. Identify local extreme values of the function $h(x) = x^3 + 3x^2 + 3x + 1$ in the interval $-\infty < x \leq 0$. Also find its absolute extrema.
17. Find the asymptotes of the graph of $f(x) = -\frac{x^2 - 3}{2x - 4}$.
18. Solve $\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}}$.

Section C

*Answer any one question.
The question carries 10 marks.*

19. At time t , the position of a body moving along the s -axis is $s = t^3 - 6t^2 + 9t$ m.
- (a) Find the body's acceleration each time the velocity is zero
 - (b) Find the body's speed each time the acceleration is zero.
 - (c) Find the total distance traveled by the body from $t = 0$ to $t = 2$.
20. (a) Show that $f(x) = x^4 + 3x + 1$ has exactly one zero in the interval $[-2, -1]$.
- (b) Suppose the derivative of the function $y = f(x)$ is $y' = (x - 1)^2 x - 2$. At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?
- (1 × 10 = 10 marks)

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FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2025

Mathematics

MAT1CJ101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours

Maximum : 70 Marks

Section A*All questions can be answered.**Each question carries 3 marks.**(Ceiling 24 marks).*

1. Draw the graph of the equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
2. Find the domain of the function $f(x) = \frac{1}{x(x-1)(x-2)}$ in real numbers.
3. Find $\lim_{x \rightarrow 0^+} \ln \frac{2}{x^2}$.
4. Discuss the continuity of the function $\sqrt{2x+3}$.
5. Find $\frac{dw}{dt}$ if $w = \tan x$ and $x = 4t^3 + t$.
6. If $y = \sin(3x)$, then find $\frac{d^2y}{dx^2}$.
7. Find the absolute maximum and minimum values of $f(x) = (x-2)^3$ on the closed interval $[1, 4]$.

Turn over

8. Verify that the hypotheses of the Mean-Value Theorem for the function $f(x) = x^2 - x$ on the interval $[-3, 5]$ and find all values of c in that interval that satisfy the conclusion of the theorem.
9. Find the relative extrema of the function $f(x) = \frac{x+1}{x-1}$.
10. Give a graph of the function $f(x) = x + \sin x$ and identify the locations of all relative extrema and inflection points.

Section B

All questions can be answered.

Each question carries 6 marks.

(Ceiling 36 marks)

11. If $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3$, then find $\lim_{x \rightarrow 2} f(x)$.
12. Find $\lim_{x \rightarrow 4} \frac{3 - x}{x^2 - 2x - 8}$.
13. If functions $f(x)$ and $g(x)$ are continuous for $0 \leq x \leq 1$, could $f(x)/g(x)$ possibly be discontinuous at a point of $[0, 1]$? Give reasons for your answer.
14. Use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, if $\sqrt{y} = 3x^2 + \tan x$.
15. Find the absolute extrema values of $F(t) = 8t - t^4$ on $[-2, 1]$.
16. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

17. Find the asymptotes of the curve $f(x) = 2 + \frac{\sin x}{x}$.

18. Solve $\lim_{x \rightarrow -\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$.

Section C

Answer any **one** questions.

Each question carries 10 marks.

19. At time $t \geq 0$, the velocity of a body moving along the s -axis is $v = t^2 - 4t + 3$.

(a) Find the body's acceleration each time the velocity is zero.

(b) When is the body moving forward? moving backward?

(c) When is the body's velocity increasing? decreasing?

20. (a) Show that $f(x) = x^3 + \frac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

(b) Suppose the derivative of the function $y = f(x)$ is $y' = (x - 1)^2(x - 2)(x - 4)$. At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection?

(1 × 10 = 10 marks)

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Name.....

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**FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION
NOVEMBER 2024**

Mathematics

MAT 1CJ 101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours

Maximum Marks : 70

Section A

*All questions can be answered.
Each question carries 3 marks.
(Ceiling 24 marks)*

1. Draw the graph of the equation $y^2 = 4x$.
2. Write any three properties of absolute value function $f(x) = |x|$.
3. Find $\lim_{x \rightarrow +\infty} \left(\frac{1 - e^x}{1 + e^x} \right)$.
4. Does $\lim_{x \rightarrow 1} \frac{1}{1-x} = \lim_{x \rightarrow 0^-} \frac{1}{x}$ exist ? Explain.
5. At what points are the function $\frac{x}{\sin x}$ continuous ?
6. Let $f(x) = x^2$ and $g(x) = x + 6$. Find $(g \circ f)(x)$ and $(g \circ f)'(x)$.
7. Find the absolute maximum and minimum values of $f(x) = 4x^2 - 12x + 10$ on the closed interval $[1, 2]$.
8. Find the two x -intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.
9. Locate the critical points of the function $f(x) = x^3 - 3x^2$ and identify which critical points are stationary points.
10. Determine whether the statement "If the graph of f has a cusp at $x = 1$, then f cannot have an inflection point at $x = 1$ " is true or false. Explain your answer.

Turn over

Section B

*All questions can be answered.
Each question carries 6 marks.
(Ceiling 36 marks)*

11. Find the center and radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$.
12. Determine whether the statement “If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then so does $\lim_{x \rightarrow a} [f(x) + g(x)]$ ” is true or false. Explain your answer.
13. Define $h(2)$ in a way that extends $h(t) = \frac{t^2 + 3t - 10}{(t - 2)}$ to be continuous at $t = 2$.
14. Find $\frac{dy}{dx}$ if $2y = x^2 + \sin y$.
15. Find the absolute extrema of $f(x) = \sqrt{x} + \cos x$ on $\{0, 2\pi\}$.
16. Find the interval on which the function $f(x) = ax^2 + bx + c$, $a \neq 0$, is increasing and decreasing. Describe the reasoning behind your answer.
17. Find the asymptotes of the graph of $f(x) = -\frac{8}{x^2 - 4}$.
18. Solve $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 4}$.

Section C

*Answer any one question.
The question carries 10 marks.*

19. (a) Suppose $\lim_{x \rightarrow 1} f(x) = 2$ and $\lim_{x \rightarrow 1} g(x) = 9$. Find $\lim_{x \rightarrow 1} \frac{f(x) - x^2 g(x)}{x - f(x)}$.
- (b) Explain the continuity of the function $f(x) = \frac{3x + 4}{x^2 - 4}$.
20. (a) Suppose that $f(-1) = 3$ and $f'(x) = 0$ for all x . Must $f(x) = 3$ for all x ? Give reasons for your answer.
- (b) Discuss the concavity and convexity of the curve $y = x^2$.

(1 × 10 = 10 marks)