D 131477 **(Pages : 3)** **Name..**

Reg. No....................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2025

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2020–2023 Admissions)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

All *questions can be attended.*
Each question carries 2 marks.
Ceiling is 25.

1. Differentiate the function $f(x) = \log\left(\frac{\log(x)}{x^2}\right)$.

2. Find the derivative of $y = \log\left(\left|\operatorname{cosec}(x) - \cot(x)\right|\right)$.

3. Find $\lim\limits_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$.

4. Show that $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$.

5. Find $\lim\limits_{n \to \infty} \frac{n + 1}{\sqrt{n^2 + n + 1}}$.

6. Determine whether the sequence $a_n = \frac{\sin(1/n)}{n}$ converges or diverges. If it converges, find the limit.

7. Express $0.784\overline{784}\ldots$ as a rational number.

Turn over

8. Show that the series $\sum_{1}^{\infty}(3)^{2n} 5^{1-n}$ is divergent.

9. By using the power series expansion of e^{-x}, show that $\frac{d}{dx}\left(e^{-x}\right)=-e^{-x}$.

10. Find the Taylor series expansion of sin x about $x=\pi/2$.

11. Find the parametric equation of the curve whose rectangular equation is $xy = 1$.

12. Show that the graph of the function $f(x)=(x-4)^{2/3}$ has a vertical tangent at $x = 4$.

13. Find the distance between the point $(1,-4,-3)$ and the plane $2x-3y+6z+1=0$.

14. Find $r(t)$ given that $r'(t)=3i+2tj$ and $r(1)=2i+5j$.

15. Find the co-ordinates of point where the line $r(t)=(2+t)i+(1-2t)j+3tk$ intersects the xz plane.

(Ceiling 25)

Section B

All *questions can be attended.*
Each question carries 5 marks.
Overall Ceiling is 35

16. Evaluate :

    (i) $\lim\limits_{x\to 0}\frac{\tan 3x}{\sin 2x}$.

    (ii) $\lim\limits_{x\to 0}\frac{e^{2x}-1}{\sin x}$.

17. Use Logarithmic differentiation to find the derivative of $y=\sqrt{x^{x}}$.

18. Find $\int \sin(x)e^{x}dx$.

19. Show that the sequence $a_n=\frac{n!}{n^n}$ converges and find its limit.

20. Find the interval of convergence and radius of convergence of the power series $\sum_0^\infty n!x^n$.

21. Find the Macluarian series for sinh$x$ and determine its interval of convergence.

22. Find the arc length of the cardioid $r = 1 + \cos\theta$.

23. Find the curvature and radius of curvature of the curve $r(t) = 2\sin(2t)i + \sin(t)j$ at $t = \pi/2$.

(Ceiling 35)

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (i) Evaluate $\int x^2 \log(x)\,dx$.

(ii) Find $\int \frac{1}{\sqrt{1+e^{-2x}}}dx$.

(iii) Find $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$.

25. (i) Let C be the circle $r(t) = \cos t + \sin t + 2k$. Find T(t) and N(t) at $t = \pi/4$.

(ii) Identify and sketch the graph of the surface $z = \frac{y^2}{4} - \frac{x^2}{9}$.

26. (i) Find $\frac{d^y}{dx^2}$ for $x = \sin(t), y = \sin(2t)$.

(ii) Find the area of the region in the first quadrant that is within the cardioid $r = 1 - \cos\theta$.

27. (i) Find the equation of the paraboloid $z = x^2 + y^2$ in cylindrical and spherical co-ordinates.

(ii) Find the spherical co-ordinates of the point that has rectangular co-ordinates $(4, -4, 4\sqrt{6})$.

(2 × 10 = 20 marks)

D 111972 **(Pages : 3)** **Name.................................**

Reg. No..............................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2023 Admissions)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

All *questions can be attended.*
Each question carries 2 marks.

1. Differentiate the function $f(x) = \log\left(\dfrac{x}{\ln(x)}\right)$.

2. Find the derivative of $y = \log\left(\left|\sec(x) + \tan(x)\right|\right)$.

3. Find $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{1 - \cos(2x)}$.

4. Show that $\cosh^2 x - \sinh^2 x = 1$.

5. Find $\lim\limits_{n \to \infty} e^{\frac{1+n}{1-n}}$.

6. Determine whether the sequence $a_n = 1 + (-1)^n / n^2$ converges or diverges. If it converges, find the limit.

7. Express $.111\overline{1}\ldots$ as a rational number.

8. State the Squeeze Theorem.

9. By using the power series expansion of sin x, show that $\dfrac{d}{dx}(\sin x) = \cos x$.

10. Find the Maclaurian series expansion of $\dfrac{1}{1+x}$.

11. Find the rectangular equation of a curve whose parametric equation is $x = t + 1$, $y = t^2 - 1$.

12. Find the equation of the tangent to the ellipse $x = 3\cos t$, $y = 2\sin t$ at $t = \pi / 4$.

13. Find an equation of the line that passes through the point (– 1, 0, 2) and is parallel to the vector $\langle 1, 5, -4 \rangle$.

Turn over

14. Find the equation of the surface $z = x^2 + y^2$ in cylindrical co-ordinates.

15. Find $r'(t)$ if $r(t) = 2\cos ti + 3\sin tj + 3tk$.

(15 × 2 = 30 marks)
Max. Ceiling : 25 marks

Section B

All *questions can be attended.*
Each question carries 5 marks.

16. Evaluate :

(i) $\lim_{x\to 0} \frac{\tan 5x}{\sin 2x}$.

(ii) $\lim_{x\to 0} \frac{x^3 - 3\sin^2 x}{x^2}$.

17. Find the derivative of $y = (\cos x)^x$.

18. Find $\int x^2 e^{-x} dx$.

19. Use the integral test to determine the series $\Sigma_1^\infty \frac{1}{n^2}$ converge or diverge.

20. Find the interval of convergence and radius of convergence of the power series $\Sigma_0^\infty \frac{x^{2n+1}}{(2n+1)!}$.

21. Find the Maclaurian series for $\frac{1}{\sqrt{1-x}}$ and determine its interval of convergence.

22. Find $\frac{d^2y}{dx^2}$ for the parametric equation $x = a\cos t,\ y = b\sin t$.

23. Identify and sketch the graph of the surface $x^2 - x^2 - y^2 = 1$.

(8 × 5 = 40 marks)
Max. Ceiling : 35 marks

Section C

Answer any **two** *questions.*
Each question carries 10 marks.

24. (i) Show that $\int \frac{dx}{\sqrt{4x^2 - 9}} dx = \frac{1}{2}\cosh^{-1}\left(\frac{2x}{3}\right),\ x > 3/2$.

(ii) Find $\int_{-\infty}^{0} \frac{e^x}{\sqrt{1+e^{2x}}} dx$.

(iii) Find $\lim_{x\to 0} (\tan x)^x$.

25. (i) Let C be the ellipse $r(t) = 3\cos t + 2\sin t.$ Find T(t) and N(t) at $t = \pi/4$.

 (ii) Find the curvature of the curve $r(t) = ti + \frac{1}{t}j$ at $t = 1$.

26. (i) Find the total arc length of the cardioid $r = 1 - \cos\theta$.

 (ii) Find the area of the cardioid $r = 1 + \cos\theta$.

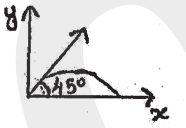
27. A shell fired from a cannon, has a muzzle speed of 80 ft/s. The barrel amkes an angle of 45° with the horizontal and, the barrel opening is assumed to be at ground level.

 (a) Find parametric equation for the shell's trajectory.

 (b) How high does the shell rise ?

 (c) How far does the shell travel horizontally ?

 (d) What is the speed of the shell at its point of impact with the ground.



(2 × 10 = 20 marks)

D 51760 **(Pages : 3)** **Name.......................................**

Reg. No..................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2023

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2022 Admissions)

Time : Two Hours and a Half   Maximum : 80 Marks

Section A

All questions can be attended
Each question carries 2 marks.
Ceiling is 25.

1. Differentiate the function $f(x) = \log(x \log(x))$.

2. Find the derivative of $y = \log(|\sec(x)|)$.

3. Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

4. Show that $\sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$.

5. Find $\lim_{n \to \infty} \ln\left(\cos\left(\frac{2n\pi + 1}{n}\right)\right)$.

6. Determine whether the sequence $a_n = 1 + (-1)^n / n$ converges or diverges. If it converges, find the limit.

7. Express $.33\overline{3}......$ as a rational number.

8. Show that the series $\sum_1^\infty \frac{1}{n(n+1)}$ is convergent.

Turn over

9. By using the power series expansion of e^x, show that $\frac{d}{dx}\left(e^x\right) = e^x$.

10. Find the Macluarian series expansion of $\frac{1}{1-x}$.

11. Find the parametric equation of the curve whose rectangular equation is $y^2 = 4x + 1$.

12. Find the equation of the tangent to the ellipse $x = 2\cos t$, $y = 3\sin t$ at $t = 3\pi/4$.

13. Find an equation of the line that passes through the points $(2, 4, -1)$ and $(5, 0, 7)$.

14. Find the rectangular co-ordinate of the point with cylindrical co-ordinate is $(4, \pi/3, -3)$.

15. Find $r'(t)$ if $r(t) = \cos t i + \sin t j + 3t^2 k$.

Section B

All questions can be attended.
Each question carries 5 marks.
Overall Ceiling is 35.

16. Evaluate :

    (i) $\lim_{x \to 0} \frac{\sin 3x}{\sin 2x}$.

    (ii) $\lim_{x \to 0} \frac{x^2 - 3\sin x}{x}$.

17. Use Logarithmic differentiation to find the derivative of $y = (\sin x)^x$.

18. Find $\int x^2 e^x dx$.

19. Show that the series $\sum_0^\infty \frac{1}{2n-1}$ is divergent.

20. Find the interval of convergence and radius of convergence of the power series $\sum_0^\infty \frac{(x-5)^n}{n^2}$.

21. Find the Macluarian series for $\frac{1}{\sqrt{1+x}}$ and determine its interval of convergence.

22. Find the arc length parametrization of the circular helix $r(t) = \cos ti + \sin tj + tk$ with $r(0) = (1, 0, 0)$.

23. Find the curvature of the ellipse $r(t) = 2\cos ti + \sin tj$ at the end points of the major and minor axis.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (i) Evaluate $\int x \log(x)\, dx$.

(ii) Find $\int \frac{e^x}{\sqrt{1+e^{2x}}}\, dx$.

(iii) Find $\lim_{x \to 0} (\sin x)^x$.

25. (i) Let C be the circle $r(t) = 5\cos t + 5\sin t + 5k$. Find $T(t)$ and $N(t)$ at $t = \pi/2$.

(ii) Identify and sketch the graph of the surface $z = x^2 + y^2 + 3$.

26. (i) Find the total arc length of the cardioid $r = 1 + \cos\theta$.

(ii) Find the area of the cardioid $r = 1 - \cos\theta$.

27. (i) Find the Taylor series expansion for $\frac{1}{1+x}$ about 3.

(ii) Prove that $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$.

(2 × 10 = 20 marks)

D 31818 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019 Admission Onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions from this section.
Each question carries 2 marks.
Maximum marks : 25.

1. Find the derivative of $\log\sqrt{x^2+1}$.

2. Find the derivative of $\tan^{-1}\sqrt{2x+3}$.

3. Evaluate $\lim\limits_{x\to\infty}\dfrac{\log x}{x}$.

4. Let $f(x)=e^x+x$. (a) find the derivative of f ; (b) find an equation to the tangent line to the graph of $f(x)$ at $x = 0$.

5. Evaluate $\int_{-1}^{\infty} e^{-x}dx$.

6. Determine whether $\left\{\dfrac{n}{n+1}\right\}$ converges or diverges.

7. Determine whether the series $\sum_{n=1}^{\infty} 3\left(\dfrac{-1}{2}\right)^{n-1}$ converges or diverges. If it converges, find the sum.

8. What is an alternating series ? Give an example.

9. Define a power series. Give an example.

Turn over

10. Find the Maclaurin's series of $f(x) = e^x$ and determine its radius of convergence.

11. Find $\frac{d^2y}{dx^2}$ if $x = t^2 - u$ and $y = t^3 - 3t$.

12. Find the parametric equation for a line L passing through the points $P(-3,3,-2)$ and $G(2,-1,4)$.

13. Find an equation in rectangular co-ordinates for the surface with the given cylindrical equation $r^2 \cos 2\theta - z^2 = 4$.

14. Find the point of tangency and unit tangent vector at the point on the curve :

$r(t) = (t^2+1)i + e^{-t} j - \sin 2tk$ at $t = 0$.

15. Find the length of the arc of the helix given by $r(t) = 2\cos ti + 2\sin tj + tk, 0 \le t \le 2\pi$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum marks : 35.

16. Find the derivative of $y = \frac{(2x-1)^3}{\sqrt{3x+1}}$.

17. Find $\int \cosh^2(3x)\sinh(3x)\,dx$.

18. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$.

19. (a) Use integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{\log n}{n}$ converges or diverges.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ converges or diverges.

20. (a) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

    (b) Find a power series representation of log ($1-x$) on (–1, 1).

21. Sketch the curve described by the parametric equations $x = t^2 - 4, y = 2t, -1 \le t \le 2$.

22. Find an equation of the plane containing the points $P(3,-1,1)$, $Q(1,4,2)$ and $R(0,1,4)$.

23. Find the curvature of the twisted cubic described by the vector function $r(t) = ti + \frac{1}{2}t^2 j + \frac{1}{3}t^3 k$.

Section C

Answer any number of questions from this section.
Each question carries 10 *marks.*
Maximum marks : 20.

24. (a) Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$.

    (b) A power line is suspended between two towers. The shape of the cable is a catenary with equation $y = 80 \cosh \frac{x}{80}, -100 \le x \le 100,$ where x is measured in feet. Find the length of the cable.

25. (a) Show that $\int_0^{\infty} e^{-x^2} dx$ is convergent.

    (b) Find $\lim_{n \to \infty} \frac{n!}{n^n}$.

26. (a) Find the Taylor series for $f(x) = \sin x \; at \; x = \pi/6$.

    (b) Find the area of the region enclosed by the cardioid $r = 1 + \cos\theta$.

27. (a) Identify and sketch the surface $12x^2 - 3y^2 + 4z^2 + 12 = 0$.

    (b) A particle moves along a curve described by the vector function $r(t) = ti + t^2 j + t^3 k$. Find the tangential scalar and normal scalar components of acceleration of the particle at time t.

D 12033 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019–2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Determine whether the function $f(x) = x^3 - 3x + 1$ has an inverse.

2. Find the derivatives of (a) $3^{\sqrt{x}}$; (b) $\cos^{-1}(3x)$.

3. Find the derivative of $\log\left[\dfrac{x^2(2x^2+1)^3}{\sqrt{5-x^2}}\right]$ when $x = 1$.

4. Evaluate $\lim\limits_{x \to 0}\left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$.

5. Find $\lim\limits_{x \to \infty} \dfrac{\log n}{n}$.

6. Determine whether the series converges. If it converges find the sum $\sum\limits_{n=1}^{\infty}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$.

7. Use integral test to determine whether $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2+1}$ converges or diverges.

8. Show that the alternating series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n}$ converges.

Turn over

9. Find the Maclaurin's series of $f(x) = \cos x$.

10. Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} n!x^n$.

11. Describe the curve represented by $x = 4\cos\theta$ and $y = 3\sin\theta, 0 \leq \theta \leq 2\pi$.

12. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

13. Find an equation in rectangular co-ordinates for the surface with the cylindrical co-ordinates $r^3 \cos 2\theta - z^2 = 4$.

14. Find a vector function that describes the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x + y + 2z = 4$.

15. Evaluate $\int_0^1 r(t)\,dt$ if $r(t) = t^2 i + \frac{1}{t+1} j + e^{-t} k.$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Use logarithmic differentiation to find the derivative of $y = \sqrt[3]{\frac{x-1}{x^2+1}}$.

17. Find the derivative of $y = x^2 \operatorname{sech}^{-1}(3x)$.

18. Evaluate $\int_0^1 \log x \, dx$.

19. Show that the series $\sum_{n=1}^{\infty}\left[\frac{2}{n(n+1)} - \frac{4}{3^n}\right]$ is convergent and find its sum.

20. Find the tangent lines of $r = \cos 2\theta$ at the origin.

21. Find the length of the Cardioid $r = 1 + \cos\theta$.

22. Find the parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

23. Find the velocity vector, acceleration vector and speed of a particle with position vector :

$r(t) = \sqrt{t}\, i + tt^2 j + e^{2t} k, t \geq 0.$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the derivative of $\sec^{-1}\left(e^{-2x}\right)$.

(b) Evaluate $\lim\limits_{x \to 0^+} \left(\frac{1}{x}\right)^{\sin x}$.

25. (a) Find the area S of the surface obtained by revolving the circle $r = \cos\theta$ about the line $\theta = \pi/2$.

(b) Show that the surface area of a sphere of radius *r* is $4\pi r^2$.

26. (a) Determine whether the series $\sum\limits_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent.

(b) Show that sequence $\left\{\frac{2^n}{n!}\right\}$ is convergent and find its limit.

27. (a) Find an equation in rectangular co-ordinates for the surface with spherical equation $\rho = 4\cos\phi$.

(b) A moving object has an initial position and an initial velocity given by the vectors $r(0) = i + 2j + k$ and $v(0) = i + 2k$. Its acceleration at time t is $a(t) = 6t\, i + j + 2k$. Find its velocity and position at time t.

(2 × 10 = 20 marks)

D 92959 (Pages : 3) Name...

Reg. No...

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Find the derivative of $y = e^{-x\cos x}$.

2. Find the derivative of $y = \log\left(e^{2x} + e^{-2x}\right)$.

3. Find $\lim\limits_{x \to 1^+} \dfrac{\sin \pi x}{\sqrt{x-1}}$.

4. Show that $\sinh^{-1} x = \log\left(x + \sqrt{x^2+1}\right)$.

5. Find $\lim\limits_{n \to \infty} e^{\sin\left(\frac{1}{n}\right)}$.

6. Express $3\cdot\overline{214}$ as a rational number.

7. Determine whether the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{3+2^n}$ converges or diverges.

8. Show that the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n}+1}$ is divergent.

9. Find a power series representation of $\dfrac{1}{(1-x)^2}$ on (–1, 1) by differentiating a power series representation of $\dfrac{1}{1-x}$.

Turn over

10. Find the Maclaurin series of $\sin x$ and determine its interval of convergence.

11. Find a rectangular equation whose graph contains the curve C with the given parametric equation $x = 2t+1, y = t-3$.

12. Find the equation of the tangent line to the curve $x = \sec t, y = \tan t, \frac{-\pi}{2} < t < \frac{\pi}{2}$ at $t = \pi/4$.

13. Find the parametric equation for the line passing through the point (–2, 1, 3) and parallel to the vector $< 1, 2, -2)$.

14. The point $\left(3, \frac{\pi}{3}, \frac{\pi}{4}\right)$ is expressed in spherical co-ordinates. Find its rectangular co-ordinates.

15. Find the antiderivative of $r^1(t) = \cos t i + e^{-t} j + \sqrt{t}\, k$ satisfying the initial condition $r(0) = i + 2j + 3k$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Use logarithmic differentiation to find the derivative of $y = \left(\sqrt{\cos x}\right)^x$.

17. Evaluate $\lim\limits_{x \to 0} \frac{x^3}{x - \tan x}$.

18. Evaluate $\int_{-\infty}^{0} xe^x dx$.

19. Show that the series $\sum_{n=1}^{\infty} \left[\frac{2}{n(n+1)} - \frac{4}{3^n}\right]$ is convergent and find its sum.

20. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 \cdot z^n}$.

21. Find the Taylor series for $f(x) = \log x$ at $x = 1$ and determine its interval of convergence.

22. Identify and sketch the surface $4x - 3y^2 - 12z^2 = 0$.

23. Let C be the helix $r(t) = 2\cos t\, i + 2\sin t\, j + t\, k, t \geq 0$. Find T (t) and N (t).

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find $\int \frac{e^x}{e^{2x}+1}\, dx$.

(b) Find $\int \cosh(2x+3)\, dx$.

(c) Evaluate $\lim_{x \to 0} x^x$.

25. (a) Evaluate $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$.

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2+n}{\sqrt{4n^7+3}}$ converges or diverges.

26. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\cos\theta$.

27. A shell is fired from a gun located on a hill 100 m above a level terrain. The muzzle speed of the gun is 500 m/sec and its angle of elevation is 30.

(a) Find the range of the shell.

(b) What is the maximum height attained by the shell ?

(c) What is the speed of the shell at impact ?

(2 × 10 = 20 marks)