TEXT-BOOK
ON
PRACTICAL ASTRONOMY

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PREFACE

The purpose of this volume is to furnish a text in Practical Astronomy especially adapted to the needs of civil-engineering students who can devote but little time to the subject, and who are not likely to take up advanced study of Astronomy. The text deals chiefly with the class of observations which can be made with surveying instruments, the methods applicable to astronomical and geodetic instruments being treated but briefly. It has been the author's intention to produce a book which is intermediate between the text-book written for the student of Astronomy or Geodesy and the short chapter on the subject generally given in text-books on Surveying. The subject has therefore been treated from the standpoint of the engineer, who is interested chiefly in obtaining results, and those refinements have been omitted which are beyond the requirements of the work which can be performed with the engineer's transit. This has led to the introduction of some rather crude mathematical processes, but it is hoped that these are presented in such a way as to aid the student in gaining a clearer conception of the problem without conveying wrong notions as to when such short-cut methods can properly be applied. The elementary principles have been treated rather elaborately but with a view to making these principles clear rather than to the introduction of refinements. Much space has been devoted to the Measurement of Time because this subject seems to cause the student more difficulty than any other branch of Practical Astronomy. The attempt has been made to arrange the text so that it will be a convenient reference book for the engineer who is doing field work.

For convenience in arranging a shorter course those subjects which are most elementary are printed in large type. The matter printed in smaller type may be included in a longer course and will be found convenient for reference in field practice, particularly that contained in Chapters X to XIII.

The author desires to acknowledge his indebtedness to those who have assisted in the preparation of this book, especially to Professor A. G. Robbins and Mr. J. W. Howard of the Massachusetts Institute of Technology and to Mr. F. C. Starr of the George Washington University for valuable suggestions and criticisms of the manuscript.

G. L. H.

BOSTON, *June*, 1910.

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PRACTICAL ASTRONOMY

CHAPTER I

THE CELESTIAL SPHERE — REAL AND APPARENT MOTIONS

1. Practical Astronomy.

Practical Astronomy treats of the theory and use of astronomical instruments and the methods of computing the results obtained by observation. The part of the subject which is of especial importance to the surveyor is that which deals with the methods of locating points on the earth's surface and of orienting the lines of a survey, and includes the determination of (1) latitude, (2) time, (3) longitude, and (4) azimuth. In solving these problems the observer makes measurements of the directions of the sun, moon, stars, and other heavenly bodies; he is not concerned with the distances of these objects, with their actual motions in space, nor with their physical characteristics, but simply regards them as a number of visible objects of known positions from which he can make his measurements.

2. The Celestial Sphere.

Since it is only the directions of these objects that are required in practical astronomy, it is found convenient to regard all heavenly bodies as being situated on the surface of a sphere whose radius is infinite and whose centre is at the eye of the observer. The apparent position of any object on the sphere is found by imagining a line drawn from the eye to the object, and prolonging it until it pierces the sphere. For example, the apparent position of S_1 on the sphere (Fig. 1) is at S_1', which is supposed to be at an infinite distance from C; the position of S_2 is S_2', etc. By means of this imaginary sphere all problems involving the angular distances between points, and angles between planes through the centre of the sphere, may readily be solved by applying the formulæ of spherical trigonometry. This device is not only convenient for mathematical purposes, but it is perfectly consistent with what we see, because all celestial objects are so far away that they appear to the eye to be at the same distance, and consequently on the surface of a great sphere.

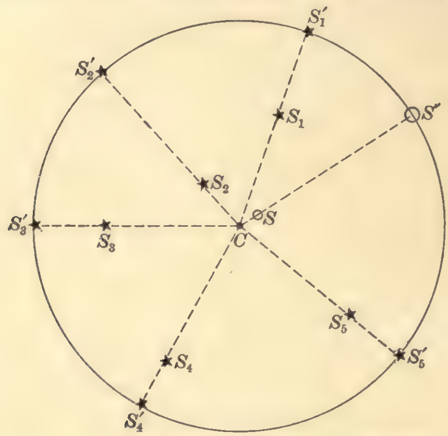

FIG. 1. APPARENT POSITIONS ON THE SPHERE

From the definition it will be apparent that each observer sees a different celestial sphere, but this causes no actual inconvenience, for distances between points on the earth's surface are so short when compared with astronomical distances that they are practically zero except for the nearer bodies in the solar system. This may be better understood from the statement that if the entire solar system be represented as occupying a field one mile in diameter the nearest star would be about 5000 miles away on the same scale; furthermore the earth's diameter is but a minute fraction of the distance across the solar system, the ratio being about 8000 miles to 5,600,000,000 miles,* or one 700,000th part of this distance.

* The diameter of Neptune's orbit.

Since the radius of the celestial sphere is infinite, all of the lines in a system of parallels will pierce the sphere in the same point, and parallel planes at any finite distance apart will cut the sphere in the same great circle. This must be kept constantly in mind when representing the sphere by means of a sketch, in which minute errors will necessarily appear to be very large. The student should become accustomed to thinking of the appearance of the sphere both from the inside and from an outside point of view. It is usually easier to understand the spherical problems by studying a small globe, but when celestial objects are actually observed they are necessarily seen from a point inside the sphere.

3. Apparent Motion of the Celestial Sphere.

If a person watches the stars for several hours he will see that they appear to rise in the east and to set in the west, and that their paths are arcs of circles. By facing to the north (in the northern hemisphere) it will be found that the circles are smaller and all appear to be concentric about a certain point in the sky called the **pole**; if a star were exactly at this point it would have no apparent motion. In other words, the whole celestial sphere appears to be rotating about an axis. This apparent rotation is found to be due simply to the actual rotation of the earth about its axis (from west to east) in the opposite direction to that in which the stars appear to move.*

4. Motions of the Planets.

If an observer were to view the solar system from a point far outside, looking from the north toward the south, he would see that all of the planets (including the earth) revolve about the sun in elliptical orbits which are nearly circular, the direction of the motion being *counter-clockwise* or left-handed rotation.

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* This apparent rotation may be easily demonstrated by taking a photograph of the stars near the pole, exposing the plate for several hours. The result is a series of concentric arcs all subtending the same angle. If the camera is pointed southward and high enough to photograph stars near the equator the star trails appear as straight lines.

He would also see that the earth rotates on its axis, once per day, in a counter-clockwise direction. The moon revolves around the earth in an orbit which is not so nearly circular, but the motion is in the same (left-handed) direction. The

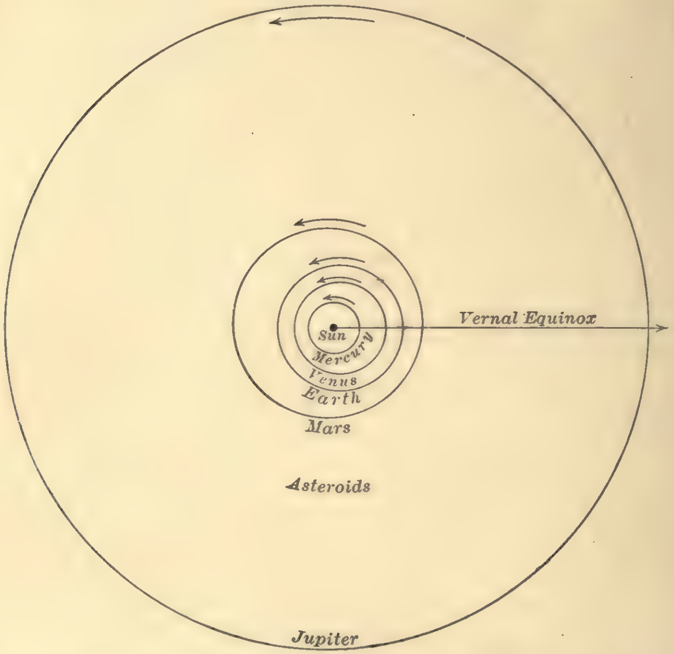

FIG. 2. DIAGRAM OF THE SOLAR SYSTEM WITHIN THE ORBIT OF SATURN

apparent motions resulting from these actual motions are as follows: The whole celestial sphere, carrying with it all the stars, sun, moon, and planets, appears to rotate about the earth's axis once per day in a *clockwise* (right-handed) direction. The stars change their positions so slowly that they appear to be fixed in position on the sphere, whereas all objects within the solar system rapidly change their apparent positions among the stars. For this reason the stars are called **fixed stars** to distinguish them from the planets; the latter, while closely resembling the stars in appearance, are really of an entirely different character. The sun appears to move slowly eastward among the stars at the rate of about 1° per day, and to make one revolution around the earth

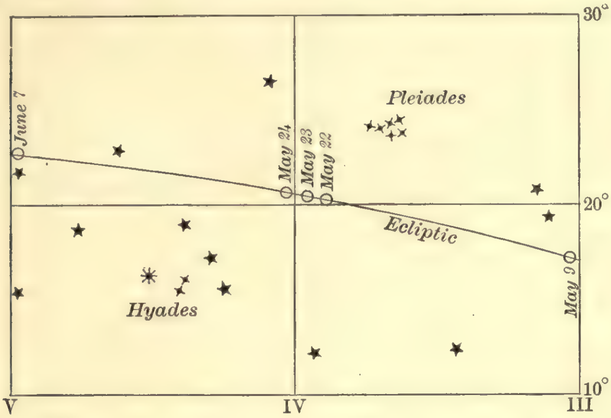

FIG. 3a. SUN'S APPARENT POSITION AT GREENWICH NOON ON MAY 22, 23, AND 24, 1910

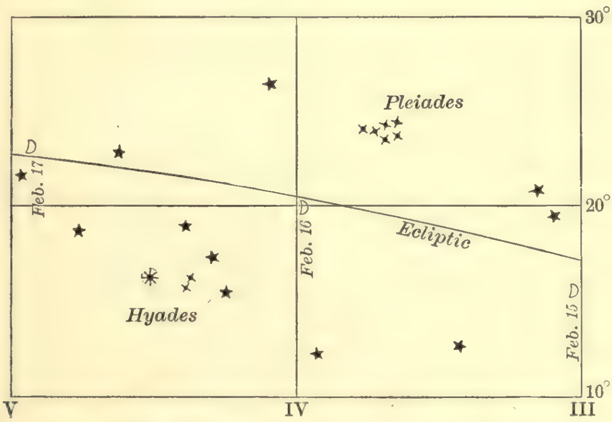

FIG. 3b. MOON'S APPARENT POSITION AT 14^h ON FEB. 15, 16, AND 17, 1910

in just one year. The moon also travels eastward among the stars, but at a much faster rate; it moves an amount equal to its own diameter in about an hour, and completes one revolution in a lunar month. Figs. 3a and 3b show the daily motions of the sun and moon respectively, as indicated by their plotted positions when passing through the constellation *Taurus*. It should be observed that the motion of the moon eastward among the stars is an actual motion, not merely an apparent one like that of the sun. The planets all move eastward among the stars, but since we ourselves are on a moving object the motion we see is a combination of the real motions of the planets around

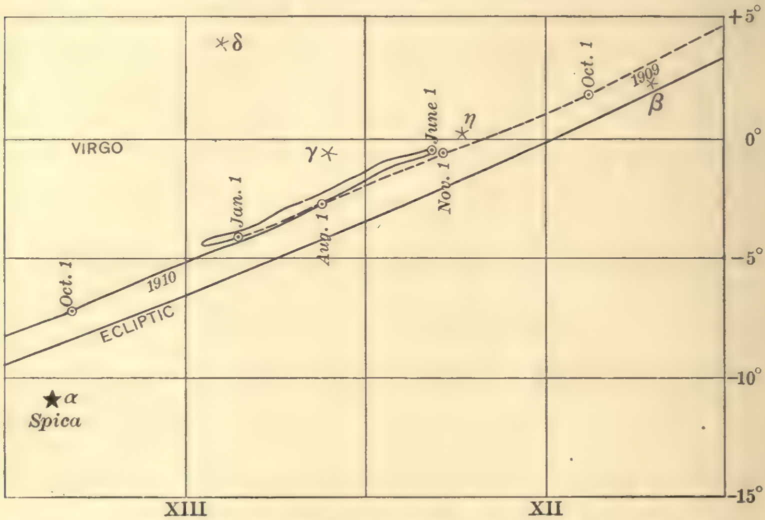

FIG. 4. APPARENT PATH OF JUPITER FROM OCT., 1909 TO OCT., 1910.

the sun and an apparent motion caused by the earth's revolution around the sun; the planets consequently appear at certain times to move westward (i.e., backward), or to **retrograde.** Fig. 4 shows the loop in the apparent path of the planet Jupiter caused by the earth's motion around the sun. It will be seen that the apparent motion of the planet was *direct* except from January to June, 1910, when it had a *retrograde* motion.

5. Meaning of Terms East and West.

In astronomy the terms " east " and " west " cannot be taken to mean the same as they do when dealing with directions in one plane. In plane surveying "east" and "west" may be considered to mean the directions perpendicular to the meridian line. If a person at Greenwich (England) and another person at the 180° meridian should both point due east, they would actually be pointing to opposite points of the sky. In Fig. 5 all four of the arrows are pointing east at the places shown. It will be seen from this figure that the terms "east" and "west" must therefore be taken to mean *directions of rotation.*

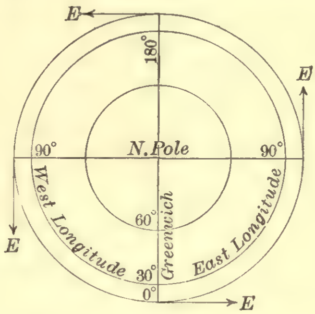

FIG. 5. ARROWS ALL POINT EASTWARD

6. The Earth's Orbital Motion. — The Seasons.

The earth moves eastward around the sun once a year in an orbit which lies (very nearly) in one plane and whose form is that

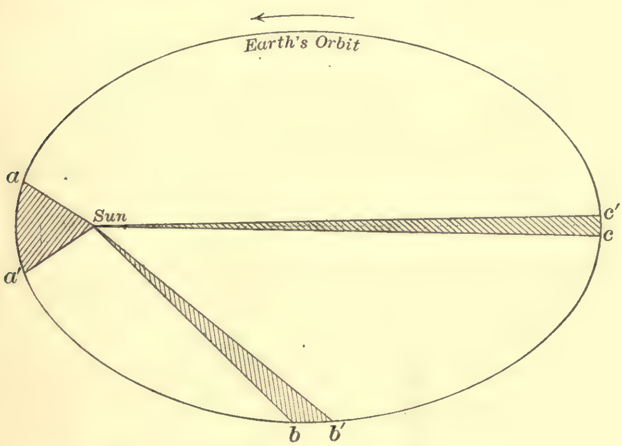

FIG. 6. THE EARTH'S ORBITAL MOTION

of an ellipse, the sun being at one of the foci. Since the earth is maintained in its position by the force of gravitation, it moves, as a consequence, at such a speed in each part of its path that the line joining the earth and sun moves over equal areas in equal times. In Fig. 6 all of the shaded areas are equal and the arcs aa', bb', cc' represent the distances passed over in the same number of days.*

The axis of rotation of the earth is inclined to the plane of the orbit at an angle of about $66°\frac{1}{2}$, that is, the plane of the earth's equator is inclined at an angle of about $23°\frac{1}{2}$ to the plane of the orbit. This latter angle is known as the **obliquity of the ecliptic.** (See Chapter II.) The direction of the earth's axis of rotation is nearly constant and it therefore points nearly to the same place in the sky year after year.

The changes in the seasons are a direct result of the inclination of the axis and of the fact that the axis remains nearly parallel

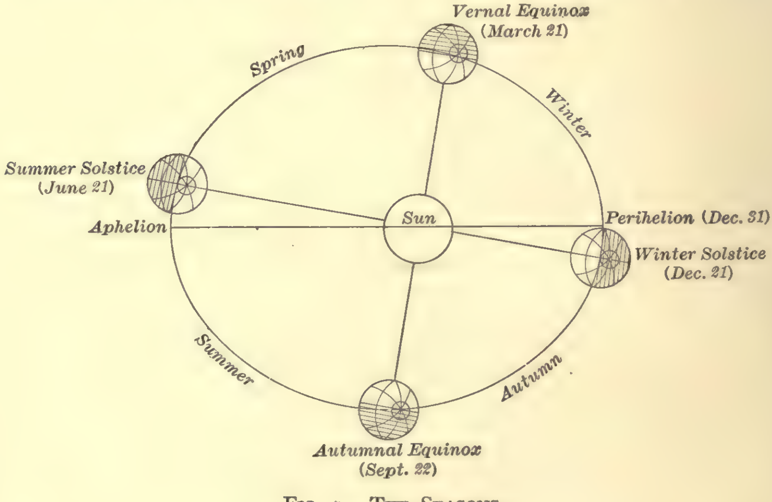

FIG. 7. THE SEASONS

to itself. When the earth is in that part of the orbit where the northern end of the axis is pointed away from the sun (Fig. 7) it is winter in the northern hemisphere. The sun appears to be

* The eccentricity of the ellipse shown in Fig. 6 is exaggerated for the sake of clearness; the earth's orbit is in reality much more nearly circular, the variation in the earth's distance from the sun being only about three per cent.

farthest south about Dec. 21, and at this time the days are shortest and the nights are longest. When the earth is in this position, a plane through the axis and perpendicular to the plane of the orbit will pass through the sun. About ten days later the earth passes the end of the major axis of the ellipse and is at its point of nearest approach to the sun, or **perihelion.** Although the earth is really nearer to the sun in winter than in summer, this has but a small effect upon the seasons; the chief reasons why it is colder in winter are that the day is shorter and the rays of sunlight strike the surface of the ground more obliquely. The sun appears to be farthest north about June 22, at which time summer begins in the northern hemisphere and the days are longest and the nights shortest. When the earth passes the other end of the major axis of the ellipse it is farthest from the sun, or at **aphelion.** On March 21 the sun is in the plane of the earth's equator and day and night are of equal length at all places on the earth (Fig. 7). On Sept. 22 the sun is again in the plane of the equator and day and night are everywhere equal. These two times are called the **equinoxes** (vernal and autumnal), and the points in the sky where the sun's centre appears to be at these two dates are called the **equinoctial points,** or more commonly the **equinoxes.**

7. The Sun's Apparent Position at Different Seasons. The apparent positions of the sun on the celestial sphere corresponding to these different positions of the earth are shown in Fig. 8. As a result of the sun's apparent eastward motion from day to day along a path which is inclined to the equator, the angular distance of the sun from the equator is continually changing. Half of the year it is north of the equator and half of the year it is south. On June 22 the sun is in its most northerly position and is visible more than half the day to a person in the northern hemisphere (*J*, Fig. 8). On Dec. 21 it is farthest south of the equator and is visible less than half the day (*D*, Fig. 8). In between these two extremes it moves back and forth across the equator, passing it about March 21 and Sept. 22 each year.

The apparent motion of the sun is therefore a *helical* motion about the axis, that is, the sun, instead of following the path which would be followed by a fixed star, gradually increases or decreases its angular distance from the pole at the same time that it revolves once a day around the earth. The sun's motion eastward on the celestial sphere, due to the earth's orbital motion,

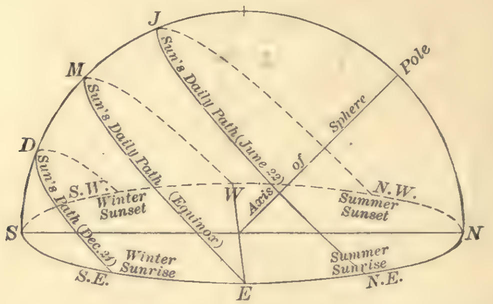

Fig. 8. Sun's Apparent Position at Different Seasons

is not noticed until the sun's position is carefully observed with reference to the stars. If a record is kept for a year showing which constellations are visible in the east soon after sunset, it will be found that these change from month to month, and at the end of a year the one first seen will again appear in the east, showing that the sun has apparently made the circuit of the heavens in an eastward direction

8. Precession and Nutation.

While the direction of the earth's rotation axis is so nearly constant that no change is observed during short periods of time, there is in reality a very slow progressive change in its direction. This change is due to the fact that the earth is not quite spherical in form but is *spheroidal*, and there is in consequence a ring of matter around the equator upon which the sun and the moon exert a force of attraction which tends to pull the plane of the equator into coincidence with the plane of the orbit. But since the earth is rotating with a high velocity and resists this attraction, the actual effect is not to permanently change the inclination of the equator to the orbit, but *first* to cause the earth's axis to describe a cone about an axis perpendicular to the orbit, and *second* to cause the inclination of the axis to go through certain periodic changes (see Fig. 9). The movement of the axis in a conical surface causes the line of intersection of the equator and the plane of the orbit to revolve slowly westward, the pole itself always moving directly toward the vernal equinox. This causes the equinoctial points to move westward in the sky, and hence the sun crosses the equator each spring earlier than it would otherwise; this is known as the

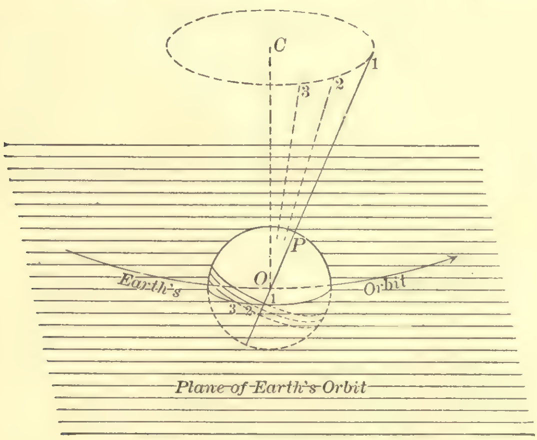

FIG. 9. PRECESSION OF THE EQUINOXES

precession of the equinoxes. In Fig. 9 the pole occupies successively the positions *1*, *2* and *3*, which causes the point *V* to move to points *1*, *2* and *3*. This motion is but 50″.2 per year, and it therefore requires about 25,800 years for the pole to make one complete revolution. The force causing the precession is not quite constant, and the motion of the equinoctial points is therefore not perfectly uniform but has a small periodic variation. In addition to this periodic change in the rate of the precession there is also a slight periodic change in the obliquity, called **Nutation.** The maximum value of the nutation is about 9″; the period is about 19 years. The phenomenon of precession is clearly illustrated by means of the apparatus called the gyroscope. As a result of the precessional movement of the axis all of the stars gradually change their positions *with reference to the plane of the equator* and the position of the equinox. The stars themselves have but a very slight angular motion, this apparent change in position being due almost entirely to the change in the positions of the circles of reference.

9. Aberration of Light.

Another apparent displacement of the stars due to the earth's motion is what is known as **aberration.** On account of the rapid motion of the earth through space, the direction in which a star is seen by an observer is a result of the combined velocities of the observer and of light from the star. The star always appears to be slightly displaced in the direction in which the observer is actually moving. In Fig. 10, if light moves from *C* to *B* in the same length of time that the observer moves from *A* to *B*, then *C* would appear to be in the direction *AC*. This

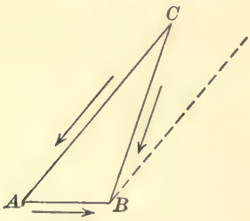

FIG. 10

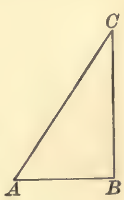

FIG. 11

may be more clearly understood by using the familiar illustration of the falling raindrop. If a raindrop is falling vertically, *CB*, Fig. 11, and while it is falling a person moves from *A* to *B*, then, considering only the two motions, it appears to the person that the raindrop has moved toward him in the direction *CA*. If a tube is to be held in such a way that the raindrop shall pass through it without touching the sides, it must be held at the inclination of AC. The apparent displacement of a star due to the observer's motion is similar to the change in the apparent direction of the raindrop.

There are two kinds of aberration, **annual** and **diurnal.** Annual aberration is that produced by the earth's motion in its orbit and is the same for all observers. Diurnal aberration is due to the earth's daily rotation about its axis, and is different in different latitudes, because the speed of a point on the earth's surface is greatest at the equator and diminishes toward the pole.

If v represents the velocity of the earth in its orbit and V the velocity of light, then when CB is at right angles to AB the displacement is a maximum and

$$\tan \alpha_0 = \frac{v}{V},$$

where α_0 is the angular displacement and is called the "constant of aberration." Its value is about 20.″5. If CB is not perpendicular to AB, then

$$\tan \alpha = \frac{v}{V} \sin B,$$

where α is the angular displacement and B is the angle ABC.

Problem

Referring to Fig. 2, make a sketch showing the path which Jupiter appears to describe, in the plane of its motion, but considering the earth as a fixed point on the diagram.

CHAPTER II

DEFINITIONS—POINTS AND CIRCLES OF REFERENCE

10. The following astronomical terms are in common use and are necessary in defining the positions of celestial objects on the sphere by means of spherical coördinates.

Vertical Line.

A **vertical line** at any point on the earth's surface is the direction of gravity at that point, and is shown by the plumb line or indirectly by means of the spirit level (*OZ*, Fig. 12).

Zenith — Nadir.

If the vertical at any point be prolonged upward it will pierce the sphere at a point called the **Zenith** (*Z*, Fig. 12). This point is of great importance because it is the point on the sphere which indicates the position of the observer on the earth's surface. The point where the vertical prolonged downward pierces the sphere is called the **Nadir** (*N′*, Fig. 12).

Horizon.

The **horizon** is the great circle on the celestial sphere cut by a plane through the centre of the earth perpendicular to the vertical (*NESW*, Fig. 12). The horizon is everywhere 90° from the zenith and the nadir. It is evident that a plane through the observer perpendicular to the vertical cuts the sphere in this same great circle. The **visible horizon** is the circle where the sea and sky seem to meet. Projected onto the sphere it is a small circle below the true horizon and parallel to it. Its distance below the true horizon depends upon the height of the observer's eye above the surface of the water.

Vertical Circles.

Vertical Circles are great circles passing through the zenith and nadir. They all cut the horizon at right angles (*HZJ*, Fig. 12).

Almucantars.

Parallels of altitude, or **almucantars,** are small circles parallel to the horizon (*DFG*, Fig. 12).

Poles.

If the earth's axis of rotation be produced indefinitely it will pierce the sphere in two points called the celestial **poles** (*PP′* Fig. 12).

Equator.

The **celestial equator** is a great circle of the celestial sphere cut by a plane through the centre of the earth perpendicular to

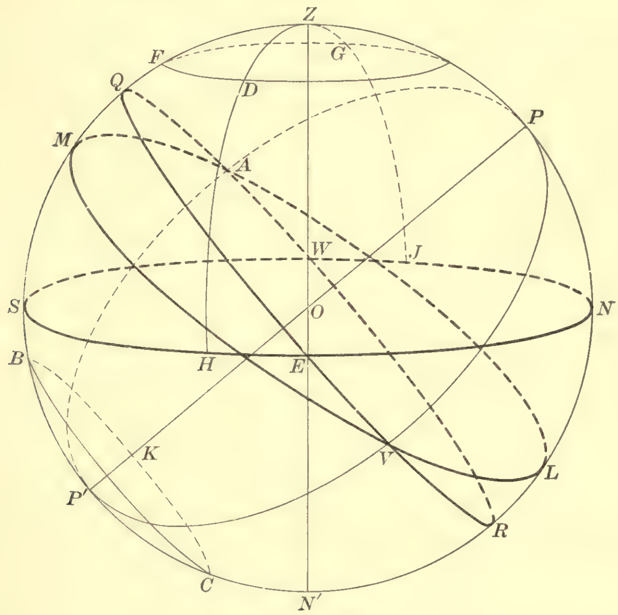

FIG. 12. THE CELESTIAL SPHERE

the axis of rotation (*QWRE*, Fig. 12). It is everywhere 90° from the poles. A parallel plane through the observer cuts the sphere in the same circle.

Hour Circles.

Hour Circles are great circles passing through the north and south celestial poles (*PVP′*, Fig. 12).

Parallels of Declination.

Small circles parallel to the plane of the equator are called **parallels of declination** (*BKC*, Fig. 12).

Meridian.

The **meridian** is the great circle passing through the zenith and the poles (*SZPL*, Fig. 12). It is at once an hour circle and a vertical circle. It is evident that different observers will in general have different meridians. The meridian cuts the horizon in the **north** and **south points** (*N*, *S*, Fig. 12). The intersection of the plane of the meridian with the horizontal plane through the observer is the **meridian line** used in plane surveying.

Prime Vertical.

The **prime vertical** is the vertical circle whose plane is perpendicular to the plane of the meridian (*EZW*, Fig. 12). It cuts the horizon in the **east** and **west points** (*E*, *W*, Fig. 12).

Ecliptic.

The **ecliptic** is the great circle on the celestial sphere which the sun's centre appears to describe during one year (*AMVL*, Fig. 12). Its plane is the plane of the earth's orbit; it is inclined to the plane of the equator at an angle of about 23° 27′, called the **obliquity of the ecliptic.**

Equinoxes.

The points of intersection of the ecliptic and the equator are called the **equinoctial points** or simply the **equinoxes.** That intersection at which the sun appears to cross the equator when going from the south side to the north side is called the **Vernal Equinox**, or sometimes the **First Point of Aries** (*V*, Fig. 12). The sun reaches this point about March 21. The other intersection is called the **Autumnal Equinox** (*A*, Fig. 12).

Solstices.

The points on the equator midway between the equinoxes are called the winter and summer **solstices.**

Colures.

The great circle through the poles and the equinoxes is called

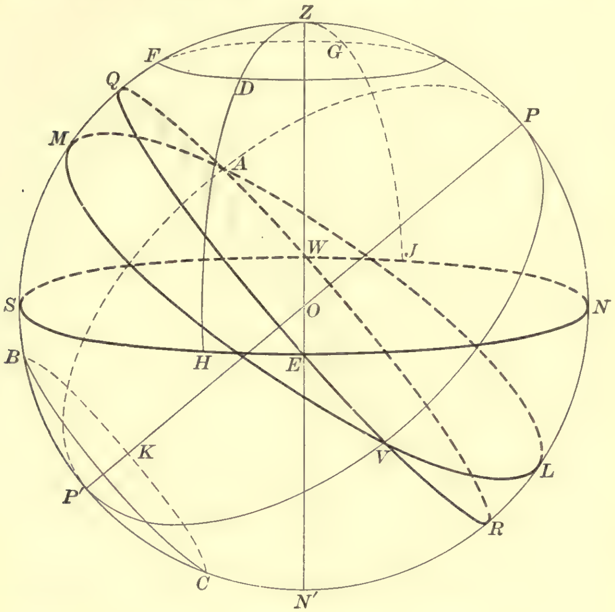

FIG. 12. THE CELESTIAL SPHERE

the **equinoctial colure** (PVP', Fig. 12). The great circle through the poles and the solstices is called the **solstitial colure.**

Questions

1. What imaginary circles on the earth's surface correspond to hour circles? To parallels of declination? To vertical circles?

2. What are the widths of the torrid, temperate and arctic zones and how are they determined?

CHAPTER III

SYSTEMS OF COÖRDINATES ON THE SPHERE

11. Spherical Coördinates.

The direction of a point in space may be defined by means of two spherical coördinates, that is, by two angular distances measured on a sphere along arcs of two great circles which cut each other at right angles. Suppose that it is desired to locate C (Fig. 13) with reference to the plane OAB and the line

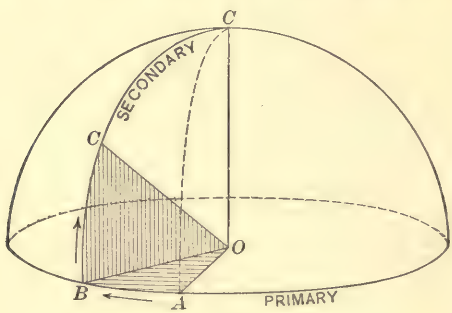

Fig. 13. Spherical Coördinates

OA, O being the origin of coördinates. Pass a plane OBC through C and perpendicular to OAB; these planes will intersect in the line OB. The two angles which fix the position of C, or the spherical coördinates, are BOC and AOB. These may be regarded as the angles at the centre of the sphere or as the arcs BC and AB. In every system of spherical coördinates the two coördinates are measured, one on a great circle called the **primary,** and the other on one of a system of great circles at right angles to the primary called **secondaries.** There are an infinite number of secondaries, each passing through the two poles of the primary. The coördinate measured from the primary is an arc of a secondary circle; the coördinate measured between the secondary circles is an arc of the primary.

12. Horizon System.

In this system the primary circle is the horizon and the secondaries are vertical circles, or circles passing through the zenith and nadir. The first coördinate of a point is its angular distance above the horizon, measured on a vertical circle; this is called the **Altitude.** The complement of the altitude is called the **Zenith distance.** The second coördinate is the angular distance on the horizon between the meridian and the vertical circle through the point; this is called the **Azimuth.** Azimuth may be reckoned either from the north or the south point and in either direction, like bearings in surveying, but the custom is to reckon it from the south point right-handed from 0° to 360° except for stars near the pole, in which case it is more convenient to reckon

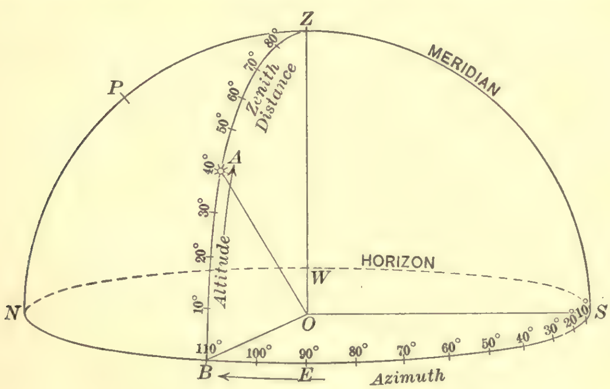

FIG. 14. THE HORIZON SYSTEM

from the north, and either to the east or to the west. In Fig. 14 the altitude of the star A is BA; its azimuth is SB.

13. The Equator Systems.

The circles of reference in this system are the equator and great circles through the poles, or hour circles. The first coördinate of a point is its angular distance north or south of the equator, measured on an hour circle; it is called the **Declination.** Declinations are considered positive when north of the equator, negative when south. The complement of the declination is called the **Polar Distance.** The second coördinate of the point is the arc of the equator between the vernal equinox and the foot of the hour circle through the point; it is called **Right Ascension.** Right ascension is measured from the equinox *eastward* to the hour circle through the point in question; it may be measured in degrees, minutes, and seconds of arc, or in hours, minutes, and

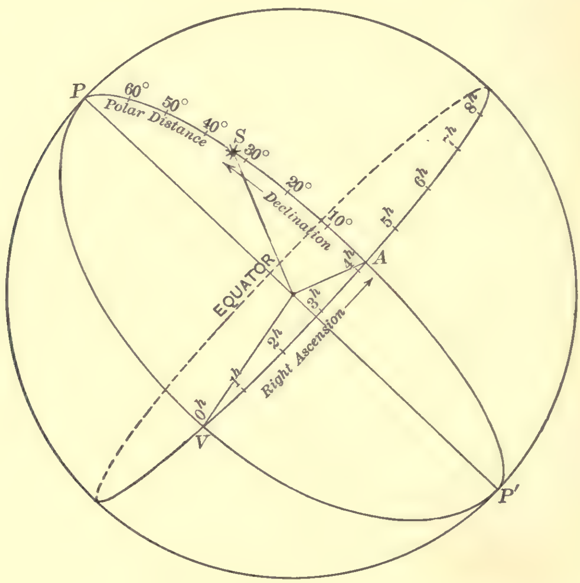

FIG. 15. THE EQUATOR SYSTEM

seconds of time. In Fig. 15 the declination of the star S is AS; the right ascension is VA.

Instead of locating a point by means of declination and right ascension it is sometimes more convenient to use declination and **Hour Angle.** The hour angle of a point is the arc of the equator between the observer's meridian and the hour circle through the point. It is measured from the meridian *westward* (clockwise) from 0^h to 24^h or from $0°$ to $360°$. In Fig. 16 the declination of the star S is AS (negative); the hour angle is

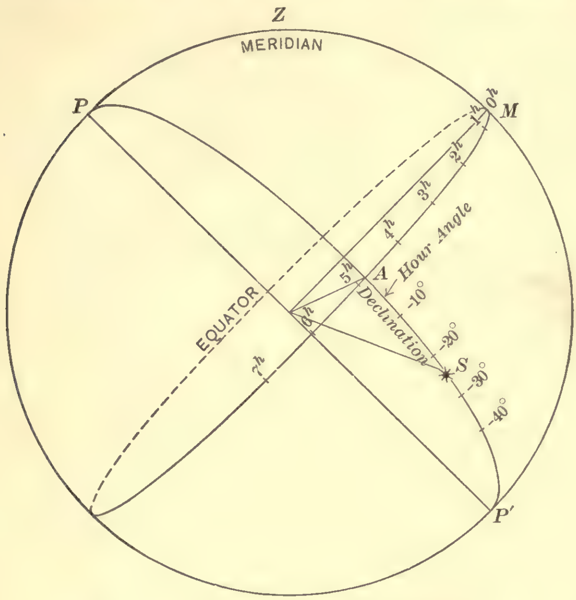

FIG. 16. HOUR ANGLE AND DECLINATION

MA. It is evident that the hour angles of all points on the celestial sphere are always increasing.

These three systems are shown in the following table.

Name.	Primary.	Secondaries.	Origin of Coördinates.	1st coörd.	2nd coörd.
Horizon System	Horizon	Vert. Circles	South point.	Altitude	Azimuth
Equator Systems	Equator	Hour Circles	Vernal Equinox.	Declin.	Rt. Ascen.
	"	" "	Intersection of Meridian and Equator.	"	Hour Angle

14. There is another system which is employed in some branches of astronomy but will not be used in this book. The coördinates are called **celestial latitude** and **celestial longitude**; the primary circle is the ecliptic. Celestial latitude is measured from the ecliptic just as declination is measured from the equator. Celestial longitude is measured eastward along the ecliptic from the equinox, just as right ascension is measured eastward along the equator. The student should be careful not to confuse celestial latitude and longitude with terrestrial latitude and longitude. The latter are the ones used in the problems discussed in this book.

15. **Coördinates of the Observer.**

The observer's position is located by means of his **latitude** and **longitude.** The latitude, which on the earth's surface is the angular distance of the observer north or south of the equator, may be defined astronomically as the declination of the observer's zenith. In Fig. 17, the terrestrial latitude is the arc *EO*,

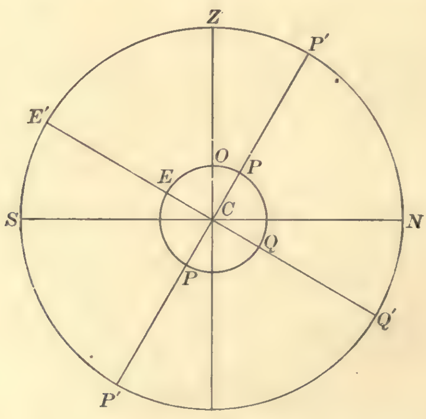

Fig. 17. The Observer's Latitude

EQ being the equator and *O* the observer. The point *Z* is the observer's zenith, so that the latitude on the sphere is the arc *E′Z*, which evidently will contain the same number of degrees as *EO*. The complement of the latitude is called the **Co-latitude.**

The terrestrial longitude of the observer is the arc of the equator between the primary meridian (usually that of Greenwich) and the meridian of the observer. On the celestial sphere the longitude would be the arc of the celestial equator contained between two hour circles whose planes are the planes of the two terrestrial meridians.

16. Relation between the Two Systems of Coördinates.

In studying the relation between different points and circles on the sphere it may be convenient to imagine that the celestial sphere consists of two spherical shells, one within the other.

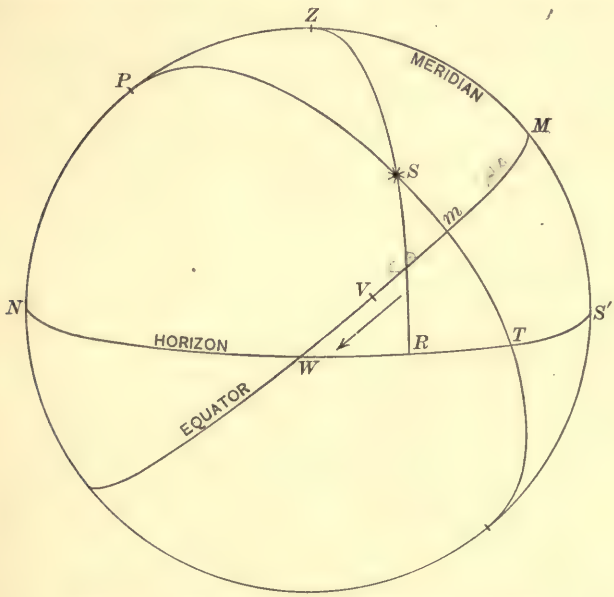

Fig. 18. The Sphere seen from the Outside

The outer one carries upon its surface the ecliptic, equinoxes, poles, equator, hour circles and all of the stars, the sun, the moon and the planets. On the inner sphere are the zenith, horizon, vertical circles, poles, equator, hour circles, and the meridian. The earth's daily rotation causes the inner sphere to revolve, while the outer sphere is motionless, or, regarding only the apparent motion, the outer sphere revolves once per day on its axis, while the inner sphere appears to be motionless. It is evident that the coördinates of a fixed star in the first equatorial system (Declination and Right Ascension) are practically always the same, whereas the coördinates in the horizon system are continually changing. It will also be seen that in the first equatorial system the coördinates are independent of the observer's position, but in the horizon system they are entirely dependent upon his position. In the second equatorial system one coördinate is independent of the observer, while the other (hour angle) is not. In making up catalogues of the positions of the stars it is necessary to use right ascensions and declinations in defining these positions. When making observations

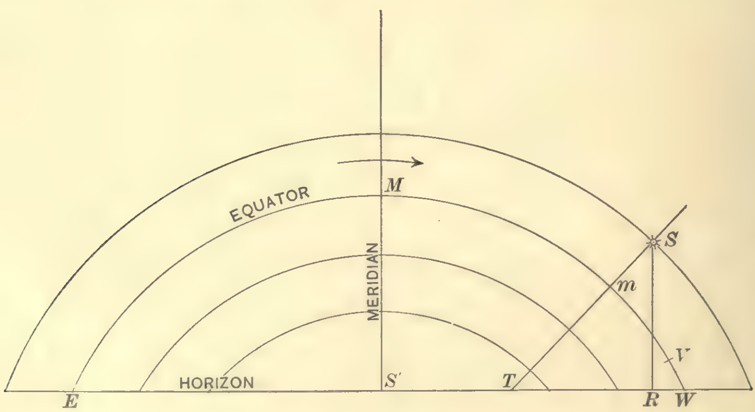

FIG. 19. THE SPHERE SEEN FROM THE EARTH (LOOKING SOUTH)

with instruments it is usually simpler to measure coördinates in the horizon system. Therefore it is necessary to be able to compute the coördinates of one system from those of another. The mathematical relations between the spherical coördinates are discussed in Chapter IV.

Figs. 18, 19, and 20 show three different views of the celestial sphere with which the student should be familiar. Fig. 18 is the sphere as seen from the outside and is the view best adapted to showing problems in spherical trigonometry. The star S has the altitude RS, azimuth $S'R$, hour angle Mm, right ascension Vm, and declination mS; the meridian is ZMS'. Fig. 19 shows a portion of the sphere as seen by an observer looking southward; the points are indicated by the same letters as in Fig. 18. Fig. 20

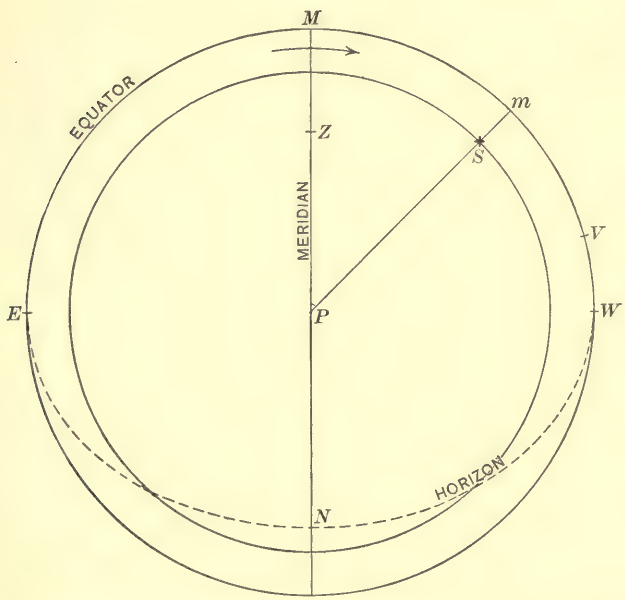

Fig. 20. The Sphere Projected onto the Plane of the Equator

shows the same points projected on the plane of the equator. In this view of the sphere the angles at the pole (i.e., the angles between hour circles) are shown their true size, and it is therefore a convenient diagram to use when dealing with right ascension and hour angles.

Questions and Problems

1. What coördinates on the sphere correspond to latitude and longitude on the earth's surface?

2. Make a sketch of the sphere and plot the position of a star having an altitude of 20° and an azimuth of 250°. Locate a star whose hour angle is 16^h and whose declination is $-10°$. Locate a star whose right ascension is 9^h and whose declination is N. 30°.

3. If a star is on the equator and also on the horizon, what is its azimuth? Its altitude? Its hour angle? Its declination?

CHAPTER IV

RELATION BETWEEN COÖRDINATES

17. Relation between Altitude of Pole and Latitude of Observer.

In Fig. 21, SZN represents the observer's meridian; let P be the celestial pole, Z the zenith, E the point of intersection of the meridian and the equator, and N and S the north and south points of the horizon. By the definitions, OZ (vertical) is perpendicular to SN (horizon) and OP (axis) is perpendicular to EO (equator). Therefore the arc PN = arc EZ. By the definitions, EZ is the declination of the zenith, or the latitude, and PN is the altitude of the celestial pole. Hence *the altitude of the pole is always equal to the latitude of the observer.* The same relation may be seen from Fig. 22, in which P is the north pole of the earth, OH is the plane of the horizon, the observer being at O, EQ is the earth's equator, and OP' is a line parallel to CP and consequently points to the celestial pole. It may readily be shown that ECO, the observer's latitude, equals HOP', the altitude of the celestial pole. A person at the equator would

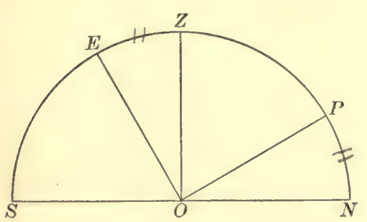

FIG. 21

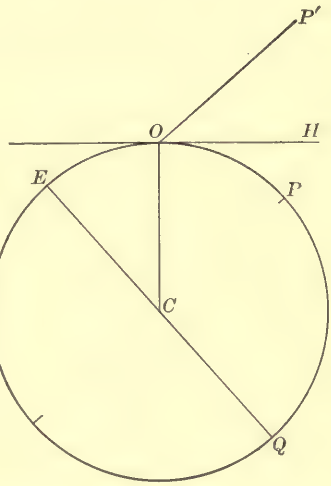

FIG. 22

see the north celestial pole in the north point of his horizon and the south celestial pole in the south point of his horizon. If he travelled northward the north pole would appear to rise, its altitude being always equal to his latitude, while the south pole would immediately go below his horizon. When the traveller reached the north pole of the earth the north celestial pole would be vertically over his head.

To a person at the equator all stars would appear to move vertically at the times of rising and setting, and all stars would be above the horizon 12^h and below 12^h during one revolution

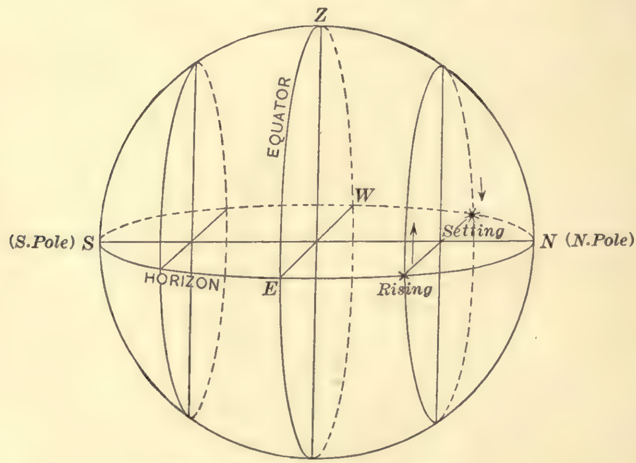

FIG. 23. THE RIGHT SPHERE

of the sphere. All stars in both hemispheres would be above the horizon at some time every day. This is called the "right sphere" (Fig. 23).

If a person were at the earth's pole the celestial equator would coincide with his horizon, and all stars in the northern hemisphere would appear to travel around in circles parallel to the horizon; they would be visible for 24^h a day, and their altitudes would not change. The stars in the southern hemisphere would never be visible. The word *north* would cease to have its usual meaning, and *south* might mean any horizontal direction. The longitude of a point on the earth and its azimuth from the Greenwich meridian would then be the same. This is called the "parallel sphere" (Fig. 24).

At all points between these two extreme latitudes the equator cuts the horizon obliquely. A star on the equator will be above

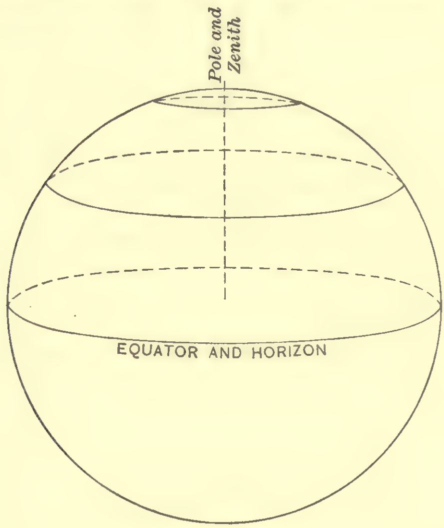

FIG. 24. THE PARALLEL SPHERE

the horizon half the time and below half the time. A star north of the equator will (to a person in the northern hemisphere) be above the horizon more than half of the day; a star south of the equator will be above the horizon less than half of the day. If the north polar distance of a star is less than the observer's north latitude, the whole of the star's diurnal circle is above the horizon, and the star will therefore remain above the horizon all of the time. It is called in this case a **circumpolar star** (Fig. 25). The south circumpolar stars are those whose south polar distances are less than the latitude; they are never visible to an observer in the northern hemisphere. If the observer travels north until he is beyond the arctic circle, latitude 66° 33′ north, then the sun becomes a circumpolar at the time of the summer solstice. At noon the sun would be at its maximum altitude; at midnight it would be at its minimum altitude but would still be above the horizon. This is called the "midnight sun."

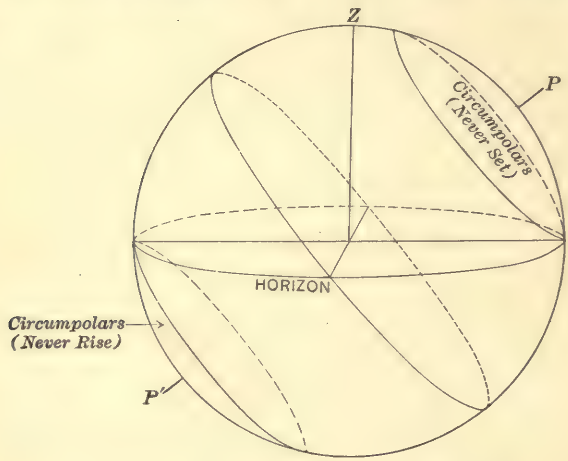

FIG. 25. CIRCUMPOLAR STARS

18. Relation between Latitude of Observer and the Declination and Altitude of a Star on the Meridian.

The relation between the latitude, altitude, and declination at the instant when a star is crossing the observer's meridian may be seen from Fig. 26. Let A be a star on the meridian, south of the zenith and north of the equator; then

$EZ = L$, the latitude,
$EA = D$, the declination,
$SA = h$, the altitude,
$ZA = z$, the zenith distance.

From the figure

$$ZA = EZ - EA$$

or

$$z = L - D$$

and

$$h = 90^\circ - (L - D); \qquad [1]$$

also

$$L = 90^\circ - (h - D). \qquad [2]$$

If A is south of the equator the declination is considered negative, so the same equation will hold true for this case.

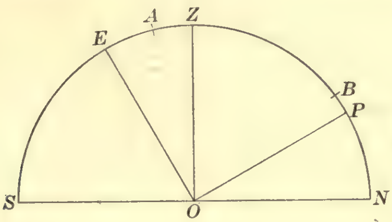

FIG. 26. STAR ON THE MERIDIAN

If the star is north of the zenith, as at B, it will be more convenient to use the polar distance, $p = 90° - D$.

In this case $$NP = NB - PB$$

or $$L = h - p. \qquad [3]$$

If B is below the pole the equation is

$$L = h + p. \qquad [4]$$

19. The Astronomical Triangle.

By joining the pole, zenith, and any star S on the sphere by arcs of great circles we obtain a triangle from which the relation existing among the spherical coördinates may be obtained. This triangle is so frequently employed in astronomy and navigation that is it called the "astronomical triangle" or the "*PZS* triangle." In Fig. 27 the arc PZ is the complement of the latitude, or co-latitude; arc ZS is the zenith distance or complement of the altitude; arc PS is the polar distance or complement of the declination; the angle P is the hour angle of the star if S is west of the meridian, or 360° minus the hour angle if S is east of the meridian; and Z is the azimuth of S, or 360° minus the azimuth, according as S is west or east of the meridian. The angle at S is called the **parallactic angle**; it is little used in practical astronomy. If any three parts of this triangle are known the other three may be calculated. The fundamental formulæ of spherical trigonometry are

$$\cos a = \cos b \cos c + \sin b \sin c \cos A, \tag{5}$$
$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A, \tag{6}$$
$$\sin a \sin B = \sin b \sin A. \tag{7}$$

If we put $A = P$, $B = S$, $C = Z$, $a = 90^\circ - h$, $b = 90^\circ - L$, $c = 90^\circ - D$, then these become

$$\sin h = \sin L \sin D + \cos L \cos D \cos P, \tag{8}$$
$$\cos h \cos S = \sin L \cos D - \cos L \sin D \cos P, \tag{9}$$
$$\cos h \sin S = \cos L \sin P. \tag{10}$$

If $A = P, B = Z, C = S, a = 90^\circ - h, b = 90^\circ - D, c = 90^\circ - L$, then

$$\cos h \cos Z = \sin D \cos L - \cos D \sin L \cos P, \tag{11}$$
$$\cos h \sin Z = \cos D \sin P. \tag{12}$$

If $A = Z, B = S, C = P, a = 90^\circ - D, b = 90^\circ - L, c = 90^\circ - h$, then

$$\sin D = \sin L \sin h + \cos L \cos h \cos Z, \tag{13}$$
$$\cos D \cos S = \sin L \cos h - \cos L \sin h \cos Z, \tag{14}$$
$$\cos D \sin S = \cos L \sin Z. \tag{15}$$

Other forms may be derived by assigning different values to the parts of the triangle ABC. The formulæ given in the following chapters may in nearly all cases be derived from equations [5] to [15].

The most common cases arising in the practice of surveying are:—

1. Given the declination, latitude, and altitude, to find the azimuth and the hour angle.

2. Given the declination, latitude, and hour angle, to find the azimuth and the altitude.

In the following formulæ

let P = the hour angle,
Z = the azimuth,*
h = the altitude,
z = the zenith distance,
D = the declination,
p = the polar distance,
L = the latitude,

and also let $s = \frac{1}{2}(h + L + p)$.

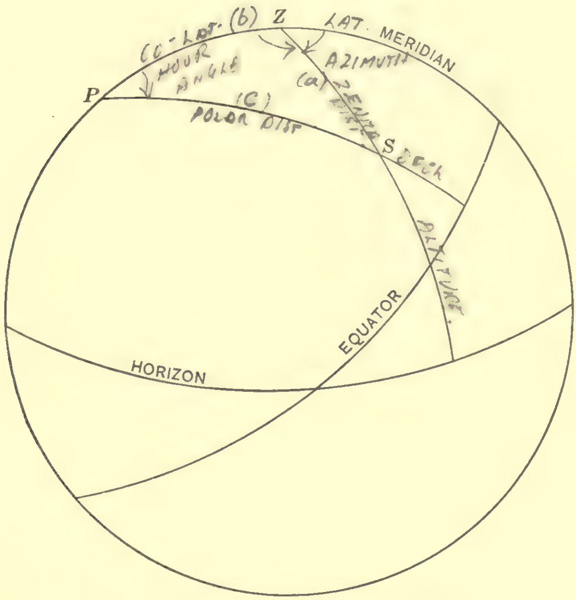

FIG. 27. THE ASTRONOMICAL TRIANGLE

For computing P any of the following formulæ may be used.

$$\sin \tfrac{1}{2} P = \sqrt{\left(\frac{\sin \frac{1}{2}[z + (L - D)] \sin \frac{1}{2}[z - (L - D)]}{\cos L \cos D}\right)}. \qquad [16]$$

* In the formulæ which follow Z is reckoned from the north (interior angle) unless otherwise designated.

$$\sin \tfrac{1}{2} P = \sqrt{\left(\frac{\cos s \sin (s - h)}{\cos L \cos D}\right)}. \qquad [17]$$

$$\cos \tfrac{1}{2} P = \sqrt{\left(\frac{\cos (s - p) \sin (s - L)}{\cos D \cos L}\right)}. \qquad [18]$$

$$\tan \tfrac{1}{2} P = \sqrt{\left(\frac{\cos s \sin (s - h)}{\sin (s - L) \cos (s - p)}\right)}. \qquad [19]$$

$$\operatorname{vers} P = \frac{\cos (L - D) - \sin h}{\cos L \cos D}. \qquad [20]$$

For computing the angle Z (measured from the north point) we have

$$\sin \tfrac{1}{2} Z = \sqrt{\left(\frac{\sin \frac{1}{2}(z + L - D) \cos \frac{1}{2}(z + L + D)}{\cos L \sin z}\right)}. \qquad [21]$$

$$\sin \tfrac{1}{2} Z = \sqrt{\left(\frac{\sin (s - h) \sin (s - L)}{\cos L \cos h}\right)}. \qquad [22]$$

$$\cos \tfrac{1}{2} Z = \sqrt{\left(\frac{\cos s \cos (s - p)}{\cos L \cos h}\right)}. \qquad [23]$$

$$\tan \tfrac{1}{2} Z = \sqrt{\left(\frac{\sin (s - L) \sin (s - h)}{\cos s \cos (s - p)}\right)}. \qquad [24]$$

$$\operatorname{vers} Z_s{}^* = \frac{\cos (L + h) + \sin D}{\cos L \cos h}. \qquad [25]$$

While any of these formulæ may be used to determine the angle sought, the choice of formula should depend somewhat upon the precision with which the angle is defined by the function. If the angle is quite small it is more accurately found through its sine than through its cosine; for an angle near 90° the reverse is the case. On account of the rapid variation of the tangent an angle is always more precisely determined by this function than by either the sine or the cosine. The versed sine formulæ require the use of both natural and logarithmic functions, but are sometimes convenient.

* In this case Z is reckoned from the south.

For computing the altitude and azimuth the following formulæ may be used:

$$\tan Z_s = \frac{\cos M \tan P}{\sin (L - M)}* \qquad [26]$$

and

$$\tan h = \frac{\cos Z_s}{\tan (L - M)}, \qquad [27]$$

where M is an auxiliary angle such that $\tan M = \dfrac{\tan D}{\cos P}$; Z_s is measured from the south point.

The altitude may also be found from the formula

$$\sin h = \cos (L - D) - 2 \cos L \cos D \sin^2 \tfrac{1}{2} P \qquad [28]$$

or

$$\sin h = \cos (L - D) - \cos L \cos D \operatorname{vers} P, \qquad [29]$$

which may be derived from Equa. [8].

If the declination, hour angle, and altitude are given, the azimuth is found from

$$\sin Z = \sin P \frac{\cos D}{\cos h}$$

$$= \sin P \cos D \sec h. \qquad [30]$$

For computing the azimuth of a star near the pole when the hour angle is known the following formula is frequently used:

$$\tan Z = \frac{\sin P}{\cos L \tan D - \sin L \cos P}. \qquad [31]$$

This equation may be derived by dividing [12] by [11] and then simplifying the result by dividing by $\cos D$.

Given the latitude and declination, find the hour angle and azimuth of a star on the horizon. Putting $h = 0$ in Equa. [8] and [13] the results are

$$\cos P = - \tan D \tan L \qquad [32]$$

and

$$\cos Z = \frac{\sin D}{\cos L}. \qquad [33]$$

---

* For the derivation of this formula see Chauvenet's *Spherical and Practical Astronomy*, Vol. I, Art. 14.

A special case of the *PZS* triangle occurs when a star near the pole (circumpolar) is at its greatest east or west position, known as its **greatest elongation.** At this time the star's bearing or azimuth is a maximum and its diurnal circle is tangent to the

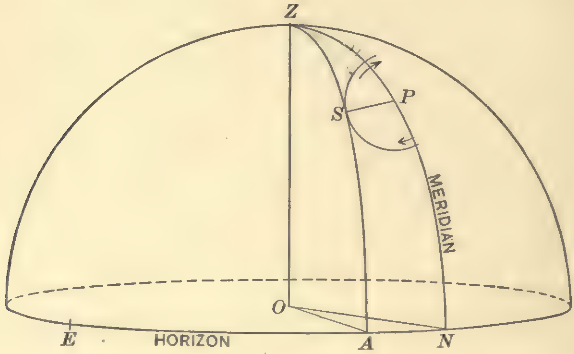

FIG. 28. STAR AT GREATEST ELONGATON (EAST)

vertical circle through the star (Fig. 28); the triangle is consequently right-angled at S.

The formulæ for this case are

$$\cos P = \frac{\tan L}{\tan D} \qquad [34]$$

and

$$\sin Z = \sin p \sec L. \qquad [35]$$

20. Relation between Right Ascension and Hour Angle.

In order to understand the relation between the right ascension and the hour angle of a point, we may think of the equator on the outer sphere as graduated into hours, minutes, and seconds of right ascension, zero being at the equinox and the numbers increasing toward the east. The equator on the inner sphere is graduated for hour angles, the zero being at the observer's meridian and the numbers increasing toward the west. (See Fig. 29.) As the outer sphere turns, the hour marks on the right ascension scale will pass the meridian in the order of the numbers. The number opposite the meridian at any instant shows how far

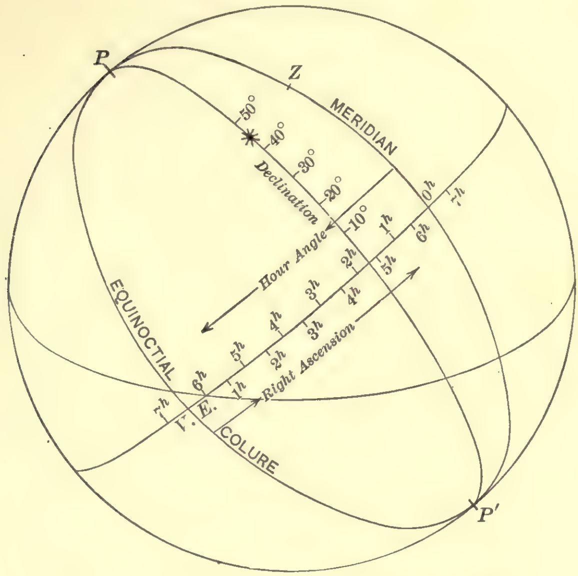

FIG. 29. RIGHT ASCENSION AND HOUR ANGLE

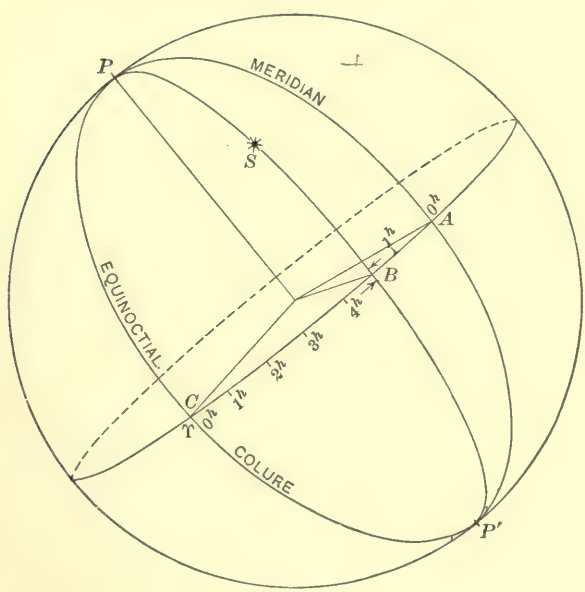

FIG. 30

the sphere has turned since the equinox was on the meridian. If we read the hour angle scale opposite the equinox, we obtain exactly the same number of hours. This number of hours (or angle) may be considered as either the *right ascension of the meridian* or the *hour angle of the equinox*. In Fig. 30 the star S has an hour angle equal to AB and a right ascension CB. The sum of these two angles is AC, or the hour angle of the equinox. The same relation will be found to hold true for all positions of S. The general relation existing between these coördinates is, then,

Hour angle of Equinox = Hour angle of Star + Right Ascension of Star. [36]

Questions and Problems

1. What is the greatest declination a star may have and culminate south of the zenith?

2. What angle does the plane of the equator make with the horizon?

3. In what latitudes can the sun be overhead?

4. What is the altitude of the sun at noon in Boston (42° 21′ N.) on December 22?

5. What are the greatest and least angles made by the ecliptic with the horizon at Boston?

6. In what latitudes is *Vega* (Decl. = 38° 42′ N.) a circumpolar star?

7. Make a sketch of the celestial sphere as it appears to an observer in latitude 20° South at the instant the vernal equinox is on the eastern horizon.

8. Derive formula [35].

CHAPTER V

MEASUREMENT OF TIME

21. The Earth's Rotation.

The measurement of intervals of time is made to depend upon the period of the earth's rotation on its axis. Although it is probable that this period is not absolutely invariable, yet the variations are too small to be measured, and the rotation is assumed to be uniform. The most natural unit of time for ordinary purposes is the solar day, or the time of one rotation of the earth with respect to the sun's direction. On account of the earth's annual motion around the sun the direction of the reference line is continually changing, and the length of the solar day is not the true time of one rotation of the earth on its axis. For this reason it is necessary in astronomical work to make use of another kind of time, based upon the actual period of rotation, called **sidereal time** (star time).

22. Transit or Culmination.

Every point on the celestial sphere crosses the meridian of an observer twice during one revolution of the sphere. The instant when any point on the celestial sphere is on the meridian of an observer is called the **transit,** or **culmination,** of that point over that meridian. When it is on that half of the meridian containing the zenith, it is called the upper transit; when it is on the other half it is called the lower transit. Except in the case of stars near the elevated pole the upper transit is the only one visible to the observer; hence when the transit of a star is mentioned the upper transit will be understood unless the contrary is stated.

23. Sidereal Day.

The **sidereal day** is the interval of time between two successive upper transits of the vernal equinox over the same meridian.

If the equinox were absolutely fixed in position, the sidereal day as thus defined would be the true period of the earth's rotation; but since the equinox has a slow westward motion caused by the precessional movement of the axis (see Art. 8), the actual interval between two transits of the equinox differs about $0^s.01$ from the true time of one rotation. The sidereal day actually used in practice, however, is the one defined above and not the true rotation period. Sidereal days are not used for reckoning long periods of time, dates always being in solar days, so this error never becomes appreciable. The sidereal day is divided into 24 hours and each hour is subdivided into minutes and seconds. When the equinox is at upper transit it is 0^h, or the beginning of the sidereal day (sidereal "noon").

24. Sidereal Time.

The **sidereal time** at a given meridian at any instant is the hour angle of the vernal equinox. It is therefore a measure of the angle through which the earth has turned since the equinox was on the meridian, and shows the position of the sphere at the given instant with respect to the observer's meridian.

25. Solar Day.

A **solar day** is the interval of time between two successive upper transits of the sun's centre over the same meridian. It is divided into 24 hours, each hour being divided into minutes and seconds. When the sun is on the upper side of the meridian (upper transit) it is **noon,** or 0^h solar time. When it is on the lower side of the meridian it is **midnight.**

26. Solar Time.

The **solar time** at a given meridian at any instant is the hour angle of the sun's centre at that instant. This hour angle is a measure of the angle through which the earth has turned with respect to the sun's direction, and consequently is a measure of the time elapsed since the sun was on the meridian.

Since the earth revolves around the sun in an elliptical orbit in accordance with the law of gravitation, the apparent angular motion of the sun is not uniform, and the days are therefore of unequal length at different seasons. In former times, when sun dials were considered sufficiently accurate for measuring time, this lack of uniformity was not important. Under modern conditions, which demand accurate measurement of time by the use of clocks, an invariable unit of time is essential. As a consequence, the time adopted for common use is that kept by a **fictitious sun,** or **mean sun,** which is conceived to move at a uniform rate along the equator,* its speed being such that it makes one apparent revolution around the earth in the same time as the true sun (i.e., one year). The fictitious sun is so placed that on the whole it precedes the true sun as much as it follows it. The time indicated by the position of the mean sun is called **mean solar time,** or simply **mean time.** The time indicated by the position of the real sun is called **apparent solar time** and is the time shown by a sun dial.

27. Equation of Time.

Since observations made on the sun for the purpose of determining the time can give *apparent time* only, it is necessary to be able to find at any instant the exact relation between apparent and mean time. The difference between the two, which varies from $+16^m$ to -16^m (nearly), is called the **equation of time.** This quantity may be found in the Nautical Almanac for each day of the year.

This difference between the two kinds of time is due to several causes, the chief of which are (1) the inequality of the earth's angular motion in the orbit, and (2) the fact that the true sun is on the ecliptic while the mean sun is on the equator. In the winter, when the earth is nearest the sun, the rate of angular motion about the sun must be greater than in summer in order that the radius vector shall describe equal areas in equal intervals of time. (See Fig. 6 and Art. 6.) The sun will then appear

* This statement is true in a general way, but the motion is not strictly uniform because the motion of the equinox itself is variable. The angle from the equinox to the "mean sun" at any instant is the sun's "mean longitude" (along the ecliptic) plus periodic terms.

to move eastward in the sky at a faster rate than in summer, and its daily revolution about the earth will be slower. This delays the instant of apparent noon, making the apparent solar days longer than their average, and therefore a sun dial will "lose time." About April 1 the sun is moving at its average speed and the sun dial ceases to lose time; from this date until about July 1 the sun dial gains on mean time, making up what it lost between Jan. 1 and April 1. During the other half of the year the process is reversed; the sun dial gains from July 1 to Oct. 1 and loses from Oct. 1 to Jan. 1. The maximum difference in time due to this cause is about 8 minutes, either + or −.

The second cause of the equation of time is illustrated by Fig. 31. Assume that point S' (sometimes called the "first

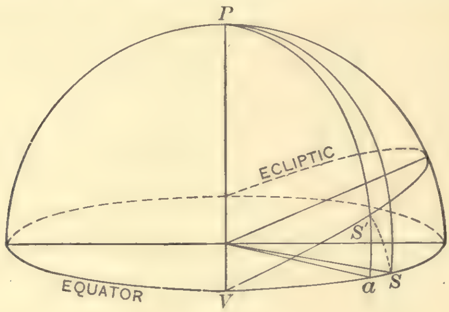

FIG. 31

mean sun") moves uniformly along the ecliptic at the average rate of the true sun; the time as indicated by this point will evidently not be affected by the eccentricity of the orbit. If the *mean sun* S (also called "the second mean sun") starts at V, the equinox, at the same instant that S' starts, then the arcs VS and VS' are equal, since both points are moving with the same speed. By drawing hour circles through these two points it will be seen that these hour circles do not coincide except when the points are at the equinoxes or at the solstices. Since the points are not on the same hour circle they will not cross the meridian at the same time, the difference in time being represented by the arc *aS*. The maximum length of *aS* is about 10 minutes of time, which may be either + or −. The combined effect of these two causes, or the *equation of time*, is shown in the following table.

TABLE A. EQUATION OF TIME FOR 1910.

	1st.	10th.	20th.	30th.
January	$+3^m\ 26^s$	$+7^m\ 27^s$	$+11^m\ 02^s$	$+13^m\ 22^s$
February	+13 41	+14 24	+13 59	. . . .
March	+12 38	+10 36	+ 7 48	+ 4 45
April	+ 4 08	+ 1 31	− 0 58	− 2 47
May	− 2 55	− 3 42	− 3 42	− 2 48
June	− 2 31	− 0 57	+ 1 08	+ 3 15
July	+ 3 27	+ 5 01	+ 6 06	+ 6 16
August	+ 6 11	+ 5 19	+ 3 26	+ 0 46
September	+ 0 09	− 2 48	− 6 20	− 9 46
October	−10 05	−12 45	−15 01	−16 13
November	−16 18	−16 02	−14 26	−11 28
December	−11 06	− 7 23	− 2 36	+ 2 21

28. Conversion of Apparent Time into Mean Time and vice versa.

Apparent time may be converted into mean time by adding or subtracting the equation of time at the instant. Since the equation of time is given in the Nautical Almanac for Greenwich noon its value at the desired instant must be found by adding or subtracting the increase or decrease since Greenwich noon.

Example 1. Find the mean time of the sun's transit over the meridian of Boston on June 30, 1910. The apparent time at Boston is $12^h\ 00^m\ 00^s$ M. at the instant of the transit of the sun's centre, and this corresponds to $4^h\ 44^m\ 18^s$ apparent time at Greenwich, since the longitude of Boston is $4^h\ 44^m\ 18^s$ west of Greenwich. The equation of time at Greenwich Apparent Noon is $3^m\ 14^s.92$ (to be added to apparent time); the hourly change is $0^s.500$ (increasing). The correction to be applied to the equation of time is $4^h.74 \times 0^s.500 = 2^s.37$, making the equation of time at Boston noon $3^m\ 17^s.29$.

$$
\begin{aligned}
\text{L. A. T.}^* &= 12^h\ 00^m\ 00^s.00 \\
\text{Equa. of. T} &= \underline{\quad\ \ 3\ \ \ 17\ .29} \\
\text{L. M. T.} &= 12^h\ 03^m\ 17^s.29
\end{aligned}
$$

* A list of abbreviations will be found on p. 191.

Example 2. Find the local apparent time at Boston at 2 P.M. (local mean time) Oct. 28, 1910. The Greenwich Mean Time corresponding to 2 P.M. local mean time is $6^h\ 44^m\ 18^s$ P.M. The equation of time at G. M. N. Oct. 28, 1910, is $16^m\ 04^s.29$ (to be added to mean time); the hourly increase is $0^s.208$. The correction to the equation of time is $6^h.74 \times 0^s.208 = 1^s.40$. The equation of time at 2 P.M. is therefore $16^m\ 05^s.69$.

$$\begin{array}{rl} \text{L. M. T.} = & 2^h\ 00^m\ 00^s.00 \\ \text{Equa. of T.} = & \underline{\quad 16 \quad 05\ .69} \\ \text{L. A. T.} = & 2^h\ 16^m\ 05^s.69 \end{array}$$

29. Astronomical and Civil Time.

For ordinary purposes it is found convenient to divide the solar day into two parts of 12^h each; from midnight to noon is called A.M. (ante meridiem), and from noon to midnight is called P.M. (post meridiem). The date changes at the instant of midnight. This mode of reckoning time is called **Civil Reckoning.** In astronomical work this subdivision of the day is not convenient. For simplicity in calculation the day is divided into 24^h, numbered consecutively from 0^h to 24^h. As it is not convenient to have the date change during the night, the astronomical date begins at *noon* or 0^h. This is called **Astronomical Time.** In using the Nautical Almanac it should be remembered that it is necessary to change the date and hours to astronomical time before taking out the desired data. In order to change from one kind of time to the other it is only necessary to remember that the astronomical day begins at noon of the civil day of the same date; that is, in the afternoon the dates and the hours will be the same, but in the forenoon the astronomical date is one day less and the hours are 12 greater than in the civil time. NOTE

Examples.

Astr. Time May 10, 15^h = Civil Time May 11, 3^h A.M.
" " Jan. 3, 7^h = " " Jan. 3, 7^h P.M.

From these examples the following rules may be derived:

To change Civil Time to Astronomical Time,

If A.M., add 12^h and drop 1 day from date, and drop the A.M.
If P.M., drop the P.M.

*To change Astronomical Time to Civil Time.**

If less than 12^h, mark it P.M.

If greater than 12^h, subtract 12^h, add 1 day to date, and mark it A.M.

30. Relation between Longitude and Time.

The hour angle of the sun at any given meridian at a given instant is the local solar time at that meridian, and will be apparent or mean time according as the true sun or the mean sun is considered. The hour angle of the sun at Greenwich at the same instant is the corresponding Greenwich solar time. The difference between the two hour angles is the longitude of the place from Greenwich, expressed either in degrees or in hours according as the hour angles themselves are expressed in degrees or in hours. Similarly the difference in local solar time of any two places at a given instant is their difference in longitude in hours, minutes, and seconds. In Fig. 32, AC is the hour angle of the sun at Greenwich (G), or the Greenwich solar time. BC is the hour angle of the sun at the meridian through P, or the local solar time at P. The difference, AB, is the longitude of P west of Greenwich. It should be observed that the reasoning is exactly the same whether C represents the true sun or the fictitious sun. The same result would also be found if the point C were to represent the vernal equinox. The arc AC would then be the hour angle of the equinox, i.e., the Greenwich Sidereal Time. BC would be the Local Sidereal Time, and AB the difference in longitude. The measurement of longitude is therefore independent of the kind of time used, because in each case the angular distances to A and B are measured from the *same point* C on the equator, and the difference in these angles does not depend upon the position of this point nor upon the speed with which this point has moved up to the position at C.

---

* The student may find it helpful to plot the time along a straight line, and to write two sets of numbers, one for Civil Dates and the other for Astronomical Dates.

The difference in the sidereal times at meridian A and meridian B (Fig. 32) is the interval of sidereal time during which a star would go from A to B. Since the star requires 24 sidereal hours to travel from meridian A to meridian A again, the time interval from A to B bears the same relation to 24^h that the longitude

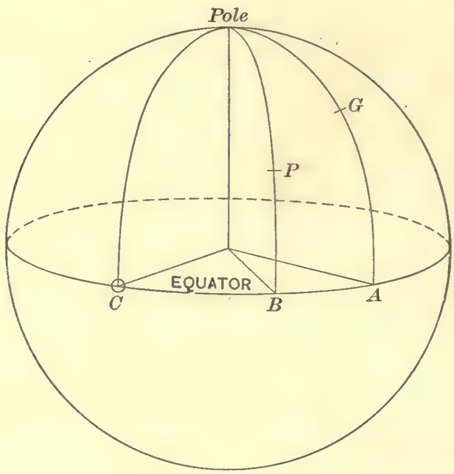

FIG. 32

difference bears to 360°. The difference in the mean solar times at A and B is the number of ~~mean~~ solar hours that the mean sun would take to go from A to B, and since the sun takes 24 solar hours to go from A to A again, the time interval from A to B bears the same ratio to 24 solar hours as when sidereal time was used. The difference in longitude is therefore correctly given when either sidereal or solar times are compared.

The method of changing from Greenwich to local time and the reverse is illustrated by the following examples.

Example 1. The Greenwich astronomical time is $7^h\ 40^m\ 10^s.0$. Required the local time at a meridian $4^h\ 50^m\ 21^s.0$ West.

$$\begin{aligned} \text{G. M. T.} &= 7^h\ 40^m\ 10^s.0 \\ \text{Long. West} &= 4\ \ 50\ \ 21\ .0 \\ \hline \text{L. M. T.} &= 2^h\ 49^m\ 49^s.0 \quad \text{(P.M.)} \end{aligned}$$

Example 2. The Greenwich mean time is $3^h\ 20^m\ 16^s.5$. Required the local mean time at a place whose longitude is $120°\ 10'$ West.

$$\begin{aligned} \text{G. M. T.} + 24^h &= 27^h\ 20^m\ 16^s.5 \\ \text{Long. West} &= \underline{\ 8^h\ 00^m\ 40^s.0} \\ \text{L. M. T.} &= 19^h\ 19^m\ 36^s.5 \\ &= \ 7^h\ 19^m\ 36^s.5 \text{ A.M.} \end{aligned}$$

Example 3. The mean time at a place 3^h East longitude is 10^h A.M. Required the Greenwich mean time.

$$\begin{aligned} \text{L. M. T.} &= 22^h\ 00^m\ 00^s.0 \\ \text{Long. East} &= \underline{\ 3^h\ 00^m\ 00^s.0} \\ \text{G. M. T.} &= 19^h\ 00^m\ 00^s.0 \\ &= \ 7^h\ 00^m\ 00^s.0 \text{ A.M.} \end{aligned}$$

Since a circle may be divided either into 24^h or into $360°$, the relation between these two units is constant. From the fact that

$$24^h = 360°$$

we have also

$$\begin{aligned} 1^h &= 15°, \\ 1^m &= 15', \\ 1^s &= 15''. \end{aligned}$$

The following equivalents are also convenient:

$$\begin{aligned} 1° &= 4^m \\ 1' &= 4^s \end{aligned}$$

By means of these two sets of equivalents time may be converted into degrees, or the contrary, without writing down the intermediate steps. In the following examples the intermediate steps are written down in order to show the process followed.

Example 1. Convert $6^h\ 35^m\ 51^s$ into degrees.

$$\begin{aligned} 6^h &= 90° \\ 35^m = 32^m + 3^m &= \ 8°\ 45' \\ 51^s = 48^s + 3^s &= \underline{\qquad 12'\ 45''} \\ \text{Total} &= 98°\ 57'\ 45'' \end{aligned}$$

Example 2. Convert $47°\ 17'\ 35''$ into hours.

$$\begin{aligned} 47° = 45° + 2° &= 3^h\ 08^m \\ 17' = 15' + 2' &= \quad 01^m\ 08^s \\ 35'' = 30'' + 5'' &= \underline{\qquad\quad 02\ .33} \\ \text{Total} &= 3^h\ 09^m\ 10^s.33 \end{aligned}$$

It should be observed that the relation $15° = 1^h$ is quite independent of the length of time that has elapsed. A star takes one sidereal hour to move over $15°$ of hour angle; the sun takes one solar hour to move over $15°$ of hour angle. In the sense in which it is used here, 1^h means an angle, and not an absolute interval of time.

31. Relation between Sidereal Time, Right Ascension, and Hour Angle of any Point at a Given Instant.

In Fig. 33 the hour angle of the equinox, or local sidereal time at the meridian through P, is the arc AV. The hour angle of

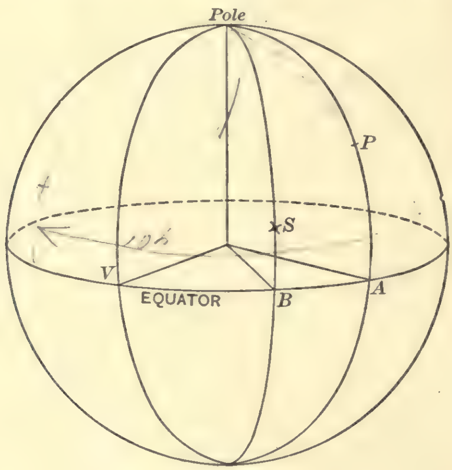

FIG. 33

the star S at the meridian through P is the arc AB. The right ascension of the star S is the arc VB. It is evident from the figure that

$$AV = VB + AB,$$

or

$$S = R + P, \qquad [37]$$

where R = the right ascension and P = the hour angle of the point S, and S = the sidereal time; or, in words,

$$\textit{Sidereal Time} = \textit{Right Ascension} + \textit{Hour Angle}. \qquad [38]$$

This relation is a perfectly general one and will be found to hold true for all points on the sphere, provided it is agreed to reckon the sidereal time beyond 24^h when necessary. For example, if the hour angle is 10^h and the right ascension is 20^h, the resulting sidereal time is 30^h. This means that the equinox has made a complete revolution and has gone 6^h, or $90°$, on the next revolution; the actual reading of the sidereal clock would be 6^h. In the reverse case, when it is necessary to subtract 20^h from 6^h to obtain the hour angle, the 6^h must first be increased by 24^h and the right ascension subtracted from the sum to obtain the hour angle, 10^h.

32. Star on the Meridian.

At the instant when the star is on the meridian its hour angle is 0^h and the equation becomes

$$\textit{Sidereal Time} = \textit{Right Ascension}; \qquad [39]$$

that is, the right ascension of a star equals the local sidereal time at which that star crosses the meridian. (See Art. 20, p. 36.)

33. Relation between Mean Solar and Sidereal Intervals of Time.

It has already been stated that on account of the earth's orbital motion the sun has an apparent eastward motion among the stars of nearly $1°$ per day. This eastward movement of the sun makes the intervals between the sun's transits greater by nearly 4^m than the intervals between the transits of the equinox, that is, the solar day is nearly 4^m longer than the sidereal day. In Fig. 34 let C and C' be the positions of the earth on two consecutive days. When the observer is at O it is local noon. After the earth makes one complete rotation, the observer will be at O', and the sidereal time will be exactly the same as it was the day before when he was at O. But the sun's direction is now $C'O''$, so the earth must turn through the angle $O'C'O''$ in order to bring the sun again on the observer's meridian. Since this angle is about $1°$ it takes about 4^m longer to complete the solar day than it does to complete the sidereal day. Since each kind of day is subdivided into hours, minutes, and seconds, all of these units in solar time will be proportionally longer than the corresponding units of sidereal time. If two clocks, one regulated to mean solar time and the other to sidereal time, were started at the same instant, both reading 0^h, the sidereal clock would immediately begin to gain on the solar clock, the gain

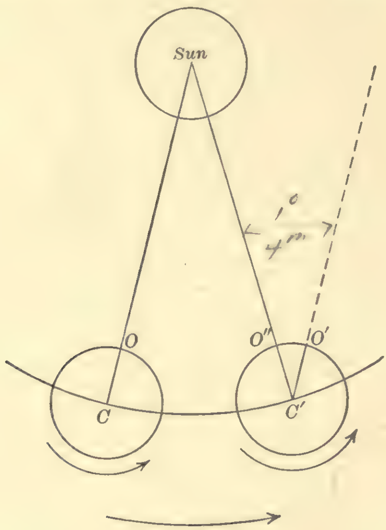

FIG. 34

being exactly proportional to the time interval, that is, about 10^s per hour, or more nearly $3^m\,56^s$ per day.

In order to find the exact relation between the two kinds of time it should be observed that the number of sidereal days in the year is exactly one greater than the number of solar days, because the sun comes back to the equinox at the end of one year. The length of the tropical* year is found to be 365.2422

* The **tropical year** is the interval of time between two successive passages of the sun over the vernal equinox. The **sidereal year** is the interval between two passages of the sun across the hour circle through a fixed star on the equator. On account of the movement of the equinox caused by precession, the tropical year is about 20^m shorter than the sidereal year.

mean solar days. The relation between the two kinds of day is therefore

$$366.2422 \text{ sidereal days} = 365.2422 \text{ solar days}, \qquad [40]$$

or

$$1 \text{ sidereal day} = 0.99726957 \text{ solar day}, \qquad [41]$$

and

$$1 \text{ solar day} = 1.00273791 \text{ sidereal days}. \qquad [42]$$

Equations [41] and [42] may be written

$$24^h \text{ sidereal time} = (24^h - 3^m\, 55^s.909) \text{ mean solar time,}$$
$$24^h \text{ mean solar time} = (24^h + 3^m\, 56^s.555) \text{ sidereal time.}$$

These equations may be put in more convenient form for computation by expressing the difference in time as a correction to be applied to any interval of time to change it from one kind of unit to the other. If I_m is a mean solar interval and I_s the corresponding number of sidereal units, then

$$I_s = I_m + .00273791 \times I_m \qquad [43]$$

and

$$I_m = I_s - .00273043 \times I_s. \qquad [44]$$

Tables II and III are constructed by multiplying different values of I_m and I_s by these constants. More extended tables may be found in the Nautical Almanac. The use of Tables II and III is illustrated by the following examples.

NOTE

Examples.

Reduce $9^h\, 23^m\, 51^s.0$ of sidereal time to the equivalent number of solar units. From Table II, opposite 9^h is the correction $-1^m\, 28^s.466$; opposite 23^m in the 4th column is $-3^s.768$; and opposite 51^s in the last column is $0^s.139$. The sum of these three partial corrections is $-1^m\, 32^s.373$, which is the amount to be subtracted from $9^h\, 23^m\, 51^s.0$ to reduce it to the equivalent solar interval, $9^h\, 22^m\, 18^s.627$.

Reduce $7^h\, 10^m$ solar time to sidereal time. The correction for 7 , Table III, is $+ 1^m\, 08^s.995$, and for 10^m is $1^s.643$. The sum, $1^m\, 10^s.638$, added to $7^h\, 10^m$ gives $7^h\, 11^m\, 10^s.638$ of sidereal time.

This reduction may be made approximately by the following rule: the correction equals 10^s per hour diminished by 1^s for every 6^h in the interval. The correction for 6^h would be $6 \times 10^s - 1^s = 59^s$. This rule is based on a change of 3^m 56^s per day. For changing solar into sidereal the error is $0^s.023$ per hour; for sidereal into solar the error is $0^s.004$ per hour.

It should be kept in mind that the conversion of time discussed in this article concerns the change from one kind of unit to another, like changing from yards to metres, and is not the same as changing from the local sidereal time to the local solar time at a particular instant.

34. Relation between Sidereal Time and Mean Solar Time at any Instant.

If in Fig. 33, Art. 31, the point B is taken to represent the **mean sun,** then equation [37] becomes

$$S = R_s + P_s, \qquad [45]$$

where R_s and P_s are the right ascension and the hour angle of the mean sun at the instant considered. P_s is the local mean time by the definition given in Art. 26. If the equation is written

$$S - P_s = R_s, \qquad [46]$$

then, since the value of the right ascension R_s does not depend upon the time at any particular meridian, but only upon the absolute instant of time considered, it is evident that the difference between sidereal time and mean time at any instant is the same for all places on the earth. The actual values of S and P_s will of course be different at different meridians, but the difference between the two is a constant for all places for the given instant. In order that Equa. [45] shall hold true it is essential that R_s and P_s shall refer to the same position of the sun, that is, to the same absolute instant of time. The right ascension of the sun obtained from the Nautical Almanac is its value at the instant of the Greenwich Mean Noon preceding, that is, at the beginning of the astronomical day at Greenwich.*

* The dates are always in mean solar days, not in sidereal days.

To reduce this right ascension to its value at the desired instant it is necessary to multiply the hourly increase in the right ascension of the mean sun by the number of solar hours elapsed since the instant of Greenwich Mean Noon. The hourly increase in the right ascension of the mean sun is constant and is evidently equal to the correction in Table III, for the difference between sidereal and solar time is caused by the sun's motion, and the amount of the difference for any number of hours is exactly equal to the increase in the right ascension. If it is desired to find the increase for any number of solar hours, Table III should be used; for sidereal hours use Table II. Equation [45] may be written

$$S = R_s + P_s + C, \qquad [47]$$

where R_s refers to the instant of the preceding local mean noon, and C is the correction (Table III) to reduce P_s to a sidereal interval, or to reduce R_s to its value at the time P_s.

In Fig. 35 suppose that the sun S and a star S' passed the meridian M at the same instant, and at the mean time P_s it is

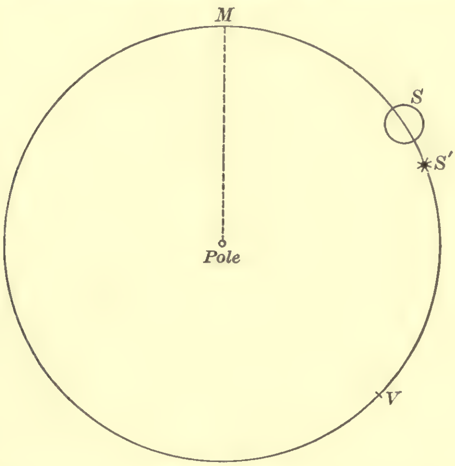

FIG. 35

desired to compute the sidereal time. Since the sun is moving at a slower rate than the star, it will describe the arc MS ($= P_s$)

while the star moves from M to S'. The arc SS', or C, represents the gain of sidereal on mean time in the mean time interval MS or P_s. But S' is the position of the sun at noon, so that VS' is the sun's right ascension at the preceding mean noon, or R_s. The right ascension desired is VS, so R_s must evidently be increased by the arc SS', or C.

If it is desired to find the mean solar time corresponding to a given instant of local sidereal time, the equation is

$$\textit{Sidereal interval from noon} = S - R_s, \qquad [48]$$

or

$$\textit{Mean time} = P_s = S - R_s - C', \qquad [49]$$

where C' is the correction from Table II to reduce $S - R_s$ to a solar interval, and represents the increase in the sun's right ascension in $S - R_s$ sidereal hours.

Examples.

To find the Greenwich Sidereal Time corresponding to Greenwich Mean Time $9^h\ 22^m\ 18^s.60$ on Jan. 7, 1907. The right ascension of the mean sun at Greenwich Mean Noon is found from the Nautical Almanac to be $19^h\ 03^m\ 36^s.38$. The correction to reduce $9^h\ 22^m\ 18^s.60$ to sidereal time (Table III) is $+1^m\ 32^s.37$. Then, applying Equa. [47],

$$\begin{aligned} R_s &= 19^h\ 03^m\ 36^s.38 \\ P_s &= 9\ \ 22\ \ 18\ .60 \\ C &= \underline{1\ \ 32\ .37} \\ S &= 28^h\ 27^m\ 27^s.35 \\ \text{Sidereal Time} &= 4^h\ 27^m\ 27^s.35 \end{aligned}$$

To find the Greenwich Mean Solar Time when the Greenwich Sidereal Time is $4^h\ 27^m\ 27^s.35$ on Jan. 7, 1907.

$$\begin{aligned} S &= 28^h\ 27^m\ 27^s.35 \\ R_s &= \underline{19\ \ 03\ \ 36\ .38} \\ S - R_s &= 9\ \ 23\ \ 50\ .97 \\ C'\ \text{(Table II)} &= \underline{-1\ \ 32\ .37} \\ \text{Mean Time} &= 9^h\ 22^m\ 18^s.60 \end{aligned}$$

If the change from sidereal to solar time (or *vice versa*) is to be made at any meridian other than Greenwich, the right ascension of the sun for local noon must be found by multiplying the increase per solar hour by the number of solar hours since Greenwich noon, that is, by the number of hours in the *longitude*, and adding this to the value of R_s from the Almanac if the place is west of Greenwich, subtracting if east.* The correction may be taken from Table III. If the sidereal time in the above example is assumed to be the time at a meridian 5^h ($75°$) west of Greenwich, the computation would be modified as follows:

$$
\begin{array}{rr}
R_s' = & 19^h\ 03^m\ 36^s.38 \\
\text{Correction for } 5^h \text{ longitude} = & \underline{49\ .28} \\
R_s = & 19\ \ 04\ \ 25\ .66 \\
S = & \underline{28\ \ 27\ \ 27\ .35} \\
S - R_s = & 9\ \ 23\ \ 01\ .69 \\
C' = & \underline{1\ \ 32\ .24} \\
\text{Local Mean Time} = & 9^h\ 21^m\ 29^s.45
\end{array}
$$

It is evident that at the instant of mean noon $P_s = 0$ and $R_s = S$. At mean noon, therefore, the sidereal time equals the right ascension of the mean sun. This quantity will be found in the Almanac under both headings, "Sidereal Time of Mean Noon" and "Right Ascension of the Mean Sun." (See p. 65.)

The reduction of mean solar time to sidereal time, or the reverse, may be made also by first changing the given local time to the corresponding instant of Greenwich time, then making the transformation as before, and finally changing back to the meridian of the place. Take, for example, the case given on page 54.

$$
\begin{array}{rr}
\text{Local Sidereal Time} = & 28^h\ 27^m 27^s.35 \\
\text{Longitude} = & \underline{5\ \ 00\ \ 00} \\
\text{Greenwich Sidereal Time} = & 33\ \ 27\ \ 27\ .35 \\
R_s \text{ at Gr. M. Noon} = & \underline{19\ \ 03\ \ 36\ .38} \\
\text{Sidereal Interval from Noon} = & 14\ \ 23\ \ 50\ .97 \\
C' = & \underline{-2\ \ 21\ .52} \\
\text{Greenwich Mean Time} = & 14\ \ 21\ \ 29\ .45 \\
\text{Longitude} = & \underline{5\ \ 00\ \ 00} \\
\text{Local Mean Time} = & 9^h\ 21^m 29^s.45
\end{array}
$$

The result agrees with that obtained by the former method. This method is quite as simple as the preceding, especially when

---

* It should be remembered that the sun's R. A. is always *increasing*.

Standard Time is to be computed, for the final correction will always be a whole number of hours. Care should be taken always to use the right ascension of the sun at the noon preceding the given time. Suppose that the instant of 10^h A.M. May 5 is to be converted into sidereal time, the longitude of the place being $4^h\ 44^m\ 18^s$ west. Civil time 10^h A.M. May 5 = Astr. time 22^h May 4. If the first method is followed, the right ascension of the sun employed should be that of noon May 4. If the reduction is made by first changing to Greenwich time, then $22^h + 4^h\ 44^m\ 18^s = 26^h\ 44^m\ 18^s$ May 4 $= 2^h\ 44^m\ 18^s$ May 5. The right ascension for the latter case would be that for noon of May 5.

35. Standard Time.

From the definition of mean solar time it will be seen that at any given instant the solar times at two places will differ from each other by an amount depending upon the difference in the longitudes. All places will have different local times except where they happen to be on the same meridian. Previous to the year 1883 it was customary in this country for each large city or town to use the mean time at its own meridian, and for all other places in the vicinity to adopt the same time. Before railroad travel became extensive this change of time from one point to another caused no great difficulty, but with the increased amount of railroad and telegraph business these frequent and irregular changes in time became so inconvenient that in 1883 a uniform system of time was adopted in the United States. The country is divided into time belts each theoretically $15°$ in width; these are known as the Eastern, Central, Mountain and Pacific time belts, and places in these belts use the mean local time of the $75°$, $90°$, $105°$ and $120°$ meridians respectively. The time at the $60°$ meridian is called Atlantic time and is used in the Eastern Provinces of Canada. The actual positions of the dividing lines between these belts depend upon the positions of the principal cities and the railroads (see Fig. 36), but the change of time from one belt to another is always exactly one hour. The

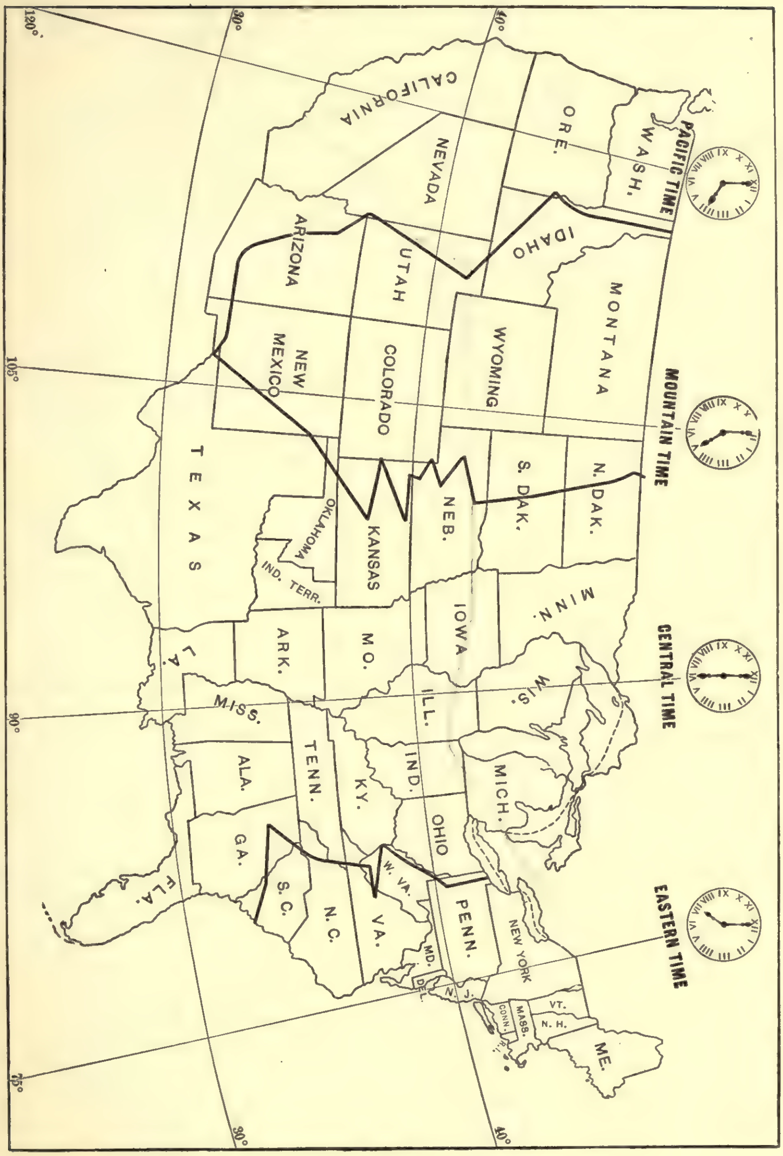

FIG. 36. MAP SHOWING THE STANDARD TIME BELTS IN THE UNITED STATES
(The clocks show the time corresponding to the instant of Greenwich mean noon.)

minutes and seconds of all clocks are the same as the minutes and seconds of the Greenwich clock. When it is noon at Greenwich it is 8 A.M. Atlantic time, 7 A.M. Eastern time, 6 A.M. Central time, 5 A.M. Mountain time, and 4 A.M. Pacific time.

The change from local to standard time, or the contrary, consists in expressing the difference in longitude between the local meridian and the standard meridian in units of time, and adding or subtracting this correction, remembering that the farther west a place is, the earlier it is in the day at any given instant of time.

Examples.

Find the standard time at a place 71° west of Greenwich when the local time is $4^h\ 20^m\ 00^s$ P.M. In longitude 71° the standard time would be that of the 75° meridian. The difference in longitude is $4^\circ = 16^m$. Since the standard meridian is west of the 71° meridian, the time is 16^m earlier than the local time. The standard time is therefore $4^h\ 04^m\ 00^s$ P.M.

Find the local time at a place 91° west of Greenwich when the Central time is $9^h\ 00^m\ 00^s$ A.M. The difference in longitude is $1^\circ = 4^m$. Since the place is west of the standard meridian, the time is earlier. The local time is therefore $8^h\ 56^m\ 00^s$ A.M.

Standard time is used not only in the United States but in a majority of the countries of the world; in nearly all cases these systems of standard time are based on the meridian of Greenwich as the prime meridian. Germany, for example, uses the local mean time at the meridian 1^h east of Greenwich; Japan uses that of the meridian 9^h east of Greenwich; Turkey, 2^h east of Greenwich, etc.

36. The Date Line.

If a person were to start at Greenwich at the instant of noon and travel westward rapidly enough to keep the sun always on his meridian he would get back to Greenwich 24^h later, but his own (local) time would not have changed but would have remained *noon* all the time. In travelling westward at a slower rate the same thing occurs, only in a longer interval of time.

The traveller has to set his watch back every day in order to keep it regulated to the meridian at which his noon occurs. As a consequence, his watch has recorded one day less than it has actually run, and his calendar is one day behind that of a person who remains at Greenwich. If the traveller goes east he has to set his watch ahead every day, and after circumnavigating the globe his calendar is one day ahead of what it should be. In order that the calendar may be everywhere uniform, it is agreed to change the date at the meridian 180° from Greenwich. Whenever a ship crosses the 180° meridian going westward, a day is omitted from the calendar, and when going eastward a day is repeated. In practice the change is made at midnight near the 180° meridian, not at the instant of crossing. The **date line** actually used does not follow the 180° meridian in all places, but is deflected so as not to separate the Aleutian islands, and in the South Pacific ocean it passes east of several groups of islands so as not to change the date formerly used in these islands.

37. The Calendar.

Previous to the time of Julius Cæsar the calendar was based upon the *lunar month*, and, as this resulted in a continual change in the date at which the seasons occurred, the calendar was frequently changed in an arbitrary manner in order to keep the seasons in their places, the result being extreme confusion in the dates. In the year 45 B.C. Julius Cæsar reformed the calendar and introduced one based upon a year of $365\frac{1}{4}$ days, since called the Julian calendar. The $\frac{1}{4}$ day was taken care of by making the year contain 365 days, except every 4th year, called **leap year,** which contained 366; the extra day was added to February in such years as were divisible by 4. The year was begun on Jan. 1; previously it had begun in March. Since the year contains actually $365^d\ 5^h\ 48^m\ 46^s$, this difference of $11^m\ 14^s$ caused a gradual change in the dates at which the seasons occurred. After many centuries the difference had accumulated to about 10 days, so in 1582 Pope Gregory XIII ordered that the calendar should be corrected by dropping ten days and that future dates should be computed by omitting the 366th day in those leap years which occurred in century years not divisible by 400; that is, such years as 1700, 1800 and 1900 should not be counted as leap years. This is the calendar used at the present time.

Questions and Problems

1. (a) Prove by direct computation of sidereal time from Fig. 37 that

$$R + P = 24^h + S,$$

in which R and P are the right ascension and hour angle of the star S, and S is the sidereal time, or hour angle of V.

(b) Prove the same relation when V is at the point V'. (See Art. 31, p. 48.)

2. Prove that the difference in longitude of two points is independent of the kind of time used, by selecting two points at which the solar time differs by say 3^h, and then converting the solar time at each place into sidereal time.

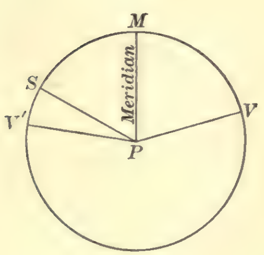

FIG. 37

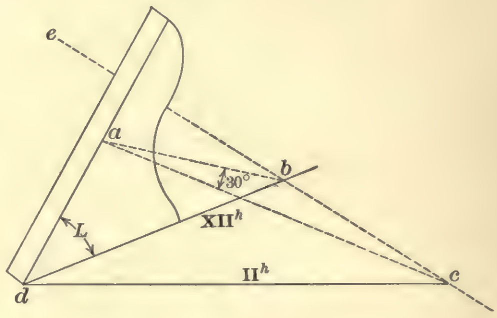

FIG. 38

3. Make a design for a horizontal sun dial for a place whose latitude is 42° 21′ N. The gnomon *ad* (Fig. 38), or line which casts the shadow on the horizontal plane, must be parallel to the earth's rotation axis; the angle which the gnomon makes with the horizontal plane therefore equals the latitude. The shadow lines for the hours (X, XI, XII, I, II, etc.) are found by passing planes through the gnomon and finding where they cut the horizontal plane of the dial. The vertical plane *adb* coincides with the meridian and therefore is the noon (XII^h) line. The other planes make, with the vertical plane, angles equal to some multiple of 15°. In finding the trace *dc* of one of these planes on the dial it should be observed that the foot of the gnomon, *d*, is a point common to all such traces. In order to find another point *c* on any trace, or shadow line, pass a plane *abc* through some point *a* on the gnomon and perpendicular to it. This plane (the plane of the equator) will cut an east and west line *ce* on the dial. If a line be drawn in this plane making an

angle of $n \times 15°$ with the meridian plane, it will cut *ce* at a point *c* which is on the shadow line. Joining *c* with the foot of the gnomon gives the required line.

In making a design for a sun dial it must be remembered that the west edge of the gnomon casts the shadow in the forenoon and the east edge in the afternoon; there will be of course two noon lines, and the two halves of the diagram will be symmetrical and separated from each other by the thickness of the gnomon. The dial may be placed in position by levelling the horizontal surface and then computing the watch time of apparent noon and turning the dial so that the shadow is on the XII^h line at the calculated time.

Prove that the horizontal angle *bdc* is given by the relation

$$\tan bdc = \tan P \sin L,$$

in which P is the sun's hour angle and L is the latitude.

4. Why are the sun's and moon's right ascension always increasing?

5. The local apparent time at a point A is $10^h\ 30^m$ A.M. If the equation of time is $+ 3^m\ 25^s.8$, what is the local mean time? What is the astronomical mean time at the given instant? Assuming the longitude of A to be 95° West, what is the Greenwich Mean Time? What is the Central Standard Time? What is the local mean time at the same instant at a point B in longitude 110° W.? If the right ascension of the mean sun at G. M. N. is $18^h\ 41^m\ 01^s.6$, what is the local sidereal time? What is the Greenwich Sidereal Time?

CHAPTER VI

THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC — STAR CATALOGUES — INTERPOLATION

38. The Ephemeris.

In the problems previously discussed it has been assumed that the coördinates of celestial objects and various other data mentioned are known to the computer. These data consist of results calculated from observations made with large instruments at the principal observatories; these results are published by the government several years in advance in the American Ephemeris and Nautical Almanac.* The Almanac contains the declinations and the right ascensions of the sun, moon, planets and stars, as well as the angular semidiameters, horizontal parallaxes, the equation of time, and other data required in astronomical calculations. Since all of these quantities vary with the time, their values are usually given for equidistant intervals of Greenwich time or of Washington time.

The Almanac is divided into three parts. Part I is computed for the meridian of Greenwich, and is arranged especially for the convenience of navigators. Part II is computed for the meridian of Washington, and is arranged chiefly for the convenience of astronomers. Part III contains the data for predicting phenomena, such as eclipses, occultations, etc. At the end of the book are certain tables computed especially for the use of the navigator and the surveyor.

The first page of Part I is headed "At Greenwich Apparent

* Similar publications by other governments are: the Nautical Almanac (Great Britain), the Berliner Astronomisches Jahrbuch (Germany), the Connaissance des Temps (France), and the Almanaque Nautico (Spain).

The word "ephemeris" means a table of coördinates of a celestial body given for equidistant intervals of time.

Noon," and contains the following data for that instant for each day in the month: right ascension and declination of the sun and their hourly changes, sun's semidiameter, time of semidiameter passing the meridian, and the equation of time with its hourly change. The second page is headed "At Greenwich Mean Noon," and contains the right ascension and declination of the sun with their hourly changes, the equation of time with its hourly change, and the right ascension of the mean sun or sidereal time at mean noon. These pages are the ones to be used by the navigator or the surveyor when making observations on the sun. Whether Table I or Table II shall be used in any given case depends upon whether apparent time or mean time is the more convenient. If the declination, right ascension, or equation of time is required for the instant of the sun's transit over any meridian, then the local apparent time is *noon* and the Greenwich Apparent Time is equal to the west longitude of the place. The desired quantity is found by taking out its value for the instant of Greenwich Apparent Noon and increasing or decreasing it by the hourly change multiplied by the number of hours in the longitude. If the quantity is to be found for some instant of local mean time or Standard Time, then the Greenwich Mean Time may be readily found, and it is therefore more convenient to compute the required value from that given for Greenwich Mean Noon. If Local Time is used, the Greenwich Time is found by adding the longitude; if Standard Time is used, the Greenwich Time is found by adding 5^h, 6^h, etc., according to the time belt indicated. The tabular quantity is then corrected for the time elapsed since Greenwich Mean Noon. Tables I and II for the month of January, 1912, are shown on pages 64 and 65. The third page of the Almanac contains data not usually required by the surveyor. On the fourth page are the semidiameter, horizontal parallax, and time of transit of the moon, with their hourly changes. On account of the rapidity with which the semidiameter and parallax vary, they are given for both Greenwich noon and midnight. The next eight pages contain the

JANUARY, 1912

AT GREENWICH APPARENT NOON

Day of the week.	Day of the month.	THE SUN'S						Sidereal time of semi-diameter passing meridian.	Equation of time, to be added to apparent time.	Diff. for 1 hour.
		Apparent right ascension.	Diff. for 1 hour.	Apparent declination.	Diff. for 1 hour.	Semi-diameter.				
		h m s	s	° ′ ″	″	′ ″		s	m s	s
Mon.	1	18 42 21.37	11.053	S. 23 5 43.7	+11.19	16 17.89		71.09	3 13.30	1.193
Tues.	2	18 46 46.49	11.039	23 1 1.5	12.34	16 17.90		71.04	3 41.79	1.179
Wed.	3	18 51 11.26	11.025	22 55 51.6	13.49	16 17.90		71.00	4 9.93	1.165
Thur.	4	18 55 35.68	11.009	22 50 14.4	+14.63	16 17.91		70.95	4 37.71	1.149
Frid.	5	18 59 59.71	10.992	22 44 10.0	15.75	16 17.91		70.90	5 5.11	1.133
Sat.	6	19 4 23.33	10.975	22 37 38.5	16.87	16 17.90		70.84	5 32.11	1.115
Sun.	7	19 8 46.53	10.957	22 30 40.3	+17.98	16 17.87		70.78	5 58.67	1.097
Mon.	8	19 13 9.28	10.939	22 23 15.3	19.09	16 17.84		70.71	6 24.78	1.077
Tues.	9	19 17 31.55	10.919	22 15 23.9	20.19	16 17.80		70.64	6 50.41	1.057
Wed.	10	19 21 53.31	10.896	22 7 6.3	+21.28	16 17.76		70.57	7 15.54	1.036
Thur.	11	19 26 14.54	10.873	21 58 22.7	22.36	16 17.72		70.50	7 40.15	1.014
Frid.	12	19 30 35.23	10.850	21 49 13.3	23.43	16 17.67		70.42	8 4.21	0.990
Sat.	13	19 34 55.33	10.826	21 39 38.4	+24.49	16 17.61		70.34	8 27.70	0.966
Sun.	14	19 39 14.83	10.800	21 29 38.3	25.53	16 17.55		70.26	8 50.59	0.941
Mon.	15	19 43 33.71	10.773	21 19 13.1	26.57	16 17.48		70.17	9 12.86	0.914
Tues.	16	19 47 51.96	10.746	21 8 23.3	+27.59	16 17.41		70.08	9 34.48	0.887
Wed.	17	19 52 9.54	10.717	20 57 9.2	28.59	16 17.34		69.98	9 55.42	0.859
Thur.	18	19 56 26.42	10.688	20 45 31.0	29.58	16 17.26		69.88	10 15.68	0.830
Frid.	19	20 0 42.58	10.658	20 33 29.1	+30.56	16 17.18		69.78	10 35.24	0.800
Sat.	20	20 4 58.02	10.628	20 21 3.9	31.53	16 17.10		69.68	10 54.08	0.769
Sun.	21	20 9 12.71	10.597	20 8 15.7	32.48	16 17.01		69.57	11 12.17	0.738
Mon.	22	20 13 26.64	10.565	19 55 4.8	+33.41	16 16.92		69.47	11 29.49	0.706
Tues.	23	20 17 39.80	10.533	19 41 31.7	34.33	16 16.82		69.36	11 46.04	0.673
Wed.	24	20 21 52.16	10.500	19 27 36.7	35.24	16 16.72		69.26	12 1.80	0.640
Thur.	25	20 26 3.72	10.465	19 13 20.0	+36.13	16 16.62		69.15	12 16.76	0.607
Frid.	26	20 30 14.46	10.430	18 58 42.3	37.01	16 16.52		69.04	12 30.91	0.573
Sat.	27	20 34 24.38	10.396	18 43 43.9	37.86	16 16.41		68.93	12 44.24	0.538
Sun.	28	20 38 33.48	10.361	18 28 25.0	+38.70	16 16.29		68.82	12 56.74	0.504
Mon.	29	20 42 41.74	10.326	18 12 46.2	39.52	16 16.16		68.71	13 18.41	0.469
Tues.	30	20 46 49.16	10.291	17 56 48.0	40.32	16 16.03		68.60	13 19.25	0.434
Wed.	31	20 50 55.74	10.256	17 40 30.7	41.11	16 15.90		68.48	13 29.25	0.400
Thur.	32	20 55 1.49	10.222	S. 17 23 54.5	+41.89	16 15.76		68.37	13 38.41	0.365

Note. — The mean time of semidiameter passing may be found by subtracting $0^s.19$ from the sidereal time. The sign + prefixed to the hourly change of declination indicates that south declinations are decreasing.

JANUARY, 1912

AT GREENWICH MEAN NOON

Day of the week.	Day of the month.	THE SUN'S Apparent right ascension.	THE SUN'S Diff. for 1 hour.	THE SUN'S Apparent declination.	THE SUN'S Diff. for 1 hour	Equation of time, to be subtracted from mean time.	Diff. for 1 hour.	Sidereal time, or right ascension of mean sun.
		h m s	s	° ′ ″	″	m s	s	h m s
Mon.	1	18 42 20.78	11.049	S. 23 5 44.3	+11.18	3 13.24	1.193	18 39 7.54
Tues.	2	18 46 45.81	11.035	23 1 2.2	12.33	3 41.72	1.179	18 43 4.10
Wed.	3	18 51 10.50	11.021	22 55 52.5	13.47	4 9.85	1.165	18 47 0.66
Thur.	4	18 55 34.84	11.006	22 50 15.5	+14.61	4 37.62	1.149	18 50 57.22
Frid.	5	18 59 58.79	10.990	22 44 11.3	15.74	5 5.01	1.133	18 54 53.78
Sat.	6	19 4 22.33	10.972	22 37 40.1	16.86	5 32.00	1.115	18 58 50.33
Sun.	7	19 8 45.45	10.953	22 30 42.0	+17.97	5 58.56	1.097	19 2 46.89
Mon.	8	19 13 8.12	10.934	22 23 17.3	19.08	6 24.66	1.077	19 6 43.45
Tues.	9	19 17 30.30	10.914	22 15 26.2	20.17	6 50.29	1.057	19 10 40.01
Wed.	10	19 21 51.99	10.893	22 7 8.8	+21.26	7 15.42	1.036	19 14 36.57
Thur.	11	19 26 13.15	10.870	21 58 25.5	22.34	7 40.02	1.014	19 18 33.13
Frid.	12	19 30 33.77	10.847	21 49 16.4	23.41	8 4.08	0.990	19 22 29.68
Sat.	13	19 34 53.81	10.822	21 39 41.8	+24.47	8 27.57	0.966	19 26 26.24
Sun.	14	19 39 13.25	10.797	21 29 42.0	25.51	8 50.45	0.940	19 30 22.80
Mon.	15	19 43 32.07	10.770	21 19 17.2	26.55	9 12.71	0.914	19 34 19.36
Tues.	16	19 47 50.25	10.743	21 8 27.7	+27.57	9 34.33	0.887	19 38 15.92
Wed.	17	19 52 7.76	10.715	20 57 13.9	28.58	9 55.28	0.859	19 42 12.48
Thur.	18	19 56 24.58	10.686	20 45 36.1	29.57	10 15.54	0.830	19 46 9.03
Frid.	19	20 0 40.70	10.656	20 33 34.6	+30.55	10 35.10	0.800	19 50 5.59
Sat.	20	20 4 56.09	10.626	20 21 9.7	31.52	10 53.94	0.769	19 54 2.15
Sun.	21	20 9 10.73	10.595	20 8 21.8	32.47	11 12.03	0.738	19 57 58.71
Mon.	22	20 13 24.62	10.563	19 55 11.2	+33.40	11 29.36	0.706	20 1 55.27
Tues.	23	20 17 37.74	10.530	19 41 38.4	34.32	11 45.91	0.673	20 5 51.82
Wed.	24	20 21 50.06	10.497	19 27 43.7	35.23	12 1.68	0.640	20 9 48.38
Thur.	25	20 26 1.58	10.463	19 13 27.4	+36.12	12 16.65	0.607	20 13 44.94
Frid.	26	20 30 12.29	10.429	18 58 50.0	37.00	12 30.80	0.573	20 17 41.50
Sat.	27	20 34 22.18	10.395	18 43 51.9	37.85	12 44.13	0.538	20 21 38.05
Sun.	28	20 38 31.25	10.360	18 28 33.4	+38.69	12 56.64	0.504	20 25 34.61
Mon.	29	20 42 39.48	10.326	18 12 55.0	39.51	13 8.31	0.469	20 29 31.17
Tues.	30	20 46 46.87	10.291	17 56 57.0	40.31	13 19.15	0.434	20 33 27.72
Wed.	31	20 50 53.44	10.256	17 40 39.9	41.10	13 29.16	0.400	20 37 24.28
Thur.	32	20 54 59.17	10.221	S. 17 24 4.0	+41.88	13 38.33	0.365	20 41 20.84

Note. — The semidiameter for mean noon may be assumed the same as that for apparent noon. The sign + prefixed to the hourly change of declination indicates that south declinations are decreasing.

Diff. for 1 Hour, $+9^s.8565$. (Table III.)

MEAN PLACES OF STARS, 1912.

WASHINGTON, JANUARY 1^d.006.

Name of star.	Magnitude.	Right Ascension.	Annual Variation.	Declination	Annual Variation.
		h m s	s	° ′ ″	″
33 Piscium	4.7	0 0 49.898	+ 3.0715	− 6 11 59.45	+20.136
α Andromedæ (*Alpheratz*)	2.2	0 3 50.162	3.0955	+28 36 16.59	19.880
β Cassiopeiæ	2.4	0 4 28.504	3.1834	+58 39 51.97	19.862
ε Phœnicis	3.9	0 4 56.832	3.0520	−46 13 58.96	19.848
22 Andromedæ	5.1	0 5 44.571	3.1088	+45 34 57.27	20.035
γ Pegasi	2.9	0 8 42.161	+ 3.0861	+14 41 39.75	+20.021
σ Andromedæ	4.5	0 13 43.613	3.1269	+36 17 50.52	19.963
ι Ceti	3.8	0 14 56.674	3.0570	− 9 18 42.04	19.974
ζ Tucanæ	4.3	0 15 29.743	3.1497	−65 23 29.80	21.172
44 Piscium	6.0	0 20 53.464	3.0742	+ 1 27 8.50	19.939
β Hydri	2.9	0 21 8.620	+ 3.2043	−77 44 59.48	+20.279
α Phœnicis	2.4	0 21 56.254	2.9731	−42 47 1.95	19.551
12 Ceti	6.0	0 25 32.885	3.0621	− 4 26 36.25	19.921
13 Ceti †	5.2	0 30 43.080	3.0871	− 4 4 37.57	19.849
ζ Cassiopeiæ	3.7	0 32 3.748	3.3272	+53 24 45.87	19.843
π Andromedæ	4.4	0 32 10.631	+ 3.1970	+33 14 6.22	+19.849
ε Andromedæ	4.5	0 33 54.131	3.1638	+28 50 2.68	19.573
δ Andromedæ	3.5	0 34 37.138	3.2014	+30 22 45.98	19.721
α Cassiop. (*Schedir*) †	*var.*	0 35 30.339	3.3852	+56 3 17.55	19.774
μ Phœnicis	4.6	0 37 10.086	2.8400	−46 34 5.92	19.751
β Ceti	2.2	0 39 10.381	+ 3.0126	−18 28 9.82	+19.795
ο Cassiopeiæ	4.7	0 39 48.961	3.3302	+47 48 10.61	19.738
21 Cassiopeiæ	5.6	0 39 48.984	3.9010	+74 30 26.07	19.718
ζ Andromedæ	4.3	0 42 40.273	3.1742	+23 47 19.04	19.622
η Cassiopeiæ †	3.6	0 43 46.125	3.6116	+57 20 59.54	19.205
δ Piscium	4.6	0 44 6.921	+ 3.1099	+ 7 6 22.84	+19.631
λ Hydri	5.0	0 45 32.752	2.1021	−75 24 7.82	19.650
20 Ceti	4.9	0 48 30.554	3.0641	− 1 37 18.43	19.595
γ Cassiopeiæ	2.2	0 51 23.243	3.5958	+60 14 25.56	19.539
μ Andromedæ	3.9	0 51 51.855	3.3202	+38 1 19.99	19.565
α Sculptoris	4.4	0 54 21.918	+ 2.8908	−29 49 59.04	+19.472
43 H. Cephei	4.5	0 56 31.244	7.5725	+85 47 8.15	19.435
ε Piscium	4.4	0 58 22.474	3.1110	+ 7 24 59.64	19.425
β Phœnicis †	3.4	1 2 9.407	2.6803	−47 11 24.48	19.288
μ Cassiopeiæ	5.3	1 2 24.358	3.9678	+54 29 21.00	17.752
η Ceti	3.6	1 4 9.775	+ 3.0175	−10 38 54.34	+19.140
β Andromedæ	2.4	1 4 48.009	3.3499	+35 9 15.21	19.133
τ Piscium	4.7	1 6 48.605	3.2965	+29 37 21.71	19.172
ζ Piscium †	5.6	1 9 7.938	3.1316	+ 7 6 36.94	19.089
κ Tucanæ †	5.0	1 12 47.111	2.0401	−69 20 36.92	19.132
f Piscium	5.3	1 13 15.525	+ 3.0924	+ 3 9 4.65	+19.005
υ Piscium	4.7	1 14 37.566	3.2901	+26 48 6.47	18.985
θ Ceti	3.8	1 19 37.455	2.9977	− 8 38 13.83	18.633
δ Cassiopeiæ	2.8	1 20 2.942	3.8983	+59 46 42.30	18.799
γ Phœnicis	3.4	1 24 32.676	2.6081	−43 46 8.58	18.472
38 Cassiopeiæ	6.0	1 24 39.740	+ 4.4111	+69 48 43.86	+18.622
η Piscium	3.7	1 26 46.307	3.2054	+14 53 32.97	18.623
α Ursæ Min. (*Polaris*) †	2.1	1 27 51.07*	+27.8225	+88 50 10.77	+18.594

13 Ceti, dup. 5^m.5, 6^m.2, 0″.3.
α Cassiop., var. irreg. 2^m.2, 2^m.8
η Cassiop., comp. 7^m.6, 5″ s. pr.
β Phœnicis, dup. 4^m.1, 4^m.1, 1″.
ζ Piscium, star 6^m.5, 24″ n.f.
κ Tucanæ, comp. 7^m, 6″ n.
α Ursæ Min., star 9^m, 18″ s. pr.

APPARENT PLACES OF STARS, 1912.

FOR THE UPPER TRANSIT AT WASHINGTON.

Mean solar date.	33 Piscium. Mag. 4.7				α Andromedæ Mag. 2.2				β Cassiopeiæ Mag. 2.4				ε Phœnicis Mag. 3.9			
	Right Ascension		Declination S.		Right Ascension		Declination N.		Right Ascension		Declination N.		Right Ascension		Declination S	
	h m		° ′		h m		° ′		h m		° ′		h m		° ′	
	0 0		− 6 11		0 3		+28 36		0 4		+58 39		0 4		−46 13	
	s		″		s		″		s		″		s		″	
Jan. 0.2	49.31	10	65.8	6	49.23	13	22.3	8	26.88	31	65.8	7	56.67	19	77.3	4
10.2	49.21	9	66.4	4	49.10	13	21.5	11	26.57	29	65.1	12	56.48	17	76.9	9
20.2	49.12	8	66.8	3	48.97	11	20.4	14	26.28	27	63.9	16	56.31	15	76.0	13
30.1	49.04	6	67.1	1	48.86	10	19.0	15	26.01	24	62.3	20	56.16	12	74.7	17
Feb. 9.1	48.98	4	67.2	1	48.76	7	17.5	15	25.77	19	60.3	24	56.04	9	73.0	21
19.1	48.94	2	67.1	2	48.69	4	16.0	16	25.58	13	57.9	25	55.95	5	70.9	23
29.1	48.92	1	66.9	5	48.65	0	14.4	15	25.45	6	55.4	27	55.90	1	68.6	27
Mar. 10.0	48.93	5	66.4	7	48.65	4	12.9	14	25.39	2	52.7	26	55.89	3	65.9	29
20.0	48.98	8	65.7	9	48.69	8	11.5	11	25.41	9	50.1	25	55.92	9	63.0	30
30.0	49.06	12	64.8	12	48.77	13	10.4	9	25.50	18	47.6	23	56.01	13	60.0	3
Apr. 9.0	49.18	16	63.6	14	48.90	18	9.5	5	25.68	25	45.3	20	56.14	19	56.8	32
18.9	49.34	20	62.2	16	49.08	22	9.0	2	25.93	32	43.3	15	56.33	24	53.6	32
28.9	49.54	24	60.6	18	49.30	26	8.8	2	26.25	38	41.8	11	56.57	28	50.4	30
May 8.9	49.78	27	58.8	20	49.56	29	9.0	6	26.63	44	40.7	6	56.85	33	47.4	29
18.8	50.05	29	56.8	21	49.85	32	9.6	10	27.07	48	40.1	0	57.18	36	44.5	27
28.8	50.34	31	54.7	21	50.17	34	10.6	13	27.55	50	40.1	4	57.54	39	41.8	24
June 7.8	50.65	32	52.6	21	50.51	35	11.9	17	28.05	52	40.5	10	57.93	41	39.4	21
17.8	50.97	33	50.5	21	50.86	35	13.6	20	28.57	51	41.5	15	58.34	42	37.3	17
27.7	51.30	32	48.4	20	51.21	35	15.6	22	29.08	50	43.0	20	58.76	42	35.6	12
July 7.7	51.62	31	46.4	19	51.56	33	17.8	24	29.58	48	45.0	23	59.18	40	34.4	8
17.7	51.93	28	44.5	16	51.89	30	20.2	25	30.06	44	47.3	27	59.58	38	33.6	3
27.7	52.21	26	42.9	15	52.19	28	22.7	25	30.50	39	50.0	30	59.96	35	33.3	2
Aug. 6.6	52.47	23	41.4	12	52.47	24	25.2	26	30.89	34	53.0	32	60.31	30	33.5	6
16.6	52.70	19	40.2	9	52.71	20	27.8	25	31.23	27	56.2	34	60.61	26	34.1	11
26.6	52.89	16	39.3	6	52.91	16	30.3	25	31.55	22	59.6	34	60.87	20	35.2	15
Sept. 5.5	53.05	11	38.7	4	53.07	12	32.8	23	31.72	15	63.0	35	61.07	14	36.7	18
15.5	53.16	8	38.3	1	53.19	8	35.1	21	31.87	9	66.5	34	61.21	9	38.5	20
25.5	53.24	4	38.2	1	53.27	4	37.2	20	31.96	3	69.9	33	61.30	3	40.5	22
Oct. 5.5	53.28	0	38.3	3	53.31	1	39.2	17	31.99	3	73.2	31	61.33	2	42.7	23
15.4	53.28	2	38.6	5	53.32	3	40.9	14	31.96	8	76.3	29	61.31	7	45.0	22
25.4	53.26	5	39.1	6	53.29	5	42.3	12	31.88	14	79.2	26	61.24	11	47.2	21
Nov. 4.4	53.21	6	39.7	7	53.24	7	43.5	9	31.74	19	81.8	22	61.13	14	49.3	19
14.4	53.15	8	40.4	7	53.17	10	44.4	5	31.55	22	84.0	17	60.9[illegible]	7	51.2	16
24.3	53.07	9	41.1	7	53.07	11	44.9	3	31.33	26	85.7	13	60.[illegible]	19	52.8	13
Dec. 4.3	52.98	10	41.8	7	52.96	12	45.2	1	31.07	28	87.0	8	60.[illegible]	20	54.1	8
14.3	52.88	10	42.5	6	52.84	13	45.1	3	30.79	30	87.8	2	60.43	20	54.9	3
24.2	52.78	10	43.1	6	52.71	13	44.8	7	30.49	30	88.0	4	60.23	19	55.2	1
34.2	52.68		43.7		52.58		44.1		30.19		87.6		60.04		55.1	
Sec δ, Tan δ	1.006		−0.109		1.139		+0.545		1.923		+1.643		1.446		−1.044	
Mean Place	$49^s.898$		59″.45		$50^s.162$		16″.59		$28^s.504$		51″.97		$56^s.832$		58″.96	
$D'\psi\alpha$, $D\omega\alpha$	0.00		+0.01		0.00		−0.04		0.00		−0.11		0.00		+0.07	
$D\psi\delta$, $D\omega\delta$	+0.4		0.0		+0.4		0.0		+0.4		0.0		+0.4		0.0	

moon's right ascension and declination for every hour of Greenwich Mean Time, together with the changes per minute of time. Following the ephemeris of the sun and moon for the twelve months are the ephemerides of the planets. The values given for Greenwich Mean Noon are for 0^h of the astronomical date or 12 M of the civil date. The word "apparent" used in these tables indicates that the correction for aberration has been applied to the coördinates, giving the position of the object as actually seen by the observer except for the effect of parallax and refraction. The "differences for 1 hour" are in all cases the rates of change at the instant, that is, the differential coefficients, not the actual differences between the consecutive tabular values.

Part II contains the following three lists of stars, the first headed "Mean places of Stars, 19 —"; the second, "Apparent Places of Circumpolar Stars, 19 —"; and the third, "Apparent Places of Stars, 19 —"; all of these are computed for the instant of transit over the meridian of Washington ($5^h\ 08^m\ 15^s.78$ West of Greenwich). A list of south circumpolar stars is also given. The tables given on pages 66 and 67 of this volume are extracts from the first and third of these tables in the Almanac. The first table contains the coördinates of about 800 stars referred to the "mean equinox" at the beginning of the year, that is, to the position that the equinox would occupy at the beginning of the solar * year if it were not affected by small periodic terms of the precession. The second table gives the coördinates of about 15 north circumpolar stars. Precession causes the coördinates of circumpolar stars to vary more rapidly than those of equatorial stars; the coördinates are therefore given for every day in the year. The hours and minutes of right ascension and the degrees and minutes of declination are at the head of the column; the column contains only the seconds. In the third table

* The year here referred to, called also the **Besselian fictitious year**, is one used in computing star places; it begins when the sun's mean longitude is 280°, that is, when the R. A. of mean sun is $18^h\ 40^m$, which occurs about Jan. 1.

are about 800 stars, the coördinates for which are given for every ten days. The only other table in Part II of particular interest to the surveyor is that headed "Moon Culminations." This table contains the data needed when determining longitude by observing transits of the moon. (See Art. 88, p. 141.)

In the latter part of the Almanac will be found the following useful tables: I, Times of Culmination and Elongation of Polaris; II, Conversion of Sidereal Time into Mean Time; III, Conversion of Mean Time into Sidereal Time; IV, Latitude by an Altitude of Polaris; V and VI, Azimuths of Polaris; VII, Intervals for δ *Cassiopeiæ* and ζ *Ursæ Majoris*. (See Art. 99, p. 161.)

39. Star Catalogues.

When it is necessary to make observations on stars not given in the list in the Ephemeris, their positions must be taken from one or more of the star catalogues. These give the **mean place** of the star at some definite epoch, such as the beginning of the year 1890, or 1900, together with the necessary data for reducing to the mean place of any other year. This data is usually obtained by combining the observations made at different observatories and at different times, so that changes in the star's coördinates are accurately determined. After the position in the catalogue has been brought up to the mean place for the desired year, the apparent place of the star for the exact date of the observation is computed by means of the formulæ and tables given in Part II of the Ephemeris. For ordinary observations made by the surveyor the list of stars given in the Ephemeris is always sufficient, but in special kinds of work, such as finding latitude by Talcott's Method, many other stars must be used.

40. Interpolation.

When taking data from the Ephemeris it is general necessary, in order to obtain the value for a particular instant, to interpolate between values of the function for stated times. In some cases this may be simple interpolation, in which the function is assumed to vary uniformly between the two values given and the desired value found by direct proportion. When the difference for one hour is given, this rate of change at the given instant may be assumed to hold good between the given value and the following one. Since this is not usually quite true, it will be more accurate to interpolate from the *nearest* given value in the table. The change for one hour is to be multiplied by the number of hours between the given time and the tabular time. This correction is either added or subtracted, according to whether the function is increasing or decreasing and whether the preceding or following tabular value is used.

Example.

At Greenwich Mean Noon.

Feb.	Sun's declination	Diff. 1^h
1	S $17° 24' 04''.0$	$+ 41''.88$
2	$17\ 07\ 09\ .8$	$42\ .64$

It is desired to find the declination at the instant 22^h G. M. T. Feb. 1. Since this is much nearer to the moon of Feb. 2 than of Feb. 1, it will be more accurate to multiply $42''.64$ by 2^h and add this to $17° 07' 09''.8$ (since the declination is decreasing). The result is S $17° 08' 35''.1$. By working forward from the value on Feb. 1 the result is S $17° 08' 32''.6$. By using a more exact formula the result is found to be S $17° 08' 35''.0$.

If the successive values of the "diff. for 1^h" or "diff. for 1^m" have large differences, and if a precise value of the function is desired, it will be necessary to interpolate between the given values of the differential coefficients to obtain the rate of change at the middle of the interval over which we are interpolating, and to use this interpolated rate of change in computing the correction.

Example.

Time	R. A. of the Moon	Diff. for 1^m
0^h	$4^h\ 46^m\ 11^s.49$	$2^s.5421$
1	$4\ 48\ 44\ .06$	$2\ .5436$

If it is desired to find the right ascension for $0^h\ 40^m$, the "diff. for 1^m" to be used is that for the instant $0^h\ 20^m$, the middle of the interval from 0^h to $0^h\ 40^m$. This value lies one third of the way from $2^s.5421$ to $2^s.5436$, or $2^s.5426$. The correction to the R. A. at 0^h is $2^s.5426 \times 40^m = 101.70^s = 1^m\ 41^s.70$, the required R. A. being $4^h\ 47^m\ 53^s.19$.

For general interpolation formulæ the student is referred to Chauvenet's *Spherical and Practical Astronomy*, Vol. I, to Doolittle's *Practical Astronomy* or to Hayford's *Geodetic Astronomy*.

Questions and Problems

1. Compute the sun's apparent declination when the M. L. T. is $8^h\ 30^m$ A.M., Jan. 16, 1912, at a place 85° west of Greenwich (see p. 65).

2. Compute the right ascension of the mean sun at local mean noon Jan. 10, 1912, at a place 96° 10′ west of Greenwich.

3. Compute the equation of time for local apparent noon Jan. 30, 1912, at a place 20° east of Greenwich.

4. Explain the relation between the sun's angular semidiameter and the time of the semidiameter passing the meridian.

5. What is the relation between the "right ascension of the mean sun" and the "apparent right ascension of the sun" on Jan. 1, 1912?

CHAPTER VII

THE EARTH'S FIGURE — CORRECTIONS TO OBSERVED ALTITUDES

41. The Earth's Figure.

The earth's form is approximately that of an oblate spheroid whose shortest axis is the axis of rotation. The actual figure deviates slightly from that of a perfect spheroid, but for most astronomical purposes these deviations may be disregarded. Each meridian may therefore be considered as an ellipse, and the equator and all parallels of latitude as circles. The semi-major axis of the meridian ellipse is about 3962.80 miles, and the semi-minor axis is 3949.56 miles in length. The length of 1° of latitude at the equator is 68.704 miles; at the pole it is 69.407 miles.

In locating points on the earth's surface by means of coördinates there are three kinds of latitude to be considered. The latitude as found by astronomical observation is dependent upon the direction of gravity as indicated by the spirit levels of the instrument, and is affected by any abnormal deviations of the plumb line* at this point; the latitude as found directly by observations is called the **astronomical latitude.** The **geodetic latitude** is the latitude that would be found by observation if the plumb line were normal to the surface of the spheroid taken to represent the earth's figure, that is, if all of the irregularities of the surface were smoothed out. Evidently the geodetic latitude cannot be directly observed but must be found by computation. The **geocentric latitude** is the angle between the plane of the equator and a line drawn from the centre of the earth to the point on the surface. In Fig. 39 the line AD is normal to

* These deviations are small, averaging about $3''$ or $4''$, but in some cases deviations of nearly $30''$ are found.

the earth's surface at A, and the angle ABE is the geodetic latitude of A. If the plumb line coincides with AD, this is also the astronomical latitude. The angle ACE is the geocentric latitude. The difference between the two, or angle BAC,

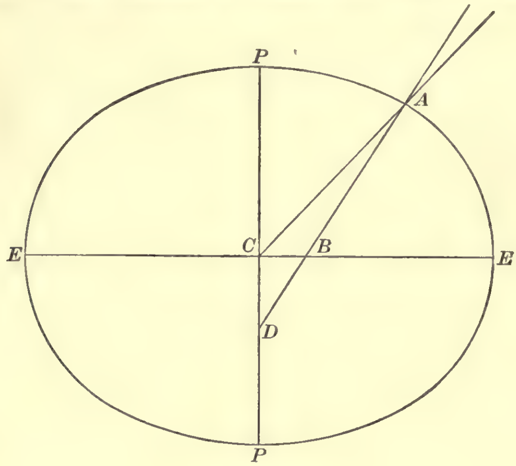

FIG. 39

is called the **angle of the vertical,** or the **reduction of latitude.** The geocentric latitude is always less than the observed latitude by an angle which varies from about $0° 11' 30''$ in latitude $45°$ to zero at the equator and the poles. Whenever observations are made at any point on the earth's surface it is necessary to reduce the observed values to their values at the earth's centre before they can be combined with other data referred to the centre. In making this reduction the geocentric latitude must be used if the exact position of the observer with reference to the centre is to be computed. For most of the observations treated in the following chapters it will not be necessary to consider the spheroidal shape of the earth; it will be sufficiently exact to regard it as a sphere.

42. Parallax.

The coördinates of a celestial object as given in the Ephemeris are referred to the centre of the earth, while the coördinates obtained by observation are necessarily measured from a point on the surface, and must be reduced to the centre. The case most frequently occurring in practice is that in which the altitude of an object is observed and the geocentric altitude is desired. For all objects except the moon the distance of the body is so great that it is sufficiently accurate to regard the earth as a

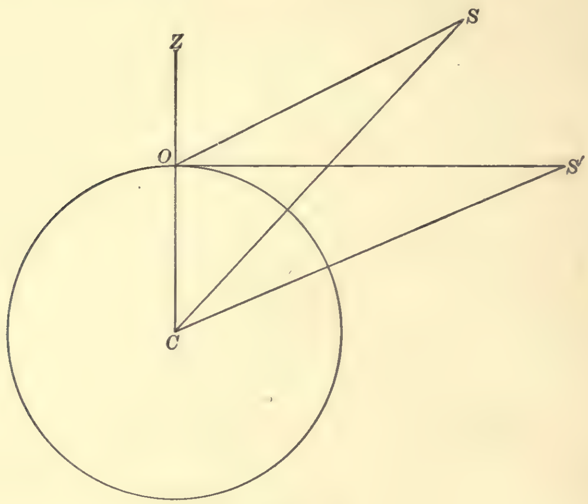

FIG. 40

sphere. In Fig. 40, the angle *ZOS* is the observed zenith distance, or the complement of the observed altitude, and *ZCS* is the true zenith distance. This apparent displacement of the object on the celestial sphere is called **parallax.** The effect of parallax is simply to decrease the altitude without altering the azimuth of the body, provided the spheroidal form of the earth be disregarded. The difference in direction between the lines *OS* and *CS*, or the angle *OSC*, is the *parallax correction.* In the triangle *OSC*, angle *COS* may be considered as known, since the altitude or complement of *ZOS* is observed. The distance *OC* is the semidiameter of the earth (3956.1 miles), and *CS* is the distance from the earth's centre to the centre of the body observed. Solving this triangle,

$$\sin S = \sin ZOS \times \frac{OC}{CS}. \qquad [50]$$

It is evident that the parallax correction will be zero at the zenith and a maximum at the horizon. For the maximum, when $ZOS = 90°$,

$$\sin S = \frac{OC}{CS}, \qquad [51]$$

which is the same for all places on the earth's surface if the earth is regarded as a sphere. If P_h represents the maximum or **horizontal parallax,** then equation [50] may be written

$$\begin{aligned}\sin S &= \sin P_h \sin z \\ &= \sin P_h \cos h, \qquad [52]\end{aligned}$$

where h is the apparent altitude of the object. But S and P_h are usually very small angles, and the error is negligible if the sines are replaced by their arcs.* Equation [52] then becomes

$$S'' = P_h'' \cos h, \qquad [53]$$

where S'' and P_h'' are both in seconds of arc.

For the moon the mean value of the horizontal parallax is about $0° 57' 02''$ †; for the sun it is $8''.8$; for the fixed stars it is

* The sine may be expressed as a series as follows:

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots \qquad [54]$$

Replacing $\sin x$ by x amounts to neglecting all terms after the first. Whether the error will be appreciable in any given case may be determined by computing the value of the first of the neglected terms. If $x = 1°$ the neglected terms are less than .005 of 1% of x. The error in an angle of $1°$ would be less than $0''.2$. The moon is the only object whose parallax is nearly as large as $1°$, so that for all other objects this approximation is usually allowable. Similarly for $\cos x = 1$, the terms neglected are those of the series

$$\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots \qquad [55]$$

† The moon's mean distance is 238,800 miles; the sun's mean distance is 92,900,000 miles.

too small to be detected. The horizontal parallaxes of objects in the solar system are given in the Nautical Almanac.* For the parallax of the sun for different altitudes see Table IV (A).

43. Refraction.

Refraction is the term applied to the bending of a ray of light by the atmosphere as it passes from a celestial object to the observer's eye. On account of the increasing density of the layers of air the rays of light coming from any object are bent downward into a curve, and consequently when the rays enter the eye they have a greater inclination to the horizon than they did before entering the atmosphere. For this reason all objects appear higher above the horizon than they actually are. In

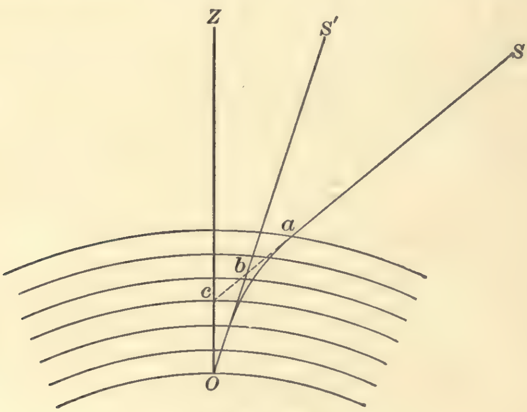

FIG. 41

Fig. 41, S is the true position of a star and S' its apparent position. The light from S is bent into a curve aO, and the star is seen in the direction of the tangent ObS'. The angle which must be subtracted from the altitude of S' to obtain the altitude of S is called the **refraction correction.** This angle is really the angle SOS', but on account of the great distance of celestial objects

* On account of the spheroidal form of the earth the equatorial diameter is the greatest and the parallax at the equator is a maximum; the parallaxes are therefore given in the Ephemeris under the heading " Equatorial Horizontal Parallax."

and the small angle of refraction the correction may be considered as the angle SbS'. From the figure it is evident that

$$ZcS = ZOS' + S'bS,$$

or

$$z' = z + r, \qquad [56]$$

where z' = the true and z = the apparent zenith distance and r = the refraction correction. The approximate law of astronomical refraction may be deduced by assuming that the bending all occurs at point b. The general law of refraction, when a ray enters a refracting medium, is expressed by the equation

$$\sin z' = n \sin z, \qquad [57]$$

where n is the index of refraction of the given medium; for air its value is roughly about 1.0003.

Substituting from Equa. [56],

$$\sin (z + r) = n \sin z, \qquad [58]$$

Expanding, $\quad \sin z \cos r + \cos z \sin r = n \sin z. \qquad [59]$

Since r is a small angle (never greater than 40′) it is allowable to put $\cos r = 1$ and $\sin r = r$; then

$$\sin z + r \cos z = n \sin z,$$

and

$$r \cos z = (n - 1) \sin z,$$

or

$$r = (n - 1) \tan z. \qquad [60]$$

Replacing n by 1.0003 and dividing by $\sin 1''$ to reduce r from circular measure to seconds of arc,

$$\begin{aligned} r'' &= \frac{(.0003)}{(.000,005)} \tan z \\ &= 60'' \tan z \\ &= 60'' \cot h, \qquad [61] \end{aligned}$$

where h is the apparent altitude.*

The value of n varies considerably with the temperature and the pressure of the air, so that equation [61] must be considered as giving only a rough approximation to the true refraction.

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* "Apparent" is used here simply to distinguish between the direction of the star as actually seen and the direction unaffected by refraction. In speaking of parallax, the word "apparent" has a different meaning, and in case of aberration, still another meaning.

For high altitudes this formula is nearly correct, but for altitudes under 10° it is not sufficiently exact. If both sides of the equation are divided by 60 so that r is reduced to minutes, we have the extremely simple relation that the refraction in minutes equals the natural cotangent of the altitude. For altitudes measured with an engineer's transit this formula is close enough for altitudes greater than about 10°. For more accurate values of the refraction Table I may be used. From the table it will be seen that the refraction correction is zero at the zenith, about 1′ at an altitude of 45°, and about 0° 34′ at the horizon.*

The following formula, due to Professor George C. Comstock, gives very accurate values of the refraction for altitudes greater than 20°, and is sufficiently accurate for all field observations made with surveyors' instruments.

$$r = \frac{983\, b}{460 + t} \cot h, \qquad [62]$$

in which b is the barometer reading in inches, and t is the temperature in Fahrenheit degrees.

Example.

Altitude 30°, barometer 29.1 in., thermometer 81° F.

	log. 983 = 2.9926
	log. 29.1 = 1.4639
460°	colog. 541 = 7.2668
81	cot h = 0.2386
541°	1.9619
	r = 91″.6
	= 1′ 31″.6

44. Semidiameters.

The discs of the sun and moon are circular, and their angular semidiameters are given for each day in the Ephemeris. Since measurements can only be taken to the edge, or limb, the altitude of the centre of the object is obtained by making a correction

* The sun's diameter is about 32′, slightly less than the refraction on the horizon; when the sun has actually gone below the horizon at sunset the entire disc is still visible on account of the 34′ increase in its apparent altitude due to atmospheric refraction.

equal to the semidiameter. The apparent angular semidiameters given in the Ephemeris may be affected in two ways, one by the change in the observer's distance because he is on the earth's surface, the other by the difference in the amount of refraction correction on the upper and lower edges of the disc.

The semidiameter given in the Ephemeris is that as seen from the centre of the earth. When the object is in the zenith the observer is nearly 4000 miles nearer than when it is in the horizon. The moon is about 240,000 miles distant from the earth, so that the semidiameter is increased by about $\frac{1}{60}$ part, or about 16″.

The vertical diameter of an object appears to be less than its horizontal diameter because the refraction lifts the lower edge more than it does the upper edge. The disc then presents the appearance of an ellipse. When the sun is rising or setting, the contraction is most noticeable. This contraction of the semidiameter does not affect the correction to an observed altitude, but must be taken into account when the distance is measured between the moon's limb and a star or a planet. (See Art. 108.)

For the angular semidiameter of the sun on the first day of each month see Table IV (B).

45. Dip.

If altitudes are taken from the sea horizon, as when observing on board ship with the sextant, the measured altitude must be diminished by the angular dip of the sea horizon below the true horizon. In Fig. 42 suppose the observer to be at O; the true horizon is OB and the sea horizon OH. Let $OP = h$, the height in feet above the surface; $PC = R$, the radius of the earth; and D, the angle of dip.

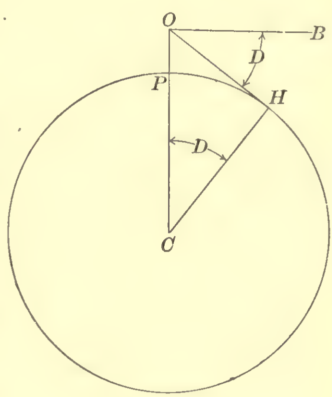


FIG. 42

Then
$$\cos D = \frac{R}{R + h}. \qquad [63]$$

Putting $\cos D = 1 - \frac{D^2}{2}$, neglecting other terms in the series,

$$\frac{D^2}{2} = \frac{h}{R + h} = \frac{h}{R} \text{ (nearly).}$$

$$\therefore D = \sqrt{\frac{2h}{R}}.$$

Replacing R by its value in feet, 20,884,000, and dividing by $\sin 1'$ to reduce D to minutes,

$$D = \frac{1}{\sqrt{\frac{R}{2}} \times \sin 1'} \times \sqrt{h}$$

$$= 1.064 \sqrt{h}. \qquad [64]$$

This shows the amount of dip unaffected by refraction. The effect of refraction is to apparently lift the horizon, and the dip affecting the observed altitude is therefore less than that given by the formula. If the coefficient 1.064 is taken as unity, the formula is nearer the truth and is simpler, although still somewhat too large. Table IV (C), based on a more exact formula, will be seen to give smaller values. For ordinary sextant observations made at sea, where the greatest precision is not required it is sufficient to take the dip in minutes equal to the square root of the height of the eye in feet, that is,

$$D' = \sqrt{h \text{ ft.}} \qquad [65]$$

46. Sequence of Corrections.

Strictly speaking, the corrections to the latitude should be made in the following order:

(1) Instrumental corrections; (2) dip (if at sea); (3) refraction; (4) semidiameter; (5) parallax. In practice, however, it is not always necessary to follow this order exactly. At sea the corrections are often taken together as a single "correction to the altitude." Care should be taken to use the refraction correction

for the limb observed, not for the centre, for if the altitude is small the two will differ appreciably.

Problems

1. Compute the sun's mean horizontal parallax. The sun's mean distance is 92,900,000 miles; for the earth's radius see Art. 41. Compute the sun's parallax at an altitude of 60°.

2. Compute the moon's mean horizontal parallax. The moon's mean distance is 238,800 miles; for the earth's radius see Art. 41. Compute the moon's parallax at an altitude of 45°.

CHAPTER VIII

DESCRIPTION OF INSTRUMENTS

47. The Engineer's Transit.

The engineer's transit is an instrument for measuring horizontal and vertical angles. For the purpose of discussing the theory of the instrument it may be regarded as a telescopic line of sight having motion about two axes at right angles to each other, one vertical, the other horizontal. The line of sight is determined by the optical centre of the object glass and the intersection of two cross hairs * placed in its principal focus. The vertical axis of the instrument coincides with the axes of two spindles, one inside the other, each of which is attached to a horizontal circular plate. The lower plate carries a graduated circle for measuring horizontal angles; the upper plate has two verniers, on opposite sides, for reading angles on the circle. On the top of the upper plate are two uprights, or standards, supporting the horizontal axis to which the telescope is attached and about which it rotates. At one end of the horizontal axis is a vertical arc, or a circle, and on the standard is a vernier, in contact with the circle, for reading the angles. The plates and the horizontal axis are provided with clamps and slow-motion screws to control the motion. On the upper plate are two spirit levels for levelling the instrument, or, in other words, for making the vertical axis coincide with the direction of gravity.

The whole instrument may be made to turn in a horizontal plane by a motion about the vertical axis, and the telescope may be made to move in a vertical plane by a motion about the horizontal axis. By means of a combination of these two

* Also called wires or threads; they are either made of spider threads or are lines ruled upon glass.

motions, vertical and horizontal, the line of sight may be made to point in any desired direction. The motion of the line of sight in a horizontal plane is measured by the angle passed over by the index of the vernier along the graduated horizontal circle. The angular motion in a vertical plane is measured by the angle on the vertical arc indicated by the vernier attached to the standard. The direction of the horizon is defined by means of a long spirit level attached to the telescope. When the bubble is central the line of sight should lie in the plane of the horizon. To be in perfect adjustment, (1) the axis of each spirit level* should be in a plane at right angles to the vertical axis; (2) the horizontal axis should be at right angles to the vertical axis; (3) the line of sight should be at right angles to the horizontal axis; (4) the axis of the telescope level should be parallel to the line of sight, and (5) the vernier of the vertical arc should read zero when the bubble is in the centre of the level tube attached to the telescope. When the plate levels are brought to the centres of their tubes, and the lower plate is so turned that the vernier reads $0°$ when the telescope points south, then the vernier readings of the horizontal plate and the vertical arc for any position of the telescope are coördinates of the horizon system (Art. 12). If the horizontal circles are clamped in any position and the telescope is moved through a complete revolution, the line of sight describes a vertical circle on the celestial sphere. If the telescope is clamped at any altitude and the instrument turned about the vertical axis, the line of sight describes a cone and traces out on the sphere a circle of equal altitudes, or an almucantar.

48. Elimination of Errors.

It is usually more difficult to measure an altitude accurately with the transit than to measure a horizontal angle. While the precision of horizontal angles may be increased by means of repetitions, in measuring altitudes the precision cannot be

* The axis of a level may be defined as a line tangent to the curve of the glass tube at the point on the scale taken as the zero point, or at the centre of the tube.

increased by repeating the angles, owing to the construction of the instrument. The vertical arc usually has but one vernier, so that the eccentricity cannot be eliminated, and this vernier often does not read as closely as the horizontal vernier. One of the errors, which is likely to be large, but which may be eliminated readily, is that known as the **index error.** The measured altitude of an object may differ from the true reading for two reasons: first, the zero of the vernier may not coincide with the zero of the circle when the telescope bubble is in the centre of its tube; second, the line of sight may not be horizontal when the bubble is in the centre of the tube. The first part of this error can be corrected by simply noting the vernier reading when the bubble is central, and applying this as a correction to the measured altitude. To eliminate the second part of the error the altitude may be measured twice, once from the point on the horizon directly beneath the object observed, and again from the opposite point of the horizon. In other words, the instrument may be reversed (180°) about its vertical axis and the vertical circle read in each position while the horizontal cross hair of the telescope is sighting the object. The mean of the two readings is free from the error in the sight line. Evidently this method is practicable only with an instrument having a complete vertical circle. If the reversal is made in this manner the error due to non-adjustment of the vernier is eliminated at the same time, so that it is unnecessary to make a special determination of it as described above. If the circle is graduated in one direction, it will be necessary to subtract the second reading from 180° and then take the mean between this result and the first altitude. In the preceding description it is assumed that the plate levels remain central during the reversal of the instrument, indicating that the vertical axis is truly vertical. If this is not the case, the instrument should be relevelled before the second altitude is measured, the difference in the two altitude readings in this case including all three errors. If it is not desirable to relevel, the error of inclination of the vertical axis may still be eliminated by reading the vernier of the vertical circle in each of the two positions when the telescope bubble is central, and applying these corrections separately. With an instrument provided with a vertical arc only it is essential that the axis of the telescope bubble be made parallel to the line of sight, and that the vertical axis be made truly vertical. To make the axis vertical without adjusting the levels themselves, bring both bubbles to the centres of their tubes, turn the instrument 180° in azimuth, and then bring each bubble *half way* back to the centre by means of the levelling screws. When the axis is truly vertical, each bubble should remain in the same part of its tube in all azimuths. The axis may always be made vertical by means of the long bubble on the telescope; this is done by setting it over one pair of levelling screws and centring it by means of the tangent screw on the standard; the telescope is then revolved about the vertical axis, and if the bubble moves from the centre of its tube it is brought half way back by means of the tangent screw, and then centred by means of the levelling screws. This process should be repeated to test the accuracy of the levelling; the telescope is then turned at right angles to the first position and the whole process repeated. This method should always be used when the greatest precision is desired, because the telescope bubble is much more sensitive than the plate bubbles.

If the line of sight is not at right angles to the horizontal axis, or if the horizontal axis is not perpendicular to the vertical axis, the errors due to these two causes may be eliminated by combining two sets of measurements, one in each position of the instrument. If a horizontal angle is measured with the vertical circle on the observer's right, and the same angle again observed with the circle on his left, the mean of these two angles is free from both these errors, because the two positions of the horizontal axis are placed symmetrically about a true horizontal line,* and

* Strictly speaking, they are placed symmetrically about a perpendicular to the vertical axis.

the two directions of the sight line are situated symmetrically about a true perpendicular to the rotation axis of the telescope. If the horizontal axis is not perpendicular to the vertical axis the line of sight describes a plane which is inclined to the true vertical plane. In this case the sight line will not pass through the zenith, and both horizontal and vertical angles will be in error. In instruments intended for precise work a striding level is provided, which may be set on the pivots of the horizontal axis. This enables the observer to level the axis or to measure its inclination without reference to the plate bubbles. The striding level should be used in both the direct and the reversed position and the mean of the two results used in order to eliminate the errors of adjustment of the striding level itself. If the line of sight is not perpendicular to the horizontal axis it will describe a cone whose axis is the horizontal axis of the instrument. The line of sight will in general not pass through the zenith, even though the horizontal axis be in perfect adjustment. The instrument must either be used in two positions, or else the cross hairs must be adjusted. Except in large transits it is not usually practicable to determine the amount of the error and allow for it.

49. Attachments to the Engineer's Transit. — Reflector. When making star observations with the transit it is necessary to make some arrangement for illuminating the field of view. Some transits are provided with a special shade tube into which is fitted a mirror set at an angle of 45° and with the central portion removed. By means of a lantern held at one side of the telescope light is reflected down the tube. The cross hairs appear as dark lines against the bright field. The stars can be seen through the opening in the centre of the mirror. If no special shade tube is provided, it is a simple matter to make a substitute, either from a piece of bright tin or by fastening a piece of tracing cloth or oiled paper over the objective. A hole about $\frac{3}{4}$ inch in diameter should be cut out, so that the light from the star may enter the lens. If cloth or paper is used, the lantern must be held so that the light is diffused in such a way as to render the cross hairs visible. The light should be held so as not to shine into the observer's eyes.

50. Prismatic Eyepiece.

When altitudes greater than about 55° to 60° are to be measured, it is necessary to attach to the eyepiece a totally reflecting prism which reflects the rays at right angles to the sight line. By means of this attachment altitudes as great as 75° can be measured. In making observations on the sun it must be remembered that the prism inverts the image, so that with a transit having an erecting eyepiece with the prism attached the apparent lower limb is the true upper limb; the positions of the right and left limbs are not affected by the prism.

51. Sun Glass.

In making observations on the sun it is necessary to cover the eyepiece with a piece of dark glass to protect the eye from the sunlight while observing. The sun glass should not be placed in front of the objective. If no shade is provided with the instrument, sun observations may be made by holding a piece of paper behind the eyepiece so that the sun's image is thrown upon it. By drawing out the eyepiece tube and varying the distance at which the paper is held, the images of the sun and the cross hairs may be sharply focussed. By means of this device an observation may be quite accurately made after a little practice.

52. The Portable Astronomical Transit.

The astronomical transit differs from the surveyor's transit chiefly in size and in the manner of support. The diameter of the object glass may be anywhere from 2 to 4 inches, and the focal length from 24 to 48 inches. The instrument is set upon a stone or brick pier. The cross hairs usually consist of several vertical hairs (say 11 or more) instead of a single one as in the surveyor's transit. The motion in altitude is controlled by means of a clamp and a tangent screw. The azimuth motion is usually very small, simply enough to allow adjustments to be made, as the transit is not used for measuring horizontal angles. The axis is levelled or its inclination measured by means of a sensitive striding level.

On account of the high precision of the work done with the astronomical transit the various errors have to be determined with great accuracy, and corresponding corrections applied to the observed results. The transit is chiefly used in the plane of the meridian for determining the times of transit of stars. The principal errors determined and allowed for are (1) azimuth, or deviation from the true meridian; (2) inclination of the horizontal axis; (3) collimation, or deviation of the sight line from the true perpendicular to the rotation axis. The corrections to reduce an observed time to the true time of transit across the meridian are given by formulæ [66] to [68]. These corrections would apply equally well to observations with the engineer's transit, and serve to show the relative magnitudes of the errors for different positions of the objects observed.

$$\text{Azimuth correction} = a \cos h \sec D, \qquad [66]$$
$$\text{Level correction} = b \sin h \sec D, \qquad [67]$$
$$\text{Collimation correction} = c \sec D, \qquad [68]$$

where a, b and c are the errors in azimuth, inclination and collimation respectively (expressed in seconds of time), and h is the altitude and D the declination of the star observed. From these formulæ Table B has been computed. It is assumed that the instrument is 1′, or 4^s, out of the meridian ($a = 4^s$); that the axis is inclined 1′, or 4^s, to the horizon ($b = 4^s$); and that the sight line defined by the middle (or the mean) wire is 1′, or 4^s, to the right or left of its true position ($c = 4^s$). The numbers in the table show the effect of these errors at different altitudes and declinations.

TABLE B. ERROR IN OBSERVED TIME OF TRANSIT (IN SECONDS OF TIME) WHERE a, b OR $c = 1'$.

Altitude (for Inclination Error)	Declinations.									Altitude (for Azimuth Error)
h	0°	10°	20°	30°	40°	50°	60°	70°	80°	h
0°	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	$0^s.0$	90°
10	0.7	0.7	0.8	0.8	0.9	1.1	1.4	2.0	4.0	80
20	1.4	1.4	1.4	1.6	1.8	2.1	2.7	4.0	7.9	70
30	2.0	2.0	2.1	2.3	2.6	3.1	4.0	5.8	11.5	60
40	2.6	2.6	2.7	3.0	3.4	4.0	5.2	7.5	14.8	50
50	3.1	3.1	3.3	3.6	4.0	4.8	6.1	9.0	17.6	40
60	3.5	3.5	3.7	4.0	4.5	5.4	6.9	10.1	19.9	30
70	3.8	3.8	4.0	4.4	4.9	5.8	7.5	11.0	21.6	20
80	3.9	4.0	4.2	4.6	5.2	6.1	7.9	11.5	22.7	10
90	4.0	4.1	4.2	4.6	5.2	6.2	8.0	11.7	23.0	0

Note. — Use the bottom line for the collimation error.

53. The Sextant.

The sextant is an instrument for measuring the angular distance between two objects, the angle always lying in the plane through the two objects and the eye of the observer. It is particularly useful at sea because it does not require a steady support like the transit. It consists of a frame carrying a graduated arc, *AB*, Fig. 43, about 60° long, and two mirrors *I* and *H*, the first one movable, the second one fixed. At the center of the arc, *I*, is a pivot on which swings an arm *IV*, 6 to 8 inches long. This arm carries a vernier *V* for reading the

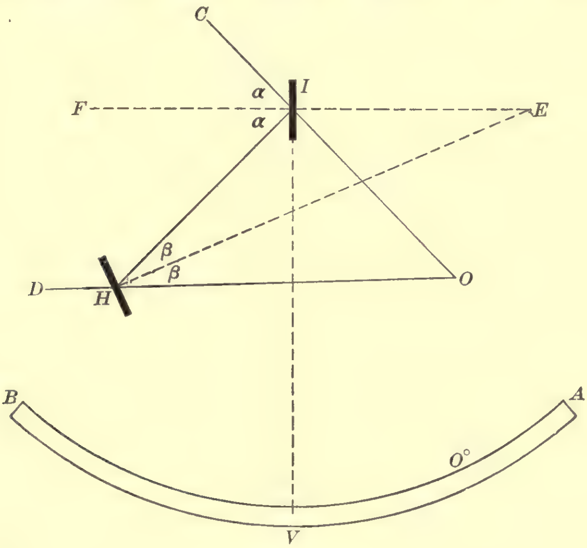


FIG. 43.

angles on the arc *AB*. Upon this arm is placed the index glass *I*. At *H* is the horizon glass. Both of these mirrors are set so that their planes are perpendicular to the plane of the arc *AB*, and so that when the vernier reads 0° the mirrors are parallel. The half of the mirror *H* which is farthest from the frame is unsilvered, so that objects may be viewed directly through the glass. In the silvered portion other objects may be seen by reflection from the mirror *I* to the mirror *H* and thence to point *O*. At a point near *O* (on the line *HO*) is a telescope of low power for viewing the objects. Between the two mirrors and also to the left of H are colored shade glasses to be used when making observations on the sun. The principle of the instrument is as follows: — A ray of light coming from an object at C is reflected by the mirror I to H, where it is again reflected to O. The observer sees the image of C in apparent coincidence with the object at D. The arc is so graduated that the reading of the vernier gives directly the angle between OC and OD. Drawing the perpendiculars FE and HE to the planes of the two mirrors, it is seen that the angle between the mirrors is $\alpha - \beta$. Prolonging CI and DH to meet at O, it is seen that the angle between the two objects is $2\alpha - 2\beta$. The angle between the mirrors is therefore half the angle between the objects that appear to coincide. In order that the true angle may be read directly from the arc each half degree is numbered as though it were a degree. It will be seen that the position of the vertex O is variable, but since all objects observed are at great distances the errors caused by changes in the position of O are always negligible in astronomical observations.

The sextant is in adjustment when, (1) both mirrors are perpendicular to the plane of the arc; (2) the line of sight of the telescope is parallel to the plane of the arc; and (3) the vernier reads $0°$ when the mirrors are parallel to each other. If the vernier does not read $0°$ when the doubly reflected image of a point coincides with the object as seen directly, the index correction may be determined and applied as follows. Set the vernier to read about $30'$ and place the shades in position for sun observations. When the sun is sighted through the telescope two images will be seen with their edges nearly in contact. This contact should be made as nearly perfect as possible and the vernier reading recorded. This should be repeated several times to increase the accuracy. Then set the vernier about $30'$ on the opposite side of the zero point and repeat the whole operation, the reflected image of the sun now being on the opposite side of the direct image. If the shade glasses are of different colors the contacts can be more precisely made. Half the difference of the two (average) readings is the index correction. If the reading *off* the arc was the greater, the correction is to be *added* to all readings of the vernier; if the greater reading was *on* the arc, the correction must be *subtracted*.

In measuring an altitude of the sun above the sea horizon the observer directs the telescope to the point on the horizon vertically under the sun and then moves the index arm until the reflected image of the sun comes into view. The sea horizon can be seen through the plain glass and the sun is seen in the mirror. The sun's lower limb is then set in contact with the horizon line. In order to be certain that the angle is measured to the point vertically beneath the sun, the instrument is tipped slowly right and left, causing the sun's image to describe an arc. This arc should be just tangent to the horizon. If at any point the sun's limb goes below the horizon the altitude measured is too great. The vernier reading corrected for index error and dip is the apparent altitude of the lower limb above the true horizon.

54. Artificial Horizon.

When altitudes are to be measured on land the visible horizon cannot be used, and the **artificial horizon** must be used instead. The surface of any heavy liquid, like mercury, molasses, or heavy oil, may be used for this purpose. When the liquid is placed in a basin and allowed to come to rest, the surface is perfectly level, and in this surface the reflected image of the sun may be seen, the image appearing as far below the horizon as the sun is above it. Another convenient form of horizon consists of a piece of black glass, with plane surfaces, mounted on a frame supported by levelling screws. This horizon is brought into position by placing a spirit level on the glass surface and levelling alternately in two positions at right angles to each other. This form of horizon is not as accurate as the mercury surface but is often more convenient. The principle of the artificial horizon may be seen from Fig. 44. Since the image seen in the horizon is as far below the true horizon as the sun is above it, the angle between the two is $2h$. In measuring this angle the observer points his telescope toward the artificial horizon and then brings the reflected sun down into the field of view by means of the index arm. By placing the apparent lower limb of the reflected sun in contact with the apparent upper limb of the image seen in the mercury surface, the angle measured is twice the altitude of the sun's lower limb. The two points in contact are really images of the same point. If the telescope inverts the image, this statement applies to the upper limb. The index correction must be applied before the angle is

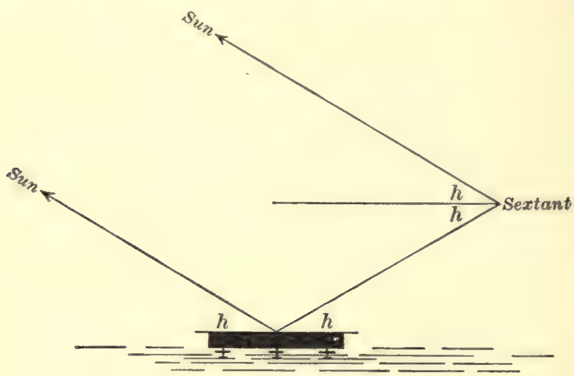


FIG. 44

divided by 2 to obtain the altitude. In using the mercury horizon care must be taken to protect it from the wind, otherwise small waves on the mercury surface will blur and distort the image. The horizon is usually provided with a roof-shaped cover having glass windows, but unless the glass has parallel faces this introduces an error into the result. A good substitute for the glass cover is one made of fine mosquito netting. This will break the force of the wind if it is not blowing hard, and does not introduce errors into the measurement.

55. Chronometer.

The chronometer is simply an accurately constructed watch with a special form of escapement. Chronometers may be regulated for either sidereal or mean time. The beat is usually a half second. Those designed to register the time on chronographs are arranged to break an electric circuit at the end of every second or every two seconds. The 60th second is distinguished either by the omission of the break at the previous second, or by an extra break, according to the construction of the chronometer. Chronometers are usually hung in gimbals to keep them level at all times; this is invariably done when they are taken to sea. It is important that the temperature of the chronometer should be kept as nearly uniform as possible, because fluctuation in temperature is the greatest source of error.

Two chronometers of the same kind cannot be directly compared with great accuracy, $0^s.1$ or $0^s.2$ being about as close as the difference can be estimated. But a sidereal and a solar chronometer can easily be compared within a few hundredths of a second. On account of the gain of the sidereal on the solar chronometer, the beats of the two will coincide once in about every $3^m\ 05^s$. If the two are compared at the instant when the beats are apparently coincident, then it is only necessary to note the seconds and half seconds, as there are no fractions to be estimated. By making several comparisons and reducing them to some common instant of time it is readily seen that the comparison is correct within a few hundredths of a second. The accuracy of the comparison depends upon the fact that the ear can detect a much smaller interval between the two beats than can possibly be estimated when comparing two chronometers whose beats do not coincide.

56. Chronograph.

The chronograph is an instrument for recording the time kept by a chronometer and also any observations the times of which it is desired to determine. A piece of paper is wrapped about a cylinder, which is revolved by a mechanism at a uniform rate. A pen in contact with the paper is held on an arm, connected with the armature of an electro-magnet, in such a way that the pen draws a continuous line which has notches in it corresponding to the breaks in the circuit made by the chronometer. By means of this instrument the time is represented accurately on the sheet as a linear distance. If it is desired to record the instant when any event occurs, such as the passage of a star over a cross hair, the observer presses a telegraph key which breaks the same circuit, and a mark is made on the chronograph sheet. The instant of the observation may be scaled from the record sheet with great precision.

57. The Zenith Telescope.

The zenith telescope is an instrument designed for making observations for latitude by a special method devised by Capt. Andrew Talcott, and which bears his name. The instrument consists of a telescope having a vertical and a horizontal axis like the transit; the telescope is attached to one end of the horizontal axis instead of at the centre. The essential features of the instrument are (1) a micrometer, placed in the focus of the eyepiece, for measuring small differences in zenith distance, and (2) a sensitive spirit level, attached to a small vertical circle on the telescope tube, for measuring small deflections of the vertical axis. The telescope is used in the plane of the meridian. There are two stops whose positions can be regulated so that the telescope may be quickly shifted, by a rotation about the vertical axis, from the north meridian to the south meridian. The observation consists in measuring with the micrometer the difference in zenith distance of two stars, one north of the zenith and one south, which culminate within a few minutes of each other, and in taking readings of the spirit level at the same time the micrometer settings are made. A diagram of the instrument in the two positions is given in Fig. 45. The inclination of the telescope to the vertical is not changed between the two observations, so it is essential that the zenith distances of the two stars should be so nearly equal that both will come within the range of the micrometer screw, usually 30′ or less. The principle involved in this method may be seen from Fig. 46. From the observed zenith distance of the star S_s the latitude is

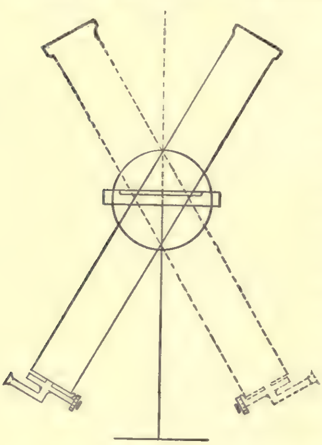


FIG. 45. THE ZENITH TELESCOPE

$$L = D_s + z_s$$

and from the star S_n

$$L = D_n - z_n.$$

Taking the mean,

$$L = \tfrac{1}{2}(D_s + D_n) + \tfrac{1}{2}(z_s - z_n). \qquad [69]$$

The latitude is therefore the mean of the declinations corrected by half the difference of the zenith distances. The declination may be computed from the star catalogues, and the difference in zenith distance may be very accurately measured with the micrometer screw. It is evidently essential that the telescope should have the same inclination to the vertical in each case. If the inclination changes, however, the amount of this change is accurately determined from the level readings already mentioned (see Art. 70).

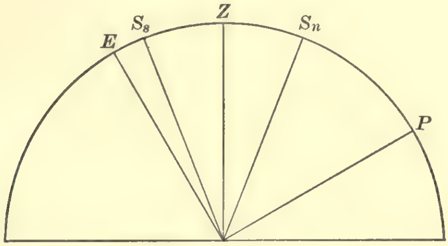


FIG. 46

58. Suggestions about Observing.

The instrument used for making such observations as are described in this book will usually be either the engineer's transit or the sextant. In using the transit care must be taken to give the tripod a firm support. It is well to set the transit in position some time before the observations are to be begun; this allows the instrument to assume the temperature of the air and the tripod legs to come to a firm bearing on the ground. The observer should handle the instrument with great care, particularly during night observations, when the instrument is likely to be accidentally disturbed. In reading angles at night it is important to hold the light in such a position that the graduations on the circle are plainly visible and may be viewed along the lines of graduation, not obliquely. By changing the position of the lantern and the position of the eye it will be found that the reading varies by larger amounts than would be expected when reading in the daylight. Care should be taken not to touch the graduated silver circles, as they soon become tarnished. The lantern should be held so as to heat the instrument as little as possible, and so as not to shine into the observer's eyes. Time may be saved and mistakes avoided if the program of observations is laid out beforehand, so that the observer knows just what is to be done and the proper order of the different steps. The observations should be arranged so as to eliminate instrumental errors, usually by means of reversals; but if this is not practicable, then the instrument must be put in good adjustment. The index correction should be determined and applied, unless it can be eliminated by the method of observing.

In observations for time it will often be necessary to use an ordinary watch. If there are two observers, one can read the time while the other makes the observations. If a chronometer is used, one observer may easily do the work of both, and at the

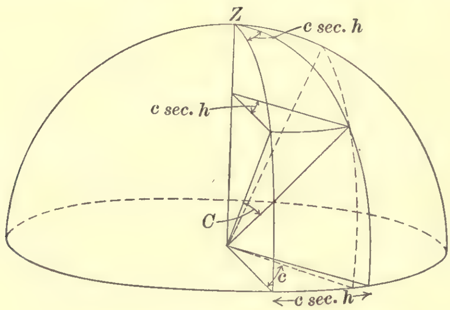


FIG. 47

same time increase the accuracy. In making observations by this method (called the "eye and ear method") the observer looks at the chronometer, notes the reading at some instant, say at the beginning of some minute, and, listening to the half-second beats, carries along the count mentally and without looking at the chronometer. In this way he can note the second and estimate the fraction without taking his attention from the star and cross hair. After making his observation he may check his count by again looking at the chronometer to see if the two agree. After a little practice this method can be used easily and accurately. In using a watch it is possible for one observer to make the observations and also note the time, but it cannot be done with any such precision as with the chronometer, because on account of the rapidity of the ticks (5 per second), the observer cannot count the seconds mentally. The observer must in this case look quickly at his watch and make an allowance, if it appears necessary, for the time lost in looking up and taking the reading.

Problems

1. Show that if the sight line makes an angle c with the perpendicular to the horizontal axis (Fig. 47) the horizontal angle between two points is in error by the angle

$$c \sec h' - c \sec h'',$$

where h' and h'' are the altitudes of the two points.

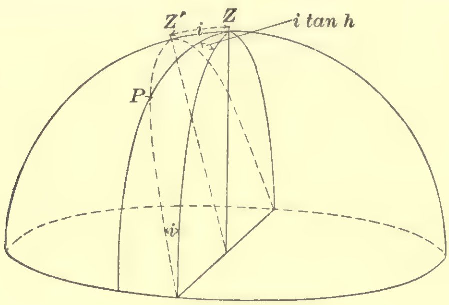


FIG. 48

2. Show that if the horizontal axis is inclined to the horizon by the angle i (Fig. 48) the effect upon the azimuth of the sight line is $i \tan h$, and that an angle is in error by

$$i\,(\tan h' - \tan h''),$$

where h' and h'' are the altitudes of the points.

CHAPTER IX

THE CONSTELLATIONS

59. The Constellations.

A study of the constellations is not really a part of the subject of Practical Astronomy, and in much of the routine work of observing it would be of comparatively little value, since the stars used can be identified by means of their coördinates and a knowledge of their positions in the constellations is not essential. If an observer has placed his transit in the meridian and knows approximately his latitude and the local time, he can identify stars crossing the meridian by means of the times and the altitudes at which they culminate. But in making occasional observations with small instruments, and where much of the astronomical data is not known to the observer at the time, some knowledge of the stars is necessary. When a surveyor is beginning a series of observations in a new place and has no accurate knowledge of his position nor the position of the celestial sphere at the moment, he must be able to identify certain stars in order to make approximate determinations of the quantities sought.

60. Method of Naming Stars.

The whole sky is divided in an arbitrary manner into irregular areas, all of the stars in any one area being called a **constellation** and given a special name. The individual stars in any constellation are usually distinguished by a name, a Greek letter,* or a number. The letters are usually assigned in the order of brightness of the stars, α being the brightest, β the next, and so on. A star is named by stating first its letter and then the name of the constellation in the (Latin) genitive form. For instance,

---

* The Greek alphabet is given on p. 190.

in the constellation *Ursa Minor* the star α is called α *Ursæ Minoris;* the star *Vega* in the constellation *Lyra* is called α *Lyræ.* When two stars are very close together and have been given the same letter, they are often distinguished by the numbers 1, 2, etc., written above the letter, as, for example, α^2 *Capricorni*, meaning that the star passes the meridian after α^1 *Capricorni.*

61. Magnitudes.

The brightness of stars is shown on a numerical scale by their **magnitudes.** A star having a magnitude 1 is brighter than one having a magnitude 2. On the scale of magnitudes in use a few of the brightest stars have fractional or negative magnitudes. Stars of the fifth magnitude are visible to the naked eye only under favorable conditions. Below the fifth magnitude a telescope is usually necessary to render the star visible.

62. Constellations Near the Pole.

The stars of the greatest importance to the surveyor are those near the pole. In the northern hemisphere the pole is marked by a second-magnitude star, called the *polestar*, *Polaris*, or α *Ursæ Minoris*, which is about $1^\circ\ 10'$ distant from the pole at the present time (1910). This distance is now decreasing at the rate of about one-third of a minute per year, so that for several centuries this star will be close to the celestial north pole. On the same side of the pole as *Polaris*, but much farther from it, is a constellation called *Cassiopeia*, the five brightest stars of which form a rather unsymmetrical letter W (Fig. 49). The lower left-hand star of this constellation, the one at the bottom of the first stroke of the W, is called δ, and is of importance to the surveyor because it is very nearly on the hour circle passing through *Polaris* and the pole; in other words its right ascension is nearly the same as that of *Polaris*. On the opposite side of the pole from *Cassiopeia* is *Ursa Major*, or the great dipper, a rather conspicuous constellation. The star ζ, which is at the bend in the dipper handle, is also nearly on the same hour circle as *Polaris* and δ *Cassiopeiæ.* If a line be drawn on the sphere between δ *Cassiopeiæ* and ζ *Ursæ Majoris*, it will pass nearly through *Polaris* and the pole, and will show at once the position of *Polaris* in its diurnal circle. The two stars in the bowl of the great dipper on the side farthest from the handle are in a line which, if prolonged, would pass near to *Polaris*. These stars are therefore called the **pointers** and may be used to find the polestar. There is no other star near *Polaris* which is likely to be confused with it. Another star which should be remembered is β *Cassiopeiæ*, the one at the upper right-hand corner of the W. Its right ascension is very nearly 0^h and therefore the hour circle through it passes nearly through the equinox. It is possible then, by simply glancing at β *Cassiopeiæ* and the polestar, to estimate approximately the local sidereal time. When β *Cassiopeiæ* is vertically above the polestar it is nearly 0^h sidereal time; when the star is below the polestar it is 12^h sidereal time; half way between these positions, left and right, it is 6^h and 18^h, respectively. In intermediate positions the hour angle of the star (= sidereal time) may be roughly estimated.

63. Constellations Near the Equator.

The principal constellations within $45°$ of the equator are shown in Figs. 50 to 52. Hour circles are drawn for each hour of R. A. and parallels for each $10°$ of declination. The approximate declination and right ascension of a star may be obtained by scaling the coördinates from the chart. The position of the ecliptic, or sun's path in the sky, is shown as a curved line. The moon and the planets are always found near this circle because the planes of their orbits have only a small inclination to the earth's orbit. A belt extending about $8°$ each side of the ecliptic is called the **Zodiac,** and all the members of the solar system will always be found within this belt. The constellations along this belt, and which have given the names to the twelve "signs of the Zodiac," are *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Pisces*. These constellations were named many centuries ago, and the

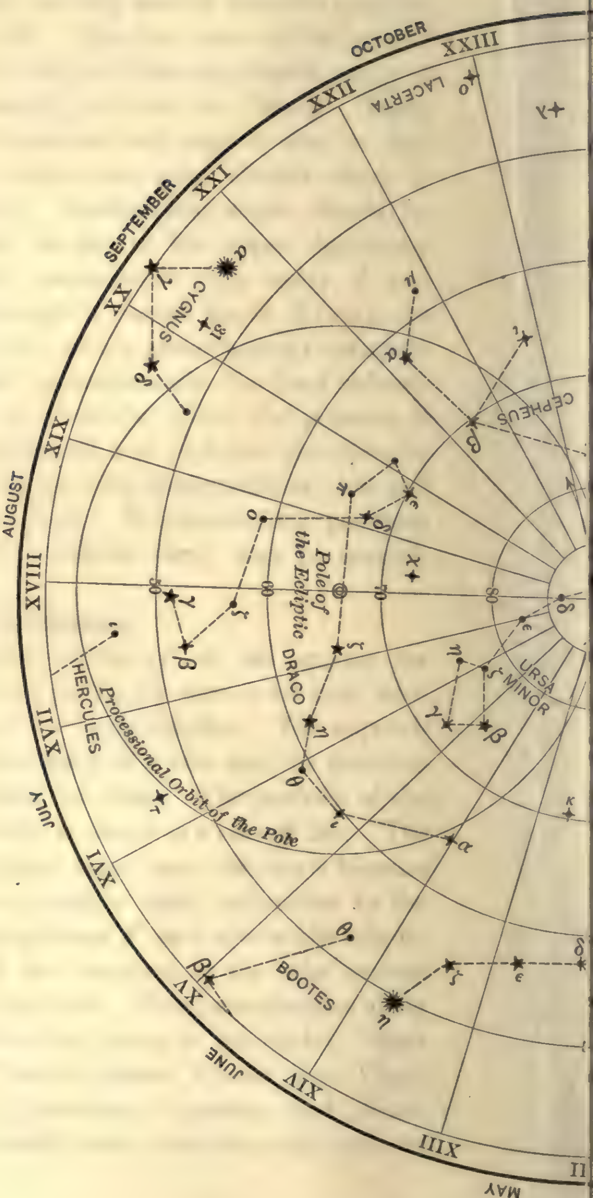
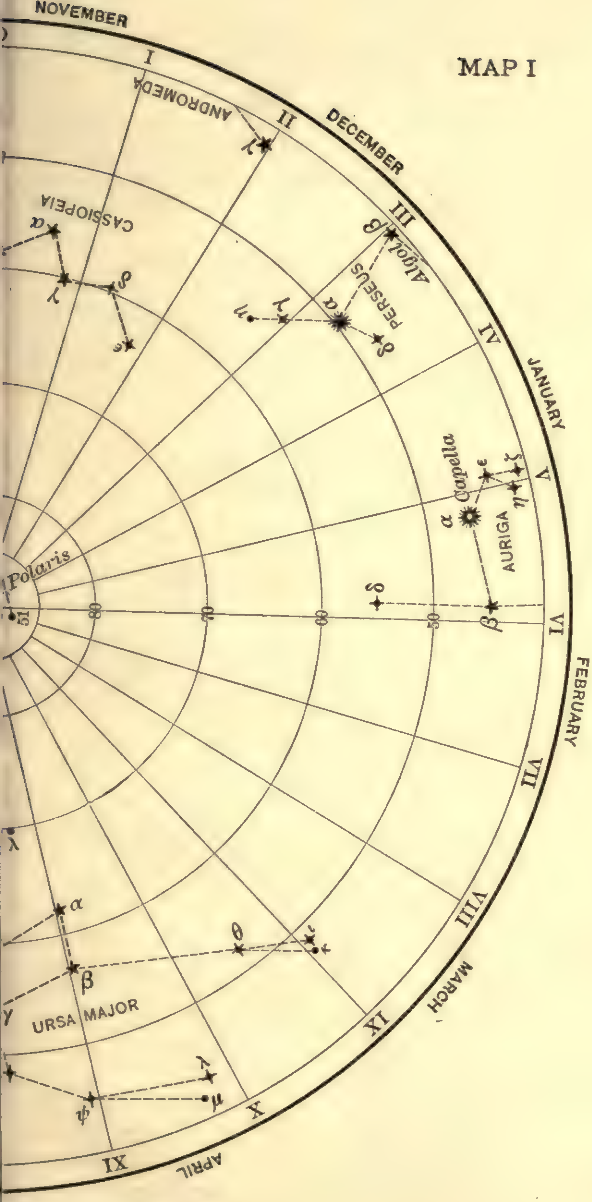


FIG. 49. CONSTELLATION

MAP I



BOUT THE NORTH POLE

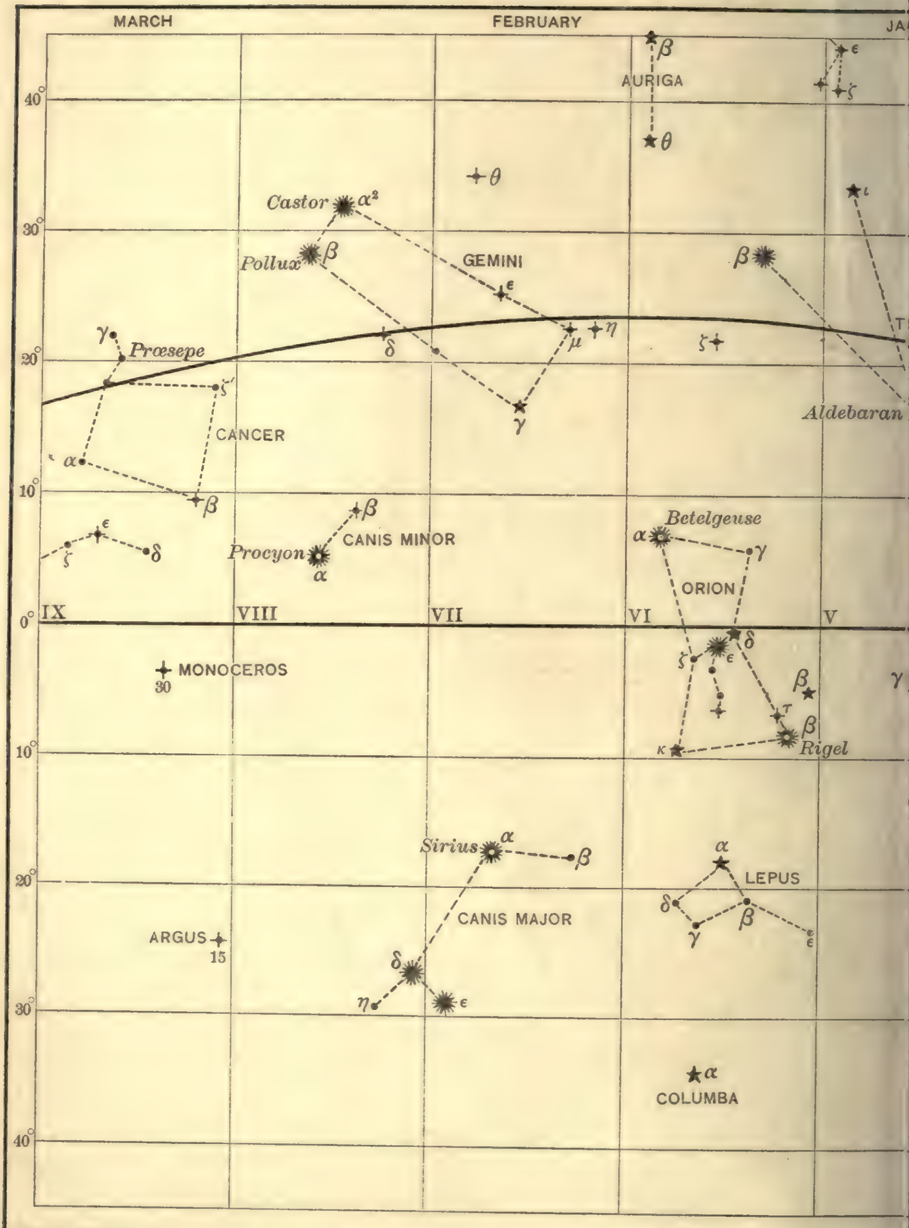
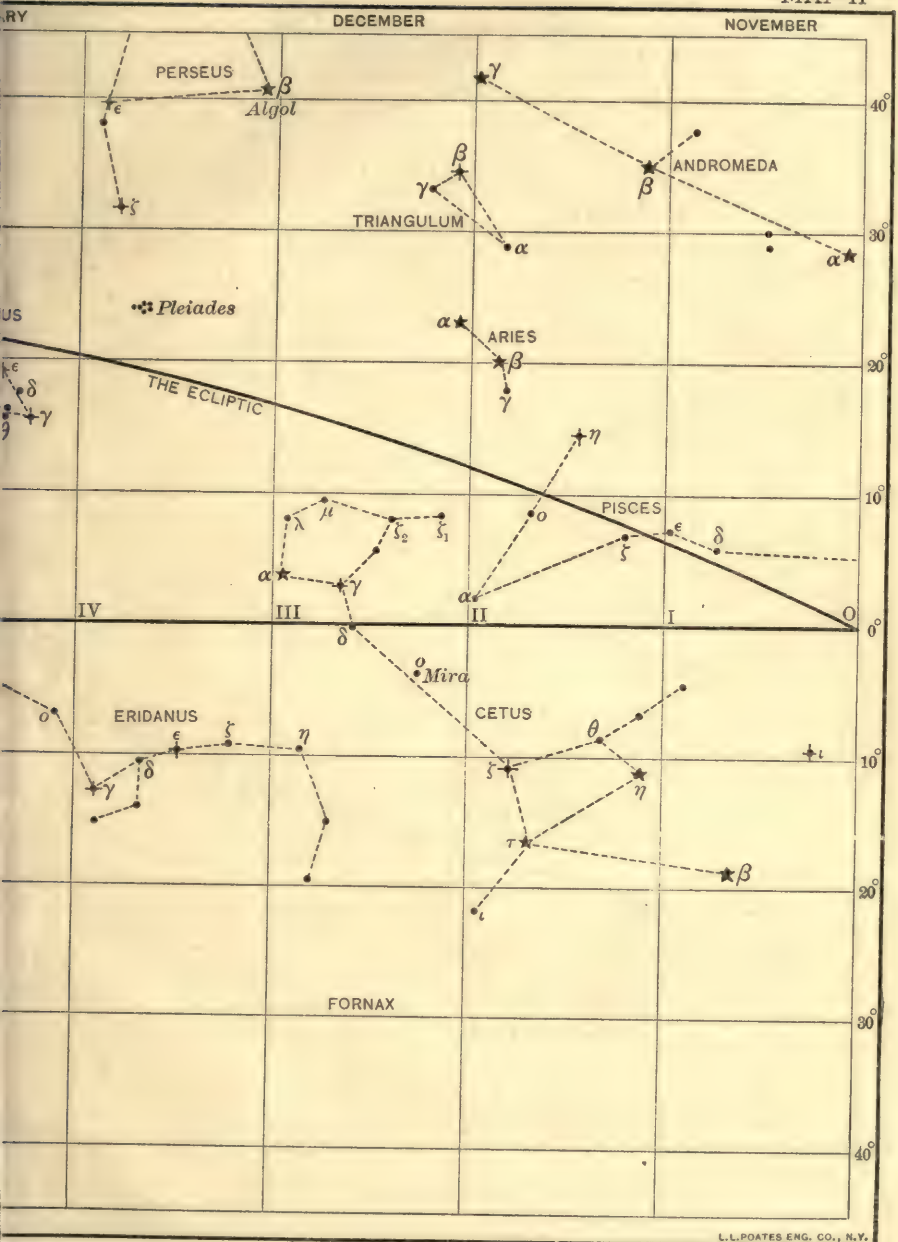


FIG. 50. PRINCIPAL FIXED STARS BETWEE

MAP II



CLINATIONS 45° NORTH AND 45° SOUTH

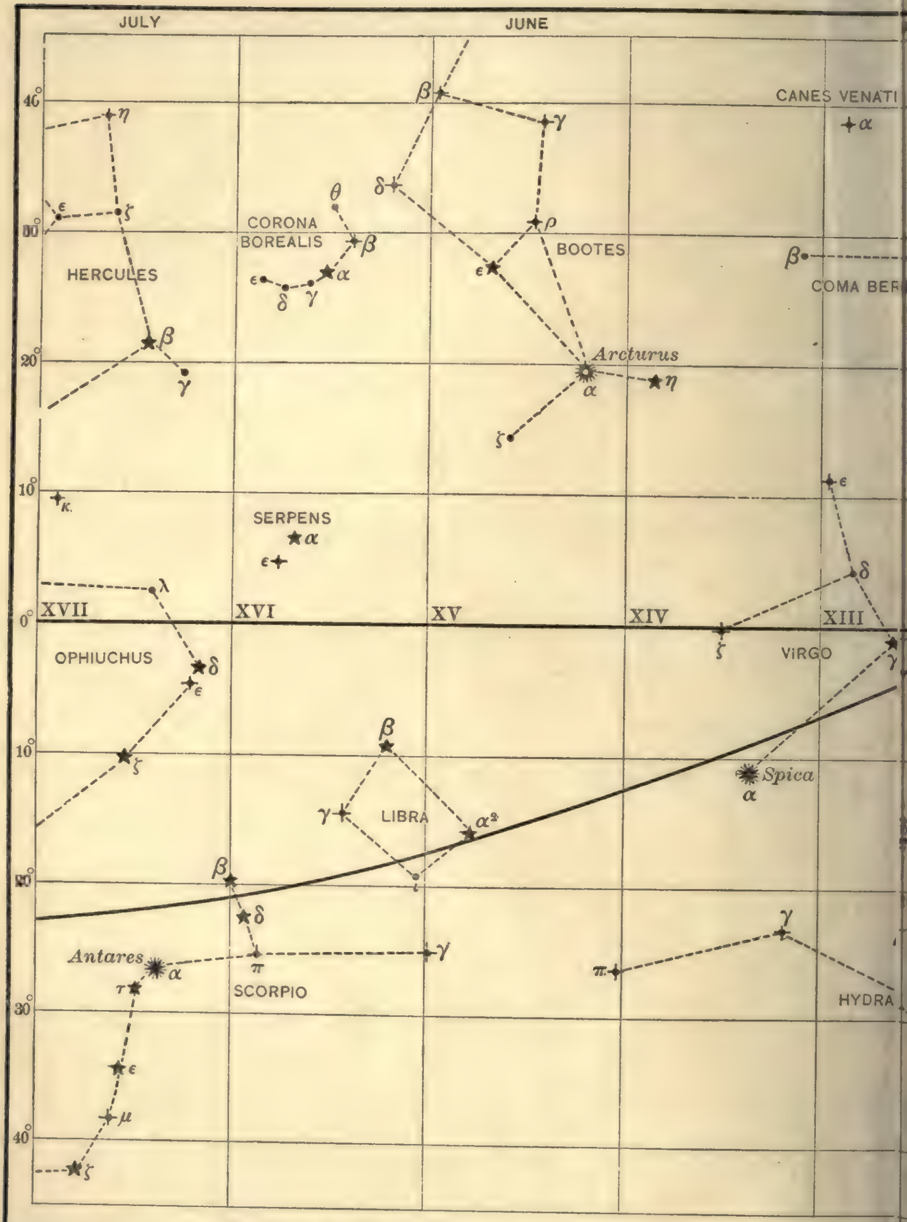
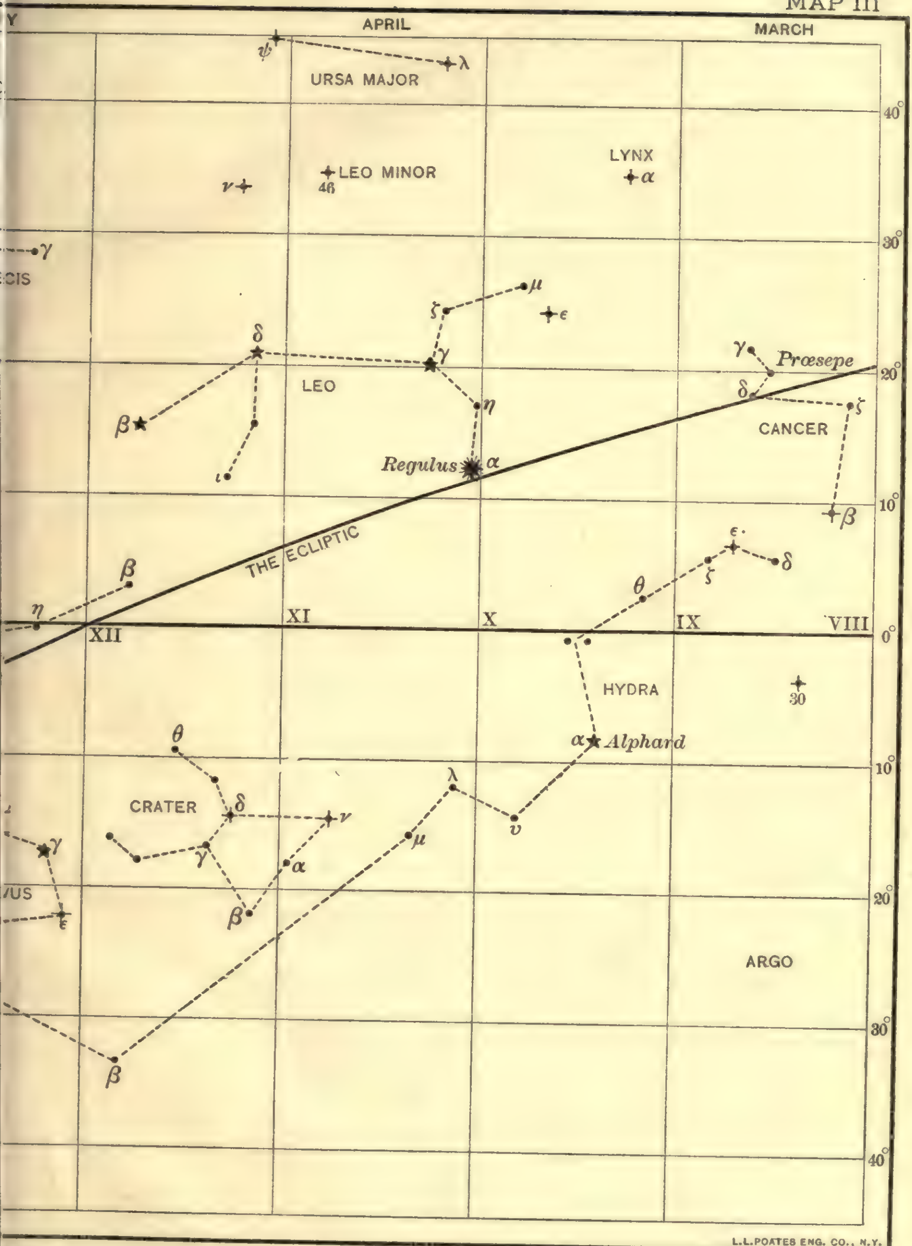


FIG. 51. PRINCIPAL FIXED STARS BETWEE

MAP III



ECLINATIONS 45° NORTH AND 45° SOUTH

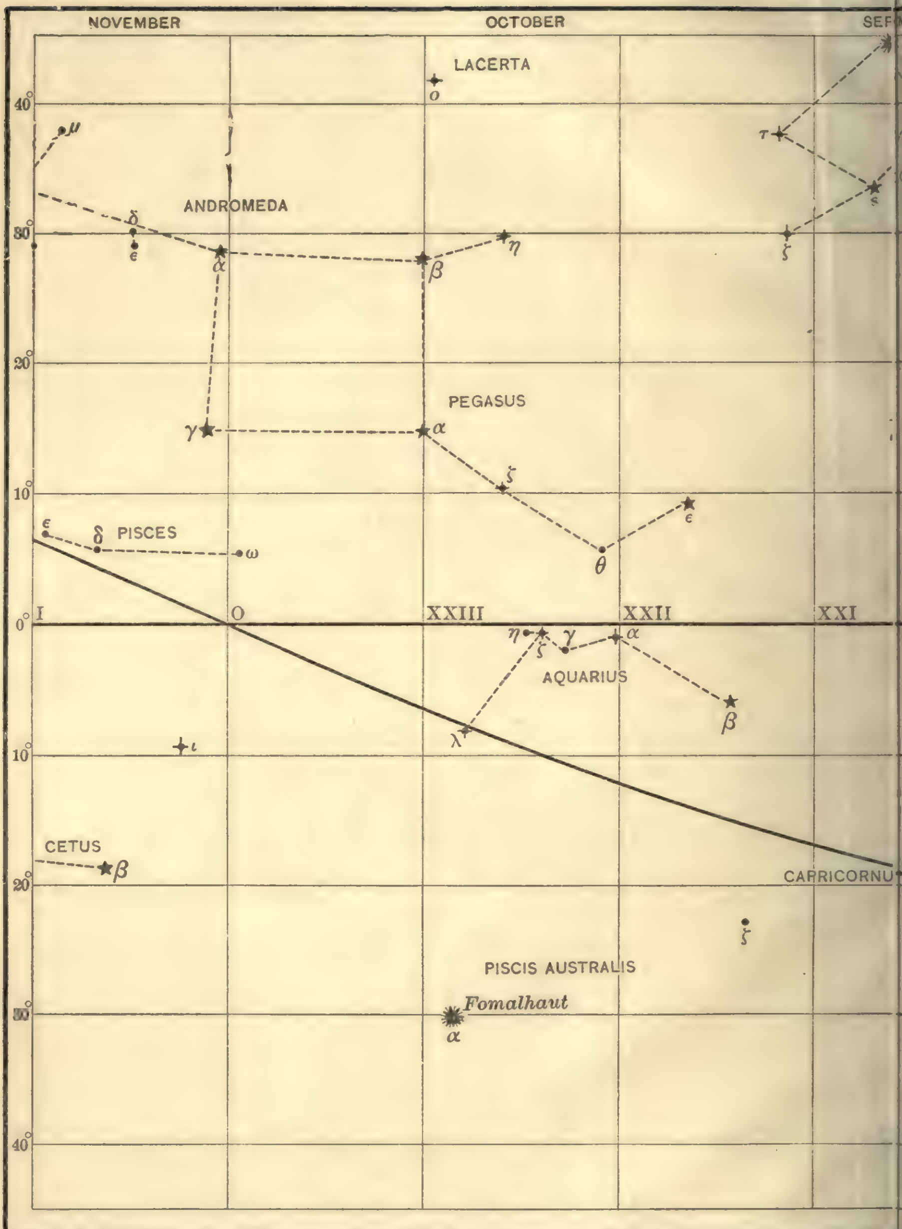
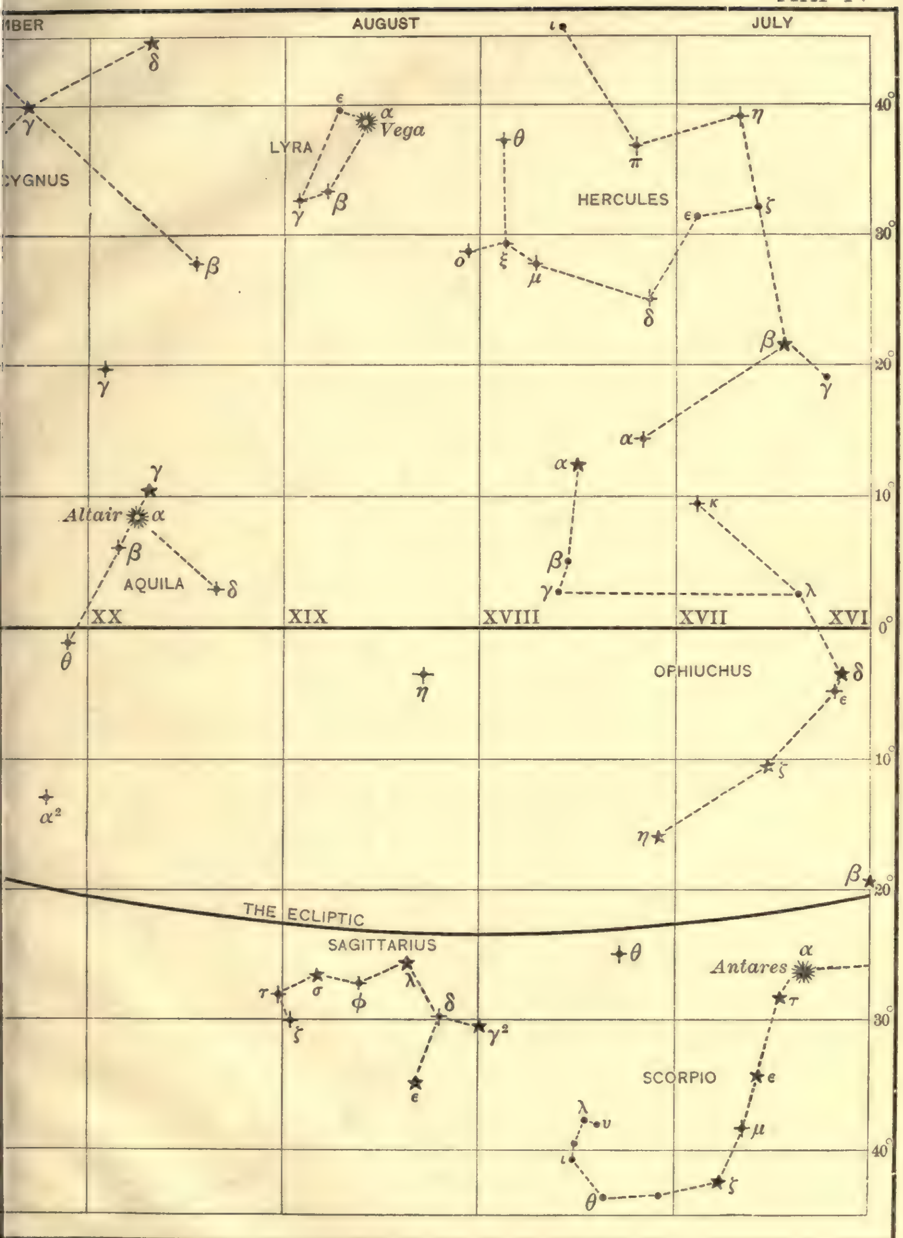


FIG. 52. PRINCIPAL FIXED STARS BETWEE

MAP IV



)ECLINATIONS 45° NORTH AND 45° SOUTH

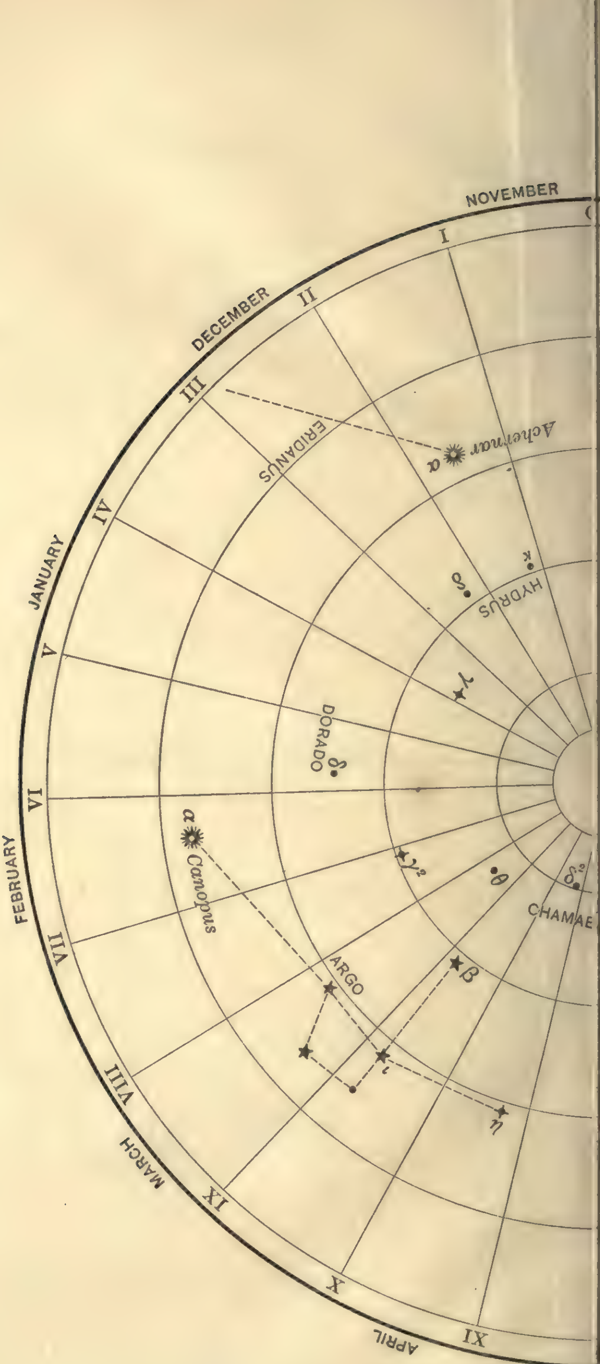
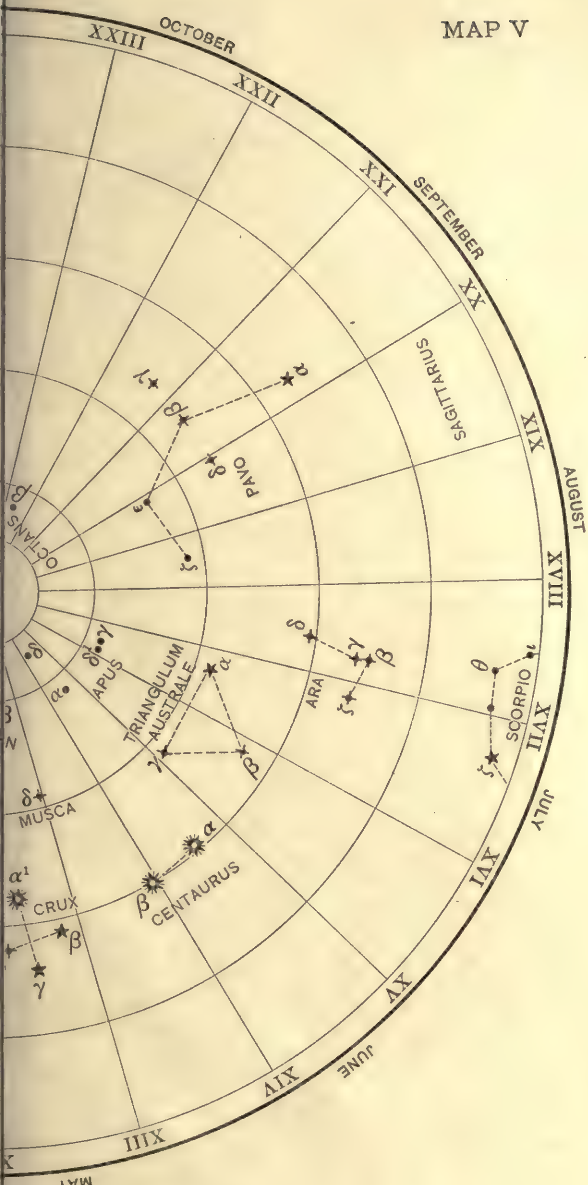


FIG. 53. CONSTELLATIO

MAP V



ABOUT THE SOUTH POLE

names have been retained, both for the constellations themselves and also for the positions in the ecliptic which they occupied at that time. But on account of the continuous westward motion of the equinox, the "signs" no longer correspond to the constellations of the same name. For example, the sign of *Aries* extends from the equinoctial point to a point on the ecliptic 30° eastward, but the constellation actually occupying this space at present is *Pisces*. In Figs. 50 to 52 the constellations are shown as seen by an observer on the earth, not as they would appear on a celestial globe. On account of the form of projection used in these maps there is some distortion, but if the observer faces south and holds the page up at an altitude equal to his colatitude, the map represents the constellations very nearly as they will appear to him. The portion of the map to be used in any month is that marked with the name of the month at the top; for example, the stars under the word "February" are those passing the meridian in the middle of February at about 9 P.M. For other hours in the evening the stars on the meridian will be those at a corresponding distance right or left, according as the time is earlier or later than 9 P.M. The approximate right ascension of a point on the meridian may be found at any time as follows: First compute the R. A. of the sun by allowing 2^h per month, or more nearly 4 per day for every day since March 23, remembering that the R. A. of the sun is always increasing. Add this R. A. to the local mean time and the result is the sidereal time or right ascension of a star on the meridian.

Example. On October 10 the R. A. of the sun is $6 \times 2^h + 17 \times 4^m = 13^h\ 08^m$. At 9^h P.M. (local mean time) the sidereal time is $13^h\ 08^m + 9^h\ 00^m = 22^h\ 08^m$. A star having a R. A. of $22^h\ 08^m$ would therefore be close to the meridian at 9 P.M.

Fig. 53 shows the stars about the south celestial pole. There is no bright star near the south pole, so that the convenient methods of determining the meridian by observations on the *polestar* are not practicable in the southern hemisphere.

64. The Planets.

In using the star maps, the student should be on the lookout for planets. These cannot be placed on the maps because their positions are rapidly changing. If a bright star is seen near the ecliptic, and its position does not correspond to that of a star on the map, it is a planet. The planet *Venus* is very bright and is never very far from the sun; it will therefore be seen a little before sunrise or a little after sunset. *Mars*, *Jupiter*, and *Saturn* are outside the earth's orbit and therefore revolve around the earth. *Jupiter* is the brightest, and when looked at through a small telescope shows a disc like that of the full moon, and four satellites can usually be seen all lying nearly in a straight line. *Saturn* is not as large as *Jupiter*, but in a telescope of moderate power its rings can be distinguished, or at least the planet looks elongated. *Mars* is reddish in color and shows a disc.

CHAPTER X

OBSERVATIONS FOR LATITUDE

In this chapter and the three immediately following are given the more common methods of determining latitude, time, longitude, and azimuth with small instruments. Those which are simple and direct are printed in large type, and may be used for a short course in the subject. Following these are given, in smaller type, several methods which, although less simple, are very useful to the engineer; these methods require a knowledge of other data which the engineer must obtain by observation, and are therefore better adapted to a more extended course of study.

65. Latitude by a Circumpolar Star at Culmination.

This method may be used with any circumpolar star, but *Polaris* is the best one to use, when it is practicable to do so, because it is of the second magnitude, while all of the other close circumpolars are quite faint. The observation consists in measuring the altitude of the star when it is a maximum or a minimum, or, in other words, when it is on the observer's meridian. This altitude may be obtained by trial, and it is not necessary to know the exact instant when the star is on the meridian. The approximate time when the star is at culmination may be obtained from Table V or by formulæ [39] and [49]. It is not necessary to know the time with accuracy, but it will save unnecessary waiting if the time is known approximately. In the absence of any definite knowledge of the time of culmination, the position of the pole star with respect to the meridian may be estimated by noting the positions of the constellations. When δ *Cassiopeiæ* is directly above or below *Polaris* the latter is at upper or lower culmination. The observation should be begun some time before one of these positions is reached. The horizontal cross hair of the transit should be set on the star* and the motion of the star followed by means of the tangent screw of the horizontal axis. When the desired maximum or minimum is reached the vertical arc is read. The index correction should then be determined. If the instrument has a complete vertical circle and the time of culmination is known approximately, it will be well to eliminate instrumental errors by taking a second altitude with the instrument reversed, provided that neither observation is made more than 4^m or 5^m from the time of culmination. If the star is a faint one, and therefore difficult to find, it may be necessary to compute its approximate altitude (using the best known value for the latitude) and set off this altitude on the vertical arc. The star may be found by moving the telescope slowly right and left until the star comes into the field of view. *Polaris* can usually be found in this manner some time before dark, when it cannot be seen with the unaided eye. It is especially important to focus the telescope carefully before attempting to find the star, for the slightest error of focus may render the star invisible. The focus may be adjusted by looking at a distant terrestrial object or, better still, by sighting at the moon or at a planet if one is visible. If observations are to be made frequently with a surveyor's transit, it is well to have a reference mark scratched on the telescope tube, so that the objective may be set at once at the proper focus.

The latitude is computed from Equa. [3] or [4]. The true altitude h is derived from the reading of the vertical circle by applying the index correction with proper sign and then subtracting the refraction correction (Table I). The polar distance is found by taking from the Ephemeris (Table of Circumpolar Stars) the *apparent declination* of the star and subtracting this from 90°.

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* The image of a star is practically a point of light; if the telescope were perfect it would be actually a point, but, owing to the imperfections in the corrections for aberration, the image, even though perfectly distinct, has an appreciable width. The image of the star should be bisected with the horizontal cross hair.

Example 1.

Observed altitude of *Polaris* at upper culmination = 43° 37′; index correction = +30″; declination = +88° 44′ 35″.

Vertical circle	= 43° 37′ 00″
Index correction	= +30
Observed altitude	= 43 37 30
Refraction correction	= 1 02
True altitude	= 43 36 28
Polar distance	= 1 15 25
Latitude	= 42° 21′ 03″

Since the vertical circle reads only to 1′ the resulting value for the latitude must be considered as reliable only to the nearest 1′.

Example 2.

Observed altitude of 51 *Cephei* at lower culmination = 39° 33′ 30″; index correction = 0″; declination = + 87° 11′ 25″.

Observed altitude	= 39° 33′ 30″
Refraction correction	= 1 10
True altitude	= 39 32 20
Polar distance	= 2 48 35
Latitude	= 42° 20′ 55″

66. Latitude by Altitude of Sun at Noon.

The altitude of the sun at noon (meridian passage) may be determined by placing the line of sight of the transit in the plane of the meridian and observing the altitude of the upper or lower limb of the sun when it is on the vertical cross hair. The watch time at which the sun will pass the meridian may be computed by converting 12^h local apparent time into Standard or local mean time (whichever is used) as shown in Arts. 28 and 35. Usually the direction of the meridian is not known, so the maximum altitude of the sun is observed and assumed to be the same as the meridian altitude. On account of the sun's changing declination the maximum altitude is not quite the same as the meridian altitude; the difference is quite small, however, usually a fraction of a second, and may be entirely neglected for observations made with the engineer's transit or the sextant. The maximum altitude of the upper or lower limb is found by trial, the horizontal cross hair being kept tangent to the limb as long as it continues to rise. When the observed limb begins to drop below the cross hair the altitude is read from the vertical arc and the index correction is determined. The true altitude of the centre of the sun is then found by applying the corrections for index error, refraction, semidiameter, and parallax. In order to compute the latitude it is necessary to know the sun's declination at the instant the altitude was taken. If the longitude of the place is known approximately (say within half a degree) the declination may be taken from the Nautical Almanac for the instant of Greenwich Apparent Noon and increased or decreased by the hourly change multiplied by the number of hours in the longitude. If the place is west of Greenwich the correction is to be added algebraically; if the place is east, it is to be subtracted. If the longitude is not known, but the Greenwich mean time is known, as would be the case if the timepiece kept either Greenwich time or Standard time, the declination may be computed by noting the watch time of the observation as nearly as possible and correcting the declination as follows: take out the declination at Greenwich Mean Noon, and increase it by the hourly change multiplied by the number of hours since Greenwich Mean Noon. The latitude is then found from Equa. [2].

Example 1.

Observed maximum altitude of sun's lower limb, Jan. 8, 1906, = $25° 06'$; index correction = $+1'$; the longitude is $4^h 44^m 18^s$ (= $71° 04' \frac{1}{2}$) west; the declination of the sun at Greenwich Apparent Noon = S $22° 19' 33''$; hourly change = $+19''.59$; the semidiameter = $16' 17''$.

Observed altitude	=	$25° 06'.0$
Index correction	=	$+1.0$
		$25\ 07.0$
Refraction correction	=	-2.0
		$25\ 05.0$
Semidiameter	=	$+16.3$
		$25\ 21'.3$
Parallax	=	$+.1$
Altitude of centre	=	$25\ 21.4$
Declination	=	$-22\ 18.0$
Colatitude	=	$47° 39'.4$
Latitude	=	$42° 20'.6$

Decl. at G. A. N.	=	$-22° 19' 33''$
$19''.59 \times 4^h.74$	=	$+1\ 33$
Decl. at L. A. N.	=	$-22° 18' 00''$

Example 2.

Observed double altitude of sun's upper limb at noon on Jan. 28, 1910 (with artificial horizon), = 59° 17′ 40″; Eastern Standard time = $11^h\,57^m$; index correction = +30″; declination at Greenwich Mean Noon = S 18° 21′ 08″; hourly change = +39″.07; semidiameter = 16′ 16″.

Decl. at G. M. N.	=	− 18° 21′ 08″
$39''.07 \times 4^h.95$	=	+3′ 13″
Decl. at $11^h\,57^m$	=	− 18° 17′ 55″

Double alt.	=	59° 17′ 40″
I. C.	=	30
		2)59 18 10
		29° 39′ 05″
Refraction	=	−1 43
		29 37 22
Semidiameter	=	−16 16
		29 21 06
Parallax	=	+8
		29 21 14
Declination	−	18 17 55
Colatitude	=	47° 39′ 09″
Latitude	=	42° 20′ 51″

67. By the Meridian Altitude of a Southern* Star.

The latitude may be found from the observed maximum altitude of a star which culminates south of the zenith, by the method of the preceding article, except that the parallax and semidiameter corrections become zero, and that it is not necessary to note the time of the observation, since the declination of the star changes so slowly. In measuring the altitude the star's image is bisected with the horizontal cross hair, and the maximum found by trial as when observing on the sun. For the method of finding the time at which a star will pass the meridian see Art. 72.

Example.

Observed meridian altitude of θ *Serpentis* = 51° 45′; index correction = 0; declination of star = + 4° 05′ 11″.

Observed altitude of θ *Serpentis*	=	51° 45′ 00
Refraction correction	=	−46
		51 44 14
Declination of star	=	+ 4 05 11
Colatitude	=	47° 39′ 03″
Latitude	=	42° 20′ 57″

* The observer is assumed to be in the northern hemisphere.

Constant errors in the measured altitudes may be eliminated by combining the results obtained from circumpolar stars with those from southern stars. An error which makes the latitude too great in one case will make it too small by the same amount in the other case.

68. Altitudes Near the Meridian.

If altitudes of the sun or a star are taken near the meridian they may be reduced to the meridian altitude provided the latitude and the times are known approximately. To derive the formula for making the reduction take the fundamental formula given in Equa. [8]

$$\sin h = \sin L \sin D + \cos L \cos D \cos P.$$

This may be transformed into

$$\sin h = \cos (L - D) - \cos L \cos D \operatorname{vers} P, \qquad [70]$$

or

$$\sin h = \cos (L - D) - \cos L \cos D \times 2 \sin^2 \frac{P}{2}. \qquad [71]$$

Transposing and denoting by h_m the meridian altitude $L - D$, the equation becomes

$$\sin h_m = \sin h + \cos L \cos D \operatorname{vers} P, \qquad [72]$$

or

$$\sin h_m = \sin h + \cos L \cos D \times 2 \sin^2 \frac{P}{2}. \qquad [73]$$

If the altitude h be measured and the corresponding time be noted, then the value of P becomes known. If L is known approximately, then the second term may be computed and h_m, or $L - D$, found through its sine. If the value of L derived from the first computation does not agree closely with the assumed value, a second computation should be made using the new value of L. When observations are taken within a few minutes of the meridian (say 15^m) the computation may be shortened by the use of the approximate formula

$$C'' = 112.5 \times P^2 \times \cos L \cos D \sec h \sin 1'', \qquad [74]$$

in which C'' is the correction in seconds of arc and P is the time from the meridian expressed in seconds of time. (Log $112.5 \times \sin 1'' = 6.7367$). If P is expressed in minutes the formula is

$$C'' = 1''.9635 \times P^2 \times \cos L \cos D \sec h. \qquad [75]$$

This formula may be derived as follows: Transposing Equa. [73]

$$\sin h_m - \sin h = 2 \cos L \cos D \sin^2 \frac{P}{2}. \qquad [76]$$

By trigonometry

$$2 \cos \tfrac{1}{2} (h_m + h) \sin \tfrac{1}{2} (h_m - h) = 2 \cos L \cos D \sin^2 \frac{P}{2}. \qquad [77]$$

Since h is nearly equal to h_m, $\cos \frac{1}{2}(h_m + h)$ may be put equal to $\cos h$; placing $C = h_m - h$, the equation becomes

$$\sin \tfrac{1}{2} C = \cos L \cos D \sin^2 \frac{P}{2} \sec h. \qquad [78]$$

C and P are both small angles and may be put in place of their sines, hence

$$C = \tfrac{1}{2} P^2 \times \cos L \cos D \sec h. \qquad [79]$$

To reduce C to seconds of arc and P to seconds of time the left member must be multiplied by $\sin 1''$ and the right by $(15 \sin 1'')^2$, giving

$$C'' = \cos L \cos D \sec h \times P^2 \times 112.5 \sin 1''. \qquad [74]$$

In using this formula it will be necessary to use an approximate value of L; a second approximation may be made if necessary.

The method of " reduction to the meridian " given above should not be applied when the object observed is far from the meridian.

Example 1.

Observed double altitude sun's lower limb Jan. 28, 1910.

	Double Alt. ⊙			Watch.
	56° 44′ 40″			$11^h\ 15^m\ 25^s$
	49 00			16 22
	52 40			17 10
	56° 48′ 47″			$11^h\ 16^m\ 19^s$
I. C.	+30″	Watch corr.		+1 19
	2) 56° 49′ 17″	E. S. T.		$11^h\ 17^m\ 38^s$
		App. Noon		11 57 21
	28° 24′ 38″			
Refr. =	−1 46	Hour angle	=	$39^m\ 43^s$
			P =	9° 55′ 45″
	28 22 52			
s.d. =	+16 16			
	28 39 08			
par. =	+ 8			
h =	28° 39′ 16″			
log cos L =	9.86763			
log cos D =	9.97745			
log vers P =	8.17546			
	8.02054			
	.01048			
nat sin h =	.47953			
nat sin h_m =	.49001			
h_m =	29° 20′ 29″			
D =	18 18 20			
Co-lat. =	47° 38′ 49″			
Lat. =	42° 21′ 11″			

Assumed lat.	=	42° 30′
L. A. N.	=	12 00 00
Eq. T	=	+13 03
L. M. T.	=	12 13 03
Red. for long.	=	15 42
E. S. T.	=	11 57 21
Sun's decl. at G. M. N.	= −	18° 21′ 08″
$39''.07 \times 4^h.3$	=	2′ 48″
Cor'd. decl.	= −	18° 18′ 20″

A recomputation, using the corrected latitude, changes this result to 42° 21′ 04″.

Example 2.

Observed altitude of γ Ceti $= 50° 33'$; index correction $= -1'$; hour angle of γ *Ceti* derived from observed time $= 3^m 14^s.2$; declination $= +2° 50' 30''$.

$$\begin{aligned}
\log\cos L &= 9.8691\\
\log\cos D &= 9.9995\\
\log\sec h &= 0.1967\\
\log\text{const} &= 6.7367\\
2\log P &= 4.5765\\
\hline
\log C'' &= 1.3785\\
C'' &= 23''.9
\end{aligned}$$

Observed altitude	=	50° 33′.0
Index correction	=	− 1.0
		50° 32.0
Refraction correction	=	− 0.8
True altitude	=	50° 31′.2
Reduction to meridian	=	+ 0.6
Meridian altitude	=	50° 31′.8
Declination	=	+ 2 50.5
Co-latitude	=	47° 41′.3
Latitude	=	42° 18′.7

The method of "ex-meridians altitudes," as it is sometimes called, may be used when the meridian observation is lost or when it is desired to increase the accuracy of the result by multiplying the number of observations.

69. Latitude by Altitude of Polaris when the Time is Known.

The altitude of *Polaris* varies slowly on account of its nearness to the pole, hence, if the sidereal time is known, the latitude may be found accurately by an altitude of this star taken at any time, because errors in the time have a relatively small effect upon the result. Several altitudes should be taken in succession, and the time noted at each pointing of the cross-hair on the star. For observations made with the surveyor's transit and covering only a few minutes' time the mean of the altitudes may be taken as corresponding to the mean of the observed times. If the instrument has a complete vertical circle, half of the observations should be made with the instrument in the reversed position. The index correction should be determined in each case. In order to compute the latitude it is necessary to know the hour angle of the star at the instant of observation. When a common watch is used for taking the time the star's hour angle is found by Equa. [47] and [37]. The latitude is then found by the formula

$$L = h - p\cos P + \tfrac{1}{2}\sin 1'\, p^2 \sin^2 P \tan h \qquad [80]$$

$$(\log \tfrac{1}{2}\sin 1' = 6.1627 - 10)$$

The derivation of the formula is rather complex and will not be given here. It is obtained by expanding the correction to h in a series in ascending powers of

p, the small terms being neglected. The sum of all terms after that containing p^2 amounts to less than $1''$ and these have therefore been omitted in Equa. [80]. In this equation p, the polar distance, is expressed in minutes of arc. Values of the last term may be taken with sufficient accuracy from Table VI. The algebraic sign of the second term is determined by the sign of cos P; the third term is always positive. In Fig. 54, P is the pole, S the star, MS the hour angle, and PDA the almucantar through P or circle of equal altitudes. It will be seen that the term $p \cos P$ is approximately the distance from S to E, a point on the six-hour circle PB; the distance desired is SD, the angular distance of S above the almucantar through P. The last term in Equa. [80] is approximately equal to DE, each term in the series giving a closer approximation to the distance SD.

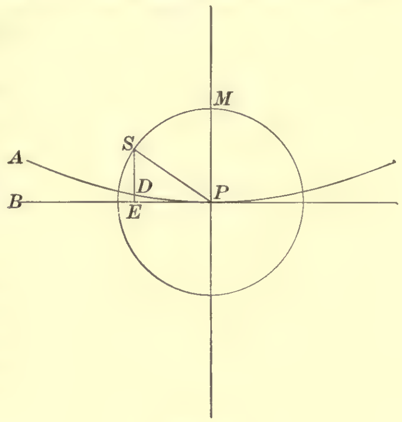

FIG. 54

Example.

Observed Altitudes of *Polaris*, Jan. 9, 1907.

Watch.	Altitudes.
$6^h\ 49^m\ 26^s$	$43°\ 28'.5$
51 45	28 .5
54 14	28 .0
56 45	28 .0

Index correction $= -1'.0$; $p = 71'.15$; P is found from the observed watch times to be $13°\ 50.7$.*

$$\log p = 1.8522 \qquad \log \text{constant} = 6.1627$$
$$\log \cos P = 9.9822 \qquad \log p^2 = 3.7044$$
$$\log \sin^2 P = 8.7578$$
$$\log p \cos P = 1.8394 \qquad \log \tan h = 9.9762$$
$$p \cos P = 69'.09 \qquad 8.6011$$
$$\text{Last term} = +0'.04$$

Observed alt. =	$43°\ 28'.25$
I. C.	$-\ 1.$
	$43°\ 27'.25$
Refraction	$1\ .01$
	$43°\ 26'.24$
1st. and 2nd terms	$1\ \ 09\ .05$
Latitude	$42°\ 17'.19$

* If the error of the watch is known the sidereal time may be found by Equa. [47]. For method of finding the sidereal time by observation see Chap. XI. The hour angle of the star is found by Equa. [38].

70. Precise Latitude Determinations. — Talcott's Method.

The most precise method of determining latitude is that known as "Talcott's Method," which requires the use of the zenith telescope. In making observations the observer selects two stars, one north of the zenith and one south of it, the two zenith distances differing by only a few minutes of angle, and the right ascensions differing by about 5 or 10 minutes of time. For the best results the zenith distances should be small and nearly equal. If the first star culminates south of the zenith the telescope is turned about its vertical axis until the stop indicates that it is in the meridian and on the south side of the zenith. The telescope is tipped until the sight line has an inclination to the vertical equal to the mean of the two zenith distances.* It is clamped in this position and great care is taken not to alter its inclination until the observations on both stars are completed. When the star appears in the field the micrometer wire is set so as to bisect the star's image; at the instant of culmination the setting of the wire is perfected and the scale of the spirit level is read at the same time. The chronometer (regulated to local sidereal time) should be read when the bisection is made, so that the reading of the micrometer may be corrected if the star was not exactly on the meridian at that instant. The micrometer screw is then read. The telescope is then turned to the north side of the meridian, the inclination remaining unchanged, and a similar observation made on the other star. When both sets of micrometer readings and level readings have been obtained, the latitude is found by the formula

$$L = \tfrac{1}{2}(D_s + D_n) + \tfrac{1}{2}(m_s - m_n) \times R + \tfrac{1}{2}(l_s + l_n) + \tfrac{1}{2}(r_s - r_n), \qquad [81]$$

in which m_s, m_n are the micrometer readings, R the value of 1 division of the micrometer, l_s, l_n the level corrections (positive when the north reading is the larger) and r_s, r_n the refraction corrections. Another correction must be added in case the observation is taken when the star is off the meridian.

In order to determine latitude by this method with the precision required in geodetic operations, observations are made on several nights, and on each night a large number of pairs of stars is observed. By this method a latitude may be determined within about $0''.05$ which is equivalent to nearly 5 feet in distance on the earth's surface.

Questions and Problems

1. Observed maximum altitude sun's lower limb, April 27, 1910, $= 61° 05'$. Index correction $= +30''$. The longitude is $4^h 44^m 18^s$ W. The sun's decl. at G. A. N. $=$ N $13° 38' 22''.3$; diff. for $1^h = +48''.07$; the semidiameter $= 15' 55''$. Compute the latitude.

2. The observed meridian altitude of δ *Crateris* $= 33° 24'$; index correction $= +30''$; declination of star $= -14° 17' 37''$. Compute the latitude.

3. Observed altitude of α *Ceti* at $3^h 08^m 49^s$ L. S. T. $= 51° 21'$; I. C. $= -1'$

* In order to compute these zenith distances it is necessary to know a rough value of the latitude, say within $1'$ or $2'$. This may be found by an observation with the zenith telescope using one of the preceding methods.

the right ascension of α *Ceti* = $2^h\ 57^m\ 24^s.8$; declination = $+ 3° 43' 22''$. Compute the latitude.

4. Observed Altitude of *Polaris*, $41° 41' 30''$; chronometer time, $9^h\ 44^m\ 38^s.5$ (Loc. Sid. Time); chronometer correction, $- 34^s$. The R. A. of *Polaris* is $1^h\ 25^m\ 42^s$; the declination is $+ 88° 49' 29''$. Compute the latitude.

5. Show by a sketch the positions of the following three points; 1. *Polaris* at greatest elongation; 2. *Polaris* on the six-hour circle; 3. *Polaris* at the same altitude as the pole.

6. What is the most favorable position of the sun for a latitude observation?

7. What is the most favorable position of *Polaris* for a latitude observation?

8. Draw a sketch showing why the sun's maximum altitude is not the same as the meridian altitude.

CHAPTER XI

OBSERVATIONS FOR DETERMINING THE TIME

71. Observation for Local Time.

Observations for determining the local time at any place at any instant usually consist in finding the error of a timepiece on the kind of time which it is supposed to keep. To find the solar time it is necessary to determine the hour angle of the sun's centre. To find the sidereal time the hour angle of the vernal equinox must be measured. In some cases these quantities cannot be measured directly, so it is often necessary to measure other coördinates and to calculate the desired hour angle from these measurements. The **chronometer correction** or **watch correction** is the amount to be added algebraically to the reading of the timepiece to give the true time at the instant. It is positive when the chronometer is slow, negative when it is fast. The **rate** is the amount the timepiece gains or loses per day; it is positive when it is losing, negative when it is gaining.

72. Time by Transit of a Star.

The most direct and simple means of determining time is by observing transits of stars across the meridian. If the line of sight of a transit be placed so as to revolve in the plane of the meridian, and the instant observed when some known star passes the vertical cross hair, then the local sidereal time at this instant is the same as the right ascension of the star given in the Nautical Almanac for the date. The difference between the observed chronometer time t and the right ascension α is the chronometer correction T,

or $$T = \alpha - t. \qquad [82]$$

If the chronometer keeps mean solar time it is only necessary to convert the true sidereal time α into mean solar time by

Equa. [49], and the difference between the observed and computed times is the chronometer correction.

The transit should be set up and the vertical cross hair sighted on a meridian mark previously established. If the instrument is in adjustment the sight line will then swing in the plane of the meridian. It is important that the horizontal axis should be accurately levelled; the plate level which is parallel to this axis should be adjusted and centred carefully, or else a striding level should be used. Any errors in the adjustment will be eliminated if the instrument is used in both the direct and reversed positions, provided the altitudes of the stars observed in the two positions are equal. It is usually possible to select stars whose altitudes are so nearly equal that the elimination of errors will be nearly complete.

In order to find the star which is to be observed, its approximate altitude should be computed beforehand and set off on the vertical arc. (See Equa. [1].) In making this computation the refraction correction may be omitted, since it is not usually necessary to know the altitude closer than 5 or 10 minutes. It is also convenient to know beforehand the approximate time at which the star will culminate, in order to be prepared for the observation. If the approximate error of the watch is already known, then the watch time of transit may be computed (Equa. [49]) and the appearance of the star in the field looked for a little in advance of this time. If the data from the Nautical Almanac are not at hand the computation may be made, with sufficient accuracy for finding the star, by the following method: Compute the sun's R. A. by multiplying 4^m by the number of days since March 22. Take the star's R. A. from any list of stars or a star map. The star's R. A. minus the sun's R. A. (Equa. [49]) will be the mean local time within perhaps 2^m or 3^m. This may be reduced to Standard Time by the method explained in Art. 35. In the surveyor's transit the field of view is usually about 1°, so the star will be seen about 2^m before it reaches the vertical cross hair. Near culmination the star's path is so nearly horizontal that it will appear to coincide with the horizontal cross hair from one side of the field to the other. When the star passes the vertical cross hair the time should be noted as accurately as possible. A stop watch will sometimes be found convenient in field obversations with the surveyor's transit. When a chronometer is used the "eye and ear method" is the best. (See Art. 58.) If it is desired to determine the latitude from this same star, the observer has only to set the horizontal cross hair on the star immediately after making the time observation, and the reading of the vertical arc will give the star's apparent altitide at culmination. (See Art. 67.)

The computation of the watch correction consists in finding the true time at which the star should transit and comparing it with the observed watch time. If a sidereal watch or chronometer is used the star's right ascension is at once the local sidereal time. If mean time is desired, the true sidereal time must be converted into local mean solar time, or into Standard Time, whichever is desired.

Example.

Observed transit of *α Hydræ* $8^h\ 48^m\ 58^s.5$, Eastern time, in longitude $5^h\ 20^m$ west; date April 5, 1902. From the almanac, the star's R. A. $= 9^h\ 22^m\ 48^s.4$, and the sun's R. A. at G. M. N. $= 0^h\ 51^m\ 24^s.6$. To reduce this to the R. A. at local mean noon take from Table III the correction for $5^h\ 20^m$ which is $+52^s.6$. The corrected R. A. $= 0^h\ 52\ \ 17^s.2$. The local sidereal time, which is $9^h\ 22^m\ 48\ .4$, is then reduced to Standard Time as follows:

R. A. Star	$=9^h$	22^m	$48^s.4$
R. A. Sun	$=0$	52	$17\ .2$
	8	30	$31\ .2$
C'	$=$	1	$23\ .6$
Mean Local Time	$=8$	29	$07\ .6$
Red. to 75° merid.	$=$	20	$00\ .0$
Eastern Time	$=8$	49	$07\ .6$
Watch time	$=8$	48	$58\ .5$
Watch slow	$=$		$9^s.1$

Transit observations for the determination of time can be much more accurately made in low than in high latitudes. Near the pole the conditions are very unfavorable.

73. Observations with Astronomical Transit.

The method just described is in principle the one in most common use for determining sidereal time with the large astronomical transit. Since the precision attainable with the latter instrument is much greater than with the engineer's transit, the method must be correspondingly more refined. The number of observations on each star is increased by using a large number of vertical threads, commonly eleven. These times of transit are recorded by electric signals on the chronograph (see Art. 56, p. 93), and are scaled from the chronograph sheet to hundredths of a second. In this class of work many errors which have been assumed to be negligible in the preceding method are important and must be carefully determined and allowed for. The instrument has to be set into the plane of the meridian by means of repeated trials, and there is always a small remaining error in the azimuth of the sight line. This error in azimuth a is measured by comparing the observed times of rapidly moving (southern) stars and slowly moving (circumpolar) stars. The correction to any observed time for the effect of azimuth error is

$$a \cos h \sec D. \qquad [66]$$

The inclination of the axis to the horizon b is measured with the spirit level and the observed times are reduced to the meridian by adding the correction

$$b \sin h \sec D. \qquad [67]$$

The error in the sight line c is found by reversing the telescope in its supports and comparing observations made in the two positions. The correction to any observation is

$$c \sec D. \qquad [68]$$

Corrections are also made for the effect of diurnal aberration and sometimes other minor corrections.

74. Selecting Stars for Transit Observations.

Before the observations are begun the observer should prepare a list of stars suitable for transit observations. This list should include the name or number of the star, its magnitude, the approximate time of culmination, and its meridian altitude or its zenith distance. The right ascensions of consecutive stars in the list should differ by sufficient intervals to give the observer time to make and record an observation and prepare for the next one. The stars used for determining time should be those which have a rapid diurnal motion, that is, stars near the equator; slowly moving stars are not suitable for time determinations. Very faint stars should not be selected unless the telescope is of high power and good definition; those smaller than the fifth magnitude are rather difficult to observe with a small transit, especially as it is difficult to reduce the amount of light used for illuminating the field of view. The selection of stars will also be governed somewhat by a consideration of the effect of the different instrumental errors. An inspection of Table B, p. 88, will show that for stars near the zenith the azimuth error is zero, while the inclination error is a maximum; for stars near the horizon the azimuth error is a maximum and the inclination error is zero. If the azimuth of the instrument is uncertain and the inclination can be accurately determined, then stars having high altitudes should be preferred. On the other hand, if the level parallel to the axis is not a sensitive one and is in poor adjustment, and if the sight line can be placed accurately in the meridian, which is usually the case with a surveyor's transit, then low stars will give the more accurate results. With the surveyor's transit the choice of stars is somewhat limited, however, because it is not practicable to sight the telescope at much greater altitudes than about 70° with the use of the prismatic eyepiece and 55° or 60° without this attachment.

Following is a sample list of stars selected for observations in a place whose latitude is 40° N., longitude 77° W., date May 5, 1910, hours between 8^h and 9^h Eastern time; the limiting altitudes chosen are 10° and 65°. The right ascension of the mean sun for the date is $2^h\ 50^m$. Adding this to $8^h - 08^m = 7^h\ 52^m$, the local mean time, the resulting sidereal time is $10^h\ 42^m$, which is approximately the right ascension of a point on the meridian at 8^h E. S. T. The limiting right ascensions are therefore $10^h\ 42^m$ and $11^h\ 42^m$. The co-latitude is 50°, which gives, for altitudes 10° and 65°, the limiting declinations + 15° and − 40°. In the table of *mean places* for 1910 the following stars are given:

MEAN PLACES FOR 1910

Star.	Magn.	Rt. Asc.	Decl.
l Leonis	5.3	$10^h\ 44^m\ 32^s$	$+11^\circ\ 01'$
δ^2 *Chameleontis*	4.7	10 44 57	−80 04
46 *Leonis Minoris*	3.9	10 48 17	+34 42
Groombridge 1706	6.3	10 52 47	+78 15
α *Ursæ Majoris*	2.0	10 58 11	+62 14
η *Octantis*	6.1	10 59 58	−84 07
p^3 *Leonis*	6.2	11 02 19	+ 2 27
ψ *Ursæ Majoris*	3.2	11 04 37	+44 59
δ *Leonis*	2.7	11 09 19	+21 01
ν *Ursæ Majoris*	3.7	11 13 37	+33 35
δ *Crateris*	3.9	11 14 50	−14 17
τ *Leonis*	5.1	11 23 19	+ 3 21
λ *Draconis*	4.0	11 26 04	+69 50
ξ *Hydræ*	3.8	11 28 34	−31 22
υ *Leonis*	4.4	11 32 20	− 0 20
χ *Ursæ Majoris*	3.9	11 41 18	+48 17
β *Leonis*	2.2	11 44 28	+15 05

From this list there are found seven stars whose declinations and right ascensions fall within or very close to the required limits. In the following list the times of transit and the altitudes have been computed roughly but with sufficient accuracy to identify the stars.

OBSERVING LIST FOR TRANSIT OBSERVATIONS

Star.	Magn.	Approx. E.S.T.	Approx. Alt.
l Leonis	5.3	$8^h\ 00^m$	$61^\circ\ 01'$
p^3 *Leonis*	6.2	8 18	52 27
δ *Crateris*	3.9	8 30	35 43
τ *Leonis*	5.1	8 39	53 21
ξ *Hydræ*	3.8	8 44	18 38
υ *Leonis*	4.4	8 48	49 40
β *Leonis*	2.2	9 00	65 05

75. Time by Transit of the Sun.

The apparent solar time may be directly determined by observing the watch times when the west and east limbs of the sun cross the meridian. The mean of the two readings is the watch time for the instant of Local Apparent Noon or 12^h M. apparent time. This apparent time is to be converted into mean time and then into Standard Time. If only one limb of the sun can be observed the time of transit of the centre may be found by adding or subtracting the "time of semidiameter passing the meridian," which is given in the Nautical Almanac.

Example.

Observed transit of sun on Jan. 28, 1910, longitude $4^h\ 44^m\ 18^s$ W. Time of transit of W. limb = $11^h\ 54^m\ 53^s$; E. limb = $11^h\ 57^m\ 11^s$; mean of two limbs = $11^h\ 56^m\ 02^s.0$.

~~L. M. T.~~	=	12^h	00^m	00^s
Equa. T.	=		+ 13	03 .0
L. M. T.	=	12	13	03 .0
Red. to 75°	=		− 15	42 .0
E. S. T.	=	11	57	21 .0
Watch time	=	11	56	02 .0
Watch slow	=		1^m	$19^s.0$

Equa. of T. at G. A. N.	=	$13^m\ 00^s.71$
$0^s.485 \times 4^h.74$	=	2 .30
Cor'd. Equa. T.	=	$13^m\ 03^s.01$

76. Time by an Altitude of the Sun.

The apparent solar time may be determined by measuring the altitude of the sun when it is not near the meridian, and then solving the *PZS* triangle for the angle at the pole, which is the hour angle of the sun east or west of the meridian. The west hour angle of the sun is the local apparent time. The observation is made by measuring several altitudes in quick succession and noting the corresponding instants of time. The mean of the observed altitudes is assumed to correspond to the mean of the observed times, that is, the curvature of the path of the sun is neglected. The error caused by neglecting the correction for curvature is very small provided the sun is not near the meridian and the series of observations extends over but a few minutes' time, say 10^m. The measurement of altitude must of course be made to the upper or the lower limb and a correction applied for the semidiameter. The observations may be made in two sets, half the altitudes being taken on the upper limb and half on the lower limb, in which case no semidiameter correction is required. The telescope should be reversed between the two sets if the instrument has a complete vertical circle. The mean of the altitudes must be corrected for index error, refraction, and parallax, and for semidiameter if but one limb is observed. The declination must be corrected by adding to the declination at G. M. N. the hourly change multiplied by the number of hours since G. M. N. It is necessary for this purpose to know the approximate Greenwich Mean Time. If the watch used is keeping Standard Time the G. M. T. is found at once. (Art. 35.) If the watch is not more than 2^m or 3^m in error the effect on the computed declination will be negligible for observations made with small instruments. If the longitude is known the declination may be corrected by first computing an approximate value of the local time and adding this to the longitude, obtaining the approximate G. M. T. With this approximate G. M. T. the declination may be corrected and the whole computation repeated. It will seldom be necessary to make a third computation. In order to compute the hour angle the latitude of the place must be known. The hour angle of the sun's centre P is then found by means of one of the formulæ of Art. 19.* When the value of P is found it is converted into hours, minutes and seconds, and if the sun is east of the meridian it is subtracted from 12^h to obtain the local (civil) apparent time; if astronomical time is desired it should be subtracted from 24^h. This apparent time is then converted into mean time by adding or subtracting the equation of time.

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* If tables of log versed sines, in addition to the usual tables, are available, then the following formula will be found convenient:

$$\text{vers } P = \frac{\cos (L - D) - \sin h}{\cos L \cos D}. \qquad [83]$$

In case P is greater than 90° the formula below may be substituted:

$$\text{vers } P' = \frac{\sin h + \cos (L + D)}{\cos L \cos D}, \qquad [84]$$

$$\text{where } P' = 180° - P.$$

The sum or difference in the numerator must be computed with natural functions and the remainder of the computation performed by logarithms.

The equation of time must be corrected for the time elapsed since G. M. N. The resulting mean time is to be converted into Standard Time, to which the watch is regulated. The difference between the computed result and the mean of the observed watch readings is the watch correction.

The most favorable conditions for an accurate determination of time by this method are when the sun is on the prime vertical and when the observer is at the equator. When the sun is east or west it is rising or falling at its most rapid rate and an error of 1′ in the altitude produces less error in the calculated hour angle than does 1′ error when the sun is near the meridian. The nearer the observer is to the equator the greater is the inclination of the sun's path to the horizon, and consequently the greater its rise or fall per second of time. If the observer were at the equator and the declination zero, the sun would rise or fall 1′ in 4^s of time. In the example given below the rise is 1′ in about 8^s of time. When the observer is near the pole the method is practically useless. Observations on the sun when it is very close to the horizon should be avoided, however, even when the sun is near the prime vertical, because the errors in the tabulated refraction correction due to variations in the temperature and pressure of the air are likely to be large. Observations should not be made when the altitude is less than about 10° if this can readily be avoided.

Example.

Observation of Sun's Altitude for Time, Nov. 28, 1905. Lat. 42° 21′ N. Long. 71° 04.5′ W.

	Altitude		Watch (Eastern Time)
Lower limb	14° 41′		8^h 39^m 42^s A.M.
Tel. dir.	15° 00′		8 42 19
Upper limb	15° 55′		8 45 34
Tel. rev.	16° 08′		8 47 34
Mean	= 15° 26′.0	Mean	= 8^h 43^m $47^s.2$ A.M.
Refraction and parallax	= 3.3	G. M. T.	= 1^h 43^m $47^s.2$ (approx.)
	h = 15° 22′.7		

$L =$	$42° 21'.0$	sec 0.13133
$h =$	$15 \ 22.7$	
$p =$	$111 \ 15.7$	csc 0.03061
$2s =$	$178° 59'.4$	
$s =$	$84° 29'.7$	cos 8.98196
$s - h =$	$69° 07'.0$	sin 9.97049
		2)9.11439

$$\log \sin \tfrac{1}{2} P = 9.55719$$
$$\tfrac{1}{2} P = 21° \ 08' \ 45''$$
$$P = 42° \ 17' \ 30''$$
$$= 2^h \ 49^m \ 10^s.0$$

L. A. T.	$= 9^h \ 10^m \ 50^s.0$
Eq. t.	$= \ 12 \ 02.8$
M. L. T.	$= 8^h \ 58^m \ 47^s.2$
	$15 \ 42.0$
Eastern time	$= 8^h \ 43^m \ 05^s.2$
Watch time	$= 8 \ 43 \ 47.2$
Watch fast	$42^s.0$

Decl. at G. M. N.	=	$-21° \ 14' \ 54''$
$-26''.81 \times 1^h.73$	=	-46
Corrected decl.	=	$-21° \ 15' \ 40''$
p	=	$111° \ 15' \ 40''$

Eq. t.	=	$12^m \ 04^s.29$
$.846 \times 1.73$	=	1.46
		$12^m \ 02^s.83$

77. Time by the Altitude of a Star.

The method of the preceding article may be applied equally well to an observation on a star. In this case the parallax and semidiameter corrections are zero. If the star is west of the meridian the computed hour angle is the star's true hour angle; if the star is east of the meridian the computed hour angle must be subtracted from 24^h. The sidereal time is then found by adding the right ascension of the star to its hour angle. If mean time is desired the sidereal time thus found is to be converted into mean solar time by Art. 34. Since it is easy to select stars in almost any position it is desirable to eliminate errors in the measured altitudes by taking two observations, one on a star which is nearly due east, the other on one about due west. The mean of these two results will be nearly free from instrumental errors, and also from errors in the assumed value of the observer's latitude. If a planet is used it will be necessary to know the G. M. T. with sufficient accuracy for correcting the right ascension and declination.

Example.

Observed altitude of *Jupiter* (east), Jan. 9, 1907. Lat. = $42^\circ\ 18'.0$; Long. = $71^\circ\ 17'.5$ Eastern time = $7^h\ 32^m\ 02^s$

Observed altitude	$= 44^\circ\ 55'$		
Index correction	$= \quad -\ 1$	Decl. at G. M. N.	$= +\ 23^\circ\ 18'\ 22''.0$
Refraction correction	$= \quad -\ 1$	Hourly change	$= \quad +\ 1''.00$
		G. M. T.	$= 12^h\ 32^m\ 02^s$ (approx.)
	$h = 44^\circ\ 53'.0$*	$12^h.53 \times 1''.00$	$= \quad +\ 12''.5$
		Corrected decl.	$= +\ 23^\circ\ 18'\ 34''.5$
$L = 42^\circ\ 18'.0$		$p =$	$66^\circ\ 41'\ 25''.5$
$h = 44\ \ 53.0$			
$p = 66\ \ 41.4$		$s - L = 34^\circ\ 38'.2$	csc 0.24537
		$s - h = 32\ \ 03.2$	sin 9.72486
$2s = 152^\circ\ 112'.4$		$s - p = 10\ \ 14.8$	sec 0.00698
$s = 76\ \ 56.2$		$s = 76\ \ 56.2$	cos 9.35416
			2)9.33137
R. A. at G. M. N.	$= 6^h\ 19^m\ 17^s.3$	$\log \tan \frac{1}{2} P =$	9.66568
Hourly change	$= \quad -\ 1^s.395$	$\frac{1}{2} P =$	$24^\circ\ 50'\ 57''$
$12^h.53 \times -\ 1^s.395$	$= \quad -17^s.5$	$P =$	$49^\circ\ 41'\ 54''$
Corrected R. A.	$= 6^h\ 18^m\ 59^s.8$	$=$	$-3^h\ 18^m\ 47^s.6$
		$=$	$20^h\ 41^m\ 12^s.4$
		R. A. =	$6^h\ 18^m\ 59^s.8$
		Sid. Time =	$27^h\ 00^m\ 12^s.2$

The local sidereal time is therefore $3^h\ 00^m\ 12^s.2$ when the watch reading is $7^h\ 32^m\ 02^s$. The error of the watch may be found by reducing the sidereal time to Eastern Time.

78. Time by Transit of Star over Vertical Circle through Polaris.†

In making observations by this method the line of sight of the telescope is set in the vertical plane through *Polaris* at any (observed) instant of time, and the time of transit of some southern star across this plane is observed immediately afterward; the correction for reducing the star's right ascension to the true sidereal time of the observation is then computed and added to the right ascension. The advantages of the method are that the direction of the meridian does not have to be established before time observations can be begun, and that the interval which must elapse between the two observed times is so small that errors due to the instability of the instrument are reduced to a minimum.

The method of making the observation is as follows: Set up the instrument and level carefully; sight the vertical cross hair on *Polaris* (and clamp) and note and record the watch reading; then revolve the telescope about the horizontal axis,

* Parallax is negligible for this planet, as it is only about $2''$.

† This method is given by Mr. George O. James in the Jour. Assoc. Eng. Soc. Vol. XXXVII, No. 2. In a later paper (Popular Astronomy No. 172) Mr. James gives the formula

$$P = p \sec L \sin (L - D) \sec (D - c) \sin (P_o - P),$$

in which c is the correction from Table IV in the Nautical Almanac. This formula is preferable to that given in the text, provided the latitude is known, since it is not necessary to make a second approximation. A discussion of the method used with large instruments is given by Professor Frederick H. Sears in Bulletin No. 5, Laws Observatory, University of Missouri.

being careful not to disturb its azimuth; set off on the vertical arc the altitude of some southern star (called the *time-star*) which will transit about 4^m or 5^m later; note the instant when this star passes the vertical cross hair. It will be of assistance in making the calculations if the altitude of each star is measured immediately after the time has been observed. The altitude of the time-star at the instant of observation will be so nearly equal to its meridian altitude that no special computation is necessary beyond what is required for ordinary transit observations. If the times of meridian transit are calculated beforehand the actual times of transit may be estimated with sufficient accuracy by noting the position of *Polaris* with respect to the meridian. If *Polaris* is near its elongation then the azimuth of the sight line will be a maximum. In latitude 40° the azimuth of *Polaris* for 1910 is about 1° 32′; a star on the equator would then pass the vertical cross hair nearly 4^m later than the computed time if *Polaris* is at eastern elongation (see Table B). If *Polaris* is near western elongation the star will transit earlier by this amount. In order to eliminate errors in the adjustment of the instrument, observations should be made in the erect and inverted positions of the telescope and the two results combined. A new setting should be made on *Polaris* just before each observation on a time-star.

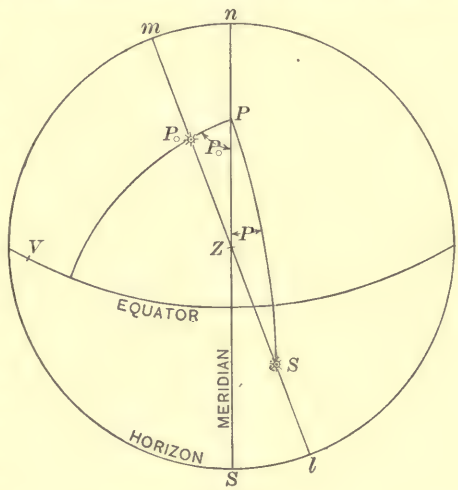

FIG. 55

In order to deduce an expression for the difference in time between the meridian transit and the observed transit let R and R_0 be the right ascensions of the stars, S and S_0 the sidereal times of transit over the cross hair, P and P_0 the hour angles of the stars, the subscripts referring to *Polaris*. Then by Equa. [37],

$$P = S - R$$

and

$$P_0 = S_0 - R_0;$$

subtracting,

$$P_0 - P = (R - R_0) - (S - S_0). \qquad [85]$$

The quantity $S - S_0$ is the observed interval of time between the two observations expressed in sidereal units. If an ordinary watch is used the interval must be reduced to sidereal units (Table III). Equa. [85] may then be written

$$P_0 - P = (R - R_0) - (T - T_0) - C, \qquad [86]$$

where T and T_0 are the actual watch readings and C is the correction to reduce this interval to sidereal time.

In Fig. 55 let P_0 be the position of *Polaris;* P, the celestial pole; Z, the zenith;

and S, the time-star. Also let Z and Z_0 represent the azimuths of the two stars; p and p_0, their polar distances; z and z_0, their zenith distances; and h and h_0, their altitudes. In the triangle PP_0Z,

$$\frac{\sin P_0}{\sin z_0} = \frac{\sin Z_0}{\sin p_0}, \tag{87}$$

and in the triangle PZS

$$\frac{\sin P}{\sin z} = \frac{\sin Z}{\sin p}. \tag{88}$$

Since the azimuth of the sight line has not changed,

$$smnl = 180° + sm$$

or $$Z = 180° + Z_0$$

and $$\sin Z = - \sin Z_0.$$

From Equa. [87] and [88], solving for sin P, there results

$$\sin P = - \sin p_0 \sin P_0 \sec D \cos h \sec h_0, \tag{89}$$

in which D is the declination of the time-star S. Since the angles P and p_0 are small they may be substituted for their sines, giving

$$P = - p_0 \sin P_0 \sec D \cos h \sec h_0. \tag{90}$$

In this equation the value of P_0 is unknown, and unless the local sidereal time is already known with accuracy it is necessary to determine P_0 by a series of approximations. A rough value of P_0 ($= P'_0$) may be found from the equation

$$P'_0 = (R - R_0) - (T - T_0) - C. \tag{91}$$

Using this value of P'_0 in Equa. [90] the result is P', an approximate value of P. A corrected value of P_0 is then obtained by the equation

$$P_0 = P'_0 + P'. \tag{92}$$

With this new value of P_0 a new value of P may be computed. If the second value of P differs much (say 5′) from the first value it will be necessary to make another computation of P_0. It is usually possible to make a rough estimate of P_0 from the known value of the local time. The watch time of the observation on *Polaris* may be converted into local sidereal time and the hour angle P_0 found by Equa. [47] and [37]. When a series of observations is made the hour angle of *Polaris* at all observations after the first may be closely estimated by adding to the value of P_0 at the first observation the time elapsed since the first observation on *Polaris*. If the altitudes of the stars have not been measured it is usually accurate enough to take $h = 90° - L - D$ (the meridian zenith distance) for the time-star, and for *Polaris* $h_0 = L + p_0 \cos P_0$, or better still $h_0 = L - c$, where c is the quantity given in Table IV at the end of the Nautical Almanac.

The final value of P, the hour angle of the time-star at the instant it was observed, is the correction to be added to the right ascension of the time-star to obtain the local sidereal time of the observation on this star. This sidereal time may then be reduced to mean time or to standard time and the watch correction obtained.

The above method is applicable to transit observations made with a small instrument. For the large astronomical transit a more refined method of making the reductions must be used.

Observation of *o Virginis* over Vertical Circle through *Polaris;* Lat., 42° 21′ N., Long., $4^h\ 44^m\ 18^s.3$ W.; Date, May 8, 1906.

$$\begin{array}{lrrr}
\text{Observed time on } Polaris & = 8^h & 35^m & 58^s \\
\text{Observed transit of } o\ Virginis & = 8 & 39 & 43 \\
\hline
\text{Diff.} & = & 3^m & 45^s
\end{array}$$

$$\begin{array}{rrrr}
R = & 12^h & 00^m & 26^s.3 \\
R_0 = & 1 & 24 & 35\ .4 \\
\hline
R - R_0 = & 10^h & 35^m & 50^s.9 \\
T - T_0 = & & 3 & 45\ .0 \\
C = & & & .6 \\
\hline
P'_0 = & 10^h & 32^m & 05^s.3 \\
= & 158^\circ & 01'.3 & \\
P' = & & -19'.8 & \\
\hline
P'_0 + P' = & 157^\circ & 41'.5 &
\end{array}
\qquad
\begin{array}{rrr}
L = & 42^\circ & 21' \\
D = + & 9 & 15 \\
\hline
L - D = & 33^\circ & 06'
\end{array}$$

$$\begin{array}{rl}
p_0 = & 71'.85 \\
\log p_0 = & 1.8564 \\
\log \sec D = & 0.0057 \\
\log \sin (L - D) = & 9.7373 \\
\log \sec (L - c) = & 0.1238 \\
\log \sin P'_0 = & 9.5732 \\
\hline
\log P' = & 1.2964\ n \\
P' = & -19'.79
\end{array}$$

The log sin of $(P'_0 + P') = 9.5793$; substituting this for log sin P'_0, the log P' is increased 61 units in the fourth place, giving $-20'.07$ for P. Converting this into time it is $-80^s.3$ or $-1^m\ 20^s.3$, the desired correction. The true sidereal time may now be found by subtracting $1^m\ 20^s.3$ from the right ascension of *o Virginis*. The complete computation of the watch correction is as follows:

$$\begin{array}{lrrr}
R = & 12^h & 00^m & 26^s.3 \\
P = & & -1 & 20\ .3 \\
\hline
S = & 11^h & 59^m & 06^s.0 \\
R_s = & 3 & 02 & 23\ .6 \\
\hline
 & 8^h & 56^m & 42^s.4 \\
C' = & & 1 & 27\ .9 \\
\hline
\text{M. L. T.} = & 8^h & 55^m & 14^s.5 \\
 & & 15 & 41\ .7 \\
\hline
\text{Eastern time} = & 8^h & 39^m & 32^s.8 \\
\text{Watch time} = & 8 & 39 & 43 \\
\hline
\text{Watch fast} = & & & 10^s.2
\end{array}$$

79. Time by Equal Altitudes of a Star.

If the altitude of a star is observed when it is east of the meridian at a certain altitude, and the same altitude of the same star again observed when the star is west of the meridian, then the mean of the two observed times is the watch reading for the instant of transit of the star. It is not necessary to know the actual value of the altitude employed, but it is essential that the two altitudes should be equal. The disadvantage of the method is that the interval between the two observations is inconveniently long.

80. Time by Two Stars at Equal Altitudes.

In this method the sidereal time is determined by observing when two stars have equal altitudes, one star being east of the meridian and the other west. If the two stars have the same declination then the mean of the two right ascensions is the sidereal time at the instant the two stars have the same altitude. As it is not practicable to find pairs of stars having exactly the same declination it is necessary to choose pairs whose declinations differ as little as possible and to introduce a correction for the effect of this difference upon the sidereal time. It is not possible to observe both stars directly with a transit at the instant when their altitudes are equal; it is necessary, therefore, to first observe one star at a certain altitude and to note the time, and then to observe the other star at the same altitude and again note the time. The advantage of this method is that the actual value of the altitude is not used in the computations; any errors in the altitude due either to lack of adjustment of the transit or to abnormal refraction are therefore eliminated from the result, provided the two altitudes are made equal. In preparing to make the observations it is well to compute beforehand the approximate time of equal altitudes and to observe the first star two or three minutes before the computed time. In this way the interval between the observations may be kept conveniently small. It is immaterial whether the east star is observed first or the west star first, provided the proper change is made in the computation. If one star is faint it is well to observe the bright one first; the faint star may then be more easily found by knowing the time at which it should pass the horizontal cross hair. The interval by which the second observation follows the time of equal altitudes is nearly the same as the interval between the first observation and the time of equal altitudes. It is evident that in the application of this method the observer must be able to identify the stars he is to observe. A star map is of great assistance in making these observations.

The observation is made by setting the horizontal cross hair a little above the easterly star 2^m or 3^m before the time of equal altitudes, and noting the instant when the star passes the horizontal cross hair. Before the star crosses the hair the clamp to the horizontal axis should be set firmly, and the plate bubble which is perpendicular to the horizontal axis should be centred. When the first observation has been made and recorded the telescope is then turned toward the westerly star, care being taken not to alter the inclination of the telescope, and the time when the star passes the horizontal cross hair is observed and recorded. It is well to note the altitude, but this is not ordinarily used in making the reduction. If the time of equal altitudes is not known, then both stars should be bright ones that are easily found in the telescope. The observer may measure an approximate altitude of first one and then the other, until they are at so nearly the same altitude that both can be brought into the field without changing the inclination of the telescope. The altitude of the east star may then be observed at once and the observation on the west star will follow by only a few minutes. If it is desired to observe the west star first, it must be observed at an altitude which is greater than when the west star is observed first. In this case the cross hair is set a little below the star.

In Fig. 56 let *nesw* represent the horizon, Z the zenith, P the pole, S_e the easterly star, and S_w the westerly star. Let P_e and P_w be the hour angle of S_e and S_w, and let HS_eS_w be an almucantar, or circle of equal altitudes.

From Equa. [37], for the two stars S_e and S_w, the sidereal time is

$$S = R_w + P_w$$
$$S = R_e - P_e.^*$$

Taking the mean value of S,

$$S = \frac{R_w + R_e}{2} + \frac{P_w - P_e}{2}, \quad [93]$$

from which it is seen that the true sidereal time equals the mean right ascension corrected by half the difference in the hour angles. To derive the equation for correcting the mean right ascension so as to obtain the true sidereal time let the fundamental equation

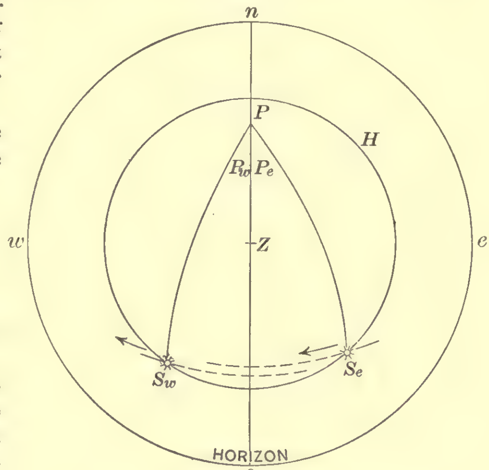

FIG. 56

$$\sin h = \sin D \sin L + \cos D \cos L \cos P \quad [8]$$

be differentiated regarding D and P as the only variables, then there results

$$0 = \sin L \cos D - \cos D \cos L \sin P \frac{dP}{dD} - \cos L \cos P \sin D,$$

from which may be obtained

$$\frac{dP}{dD} = \frac{\tan L}{\sin P} - \frac{\tan D}{\tan P}. \quad [94]$$

If the difference in the declination is small, dD may be replaced by $\frac{1}{2}(D_w - D_e)$, in which case dP will be the resulting change in the hour angle, or $\frac{1}{2}(P_w - P_e)$.

The equation for the sidereal time then becomes

$$S = \frac{R_w + R_e}{2} + \frac{D_w - D_e}{2}\left[\frac{\tan L}{\sin P} - \frac{\tan D}{\tan P}\right], \quad [95]$$

in which $(D_w - D_e)$ must be expressed in seconds of time. D may be taken as the mean of D_e and D_w. The value of P would be the mean of P_e and P_w if

* P_e is here taken as the actual value of the hour angle east of the meridian.

the two stars were observed at the same instant, but since there is an appreciable interval between the two times P must be found by

$$P = \frac{R_e - R_w}{2} + \frac{T_w - T_e}{2}. \qquad [96]$$

If the west star is observed first, then the last term becomes a negative quantity. Strictly speaking this last term should be converted into sidereal units, but the effect upon the result is usually very small. In regard to the sign of the correction to the mean right ascension it should be observed that if the west star has the greater declination the time of equal altitudes is later than that indicated by the mean right ascension. In selecting stars for the observation the members of a pair should differ in right ascension by 6 to 8 hours, or more, according to the declinations. Stars above the equator should have a longer interval between them than those below the equator. On account of the approximations made in deriving the formula the declinations should differ as little as possible. If the declinations do not differ by more than about 5°, however, the result will usually be close enough for observations made with the engineer's transit. From the extensive star list now given in the Nautical Almanac it is not difficult to select a sufficient number of pairs at any time for making an accurate determination of the local time. Following is a short list taken from the American Ephemeris and arranged for making an observation on April 30, 1912.

LIST FOR OBSERVING BY EQUAL ALTITUDES

Lat., 42° 21′ N. Long., $4^h\ 44^m\ 18^s$ W. Date, Apr. 30, 1912.

Stars.	Magn.	Sidereal time of equal altitudes.	Eastern time of equal altitudes.	Observed times.
α Corona Borealis	2.3	$10^h\ 28^m$	$7^h\ 38^m$	
β Tauri	1.8			
α Boötis	0.2	10 37	7 47	
ζ Geminorum	4			
α Boötis	0.2	10 48	7 58	
δ Geminorum	3.5			
ρ Boötis	3.6	11 00	8 10	
α² Geminorum	1.9			
π Hydræ	3.5	11 10	8 20	
ρ Argus	2.9			
β Herculis	2.8	11 19	8 29	
η Geminorum	3.5			
α Serpentis	2.7	11 35	8 45	
α Canis Minoris	0.5			
β Herculis	2.8	11 51	9 01	
δ Geminorum	3.5			
α Serpentis	2.7	12 02	9 12	
β Cancri	3.8			
α Serpentis	2.7	12 11	9 21	
ε Hydræ	3.5			
β Libræ	2.9	12 20	9 30	
α Hydræ	2.1			
β Herculis	2.8	12 32	9 42	
γ Cancri	4.9			

Following is an example of an observation for time by the method of equal altitudes.

Example.

Lat., 42° 21′ N. Long., $4^h\ 44^m\ 18^s$ W. Date, Apr. 14, 1905.

Star.	Rt. Asc.	Decl.	Watch.
α Ceti (E)	$2^h\ 57^m\ 22^s.1$	$+3°\ 43'\ 69''.1$	$5^h\ 18^m\ 00^s$
δ Aquilæ (W)	19 20 43 .6	+2 55 44 .0	5 22 13
Mean	$23^h\ 09^m\ 02^s.8$	$+3°\ 19'\ 56''.6$	$5^h\ 20^m\ 06^s.5$
Diff.	7 36 38 .5	2) $-0°\ 48'\ 25''.1$	04 13
	4 13 .7	$\frac{D_w - D_e}{2} = -24'\ 12''.6$	
	2) $7^h\ 40^m\ 52^s.1$	$= -96^s.84$	
$P =$	$3^h\ 50^m\ 26^s.1$		
	$= 57°\ 36'\ 31''.5$		

Mean R. A.	$= 23^h\ 09^m\ 02^s.8$		
Corr.	= −01 41 .0		
Sid. Time	$= 23^h\ 07^m\ 21^s.8$	$\log \frac{D_w - D_e}{2} = 1.9861$ (n)	1.9861 (n)
R_s	= 17 30 43 .2		
	$5^h\ 36^m\ 38^s.6$	$\log \tan L = 9.9598$	$\log \tan D = 8.7650$
$C' =$	55 .2	$\log \csc P = 0.0735$	$\log \cot P = 9.8024$
M. L. T.	$= 5^h\ 35^m\ 43^s.4$	= 2.0194 (n)	0.5535 (n)
	15 42 .0	$-104^s.6$	$-3^s.6$
		− 3 .6	
Eastern time	$= 5^h\ 20^m\ 01^s.4$	Corr. $= -101^s.0$	
Watch time	= 5 20 06 .5		
Watch fast	$= 5^s.1$		

81. Formula [94] may be made practically exact by means of the following device. Applying Equa. [8] to each star separately and subtracting one result from the other we obtain the equation *

$$\sin \Delta P = \frac{\tan L \tan \Delta D}{\sin P} - \frac{\tan D \tan \Delta D}{\tan P} + \frac{\tan D \tan \Delta D}{\tan P} \text{ vers } \Delta P, \quad [97]$$

where ΔD is half the difference in the declinations and ΔP is the correction to the mean right ascension. If $\sin \Delta P$ and $\tan \Delta D$ are replaced by their arcs and the third term dropped, this reduces to Equa. [94], except that ΔD and ΔP are finite differences instead of infinitesimals. In order to compensate for the errors thus produced let ΔD be increased by a quantity equal to the difference between the arc and the tangent (Table C); and let a correction be added to the sum of the first two terms to allow for the difference between the arc and sine of ΔP (Table C). With the approximate value of ΔP thus obtained the third

* Chauvenet, *Spherical and Practical Astronomy*, Vol. I, p. 199.

term of the series may be taken from Table D. By this means the precision of the computed result may be increased, and the limits of ΔD may therefore be extended without increasing the errors arising from the approximations.

TABLE C. CORRECTIONS TO BE ADDED TO ΔD AND ΔP

(Equa. [99], Art. 81)

Arc or sine.	Correction to ΔD.	Correction to ΔP.	Arc or sine.	Correction to ΔD.	Correction to ΔP.
s	s	s	s	s	s
100	0.00	0.00	800	0.90	0.45
200	0.01	0.01	850	1.08	0.54
300	0.05	0.02	900	1.29	0.64
400	0.11	0.06	950	1.51	0.76
500	0.22	0.11	1000	1.77	0.88
600	0.38	0.19	1050	2.05	1.02
650	0.48	0.24	1100	2.35	1.17
700	0.60	0.30	1150	2.69	1.34
750	0.74	0.37	1200	3.06	1.52

TABLE D. CORRECTION TO BE ADDED TO ΔP *

(Equa. [99], Art. 81)

	ΔP (in seconds of time).									
2d term.	100^s	200^s	300^s	400^s	500^s	600^s	700^s	800^s	900^s	1000^s
s	s	s	s	s	s	s	s	s	s	s
100	0.00	0.01	0.02	0.04	0.07	0.10	0.13	0.17	0.21	0.26
200	0.01	0.02	0.05	0.08	0.13	0.19	0.26	0.34	0.43	0.53
300	0.01	0.03	0.07	0.13	0.20	0.29	0.39	0.51	0.64	0.79
400	0.01	0.04	0.10	0.17	0.26	0.38	0.52	0.68	0.86	1.06
500	0.01	0.05	0.12	0.21	0.33	0.48	0.65	0.85	1.07	1.32
600	0.02	0.06	0.14	0.25	0.40	0.57	0.78	1.02	1.28	1.59
700	0.02	0.07	0.17	0.30	0.46	0.67	0.91	1.18	1.50	1.85
800	0.02	0.08	0.19	0.34	0.53	0.76	1.04	1.35	1.71	2.11
900	0.02	0.10	0.21	0.38	0.59	0.86	1.17	1.52	1.93	2.38
1000	0.03	0.11	0.24	0.42	0.66	0.95	1.30	1.69	2.14	2.64
1100	0.03	0.12	0.26	0.47	0.73	1.05	1.42	1.86	2.36	2.91
1200	0.03	0.13	0.29	0.51	0.79	1.14	1.55	2.03	2.57	3.17

* The algebraic sign of this term is always opposite to that of the second term.

Example.

Compute the time of equal altitudes of α *Boötis* and ι *Geminorum* on Jan. 1, 1912, in latitude $42° 21'$. R. A. α *Boötis* = $14^h\ 11^m\ 37^s.98$; decl. = $+19° 38' 15''.2$. R. A. ι *Geminorum* = $7^h\ 20^m\ 16^s.85$; decl. = $+27° 58' 30''.8$.

$$\begin{array}{r}
14^h\ 11^m\ 37^s.98 \\
7\quad 20\quad 16\ .85 \\
\hline
2)\ \underline{6^h\ 51^m\ 21^s.13} \\
3^h\ 25^m\ 40^s.56 \\
P = 51° 25'\ \ 08''.4
\end{array}
\qquad
\begin{array}{r}
27°\ 58'\ 30''.8 \\
19\quad 38\quad 15\ .2 \\
\hline
2)\ \underline{8°\ 20'\ \ 15''.6} \\
\Delta D = 4°\ 10'\ \ 07''.8 \\
= 1000^s.52 \\
\text{Corr., Table C} = \quad 1\ .77 \\
\hline
\Delta D = 1002^s.29
\end{array}$$

$$\begin{array}{lr}
\log \Delta D & = 3.000993 \\
\log \tan L & = 9.959769 \\
\log \csc P & = 0.106945 \\
\hline
 & 3.067707 \\
 & 1168^s.71 \\
 & 352\ .76 \\
\hline
 & 815^s.95 \\
\text{Corr., Table C} & = \ +\ .48 \\
\text{Corr., Table D} & = \ +\ .63 \\
\hline
\Delta P & = +\ 817^s.06 \\
 & = +\quad 13^m\ 37^s.06 \\
\text{Mean R. A.} & = 10\quad 45\quad 57.42 \\
\hline
\text{Sid. Time of Equal Alt.} & = 10^h\ 59^m\ 34^s.48
\end{array}
\qquad
\begin{array}{lr}
\log \Delta D & = 3.00099 \\
\log \tan D & = 9.64462 \\
\log \cot P & = 9.90187 \\
\hline
 & 2.54748 \\
 & 352.76
\end{array}$$

For refined observations the inclination of the vertical axis should be measured with a spirit level and a correction applied to the observed time. With the engineer's transit the only practicable way of doing this is by means of the plate-level which is parallel to the plane of motion of the telescope. If both ends of this level are read at each observation, O denoting the reading of the object end and E the eye end of the bubble, then the change in the inclination is expressed by

$$i = \Big((O - E) - (O' - E') \Big) \times \frac{d}{2},$$

where d is the angular value of one scale division in seconds of arc. The correction to the mean watch reading is

$$\text{Corr.} = \frac{i}{30 \sin S \cos D} = \frac{i}{30 \cos L \sin Z},$$

in which S may be taken from the Azimuth* tables or Z may be found from the measured horizontal angle between the stars. If the west star is observed at a higher altitude than the east star (bubble nearer objective), the correction must be added to the mean watch reading. If it is applied to the mean of the right ascensions the algebraic sign must be reversed.

---

* See Arts. 82 and 109 for the method of using these tables.

82. The correction to the mean right ascension of the two stars may be conveniently found by the following method, provided the calculation of the parallactic angle, S in the PZS triangle, can be avoided by the use of tables. Publication No. 120 of the U. S. Hydrographic Office gives value of the azimuth angle for every whole degree of latitude and declination and for every 10^m of hour angle. The parallactic angle may be obtained from these tables (by interpolation) by interchanging the latitude and the declination, that is, by looking up the declination at the head of the page and the latitude in the line marked "Declination." For latitudes under 23° it will be necessary to use Publication No. 71.

In taking out the angle the table should be entered with the next less whole degree of latitude and of declination and the next less 10^m of hour angle, and the corresponding tabular angle written down; the proportional parts for minutes of latitude, of declination, and of hour angle are then taken out and added algebraically to the first angle. The result may be made more accurate by working

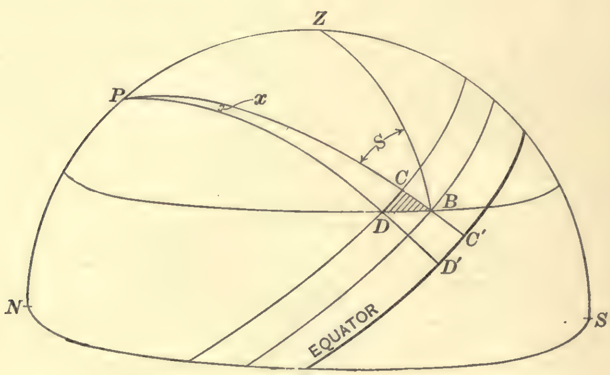

FIG. 57

from the nearest tabular numbers instead of the next less. The instructions given in Pub. 120 for taking out the angle when the latitude and declination are of opposite sign should be modified as follows. Enter the table with the **supplement** of the hour angle, the latitude and declination being interchanged as before, and the tabular angle is the value of S sought.

Suppose that two stars have equal declinations and that at a certain instant their altitudes are equal, A being east of the meridian and B west of the meridian. If the declination of B is increased so that the star occupies the position C, then the star must increase its hour angle by a certain amount x in order to be again on the almucantar through B. Half of the angle x is the desired correction. In Fig. 57 BC is the increase in declination; BD is the almucantar through A, B and D; and CD is the arc of the parallel of declination through which the star must move in order to reach BD. The arcs BD and CD are not arcs of great circles, and the triangle BCD is not strictly a spherical triangle, but it may be shown that the error is usually negligible in observations made with the engineer's transit if BCD is computed as a spherical triangle or even as a plane triangle. The angle ZBP is the angle S and DBC is $90° - S$. The length of the arc CD is then BC cot S, or $(D_w - D_e)$ cot S. The angle at P is the same as the arc $C'D'$ and equals CD sec D. If $(D_w - D_e)$ is expressed in minutes of arc and the correction is to be in seconds of time, then, remembering that the correction is half the angle x,

$$\text{Correction} = 2\,(D_w - D_e) \cot S \sec D. \tag{98}$$

D should be taken as the mean of the two declinations, and the hour angle, used in finding S, is half the difference in right ascension corrected for half the watch interval.

The trigonometric formula for determining the correction for equal altitudes is

$$\tan \frac{\Delta P}{2} = \sin \frac{\Delta D}{2} \cot \tfrac{1}{2}\,(S_1 + S_2) \sec \tfrac{1}{2}\,(D_1 + D_2). \tag{99}$$

By substituting arcs for the sine and tangent this reduces to the equation given above, except that the mean of S_1 and S_2 is not exactly the same as the value of S obtained by using the mean of the hour angles.

The example on p. 131 worked by this method is as follows. From the azimuth tables, using a declination of 42°, latitude 3°, and hour angle $3^h\ 50^m$, the approximate value of S is 44° 05′. Then from the tabular differences,—

Correction for 21′ decl. $= -\ 22'$
Correction for 20′ lat. $= +\ 07$
Correction for 26^s h. a. $= +\ 02$

The corrected value of S is therefore 43° 52′

$$
\begin{aligned}
2\,(D_w - D_e) = -\ 96'.84 \quad \log &= 1.9861\ (n)\\
\log \cot S &= 0.0172\\
\log \sec D &= 0.0007\\
\hline
\log \text{corr.} &= 2.0040\ (n)\\
\log \text{corr.} &= -\ 100^s.9
\end{aligned}
$$

This solution is sufficiently accurate for observations made with the engineer's transit, provided the difference in the declinations of the two stars is not greater than about 5° and the other conditions are favorable. For larger instruments and for refined work this formula is not sufficiently exact.

The equal-altitude method, like all of the preceding methods, gives more precise results in low than in high latitudes.

83. Rating a Watch by Transit of a Star over a Range.

If the time of transit of a fixed* star across some well-defined range can be observed, the rate of a watch may be quite accurately determined without knowing its actual error. The disappearance of the star behind a building or other object

* A planet should not be used for this observation.

when the eye is placed at some definite point will serve the purpose. The star will pass the range at the same instant of sidereal time every day. If the watch keeps sidereal time, then its reading should be the same each day at the time of the star's transit over the range. If the watch keeps mean time it will lose $3^m\ 55^s.91$ per sidereal day, so that the readings on successive days will be less by this amount. If, then, the passage of the star be observed on a certain night, the time of transit on any subsequent night is computed by multiplying $3^m\ 55^s.91$ by the number of days intervening and subtracting this correction from the observed time. The difference between the observed and computed times divided by the number of days is the daily gain or loss. After a few weeks the star will cross the range in daylight, and it will be necessary before this occurs to transfer to another star which transits later in the same evening. In this way the observations may be carried on indefinitely.

84. Time Service.

The Standard Time used for general purposes in this country is determined by observations at Washington and is sent out to all parts of the country east of the Rocky Mountains by means of electric signals transmitted over the lines of the telegraph companies. For the territory west of the Rocky Mountains the time is determined at the Mare Island Navy Yard and distributed by telegraphic signals. The error of the sidereal clock of the observatory is determined at frequent intervals by observing star transits. The sidereal clock is then compared with a mean-time clock, by means of a chronograph, and the error of this clock on mean time is computed. The mean-time clock is then compared with another mean-time clock especially designed for sending the automatic signals. When the error of this sending clock is found it is "set" (to Eastern Standard Time) by accelerating or retarding the motion of the pendulum until the error is reduced to a negligible quantity. The series of signals sent out each day begins at $11^h\ 55^m$ A.M., Eastern time, and continues for five minutes. The clock mechanism is arranged to break the circuit at the end of each second; this makes a click on every telegraph sounder on the line, or a notch on the sheet of a chronograph placed in the circuit. The end of each minute is shown by the omission of the 55th to 59th seconds inclusive, except for the noon signal, which is preceded by a ten-second interval. During this ten-second interval the local circuits controlling the time-balls,* which are dropped by this same signal, are thrown into the main circuit. The signals sent out in this way are seldom in error by an amount greater than one or two tenths of a second. The break in the circuit which occurs at the instant of noon, Eastern time, drops all the time-balls, corrects the clocks placed in the circuit, and gives a click on every telegraph sounder on the line. In many seaports the wireless telegraph lines are also thrown into the circuit and the signal thus made available at sea.

Questions and Problems

1. Compute the approximate Eastern time of transit of *Regulus* over the meridian $71° \ 04'.5$ West of Greenwich on March 21, 1908. The R. A. of *Regulus* is $10^h \ 03^m \ 29^s.1$; R_s at G. M. N. = $23^h \ 54^m \ 23^s.99$.

2. Compute the error of the watch from the data given in prob. 6, p. 169.

3. Observed time of transit of δ *Capricorni* over the vertical circle through *Polaris*, Oct. 26, 1906. Latitude = $42° \ 18'.5$; longitude = $4^h \ 45^m \ 07^s$. Observed watch time of transit of *Polaris* = $7^h \ 10^m \ 20^s$; of δ *Capricorni* = $7^h \ 13^m \ 28^s$, Eastern Time. Declination of *Polaris* = $+ \ 88° \ 48' \ 31''.3$; right ascension = $1^h \ 26^m \ 37^s.9$. Declination of δ *Capricorni* = $- \ 16° \ 3.3' \ 02''.8$; right ascension = $21^h \ 41^m \ 53^s.3$. The right ascension of the Mean Sun at Local Mean Noon = $14^h \ 16^m \ 34^s.6$. Compute the error of the watch on Eastern Time.

4. Time observation on May 3, 1907, in latitude $42° \ 21'.0$, longitude $4^h \ 44^m \ 18^s.0$. Observed transit of *Polaris* = $7^h \ 16^m \ 17^s.0$; of μ *Hydræ* = $7^h \ 18^m \ 50^s.5$. Decl. of *Polaris* = $+ \ 88° \ 48' \ 28''.3$; R. A. = $1^h \ 24^m \ 50^s.2$. Decl. of μ *Hydræ* = $- \ 16° \ 21' \ 53''.2$; R. A. = $10^h \ 21^m \ 36^s.1$. R. A. of Mean Sun at G. M. N. = $2^h \ 40^m \ 56^s.63$. Find the error of the watch.

5. Observation for time by equal altitudes, Dec. 18, 1904.

	R. A.	Decl.	Watch.
α *Tauri* (E)	$4^h \ 30^m \ 29^s.01$	$+ \ 16° \ 18' \ 59''.9$	$7^h \ 34^m \ 56^s$
α *Pegasi* (W)	22 59 61 .12	+ 14 41 43 .7	7 39 45

Lat. = $42° \ 28'.0$; long. = $4^h \ 44^m \ 15^s.0$. R. A. Mean Sun at G. M. N. = $17^h \ 46^m \ 40^s.38$.

---

* Time-balls are now in use in the principal ports on the Atlantic, Pacific, and Gulf coasts and on the Great Lakes.

6. Time by equal altitudes, Oct. 13, 1906.

	R. A.	Decl.	Watch
ν *Ophiuchi* (W)	$17^h\ 53^m\ 52^s.15$	$-9°\ 45'\ 34''.6$	$7^h\ 13^m\ 49^s$
ι *Ceti* (E)	$0\ \ 14\ \ 40\ .99$	$-9\ \ 20\ \ 25\ .7$	$7\ \ 28\ \ 25$

Lat. = $42°\ 18'$; long. = $4^h\ 45^m\ 06^s.8$. R. A. of Mean Sun at G. M. N. = $13^h\ 24^m\ 32^s.56$.

7. Show by differentiating Equa. [8] that the most favorable position of the sun for a time observation is on the prime vertical. The differential coefficients $\frac{dP}{dh}$ and $\frac{dP}{dL}$ should be a minimum to give the greatest accuracy. The expressions obtained may be simplified by means of Equa. [12].

8. Compute the watch correction from the observation given on p. 124. The R. A. of the mean sun on Jan. 9, 1907, was $19^h\ 11^m\ 29^s.49$.

CHAPTER XII

OBSERVATIONS FOR LONGITUDE

85. Method of Measuring Longitude.

The measurement of the difference in longitude of two places depends upon a comparison of the local times of the places at the same absolute instant of time. One important method is that in which the timepiece is carried from one station to the other and its error on local time determined in each place. The most precise method, however, and the one chiefly used in geodetic work, is the telegraphic method, in which the local times are compared by means of electric signals sent through a telegraph line. Other methods, most of them of inferior accuracy, are those which depend upon a determination of the moon's position (moon culminations, eclipses, occultations) and upon eclipses of Jupiter's satellites, and those in which terrestrial signals are employed.

86. Longitude by Transportation of Timepiece.

In this method the error of the watch or chronometer with reference to the first meridian is found by observing the local time at the first station. The rate of the timepiece should be determined by making another observation at the same place at a later date. The timepiece is then carried to the second station and its error determined with reference to this meridian. If the watch runs perfectly the two watch corrections will differ by just the difference in longitude. Assume that the first observation is made at the easterly station and the second at the westerly station. To correct for rate, let r be the daily rate in seconds, $+$ when losing $-$ when gaining, c the watch correction at the east station, c' the watch correction at the west station, d the number of days between the observations, and T the watch reading at the second observation. Then the difference in the longitude is found as follows:

$$\text{Local time at W. station} = T + c'$$
$$\text{Local time at E. station} = T + c + dr$$
$$\text{Diff. in time} = \text{Diff. in Long.} = c + dr - c'. \qquad [100]$$

The same result will be obtained if the stations are occupied in the reverse order.

If the error of a mean-time chronometer or watch is found by star observations, it is necessary to know the longitudes accurately enough to correct the sun's right ascension. If a sidereal chronometer is used and its error found on L. S. T. this correction is rendered unnecessary.

In order to obtain a check on the rate of the timepiece the observer should, if possible, return to the first station and again determine the local time. If the rate is uniform the error in its determination will be eliminated by taking the mean of the results. This method is not as accurate as the telegraphic method, but if several chronometers are used and several round trips between stations are made it will give good results. It is useful at sea and in exploration surveys.

Example.

Observations for local mean time at meridian A indicate that the watch is $15^m\ 40^s$ slow. At a point B, west of A, the watch is found to be $14^m\ 10^s$ slow on local mean time. The watch is known to be gaining 8^s per day. The second observation is made 48 hours after the first. The difference in longitude is therefore

$$+ 15^m\ 40^s - 2 \times 8^s - 14^m\ 10^s = 1^m\ 14^s.$$

The meridian B is therefore $1^m\ 14^s$ or $18'\ 30''$ west of meridian A.

87. Longitude by the Electric Telegraph.

In the telegraphic method the local sidereal time is accurately determined by star transits observed at each of the stations. The observations are made with large portable transits and are recorded on chronographs which are connected with break-circuit chronometers. The stars observed are chosen in such a manner as to determine the errors of the instruments so that these may be eliminated from the results as completely as possible. Some of the stars are slowly moving (circumpolar) stars and others are more rapidly moving stars near the zenith; a comparison of these two makes it possible to compute the azimuth of the line of collimation. Half of the stars are observed with the instrument in one position, half in the reversed position; this determines the error in the sight line. The inclination error is measured with the striding level.

After the corrections to the two chronometers have been accurately determined the two chronographs are switched into the main-line circuit and signals are sent by breaking the circuit a number of times by pressing a telegraph key. These signals are recorded on both chronographs. In order to eliminate the error due to the time required in transmitting a signal,* these signals are sent first in one direction (E - W) and afterward in the opposite direction (W - E). In this manner the transmission time is eliminated, provided it is constant. The personal errors of the observers are eliminated by the observers exchanging places in the middle of the series; i.e., the above operation would be repeated for about five nights with the observers in one position and then for five nights after the observers have exchanged positions. After all of the observations have been corrected for instrumental errors, and the error of the chronometer on local sidereal time is known, each signal sent over the main line will be found to correspond to a certain instant of sidereal time at the east station and a different instant of sidereal time at the west station. This difference is the difference in longitude. The mean of all these values is the final difference free from errors in transmission time and personal errors. By this method the difference in longitude may be determined with an error of perhaps 10 to 20 feet on the earth's surface.

88. Longitude by Transit of the Moon.

A method which is easily used with the surveyor's transit and which, although not precise, may be of use in exploration surveys, is that of determining the moon's right ascension by observing its transit over the meridian. The right ascension of the moon's centre is tabulated in the Nautical Almanac for every hour of Greenwich Mean Time; hence, if the right ascension can be determined, the Greenwich time can be computed. A comparison of this with local time gives the longitude.

The observation consists in placing the instrument in the plane of the meridian and noting the time of transit of the bright limb † of the moon and also of several stars whose declinations are nearly the same as that of the moon. The observed time interval between the moon's transit and that of a star (reduced to sidereal time if necessary), added to or subtracted from the star's right ascension, gives the right ascension of the moon's limb. A value of the right ascension is obtained

* In a test made in 1905 it was found that the time signal sent from Washington reached Lick Observatory, Mt. Hamilton, Cal., in $0^s.05$.

† The table of moon culmination in the Ephemeris shows which limb (I or II) may be observed. See also note, p. 143.

from each star and the mean value used. To obtain the right ascension of the centre of the moon it is necessary to apply to the right ascension of the limb a correction taken from the Ephemeris called "sidereal time of semidiameter passing meridian." In computing this correction the increase in the right ascension during this short interval has been allowed for; so the result is not the right ascension of the centre at the instant of the observation, but its right ascension at the instant of the transit of the centre over the meridian. If the west limb was observed this correction must be added; if the east limb was observed it must be subtracted. The result is the right ascension of the centre at the instant of transit, which is also the *local sidereal time* at that instant. Then the Greenwich Mean Time corresponding to this instant is found by interpolating in the table giving the moon's right ascension for every hour. To obtain the G. M. T. by simple interpolation find the next less right ascension in the table and the "diff. for 1^m" on the same line; subtract the tabular right ascension from the given right ascension (found from the observation) and divide this difference by the "diff. for 1^m." The result is the number of minutes and decimals of minutes to be added to the hour of G. M. T. opposite the tabular right ascension used. If the "diff. for 1^m" is varying rapidly it will be more accurate to interpolate as follows. Interpolate between the two values of the "diff. for 1^m" and obtain a "diff. for 1^m" which corresponds to the *middle* of the interval over which the interpolation is carried. In observations made with the surveyor's transit this more accurate interpolation is seldom necessary.

In order to compare the Greenwich time with the local time it is necessary to convert the G. M. T. just obtained into the corresponding instant of Greenwich Sidereal Time. The difference between this and the local sidereal time is the longitude from Greenwich.

In preparing for observations of the moon's transit the Nautical Almanac should be consulted (Table of Moon Culminations) to see whether an observation can be made and to find the approximate time of transit. The civil date should be converted into astronomical before entering the Almanac. The time of the moon's transit may be taken from the column headed "Mean time of transit" and corrected for longitude, or it may be computed from the approximate right ascension. The altitude of the moon should be computed as for a star, and in addition the parallax correction should be applied. The moon's parallax is so large that the moon probably would not be in the field of the telescope at all if this correction were neglected. The horizontal parallax multiplied by the cosine of the altitude is the correction to be applied; the moon will appear lower than it would if seen at the centre of the earth, so the correction is negative.

Since the moon increases its right ascension about 2^s in every 1^m of time it is evident that any error in determining the right ascension will produce an error about thirty times as great in the longitude, so that this method cannot be made to give very precise results.

Following is an example of an observation for longitude by the method of moon culminations made with an engineer's transit.

Example.

Observed transit of Moon on Jan. 9, 1900, for longitude. Moon's west limb passed cross hair at $6^h\ 59^m\ 37^s.7$; δ *Ceti* passed at $7^h\ 02^m\ 57^s.0$; and γ *Ceti* passed at $7^h\ 06^m\ 42^s.0$.

δ *Ceti*	$7^h\ 02^m\ 57^s.0$	γ *Ceti*	$7^h\ 06^m\ 42^s.0$
Moon's Limb	$6\ \ 59\ \ 37\ .7$	Moon's Limb	$6\ \ 59\ \ 37\ .7$
	$03^m\ 19^s.3$		$07^m\ 04^s.3$
	$.55$		$1\ .16$
Sid. int.	$=\ 03\ \ 19\ .85$	Sid. int.	$=\ 07^m\ 05^s.46$
R. A. δ *Ceti*	$= 2\ \ 34\ \ 23\ .02$	R. A. γ *Ceti*	$= 2\ \ 38\ \ 08\ .77$
R. A. M.'s Limb	$= 2^h\ 31^m\ 03^s.17$	R.A.M.'s Limb	$= 2^h\ 31^m\ 03^s.31$
	$2\ \ 31\ \ 03\ .31$		
Mean	$= 2^h\ 31^m\ 03^s.24$		
Time of s. d. passing merid.	$=\ 1\ \ 08\ .86$		
R. A. M.'s Centre	$= 2^h\ 32^m\ 12^s.10$	= L. S. T.	

From the Nautical Almanac.

G. M. T.	R. A. Moon	Diff. for 1^m
11^h	$2^h\ 29^m\ 55^s.77$	2.2748
12	$2\ \ 32\ \ 12\ .32$	2.2767

	$32^m\ 12^s.10$			
	$29\ \ 55\ .77$			
	$2^m\ 16^s.33$	$= 136^s.33$	log 2.13459	G. M. T. $= 11^h\ 59^m\ 52^s.86$
Interpolated Diff. 1^m		$= 2.2767$	log 0.35730	R.A.M.S. $= 19\ \ 14\ \ 15\ .92$
				$1\ \ 52\ .86$
M. T. Interval		$= 59^m.881$	log 1.77729	
				G. S. T. $= 7^h\ 16^m\ 01^s.64$
G. M. T.		$= 11^h\ 59^m\ 52^s.86$		L. S. T. $= 2\ \ 32\ \ 12\ .10$
				Long. W. $= 4^h\ 43^m\ 49^s.54$

NOTE. It has already been stated that the moon moves eastward on the celestial sphere at the rate of about 13° per day; as a result of this motion the time of meridian passage occurs about 51^m later (on the average) each day. On account of the eccentricity of its orbit, however, the actual retardation may vary considerably from the mean. The moon's orbit is inclined at an angle of about 5° 08′ to the plane of the earth's orbit. The line of intersection of these two planes rotates in a similar manner to that described under the precession of the equinoxes, except that its period is only 19 years. The moon's maximum declination, therefore, varies from 23° 27′ + 5° 08′ to 23° 27′ − 5° 08′, that is, from 28° 35′ to 18° 19′, according to the relative position of the plane of the moon's orbit and the plane of the equator. The rapid changes in the relative position of the sun, moon, and earth, and the consequent changes in the amount of the moon's surface that is visible from the earth, cause the moon to present the different aspects known as the moon's **phases.** Fig. 58 shows the relative positions of the three bodies at several

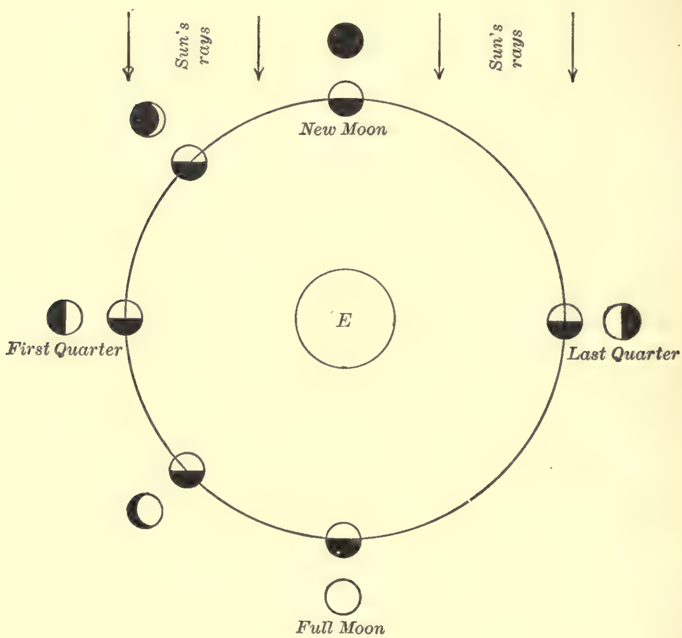

FIG. 58. THE MOON'S PHASES

different times in the month. The appearance of the moon as seen from the earth is shown by the figures around the outside of the diagram.

It may easily be seen from the diagram that at the time of first quarter the moon will cross the meridian at about 6 P.M.; at full moon it will transit at midnight; and at last quarter it will transit at about 6 A.M. Although the part of the illuminated hemisphere which can be seen from the earth is continually changing, the part of the moon's surface that is turned toward the earth is always the same, because the moon makes but one rotation on its axis in one lunar month. Nearly half of the moon's surface is never seen from the earth.

Questions and Problems.

1. Compute the longitude from the following observed transits: θ *Aquarii*, $5^h\ 16^m\ 04^s$; π *Aquarii*, $5^h\ 24^m\ 40^s$; moon's W. limb, $5^h\ 32^m\ 27^s$; λ *Aquarii*, $5^h\ 51^m\ 47^s$. R. A. θ *Aquarii* $= 22^h\ 11^m\ 27^s.6$; R. A. π *Aquarii*, $= 22^h\ 20^m\ 04^s.6$; R. A. λ *Aquarii* $= 22^h\ 47^m\ 18^s.3$; sidereal time of semidiameter passing meridian $= 60^s.3$; at G. M. T. 10^h, moon's R. A. $= 22^h\ 27^m\ 53^s.3$; diff. for $1^m = 1^s.9800$; R. A. mean sum at G. M. N. $= 16^h\ 38^m\ 28^s.0$.

2. Can the longitude be computed by comparing G. M. T. and L. M. T.?

3. Which limb can be observed in a P. M. observation of a moon culmination ?

4. At about what time (mean local) will the moon transit at first quarter ?

CHAPTER XIII

OBSERVATIONS FOR AZIMUTH

89. Determination of Azimuth.

The determination of the azimuth of a line is of frequent occurrence in the practice of the surveyor, and is the most important to him of all the astronomical problems. On account of the high altitudes of the objects observed, as compared with those observed in surveying, the adjustments of the instrument and the elimination of errors are of unusual importance in these observations. All of the precautions mentioned in Chapter VIII in regard to stability of the instrument, etc., should be carefully observed: the instrument should be allowed to stand for some time before observations are begun; temperature changes from any source, such as heat from the lamp or from the hand, are to be avoided; the clamps and tangent screws should be used with the same care as in triangulation work if the greatest accuracy is desired in the results.

90. Azimuth Mark.

When the observation is made at night it is frequently inconvenient to sight directly at the object whose azimuth is to be determined; it is necessary in such cases to determine the azimuth of a special mark called the **azimuth mark,** which can be seen both at night and in daylight, and then to measure the angle between this mark and the first object during the day. The azimuth mark usually consists of a lamp set inside of a box having a small hole cut in the side, through which the light may shine. The size of the opening should be determined by the distance of the mark; for accurate work it should subtend an angle not greater than about $0''.5$ to $1''.0$. If possible, the mark should be a mile or more distant, so that the focus of the telescope will not have to be altered when changing from the star to the mark. It is frequently necessary, however, to set the mark nearer on account of the topographic and other conditions.

91. Azimuth of Polaris at Elongation.

The simplest method of determining the direction of the meridian with accuracy is by means of an observation of the *polestar*, or any other close circumpolar, when it is at its greatest elongation. (See Art. 19, p. 31.) The appearance of the constellations at the time of this observation on *Polaris* may be seen by referring to Fig. 49. When the *polestar* is west of the pole the *Great Dipper* is on the right and *Cassiopeia* on the left. The exact time of elongation may be found by computing the sidereal time when the star is at elongation, and converting this into mean solar time (local or standard) by the methods of Arts. 34 and 35. To find the sidereal time of elongation first compute the hour angle P by Equa. [34] and then convert it into time. If western elongation is desired, then P is the hour angle; if eastern elongation is desired, then $24^h - P$ is the true hour angle. The sidereal time is then found by Equa. [37]. An average value of P for *Polaris* for latitudes between $30°$ and $50°$ is about $5^h\,55^m$; this is sufficiently accurate for computing the time of elongation for many purposes. Approximate values of the times of elongation of *Polaris* may be taken from Table V.

Example.

Find the Eastern time of Elongation of *Polaris* on April 6, 1904, in lat. $42°\,21'$; long. $4^h\,44^m\,18^s$ W. The right ascension is $1^h\,23^m\,48^s.3$; the declination is $+\,88°\,47'\,43''.6$; the sun's right ascension is $0^h\,57^m\,22^s.44$.

$$\begin{aligned}
\log\tan L &= 9.95977\\
\log\tan D &= 1.67723\\
\hline
\log\cos P &= 8.28254\\
P &= 88°\,54'\,07''+\\
&= 5^h\,55^m\,36^s.5
\end{aligned}$$

$$\begin{aligned}
P &= 5^h\;55^m\;36^s.5\\
R &= 1\;\;\;23\;\;\;48\;.3\\
\hline
S &= 7^h\;19^m\;24^s.8
\end{aligned}$$

$$\begin{aligned}
S &= 7^h\;19^m\;24^s.8\\
R_s &= 0\;\;\;58\;\;\;09\;.1\\
\hline
&\;6^h\;21^m\;15^s.7\\
C' &= \;-1\;\;\;02\;.5\\
\hline
\text{M. L. T.} &= 6^h\;20^m\;13^s.2\\
&\;\;\;\;15\;\;\;42\\
\hline
\text{E. S. T.} &= 6^h\;04^m\;31^s.2
\end{aligned}$$

The transit should be set in position half an hour or so before elongation. The star is bisected by the vertical cross hair, and as it moves out toward its greatest elongation its motion is followed by means of the tangent screw of the upper or the lower plate. Near the time of elongation the star will appear to move almost vertically, so that no motion in azimuth can be detected for five minutes or so before or after elongation. About 5^m before elongation, centre the plate levels, set the cross hair carefully on the star, lower the telescope without disturbing its azimuth, and set a stake or a mark carefully in line at a distance of several hundred feet north of the transit. Reverse the telescope, recentre the levels if necessary, bisect the star again, and set another point beside the first one. If there are errors of adjustment the two points will not coincide; the mean of the two is the true point. The angle between the meridian and the line to the stake (the star's azimuth) is found by the equation

$$\sin Z = \sin p \sec L \qquad [35]$$

where Z is the azimuth from the north; p, the polar distance of the star; and L, the latitude of the place. L does not have to be known with great precision; an error of $1'$ in L produces only about $1''$ error in the azimuth of *Polaris* for latitudes within the United States. The above method may be applied to any close circumpolar star. For *Polaris*, whose polar distance is about $1° 10'$, it is usually accurate enough to use the formula

$$Z'' = p'' \sec L, \qquad [101]$$

in which Z'' and p'' are expressed in seconds of arc. This computed angle may be laid off in the proper direction with a transit (by daylight), using the method of repetitions, or with a tape, by means of a perpendicular offset calculated from the measured distance to the stake and the calculated azimuth angle. (Fig. 59.) The result is the true north and south line.

It is often desirable to measure the horizontal angle between the star at elongation and some fixed point instead of marking the meridian itself. On account of the slow change in azimuth there is ample time to measure several repetitions before the error in azimuth amounts to more than 1″ or 2″.* The errors of adjustment of the transit will be eliminated if half of the angles are taken with the telescope erect and half inverted. The plate levels should be recentred for each position of the instrument before the measurements are begun and while the telescope is pointing toward the star.

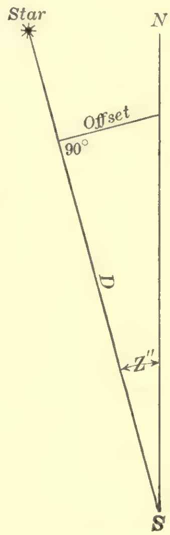

FIG. 59

Example.

Compute the azimuth of *Polaris* at greatest elongation on April 6, 1904, in latitude 42° 21′ N. The declination of the star for the given date is + 88° 47′ 43″.6.

$$\begin{aligned} \log \sin p &= 8.32267 \\ \log \sec L &= 0.13133 \\ \hline \log \sin Z &= 8.45400 \\ Z &= 1°\ 37'\ 47''.9 \end{aligned}$$

By using the angles in place of the sines, neglecting fractions of a second, the following result is obtained:

$$\begin{aligned} p &= 4336'' \\ \log p &= 3.6371 \\ \log \sec L &= 0.1313 \\ \hline \log Z'' &= 3.7684 \\ Z'' &= 5867 \\ &= 1°\ 37'\ 47'' \end{aligned}$$

92. Observations Near Elongation.

If the observation is made on *Polaris* at any time within half an hour of elongation, the azimuth of the star at each pointing

* In latitude 40° the azimuth changes about 1′ in half an hour before or after elongation; the change in azimuth varies approximately as the square of the time from elongation.

of the telescope may be reduced to its value at elongation, provided the time is known. The formula for this reduction is

$$C = 112.5 \times 3600 \times \sin 1'' \times \tan Z_e \times (T - T_e)^2 * \quad [102]$$

in which Z_e is the azimuth at elongation; T, the observed time; T_e, the time of elongation; and C, the correction in seconds of

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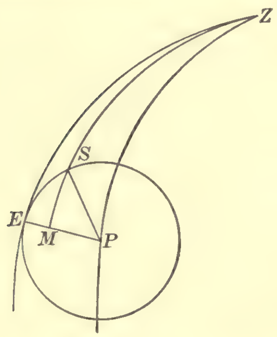

FIG. 60

* For the rigorous demonstration of this formula, which is rather complex, see Doolittle's *Practical Astronomy*. The following proof, although inexact, gives substantially the same result. In Fig. 60 S is the position of *Polaris* and E its position when at greatest elongation, the angle SPE, or i, being not greater than about 8°. In the triangle SPM,

$$\tan MP = \tan PS \cos SPM$$

Since the arcs are small, we may put

$$MP = SP \cos SPM,$$

or

$$MP = p \cos i.$$
$$EM = EP - PM$$
$$= p - p \cos i.$$

Replacing $\cos i$ by the series $1 - \frac{i^2}{2} + \cdots$,

$$EM = p\frac{i^2}{2}.$$

In the triangle ZME, $\angle E = 90°$, and $ZM = ZP$ (nearly), whence

$$\sin MZE = \frac{\sin\left(p\frac{i}{2}\right)}{\cos L},$$
$$= \frac{\sin p}{\cos L} \times p\frac{i^2}{2} \text{ (nearly)},$$
$$= \sin Z_e \times p\frac{i^2}{2},$$

in which Z_e is the azimuth at elongation. Replacing $\sin MZE$ by its arc in seconds (C'') and reducing i to seconds of time,

$$C'' = \frac{i^2}{2} \times \sin 1'' \times (60)^2 \times (15)^2 \times \sin Z_e. \quad [103]$$

Replacing $\sin Z_e$ by $\tan Z_e$ produces an error of only about 0''.02 for *Polaris* in latitude 40° and reduces [103] to [102].

arc. $T - T_e$ must be in minutes of (sidereal) time. The factor $112.5 \times 3600 \times \sin 1'' \times \tan Z_e$ may be computed, and then all observations made at the same place at about the same date may be reduced by multiplying the square of the time intervals in minutes by the factor computed. Table VII gives values of the factor for values of Z_e ranging from 1° to 2°. These corrections will also be found in Table VIa at the end of the Nautical Almanac.

Example.

Three repetitions of the angle between *Polaris* at western elongation and a mark supposed to be on the meridian, April 6, 1904. Lat. 42° 21′; long. 71° 04′.5 W. The observed times are $6^h\ 28^m\ 30^s$, $6^h\ 31^m\ 20^s$ and $6^h\ 34^m\ 20^s$. First reading of vernier = 0° 00′; last reading of vernier = 4° 51′ 00″. The R. A. of *Polaris* = $1^h\ 23^m\ 48^s.3$; its declination = + 88° 47′ 43″.6. R. A. Mean Sun at G. M. N. = $0^h\ 57^m\ 22^s.44$.

From this data the Eastern time of elongation is found to be $6^h\ 04^m\ 31^s.2$. The intervals $(T - T_e)$ are $23^m\ 58^s.8$, $26^m\ 48^s.8$ and $29^m\ 48^s.8$. The azimuth of the star at elongation is 1° 37′ 48″. From Table VII the factor is found to be .0559. The resulting corrections are 32″, 40″ and 50″. Adding these to the third reading, the sum is 4° 53′ 02″. One third of this is 1° 37′ 41″, the measured angle between the mark and the star *at elongation*. The meridian mark is therefore 7″ west of north, according to this observation.

93. Azimuth by an Altitude of the Sun.

In order to determine the azimuth of a line by means of an observation on the sun the instrument should be set up over one of the points marking the line and carefully levelled. The plate vernier is first set at 0° and the vertical cross hair sighted on the other point marking the line. The colored shade glass is then screwed on to the eyepiece, the upper clamp loosened, and the telescope turned toward the sun. The sun's disc should be sharply focussed before beginning the observations. In making the pointings on the sun great care should be taken not to mistake one of the stadia hairs for the middle hair. If the observation is to be made, say, in the forenoon (in the northern hemisphere), first set the cross hairs so that the vertical hair is tangent to the right edge of the sun and the horizontal hair cuts off a small segment at the lower edge of the disc. (Fig. 61.)* The arrow in the figure shows the direction of the sun's apparent motion. Since the sun is now rising it will in a few seconds be tangent to the horizontal hair. It is only necessary to follow the right edge by means of the upper plate tangent screw until both cross hairs are tangent. At this instant, stop following the sun's motion and note the time. If it is desired to determine the time accurately, so that the watch correction may be found from this same observation, it can be read more closely by a second observer. Both the horizontal and the vertical circles are read, and both angles and the time are recorded. The same observation may be repeated three or four times to increase the accuracy. The instrument should then be reversed and the set of observations repeated, except that the horizontal cross hair is set tangent to the upper edge of the sun and the vertical cross hair cuts a segment from the left edge (Fig. 62). The same number of pointings should be taken in each position of the instrument. After the pointings on the sun are completed the telescope should be turned to the mark again and the vernier reading checked. If the transit has a vertical arc only, the telescope cannot be used in the reversed position and the index correction must therefore be determined. If the observation is to be made in the afternoon the positions will be those indicated in Fig. 63.†

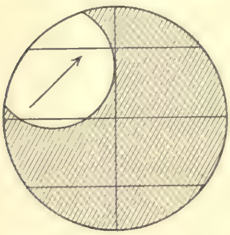

FIG. 61. POSITION OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION
(A. M. Observation in Northern Hemisphere.)

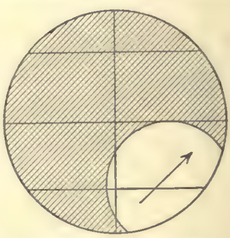

FIG. 62. POSITION OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION
(A. M. Observation in Northern Hemisphere.)

* In the diagram only a portion of the sun's disc is visible; in a telescope of low power the entire disc can be seen.

† It should be kept in mind that if the instrument has an inverting eyepiece the direction of the sun's apparent motion is reversed. If a prism is attached to the eyepiece, the upper and lower limbs of the sun are apparently interchanged, but the right and left limbs are not.

In computing the azimuth it is customary to neglect the curvature of the sun's path during the short interval between the first and last pointings, unless the series extends over a longer period than is usually required to make such observations. If the observation is taken near noon the curvature is greater than when it is taken near the prime vertical. The mean of the altitudes and the mean of the horizontal angles are assumed to correspond to the position of the sun's centre at the instant shown by the mean watch reading. The mean altitude reading corrected for refraction and parallax is the true altitude of

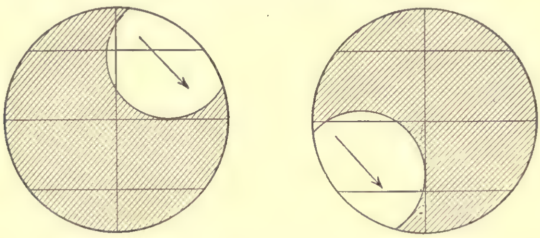

FIG. 63. POSITIONS OF SUN'S DISC A FEW SECONDS BEFORE OBSERVATION (P. M. Observation in Northern Hemisphere.)

the sun's centre. The azimuth is then computed by any one of the formulæ on page 34. The resulting azimuth combined with the mean horizontal circle reading gives the azimuth of the mark. Five-place logarithmic tables will give the azimuth within $5''$ to $10''$, which is as precise as the azimuth can be determined by this method.

If for any reason only one limb of the sun has been observed, the azimuth observed may be reduced to the centre of the sun by applying the correction $S \sec h$, where S is the semidiameter and h is the altitude of the centre.

If one has at hand a set of tables containing log versed sines (such as are included in railroad engineering tables) the following formulæ will sometimes be found useful.

$$\text{vers } Z_s = \frac{\cos (L + h) + \sin D}{\cos L \cos h} \qquad [104]$$

and

$$\text{vers } Z_n = \frac{\cos (L - h) - \sin D}{\cos L \cos h} \qquad [105]$$

The sum or difference in the numerator must be computed by natural functions and the remainder of the work performed by means of logarithms.

Example.

Observation on Sun for Azimuth.

Lat. $42° 21'$ N. Long. $4^h 44^m 18^s$ W. Date, Nov. 28, 1905.

	Hor. Circle. Ver. A.	B.	Vert. Circle.	Watch. A. M.
Mark	238° 14′	14′		
R & L limbs	311 48	48.5	14° 41′	$8^h 39^m 42^s$
R & L limbs	312 20	20	15 00	8 42 19
(instrument reversed)				
L & U limbs	312 27	26.5	15 55	8 45 34
L & U limbs	312 52	51.5	16 08	8 47 34
Mark	238 14	14		

Mean reading on mark	238° 14′.0	Mean = 15° 26′	Mean = $8^h 43^m 47^s$
Mean reading on sun	312 21.7	R & P = 3.3	
Mark N. of sun	74° 07′.7	h = 15° 22′.7	

$L = 42° 21'.0$
$h = 15\ 22.7$
$p = 111\ 15.7$

$2s = 168° 59'.4$
$s = 84° 29'.7$

G. M. T = $1^h 43^m 47^s$
Sun's Decl. at G. M. N. = $-$ 21° 14′ 54″.4
$-$ 26″.81 × 1^m.73 = $-$ 46 .4
Declination = $-$ 21° 15′ 40″.8
N. Polar distance = 111° 15′ 40″.8

$s - L =$	42° 08′.7	log sin 9.82673
$s - h =$	69 07.0	log sin 9.97049
$s - p =$	$-$ 26 46.0	log sec 0.04922
$s =$	84 29.7	log sec 1.01804
		2)0.86448

log tan $\frac{1}{2} Z_n$ = 0.43224
$\frac{1}{2} Z_n$ = 69° 42′.9
Z_n = 139° 25′.8
Mark N. of sun = 74 07.7

Bearing of Mark = N 65° 18′.1 E

By differentiating Equa. [13] it may be shown that when the latitude is greater than the sun's declination the greatest accuracy in the azimuth, so far as errors in altitude are concerned, is secured when the sun is somewhere between the prime vertical and the six-hour circle; the exact position for maximum accuracy depends upon the latitude and upon the parallactic angle. If an observer were on the equator and the sun's declination zero, the motion would be vertical and the change in azimuth would be zero. In the preceding example the azimuth increases about 1′ 50″ for an increase of 1′ in the altitude. Errors in the azimuth due to errors in the assumed value of the latitude are a minimum when the sun is on the six-hour circle. Observations very near the horizon, however, are subject to errors in the refraction, since the tabular values of the mean refraction may be largely in error for very low altitudes under the temperature and pressure conditions existing at the time of the observation. The general rule is therefore to avoid observations near the meridian and also those within 10° or less of the horizon.

If it is desired to compute the hour angle of the sun from the same observations used in determining the azimuth, it may be found by formula [19], in which case no new logarithms have to be taken from the tables; or it may be found by the equation

$$\sin P = \sin Z \cos h \sec D. \qquad [12]$$

The value of P and the error of the watch obtained by the use of this formula are given below.*

$\log \sin Z$	= 9.81317			
$\log \cos h$	= 9.98416			
$\log \sec D$	= 0.03061			
$\log \sin P$	= 9.82794			
P	= 42°	17′	26″	
	=	2^h	49^m	$09^s.7$
L. A. T.	=	9	10	50 .3
Eq. T.	=		12	02 .8
L. M. T.	=	8^h	58^m	$47^s.5$
			15	42
E. S. T.	=	8^h	43^m	$05^s.5$
Watch	=	8	43	47 .2
Watch fast	=			$41^s.7$

94. Azimuth by an Altitude of a Star.

The method described in the preceding article applies equally well to an observation on a star, except that the star's image is bisected with both cross hairs and the parallax and semidiameter corrections become zero. The declination of the star changes so little during one day that it may be regarded as constant, and consequently the time of the observation is not required. Errors in the altitude and the latitude may be partially eliminated by combining two observations, one on a star about due east and the other on one about due west.

95. Azimuth Observation on a Circumpolar Star at any Hour Angle.

The most precise determination of azimuth may be made by measuring the horizontal angle between a circumpolar star and an azimuth mark, the hour angle

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* See also Art. 102, p. 166, and Art. 110, p. 175.

of the star at each pointing being known. If the sidereal time is determined accurately, by any of the methods given in Chapter XI, the hour angle of the star may be found at once by Equa. [37] and the azimuth of the star at the instant may be computed. Since the close circumpolar stars move very slowly and errors in the observed times will have a small effect upon the computed azimuth, it is evident that only such stars should be used if precise results are sought. The advantage of observing the star at any hour angle, rather than at elongation, is that the number of observations may be increased indefinitely and greater accuracy thereby secured.

The angles may be measured either with a repeating instrument (like the engineer's transit) or with a direction instrument in which the circles are read with

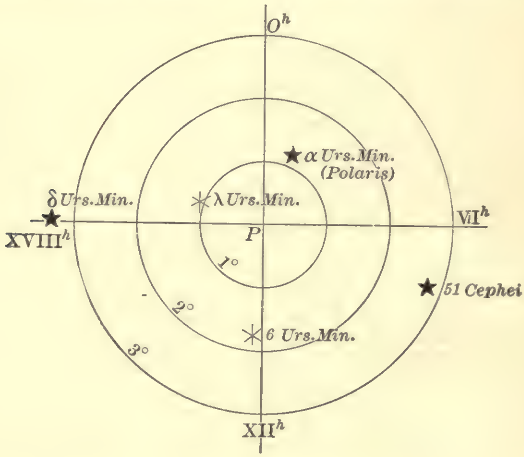

FIG. 64

great precision by means of micrometer microscopes. For refined work the instrument should be provided with a sensitive striding level. If there is no striding level provided with the instrument* the plate level which is parallel to the horizontal axis should be a sensitive one and should be kept well adjusted. At all places in the United States the celestial pole is at such high altitudes that errors in the adjustment of the horizontal axis and of the sight line have a comparatively large effect upon the results.

The star chosen for this observation should be one of the close circumpolar stars given in the special list in the Nautical Almanac. (See Fig. 64.) *Polaris* is the only bright star in this group and should be used in preference to the others when it is

* The error due to inclination of the axis may be eliminated by taking half of the observations direct and half on the image of the star reflected in a basin of mercury.

practicable to do so. If the time is uncertain and *Polaris* is near the meridian, in which case the computed azimuth would be uncertain, it is better to use 51 *Cephei*,* because this star would then be near its elongation and comparatively large errors in the time would have but little effect upon the computed azimuth. If a repeating theodolite or an ordinary transit is used the observations consist in repeating the angle between the star and the mark a certain number of times and then reversing the instrument and making another set containing the same number of repetitions. Since the star is continually changing its azimuth it is necessary to read and record the time of each pointing on the star with the vertical cross hair. The altitude of the star should be measured just before and again just after each half-set so that its altitude for any desired instant may be obtained by simple interpolation. If the instrument has no striding level the cross-level on the plate should be recentred before the second half-set is begun. If a striding level is used the inclination of the axis may be measured, while the telescope is pointing toward the star, by reading both ends of the bubble, with the level first in the direct position and then in the reversed position.

In computing the results the azimuth of the star might be computed for each of the observed times and the mean of these azimuths combined with the mean of the measured horizontal angles. The labor involved in this process is so great, however, that the practice is first to compute the azimuth corresponding to the *mean* of the observed times, and then to correct this result for the effect of the curvature of the star's path, i.e., by the difference between the mean azimuth and the azimuth at the mean of the times. The formula for the azimuth is

$$\tan Z = \frac{\sin P}{\cos L \tan D - \sin L \cos P}. \qquad [31]$$

The formula given below, although not exact, is sufficiently accurate for all work except refined geodetic observations.

$$Z'' = p'' \sin P \sec h, \qquad [106]$$

in which Z'' and p'' are in seconds of arc. In this formula the arcs have been substituted for their sines. The precision of the computed azimuth depends chiefly upon the precision with which h can be determined. If the vertical arc cannot be relied upon, and the latitude is known accurately, the first formula may be preferred. If desired, the altitude of *Polaris* may be computed by formula [80] and its value substituted in [106].

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* 51 *Cephei* may be found by first pointing on *Polaris* and then changing the altitude and the azimuth by an amount which will bring 51 *Cephei* into the field. The difference in altitude and in azimuth may be obtained with sufficient accuracy by holding Fig. 64 so that *Polaris* is in its true position with reference to the meridian (as indicated by the position of δ *Cassiopeiæ*) and then estimating the difference in altitude and the difference in azimuth. It should be remembered that the distance of 51 *Cephei* east or west of *Polaris* has nearly the same ratio to the difference in azimuth that the polar distance of *Polaris* has to its azimuth at elongation, i.e., 1 to sec L.

96. The Curvature Correction.

If we let T_1, T_2, T_3, etc. = the observed times, T_o = the mean of these times, Z_1, Z_2, Z_3, etc, = corresponding azimuths, and Z_o the azimuth at the instant T_o, then

$$\frac{Z_1 + Z_2 + \cdots Z_n}{n} = Z_o - \tan Z_o\,[0.2930]\,\frac{1}{n}\,\Sigma\,(T - T_o)^2.\text{*} \qquad [107]$$

The quantity in brackets is the logarithm of a constant; $\Sigma\,(T - T_o)^2$ is the sum of the squares of the time-intervals (in minutes and decimals) reduced to sidereal intervals. The azimuth is therefore computed by first finding Z_o by Equa. [31] and then correcting it by means of the last term of Equa. [107].

If it is desired to express $(T - T_o)$ in seconds of time the constant log becomes [6.73672]. When the star is near culmination the curvature correction is very small; near elongation it is a maximum.

97. The Level Correction.

The inclination i of the axis as determined by the striding level is given in seconds of arc by

$$i = [(w + w') - (e + e')]\,\frac{d}{4}, \qquad [108]$$

where w and e are the readings of the west and east ends of the bubble for the direct position, and w' and e' are the same for the reversed positions, and d is the angular value of one division of the level scale. The correction to the measured horizontal angle is

$$C = i \tan h. \qquad [109]$$

If the west end of the axis is too high (i positive) the telescope has to be turned too far west in pointing at the star; the correction must therefore be added to the measured angle if the mark is west of the star, subtracted if east. If the instrument has no striding level the error must be eliminated as completely as possible by relevelling between the half-sets.

98. Diurnal Aberration.

Strictly speaking, the computed azimuth of the star should be corrected for diurnal aberration, the effect of which is to make the star appear farther east than it actually is, because the observer is being carried due east by the diurnal motion of the earth. The correction is

$$0''.319 \times \frac{\cos L \cos Z}{\cos h}. \qquad [110]$$

For all but the most precise observations it may be taken as $0''.32$, since the factor $\frac{\cos L \cos Z}{\cos h}$ is never far from unity.

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* For the derivation of the formula see Doolittle's *Practical Astronomy* and Hayford's *Geodetic Astronomy*.

Example 1.

RECORD OF AZIMUTH OBSERVATIONS

Instrument (B. & B. No. 3441) at South Meridian Mark. Boston, May 16, 1910.
(One division of level = 15″.0.)

Object.	Pos. of tel.	No. of rep.	Chronometer.	Horizontal circle. Vernier A.	B.	Level readings and angles.
Polaris..	Direct		$11^h\ 24^m\ 35^s.0$	0° 00′ 00″	00″	W E 7.0 3.9 5.8 5.1
			27 15.0			12.8 9.0 9.0
			28 31.5			3.8
			30 00.0			Corr. = 12″.5
			31 20.5			Alt. *Polaris* at $11^h\ 34^m\ 20^s.5$ = 41° 20′ 30″
			32 27.0			Alt. *Polaris* at $11^h\ 51^m\ 04^s.0$ = 41° 18′ 40″
Mark...		6		*39° 33′ 30″	30″	Mean horizontal angle = 66° 35′ 35″.0
Polaris..	Reversed		11 42 45.5	39° 33′ 30″	30″	W E 5.1 5.8 3.3 7.6
			44 09.0			8.4 13.4 8.4
			45 15.0			
			46 29.5			5.0 Corr. = 16″.5
			47 25.0			
			48 54.5			
Mark...		6		*78° 27′ 30″	20″	Mean horizontal angle = 66° 28′ 59″.2 Alt. *Polaris* at $12^h\ 09^m\ 31^s.5$ = 41° 15′ 40″

* Passed 360°.

RECORD OF TIME OBSERVATIONS

Polaris: — Chronometer, $12^h\ 09^m\ 31^s.5$; alt., $41°\ 15'\ 40''$
ε Corvi: — Chronometer, $12\ \ 13\ \ 37\ .5$; alt., $25\ \ 34\ \ 00$

Polaris: R. A. = $1^h\ 25^m\ 51^s.1$; decl. = $+88°\ 49'\ 24''.8$
ε Corvi: R. A. = $12^h\ 5^m\ 30^s.5$; decl. = $-22°\ 07'\ 21''.0$

	Chronometer	R. A.	Decl.
α Serpentis (E)	$12^h\ 24^m\ 15^s.7$	$15^h\ 39^m\ 51^s.6$	$+6°\ 42'\ 20''.7$
ε Hydræ: (W)	12 18 32 .0	8 42 00 .5	+6 44 58 .9

(Lat. = $42°\ 21'\ 00''$ N.; Long. = $4^h\ 44^m\ 18^s.0$ W.)

From these observations the chronometer is found to be $10^m\ 22^s.1$ fast.

COMPUTATION OF AZIMUTH

Mean of Observed times	=	$11^h\ 37^m\ 25^s.6$
Chronometer correction	=	− 10 22 .1
Sidereal time	=	11 27 03 .5
R. A. of *Polaris*	=	1 25 51 .1
Hour Angle of *Polaris*	=	10 01 12 .4
P	=	$150°\ 18'\ 06''$
log cos L	=	9.868670
log tan D	=	1.687490
log cos L tan D	=	1.556160
cos L tan D	=	35.9882
log sin L	=	9.82844
log cos P	=	9.93884
log sin L cos P	=	9.76728
sin L cos P	=	.5852
denominator	=	36.5734
log sin P	=	9.694985
log denom.	=	1.563165
log tan Z	=	8.131820
Z	=	$0°\ 46'\ 34''.2$
Curvature correction	=	2.1
Azimuth of star	=	0 46 32 .1
Measured angle, first half	=	$66°\ 35'\ 35''.0$
Level correction	=	−12 .5
Corrected angle	=	66 35 22 .5
Measured angle, second half	=	66 28 59 .2
Level correction	=	+16 .5
Corrected angle	=	66 29 15 .7
Mark east of star	=	66 32 19 .1
Mark east of North	=	$65°\ 45'\ 47''.0$

Example 2.

Observed altitudes of *Regulus* (east), Feb. 11, 1908, in lat. 42° 21′.

Altitude	Watch
17° 05′	7^h 12^m 16^s
17 31	14 31
17 49	16 07
18 02	17 20

The right ascension of *Regulus* is 10^h 03^m $29^s.1$; the declination is + 12° 24′ 57″. From these data the sidereal time corresponding to the mean watch reading (7^h 15^m $03^s.5$) is found to be 4^h 53^m $42^s.7$.

Observed horizontal angles from azimuth mark to *Polaris*.
(Mark east of north.)

Telescope Direct		Time of pointing on *Polaris*
Mark,	0° 00′	7^h 20^m 38^s
		23 00
Third repetition	201° 48′	23 56
Mean =	67° 16′.0	7^h 22^m $31^s.3$
Telescope Reversed		
Mark =	0° 00′	7 27 09
		28 17
Third Repetition	201° 54′	29 21
Mean =	67° 18′.0	7^h 28^m $15^s.7$

Altitude of *Polaris* at 7^h 20^m 38^s = 43° 03′
Altitude of *Polaris* at 7 29 21 = 43 01
Mean watch reading for *Polaris* = 7^h 25^m $23^s.5$
Corresponding sidereal time = 5 04 04 .4
Right Ascension of *Polaris* = 1 25 32 .3
Hour-angle of *Polaris* = 3 38 32 .1
P = 54° 38′

p = 4251
log p = 3.62849
log sin P = 9.91141
log sec h = 0.13611

log azimuth = 3.67601
azimuth = 4743″
= 1° 19′.0
Mean angle = 67 17 .0

Mark East of North = 65° 58′.0

99. Meridian by Polaris at Culmination.

The following method is given in Lalande's Astronomy and was practiced by Andrew Ellicott, in 1785, on the Ohio and Pennsylvania boundary survey. The direction of the meridian is determined by noting the instant when *Polaris* and some other star are in the same vertical plane, and then waiting a certain interval of time, depending upon the date and the star observed, when *Polaris* will be in the meridian. At this instant *Polaris* is sighted and its direction then marked on the ground by means of stakes. The stars selected for this observation should be near the hour circle through the polestar; that is, their right ascensions should be nearly equal to that of the polestar, or else nearly 12^h greater. The stars best adapted for this purpose at the present time are *δ Cassiopeiæ* and *ζ Ursæ Majoris*.

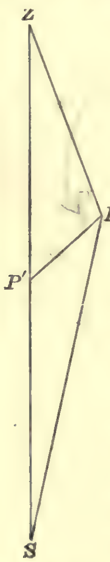

FIG. 65

The interval of time between the instant when the star is vertically above or beneath *Polaris* and the instant when the latter is in the meridian is computed as follows: In Fig. 65 P is the pole, P' is *Polaris*, S is the other star (*δ Cassiopeiæ*) and Z is the zenith. At the time when S is vertically under P', $ZP'S$ is a vertical circle. The angle desired is ZPP', the hour angle of *Polaris*. PP' and PS, the polar distances of the stars, are known quantities; $P'PS$ is the difference in right ascension, and may be obtained from the Ephemeris. The triangle $P'PS$ may therefore be solved for the angle at P'. Subtracting this from 180° gives the angle $ZP'P$; PP' is known, and PZ is the colatitude of the observer. The triangle $ZP'P$ may then be solved for ZPP', the desired angle. Subtracting ZPP' from 180° or 12^h gives the sidereal interval of time which must elapse between the two observations. The angle SPP' and the side PP' are so small that the usual formulæ may be simplified, by replacing sines by arcs, without appreciably diminishing the accuracy of the result. A similar solution may be made for the upper culmination of *δ Cassiopeiæ* or for the two positions of the star *ζ Ursæ Majoris*, which is on the opposite side of the pole from *Polaris*. The above solution, using the right ascensions and declinations for the date, gives the exact interval required; but for many purposes it is sufficient to use a time interval calculated from the mean places of the star and for a mean latitude of the United States. The interval for the star *δ Cassiopeiæ* for the year 1901 is $3^m.0$; for 1910 it is $6^m.1$, the annual increase being $0^m.35$. For *ζ Ursæ Majoris* the interval for 1901 is $3^m.7$; for 1910 it is $6^m.7$, the annual increase being $0^m.33$. Beginning with the issue for 1910 the American Ephemeris and Nautical Almanac contains values of these intervals (Table VII) for different latitudes and for different dates. Within the limits of the United States it will generally be necessary to observe on *δ Cassiopeiæ* when *Polaris* is at lower culmination and on *ζ Ursæ Majoris* when *Polaris* is at upper culmination.

The determination of the instant when the two stars are in the same vertical plane is necessarily approximate, since there is some delay in changing the telescope from one star to the other. The motion of *Polaris* is so slow, however, that a very fair degree of accuracy may be obtained by first sighting on *Polaris*, then pointing the telescope to the altitude of the other star (say *δ Cassiopeiæ*) and waiting until it appears in the field; when *δ Cassiopeiæ* is seen, sight again at *Polaris* to allow for its motion since the first pointing, turn the telescope again to *δ Cassiopeiæ* and observe the instant when it crosses the vertical hair. The motion of the polestar during this short interval may safely be neglected. The tabular interval of time corrected to date must be added to the watch reading. When this computed time arrives, the cross hair is to be set accurately on *Polaris* and then the telescope lowered in this vertical plane and a mark set in line with the cross hairs. The change in the azimuth of *Polaris* in 1^m of time is not far from half a minute of angle, so that an error of a few seconds in the time of sighting at *Polaris* has but little effect upon the result. It is evident that the actual error of the watch on local time has no effect whatever upon the result, because the only requirement is that the *interval* should be correctly measured.

100. Azimuth by Equal Altitudes of a Star.

The meridian may be found in a very simple manner by means of two equal altitudes of a star, one east of the meridian and one west. This method has the advantage that the coördinates of the star are not required, so that the Almanac or other table is not necessary. The method is inconvenient because it requires two observations at night several hours apart. It is of special value to surveyors in the southern hemisphere, where there is no bright star near the pole. The star to be used should be approaching the meridian (in the evening) and about 3^h or 4^h from it. The altitude should be a convenient one for measuring with the transit, and the star should be one that can be identified with certainty 6^h or 8^h later. Care should be taken to use a star which will reach the same altitude on the opposite side of the meridian before daylight interferes with the observation. In the

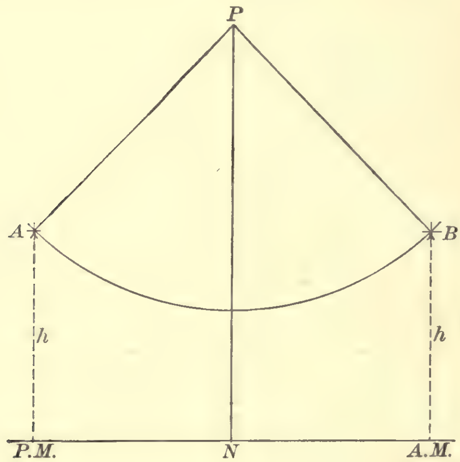

FIG. 66

northern hemisphere one of the stars in *Cassiopeia* might be used. The position at the first (evening) observation would then be at A in Fig. 66. The star should be bisected with both cross hairs and the altitude read and recorded. A note or a sketch should be made showing which star is used. The direction of the star should be marked on the ground, or else the horizontal angle measured from some reference mark to the position of the star at the time of the observation. When the star is approaching the same altitude on the opposite side of the meridian (at B) the telescope should be set at exactly the same altitude as was read at the first observation. When the star comes into the field it is bisected with the vertical cross hair and followed in azimuth until it reaches the horizontal hair. The motion in azimuth should be stopped at this instant. Another point is then set on the ground (at same distance from the transit as the first) or else another angle is turned to the same reference mark. The bisector of the angle between the two directions is the meridian line through the transit point. It is evident that the index and refraction errors are eliminated, because they are alike for the two observations. If one observation is made with the telescope direct and the other with the telescope reversed, the other instrumental errors will be eliminated. Care should be taken to level the instrument just before the observations. The accuracy of the final result may be increased by observing the star at several different altitudes and using the mean value of the different results.

101. Observation for Meridian by Equal Altitudes of the Sun in the Forenoon and in the Afternoon.

This observation consists in measuring the horizontal angle between the mark and the sun when it has a certain altitude in the forenoon and measuring the angle again to the sun when it has an equal altitude in the afternoon. Since the sun's declination will change during the interval, the mean of the two angles will not be the true angle between the meridian and the mark, but will require a small correction. The angle between the south point of the meridian and the point midway between the two directions of the sun is given by the equation

$$\text{Correction} = \frac{d}{\cos L \sin P}, \qquad [111]$$

in which d is the hourly change in declination multiplied by the number of hours elapsed between the two observations, L is the latitude, and P is the hour angle of the sun, or approximately half the elapsed interval of time. The correction depends upon the change in the declination, not upon its absolute value, so that the hourly change may be taken with sufficient accuracy from the Almanac for any year for the corresponding date.

In making the observation the instrument is set up at one end of the line whose azimuth is to be determined, and the plate vernier set at 0°. The vertical cross hair is set on the mark and the lower clamp tightened. The sun glass is then put in position, the upper clamp loosened, and the telescope pointed at the sun. It is not necessary to observe on both edges of the sun, but is sufficient to sight, say, the lower limb at both observations, and to sight the vertical cross hair on the opposite limb in the afternoon from that used in the forenoon. The horizontal hair is therefore set on the *lower* limb and the vertical cross hair on the *left* limb. When the instrument is in this position the time should be noted as accurately as possible. The altitude and the horizontal angle are both read. In the afternoon the instrument is set up at the same point, and the same observation is made, except that the vertical hair is now sighted on the *right* limb; the horizontal hair is set on the *lower* limb as before. A few minutes before the sun reaches an altitude equal to that observed in the morning the vertical arc is set to read exactly the same altitude as was read at the first observation. As the sun's altitude decreases the vertical hair is kept tangent to the right limb until the lower edge of the sun is in contact with the horizontal hair. At this instant the time is again noted accurately; the horizontal angle is then read. The mean of the two circle readings, corrected for the effect of change in declination, is the angle from the mark to the south point of the horizon. The algebraic sign of the correction is determined from the fact that if the sun is going north the mean of the two vernier readings lies to the west of the south point, and vice versa. The precision of the result may be increased by taking several forenoon observations in succession and corresponding observations in the afternoon.

Example.

Lat. 42° 18′ N. Apr. 19, 1906.

A.M. Observations.	P.M. Observations.
Reading on Mark, 0° 00′ 00″	Reading on Mark, 0° 00′ 00″
U & L limbs { Alt., 24° 58′; Hor. Angle, 357° 14′ 15″; Time, $7^h\ 19^m\ 30^s$	U & R limbs { Alt., 24° 58′; Hor. angle, 162° 28′ 00″; Time $4^h\ 12^m\ 15^s$

$$\tfrac{1}{2}\text{ elapsed time} = 4^h\ 26^m\ 22^s$$

$$\begin{aligned} P &= 66°\ 35'\ 30'' \\ \log\sin P &= 9.96270 \\ \log\cos L &= 9.86902 \\ \hline & \ 9.83152 \\ \log 230''.9 &= 2.36342 \\ \hline & \ 2.53170 \\ \text{Corr.} &= 340''.2 \end{aligned}$$

$$\begin{aligned} \text{Incr. in decl.} &= +52'' \times 4^h.44 \\ &= +230''.9 \end{aligned}$$

$$\begin{aligned} \text{Mean Circle Reading} &= 79°\ 51'\ 08'' \\ \text{Correction} &= 5\ \ 40 \\ \hline \text{True Angle} &= \text{S } 79°\ 45'\ 28''\text{ E.} \\ \text{Azimuth} &= 280°\ 14'\ 32'' \end{aligned}$$

102. Azimuth of Sun near Noon.

The azimuth of the sun near noon may be determined by means of Equa. [30], provided the local apparent time is known or can be computed. If the longitude and the watch correction on Standard Time are known within one or two seconds the local apparent time may be readily calculated. This method may be useful when it is desired to obtain a meridian during the middle of the day, because the other methods are not then applicable.

If, for example, an observation has been made in the forenoon from which a reliable watch correction may be computed, then this correction may be used in the azimuth computation for the observation near noon; or if the Standard Time can be obtained accurately by a comparison at noon and the longitude can be obtained from a map within about 1000 feet, the local apparent time may be found with sufficient accuracy. This method is not usually convenient in midsummer, on account of the high altitude of the sun, but if the altitude is not greater than about 50° the method may be used without difficulty. The observations are made exactly as in Art. 93, except that the *time* of each pointing is determined more precisely; the accuracy of the result depends very largely upon the accuracy with which the hour angle of the sun can be computed, and great care must therefore be used in determining the time. The observed watch reading is corrected for the known error of the watch, and is then converted into local apparent time. The local apparent time converted into degrees is the angle at the pole, P. The azimuth is then found by the formula

$$\sin Z = \sin P \sec h \cos D. \qquad [30]$$

Errors in the time and the longitude produce large errors in Z, so this method should not be used unless both can be determined with certainty.

Example.

Observation on the sun for azimuth.

Lat. $42^\circ\ 21'$. Long. $4^h\ 44^m\ 18^s$ W. Date, Feb. 5, 1910.

	Hor. Circle.	Vert. Circle.		Watch.
Mark,	$0^\circ\ 00'$			(30^s fast)
app. L & L limbs,	29 01	$31^\circ\ 49'$		$11^h\ 43^m\ 22^s$
app. U & R limbs,	28 39	31 16		11 44 20
Mean,	$28^\circ\ 50'$	$31^\circ\ 32'.5$		$11^h\ 43^m\ 51^s$
	Refr.,	1 .6	Watch corr. =	-30
		$h = 31^\circ\ 30'.9$	E. S. T. =	$11^h\ 43^m\ 21^s$
				15 42
			L. M. T. =	$11^h\ 59^m\ 03^s$
			Eq. t. =	14 09 .1
			L. A. T. =	$11^h\ 44^m\ 53^s.9$
			P =	$15^m\ 06^s.1$
			=	$3^\circ\ 46'.5$

$$D = -16^\circ\ 06'\ 04''.5$$
$$45''.17 \times 4^h.7 = 3\ \ 32\ .3$$
$$D = -16^\circ\ 02'\ 32''.2$$

$$\text{Eq. t.} = 14^m\ 08^s.05$$
$$.217 \times 4^h.7 = 1\ .02$$
$$\text{Eq. t.} = 14^m\ 09^s.07$$

$$\log \sin P = 8.81847$$
$$\log \cos D = 9.98275$$
$$\log \sec h = 0.06930$$

$$\log \sin Z = 8.87052$$
$$Z = 4^\circ\ 15'.4$$
$$\text{Hor. Circle} = 28\ \ 50$$

$$\text{Azimuth} = \text{S } 33^\circ\ 05'.4 \text{ E}$$
$$= 326^\circ\ 54'.6$$

103. Combining Observations.

From the foregoing descriptions of field methods of observing, it will be seen that but few of these methods are quite independent of the data obtained by other observations, and in the practice of the engineer it often happens that no one of the quantities which he desires can be completely determined until some or all of the others are known approximately. The latitude may be determined directly by means of a star at culmination, but it may be inconvenient or impracticable in many cases to wait until either *Polaris* or a southern star comes to the meridian. In all of the methods of determining time it is necessary to know either the latitude or the direction of the meridian before the time can be directly computed. In all of the methods of determining azimuth either the time, the latitude, or both must be known. Where all of these quantities are entangled it is usually necessary to obtain the true values by a series of approximations. In most cases, however, very few approximations are necessary to give the greatest accuracy afforded by the observations.

If it is necessary to determine a precise azimuth, and nothing whatever is known in regard to the latitude of the point or the local time, then all three may be accurately determined by making observations of transits across the vertical circle through *Polaris*, measuring the altitudes of all the stars, and then repeating the horizontal angle between *Polaris* and an azimuth mark. The measured altitudes of *Polaris* and the time-star make it possible to compute the sidereal time by two approximations (Art. 78) without knowing the latitude. When the time is known the latitude may be found by Arts. 68 and 69. If the instrument has only a vertical arc (180°), then the altitudes of the southern stars may be measured and the first approximation to the latitude found from these observations. The altitudes of *Polaris* may then be calculated closely enough for computing Equa. [106]. After the time and the latitude are known the azimuth is found directly. By using the instrument in the two positions and increasing the number of observations the precision of all of the results may be increased.

The same results may be obtained by using the method of equal altitudes (Arts. 80–82), combined with measured altitudes of the polestar and observations for azimuth. By selecting a pair of stars having a large difference in right ascension or a small difference in declination, the time may be fairly well determined by using an estimated latitude obtained by estimating a correction to the observed altitude of *Polaris*. When the time is known approximately, a new value of the latitude may be obtained, and with this new latitude the time may be recomputed. The azimuth may then be found as before.

A very rapid but not very precise way of determining these three quantities and also checking the azimuth is to sight on the mark, then to sight on the polestar, reading both the horizontal and vertical angles, and finally to sight on a prime vertical star, reading both angles. Using an estimated latitude the *PZS* triangle may be solved for *P*; with this value of *P* a close value of the latitude is found, and the hour angle is then recomputed. If the latitude and the time are known the azimuth may be determined from the polestar and checked by the azimuth from the star near the prime vertical.

It is well when determining azimuths for surveying purposes to obtain checks by methods which are independent of one another. For example, if the azimuth is being found by angles measured to *Polaris*, a check may be obtained by turning an angle from some star near the prime vertical (Art. 77) and measuring its altitude simultaneously. Observations made on both east and west stars will increase the accuracy. The azimuth thus computed is inferior in accuracy to that found from *Polaris*, but the fact that it is independent makes it a valuable check against mistakes or large errors in the *Polaris* observations. A sun observation made late in the afternoon may be used in a similar way to check an evening observation on *Polaris*.

Questions and Problems

1. What error is caused by making the approximations in deriving formula [35]?

2. Derive formula [106].

3. Show that if the declination is less than the latitude the most favorable conditions for determining azimuth by an altitude of the sun occur when the sun

is between the six-hour circle and the prime vertical. For greatest accuracy $\frac{dZ}{dh}$ and $\frac{dZ}{dL}$ should be a minimum. Differentiate Equa. [13] and simplify by means of [12] and [15].

4. Show that the factor $\cos L \cos Z \sec h$ (Equa. [110]) is always nearly equal to unity.

5. Compute the approximate local mean time of eastern elongation of *Polaris* on Sept. 10. R. A. of *Polaris*, $1^h\ 25^m$. See Art. 63, p. 101, for an approximate method of finding the R. A. of the mean sun. Use $5^h\ 55^m$ for the hour angle of *Polaris* at elongation (see Art. 91, p. 147).

6. Observation on sun May 15, 1906, for azimuth. Vernier A, on mark, read $0°\ 00'$. On the sun, right and lower limbs, vertical circle read $43°\ 36'$; vernier A read $168°\ 59'$ (right-handed); E. S. T., $2^h\ 52^m\ 45^s$ P.M. Upper and left limbs, vernier A read $169°\ 52'$; vertical circle read $42°\ 33'$; E. S. T., $2^h\ 55^m\ 37^s$ P.M. Declination at G. M. N. $= +\ 18°\ 42'\ 43''.6$; diff. for $1^h = +\ 35''.94$. The latitude of the place is $42°\ 21'$ N; longitude $71°\ 05'$ W. Compute the azimuth of the mark.

7. Compute the azimuth of *Jupiter* from the data given in Art. 77, p. 124.

8. Prove that the horizontal angle between the centre of the sun and the right or left limb is $S \sec h$ where S is the apparent angular semidiameter and h is the apparent altitude.

9. Prove that the level correction (Art. 97) is $i \tan h$.

10. Why could not Equa. [106] be used in place of Equa. [30] in the method of Art. 102?

11. If there is an error of 4^s in the assumed value of the watch correction and an azimuth is determined by the method of Art. 102 (near noon), what would be the relative effect of this error when the sun is on the equator and when it is $23°$ South? Assume the latitude to be $45°$ N. (See Table B.)

12. Make a set of azimuth observations by the method of Art. 93 (three pointings in each position of the instrument), and plot a curve using altitudes for ordinates and horizontal angles for abscissæ; also plot a curve using altitudes and times for the two coördinates.

CHAPTER XIV

NAUTICAL ASTRONOMY

104. Observations at Sea.

The problems of determining a ship's position at sea and the bearing of a celestial object at any time are based upon exactly the same principles as the surveyor's problems of determining his position on land and the azimuth of a line of a survey. The method of making the observations, however, is different, since the use of instruments requiring a stable support, such as the transit and the artificial horizon, is not practicable at sea. The sextant does not require a stable support and is well adapted to making observations at sea. Since the sextant can be used only to measure the angle between two visible points, it is necessary to measure all altitudes from the sea-horizon and to make the proper correction for dip.

Determination of Latitude at Sea

105. Latitude by Noon Altitude of Sun.

The determination of latitude by measuring the maximum altitude of the sun's lower limb at noon is made in exactly the same way as described in Art. 66. The observation should be begun a little before local apparent noon and altitudes measured in quick succession until the maximum is reached. In measuring an altitude above the sea-horizon the observer should bring the sun's image down until the lower limb appears to be in contact with the horizon line. The sextant should then be tipped by rotating right and left about the axis of the telescope so as to make the sun's image describe an arc; if the lower limb of the sun drops below the horizon at any point, the measured altitude is too great, and the index arm should be moved until the sun's image is just tangent to the horizon when at the lowest point of the arc. (Fig. 67.) This method is illustrated by the following example.

Example.

Observed altitude of sun's lower limb 69° 21′ 30″, bearing north. Index correction = − 1′ 10″; height of eye = 18 feet; sun's declination at G. A. N. = N 8° 59′ 32″; diff. 1^h = + 54″.43. Approx. lat. = 11° 30′ S; approx. long. = $1^h\ 00^m$ W.

Obs'd alt. = 69° 21′ 30″	Corrections	Decl + 8° 59′ 32″
Corr. = + 10 16	I. C. = − 1′ 10″	+ 54
	Dip = − 4 12	
Alt. centre = 69° 31′ 46″	r & p = − 0 20	9° 00′ 26″
Declination = 9 00 26	S. D. = + 15 58	
Colatitude = 78° 32′ 12″	Corr. = + 10′ 16″	
Latitude = 11° 27′ 48″ S		

FIG. 67

106. Latitude by Ex-Meridian Altitudes.

If for any reason the noon altitude has been lost, an altitude may be measured near noon and this altitude corrected to the corresponding noon altitude by Equa. [72]. In order to make this "reduction to the meridian" it is necessary to know the sun's hour angle. If the altitude is taken within a few minutes of noon the reduction may be made by the more convenient formula, [74]; in practice this is done by means of tables.

Example.

Observed altitude ☉ Jan. 20, 1910 = 20° 05′; I. C. = 0; G. M. T. = $1^h\ 35^m\ 28^s$; lat. by dead reckoning = 49° 20′ N; longitude by dead reckoning = $1^h\ 05^m\ 20^s$; height of eye = 16 feet; decl. at G. M. N. = 20° 15′ 02″ S; diff. for 1^h = + 32″.0; S. D. = 16′ 17″.

		$\cos L$ = 9.8140
G. M. T. = $1^h\ 35^m\ 28^s$	S. D. = + 16′ 17″	$\cos D$ = 9.9723
Long. W = 1 05 20	I. C. = 00	vers P = 7.9361
	Dip. = − 3 55	
H. A. = $0^h\ 30^m\ 08^s$	r & p = − 2 30	7.7224
= 7° 32′		Corr. = .0053
	Corr. = + 9′ 52″	$\sin h$ = .3461
	Obs. Alt. = 20° 05	
Decl. G. M. N. = 20° 15′ 02″		$\sin h_m$ = .3514
Diff. for $1^h.1$ = 35	True Alt. = 20° 14′ 52″	h_m = 20° 34′ ½
		Decl. = 20 14 ½
Decl. = 20° 14′ 27″ S		Colat. = 40° 49′
		Lat. = 49° 11′

Determination of Longitude at Sea

107. By the Greenwich Time and the Sun's Altitude.

The usual method of finding the longitude at sea is to determine the local mean time from an observed altitude of the sun (Art. 76) and to compare this with the Greenwich Mean Time as shown by the chronometer. The error of the chronometer at some previous date and its daily gain or loss are supposed to be known. This is the same in principle as the method of Art. 86. The value of the latitude used in solving the *PZS* triangle must be that of the ship at the time the observation is made; this latitude must be found by correcting the latitude by observation at the previous noon for the run of the ship in the interval. This is called the latitude by "dead reckoning." On account of the large errors which may enter into this estimated latitude it is important that the observation ("time-sight") should be made when the sun is near the prime vertical.

Example.

True alt. ⊙ May 19, 1910 (P.M.) = $44° 05'$; G. M. T. = $6^h\ 55^m\ 10^s$. Lat. by dead reckoning = $42° 00'$ N; decl. at G. M. N. = $19° 38' 20''$ N; diff. $1^h = +32''.7$; equa. of time = $-3^m\ 44^s.1$; decr. per $1^h = 0^s.1$.

$$
\begin{array}{llll}
 & L = 42° 00' \quad \sec = .1289 & \text{Decl. G. M. N.} & 19° 38' 20'' \text{ N} \\
 & D = 19\ \ 42 \quad \sec = .0262 & +32''.7 \times 6^h.9 = & 3\ \ 46 \\
\hline
\cos = .9252 & L - D = 22\ \ 18 & \text{Cor'd. decl.} = & 19° 42' 06'' \text{ N} \\
\sin = .6957 & h = 44\ \ 05 & & \\
\hline
\text{diff.} = .2295 & \log = 9.3608 & &
\end{array}
$$

$$
\begin{array}{rll}
\log \text{vers} = & 9.5159 & \text{Equa.G.M.N.} = -3^m\ 44^s.1 \\
\text{H. A.} = & 3^h\ 11^m\ 07^s & 0^s.1 \times 6^h.9 = \quad .7 \\
\text{Eq. t.} = & -\ \ 3\ \ 43 & \\
\hline
 & & \text{Cor'd Eq. t.} = -3^m\ 43^s.4 \\
\text{L. M. T.} = & 3^h\ 07^m\ 24^s & \\
\text{G. M. T.} = & 6\ \ 55\ \ 10 & \\
\hline
\text{Long. W} = & 3^h\ 47^m\ 46^s & \\
 = & 56° 56' \tfrac{1}{2} \text{ W.} &
\end{array}
$$

108. By a Lunar Distance.

The accuracy of the preceding method is wholly dependent upon the accuracy of the chronometer giving the Greenwich time. With steam vessels making short trips and carrying several chronometers this method gives the longitude with sufficient accuracy. In the days when commerce was carried on chiefly by means of sailing vessels the voyages were of long duration, and consequently the error of the chronometer could be verified only at long intervals; furthermore, the chronometers of that time were far less perfect than those of to-day, and their rates were subject to greater irregularities. Under these circumstances the method just described sometimes became wholly unreliable; in such cases the method of "lunar distance" was used. Although this method is necessarily of inferior accuracy it has the advantage of being entirely independent of the chronometer time. In the Nautical Almanac previous to the issue for 1912 there were given the geocentric distances of the moon from several bright stars, planets, and the sun, for every 3^h of Greenwich Mean Time. If a lunar distance were measured at sea and this distance reduced to the centre of the earth, the corresponding instant of G. M. T. could be found by interpolation in these tables.

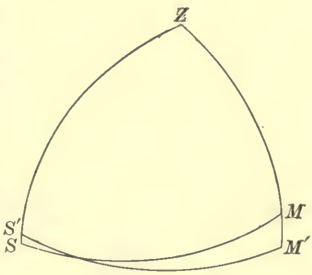

FIG. 68

The observation requires that the altitudes of the moon and the sun or star should be measured simultaneously with the distance, and that the chronometer should be read at the same instant. In Fig. 68 let Z be the observer's zenith, M' the apparent and M the true position of the moon, and S' and S the apparent and true positions of the sun. The sun's apparent position is higher than its true position because its refraction is greater than its parallax. The moon's true position is higher than its apparent position because the parallax correction is the greater. The measured distance $S'M'$ is to be reduced to the true distance SM. In the triangle $ZS'M'$ the three sides have been measured and the angle Z may be computed. Then in the triangle ZSM the angle Z and the sides ZS and ZM are known, because the refraction and parallax corrections are known, and MS may be computed. By interpolating in the tables, the true G. M. T. corresponding to the instant of this observation may be obtained, the difference between this and the observed chronometer time being the error of the chronometer on G. M.T. The longitude may then be found by comparing the true G. M. T. with the local time computed from the sun's altitude.

In the Ephemeris for 1912 the tables of lunar distances have been omitted, as lunar observations are no longer considered to be of practical value to the navigator.

109. Azimuth of the Sun at a Given Time.

For determining the error of the compass and for other purposes it is frequently necessary at sea to know the sun's azimuth at an observed instant of time. If the observed time be converted into local apparent time the azimuth Z may be computed by the following formulæ.*

$$\tan \tfrac{1}{2}(Z+S) = \cot \tfrac{1}{2} P \sec \tfrac{1}{2}(p+\text{co-}L) \cos \tfrac{1}{2}(p-\text{co-}L), \qquad [112]$$
$$\tan \tfrac{1}{2}(Z-S) = \cot \tfrac{1}{2} P \csc \tfrac{1}{2}(p+\text{co-}L) \sin \tfrac{1}{2}(p-\text{co-}L). \qquad [113]$$

In these formulæ co-L is the co-latitude. In practice the azimuth is taken from tables computed by use of these formulæ. Burdwood's and Davis's Azimuth Tables give the azimuth for each degree of P, L, and p, the former ranging from Lat. 30° to Lat. 60° and the latter from 30° N to 30° S. Publication No. 71 of the U. S. Hydrographic Office gives azimuths of the sun for latitudes up to 61°. For finding the azimuth of an object having a declination greater than 24° publication No. 120 of the Hydrographic Office may be used.

Example.

Find the sun's azimuth when $L = 42° 01'$ N, $D = 22° 47'$ S, $P = 9^h 25^m 18^s$. From Publ. No. 71 for $L = 42°$, $D = 22°$, $P = 9^h 20^m$, the azimuth is N 141° 40′ E. The corresponding azimuth for $L = 43°$ is 141° 50′, that is, an increase of 10′ for 1°; the azimuth for $L = 42°$ $D = 23°$, and $P = 9^h 20^m$, is 142° 11′, or an increase of 31′ for 1° of declination; for $L = 42°$, $D = 22°$, and $P = 9^h 30^m$ the

* Napier's Analogies.

azimuth is 143° 47′, or an increase of 2° 07′ for 10^m, or 12′.7 for 1^m. The desired azimuth is therefore $141° \; 40' + \frac{1}{60} \times 10' + \frac{47}{60} \times 31' + 5.3 \times 12'.7 = 143° \; 12'$. The azimuth from the south point is therefore S 36° 48′ E.

When the azimuth is determined for the purpose of finding the error of the compass the observation is usually taken near sunrise or sunset, which is not only a convenient time for making the pointings at the sun but is a favorable time for accurate determination of the azimuth.

110. Azimuth of the Sun by Altitude and Time.

When the altitude of the sun is observed for the purpose of finding the local time, the azimuth at the same instant may be computed by the formula

$$\sin Z = \sin P \cos D \sec h. \qquad [12]$$

Example.

Find the sun's azimuth when $P = 34° \; 46'.4$ (P.M.), $D = -22° \; 45' \; 50''$, $h = 17° \; 41'$

$$\begin{aligned} \log \sin P &= 9.75612 \\ \log \cos D &= 9.96478 \\ \log \sec h &= 0.02102 \\ \hline \log \sin Z &= 9.74192 \\ Z &= \text{S } 33° \; 30'.2 \text{ W} \end{aligned}$$

111. Sumner's Method of Determining a Ship's Position.*

If the declination of the sun and the Greenwich Apparent Time are known at any instant, these two coördinates are the latitude and longitude respectively of a point on the earth's surface which is vertically under the sun's centre and which may be called the "sub-solar" point. If an observer were at the sub-solar point he would have the sun in his zenith. If he were located 1° from this point, in any direction, the sun's zenith distance would be 1°; if he were 2° away, the zenith distance would be 2°. It is evident, then, that if an observer measures an altitude of the sun he locates himself on the circumference of a circle whose centre is the sub-solar point and whose radius (in degrees) is the zenith distance of the sun. This circle could be drawn on a globe by first plotting the position of the sub-solar point by means of its coördinates, and

---

* This method was first described by Captain Sumner in 1843.

then setting a pair of dividers to subtend an arc equal to the zenith distance (by means of a graduated circle on the globe) and describing a circle about the sub-solar point as a centre. The observer is somewhere on this circle because all positions on the earth where the sun has this measured altitude are located on this same circle. This is called a **circle of position**, and any portion of it a **line of position** or a **Sumner line.**

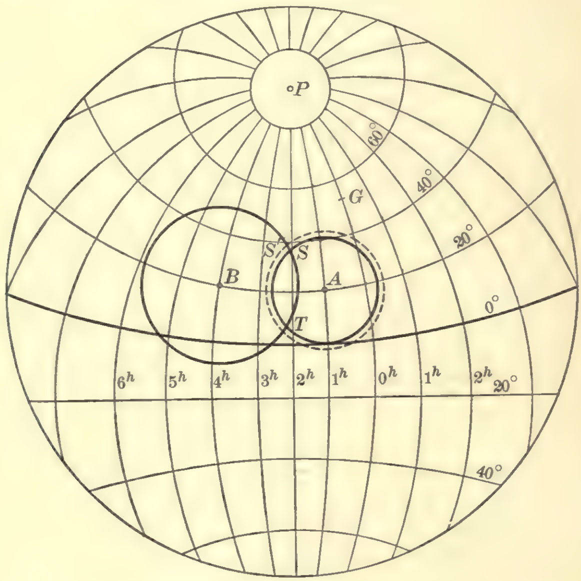

FIG. 69

Suppose that at Greenwich Apparent Time 1^h the sun is observed to have a zenith distance of 20°, the declination being 20° N. The sub-solar point is then at A, Fig. 69, and the observer is somewhere on the circle described about A with a radius 20°. If he waits until the G. A. T. is 4^h and again observes the sun, obtaining 30° for his zenith distance, he locates himself on the circle whose centre is B, the coördinates being 4^h and (say) 20° 02′ N, and the radius of which is 30°. If the ship's position has not changed between the observations it is either at S or at T; in practice there is no difficulty in deciding which is the correct point, on account of their great distance apart. A knowledge of the sun's bearing also shows which portion of the circle contains the point. If, however, the ship has changed its position since the first observation, it is necessary to allow for its "run" as follows. Assuming that the ship has sailed directly away from the sun, say a distance of 60 miles or $1°$, then, if the first observation had been made while the ship was in the second position, the point A would be the same, but the radius of the circle would be $21°$, locating the ship on the dotted circle. The true position of the ship at the second observation is, therefore, at the intersection S'. If the vessel does not actually sail directly away from or directly toward the sun it is a simple matter to calculate the increase or decrease in radius due to the change in the observer's zenith.

This is in principle Sumner's method of locating a ship. In practice the circles would seldom have as short radii as those in Fig. 69; small circles were chosen only for convenience in illustrating the method. On account of the long radius of the circle of position only a small portion of this circle can be shown on an ordinary chart; in fact, the portion which it is necessary to use is generally so short that the curvature is negligible and the line of position appears on the chart as a straight line. In order to plot a Sumner line on the chart, two latitudes may be assumed between which the actual latitude is supposed to lie; and from these, the known declination, the observed altitude, and the chronometer reading, two longitudes may be computed (Art. 107), one for each of the assumed latitudes. This gives the coördinates of two points on the line of position by means of which it may be plotted on the chart. Another observation may be made a few hours later and the new line plotted in a similar manner. In order to allow for the change in the radius of the circle due to the ship's run between observations, it is only necessary to move the first position line parallel to itself in the direction of the ship's course and a distance equal to the ship's run. In Fig. 70, AB is a line obtained from a 9 A.M observation on the sun and by assuming the latitudes 42° and 43°. A second observation is made at 2 P.M.; between 9^h and 2^h the ship has sailed S 75° W, 67′.* Plotting this run on the chart so as to move any point on AB, such as x, in the direction S 75° W and a distance of 67′, the new position line for the first

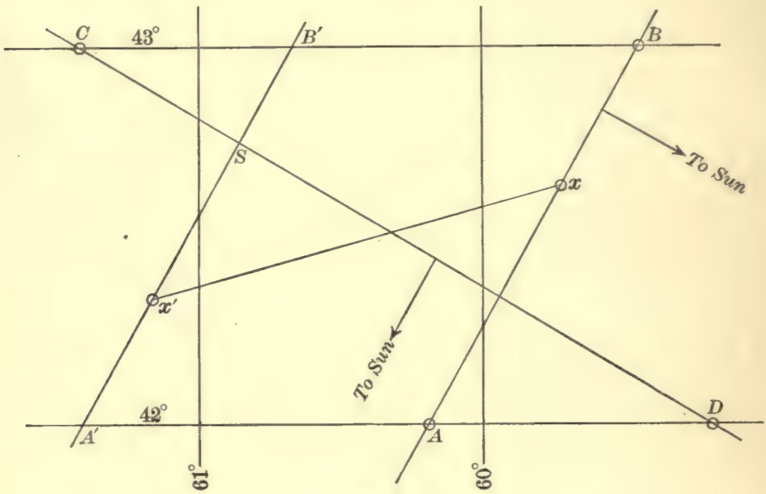

FIG. 70

observation is $A'B'$. The P.M. line of position is located by assuming the same latitudes, 42° and 43°, the result being the line CD. The point of intersection S is the position of the ship at the time of the second observation. Since the bearing of the sun is always at right angles to the bearing of the Sumner line, it is evident that one point and the bearing would be sufficient to locate the line on the chart.

112. Position by Computation.

The coördinates of the point of intersection of the lines of position may be calculated more precisely than they can be taken from the chart. When the first

* The nautical mile (6080.27 feet) is assumed to be equal to an arc of 1′ on any part of the earth's surface.

altitude is measured the navigator assumes a latitude which is near the true latitude, and from this calculates the corresponding longitude. The approximate azimuth of the sun is also calculated from the same data. (Equa. [30].) The run of the ship up to the time of the second observation is reduced to the difference in latitude and the difference in longitude from the known course and speed of the vessel. These two differences are applied as corrections to the assumed latitude and the calculated longitude. This places the ship on the new Sumner line (corresponding to $A'B'$, Fig. 70). When the P.M. observation is made the *corrected* latitude is used in computing the new longitude. The result of these two observations is shown in Fig. 71. Point A is the first position; A' is the position of A

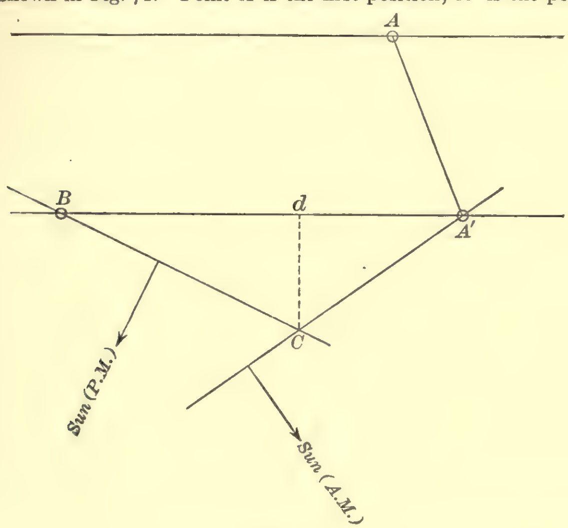

FIG. 71

corrected for the run of the ship; B is the position obtained by the P.M. observation using the latitude of A'. $A'B$ is then the discrepancy in the longitudes, due to the fact that a wrong latitude has been chosen, and is the base of a triangle the vertex of which, C, is the true position of the ship. The base angles A' and B are the azimuths of the sun at the times of observation. In practice this triangle is often solved as follows:* Dropping a perpendicular from C to $A'B$, forming two right triangles,

$$Bd = Cd \cot Z_2,$$

and

$$A'd = Cd \cot Z_1,$$

or

$$\Delta p_2 = \Delta L \cot Z_2,$$
$$\Delta p_1 = \Delta L \cot Z_1,$$

* See A. C. Johnson's "On Finding the Latitude and Longitude in Cloudy Weather."

where ΔL = the error in latitude and Δp the difference in departure. In order to express Bd and $A'd$ as differences in longitude (ΔM) it is necessary to introduce the factor sec L, giving

$$\Delta M_2 = \Delta L \sec L \cot Z_2, \qquad [114]$$
$$\Delta M_2 = \Delta L \sec L \cot Z_1. \qquad [115]$$

To find ΔL, the correction to the latitude, the distance $A'B = \Delta M_2 + \Delta M_1$ is known, the factors sec L cot Z may be found from the approximate latitude and the sun's azimuths, therefore

$$\Delta L = \frac{A'B}{\sec L \cot Z_1 + \sec L \cot Z_2}. \qquad [116]$$

Having found ΔL, the corrections ΔM_1 and ΔM_2 are found by [114] and [115]. Since the factors sec L cot Z are, in practice, taken from a table and the operations indicated in Equa. [114], [115], and [116] are easily performed with the slide rule the method is in reality a rapid one.

In the above description the observations are taken one in the forenoon and one in the afternoon, but any two observations, provided the position lines intersect at an angle over 30°, will give good results. If the observations are both on the same side of the meridian the denominator of [116] becomes the difference of the factors instead of the sum. If two objects can be observed at the same time, and their bearings differ by 30° or more, the position of the ship is obtained at once, since there is no run of the ship to be applied. This observation might be made upon two bright stars or planets at twilight. It should be observed that the accuracy of this method depends upon the accuracy of the chronometer, just as in the methods of Art. 107.

One of the great advantages of this method is that even if only one observation can be taken it may be utilized to locate the ship along a (nearly) straight line; and this is often of great value. For example, if the first position line is found to pass directly through some point of danger, then the navigator knows the bearing of the point, although he does not know his distance from it; but with the single observation he is able to avoid the danger. In case it is a point which it is desired to reach, the true course which the ship should steer is at once known. The following example illustrates the method of computing the coördinates of the point of intersection.

Example.

Location of ship by Sumner's Method, Jan. 4, 1910.

At chronometer time $1^h\ 12^m\ 48^s$ the sun's lower limb is observed to be $15°\ 53'\ 30''$; index corr. $= 0''$; height of eye $= 36$ ft.; chronometer is 15^s fast of G. M. T. Latitude by dead reckoning, $42°\ 00'$ N. At chronometer time $6^h\ 05^m\ 46^s$ the altitude of the sun's lower limb $= 17°\ 33'\ 30''$; index corr. $= 0''$; height of eye, 36 ft; chronometer, 15^s fast. The run between the observations was $1'$ N and $60'$ W.

First Observation

$$\begin{aligned}
\text{Semidiam.} &= +16'\ 17'' \\
\text{dip} &= -\ \ 5\ \ 53 \\
r\ \&\ p &= -\ \ 3\ \ 14 \\
\hline
\text{Corr.} &= +\ \ \ 07'\ 10'' \\
\text{Obs. Alt.} &= \ 15°\ 53'\ 30'' \\
\hline
\text{True Alt.} &= \ 16°\ 00'\ 40''
\end{aligned}$$

$$\begin{aligned}
\text{Declination at G. M. N.} &= -\ 22°\ 47'\ 22''.3 \\
+15''.15 \times 1^h.2 &= \qquad\qquad +18\ .2 \\
\hline
\text{Decl.} &= -\ 22°\ 47'\ 04''.1 \\
p &= \ 112°\ 47'\ 04''.1
\end{aligned}$$

$$\begin{aligned}
L &= 42°\ 00' && \sec 0.12893 \\
p &= 112\ \ 47 && \csc 0.03528 \\
h &= 16\ \ 00.7 \\
\hline
& \ \ 170\ \ 47.7 \\
s &= 85°\ 23'.8 && \cos 8.90448 \\
s-h &= 69°\ 23.1 && \sin 9.97126 \\
\hline
& && 2)9.03995
\end{aligned}$$

$$\begin{aligned}
\text{Equa. t.} &= 4^m\ 49^s.80 \\
1^s.145 \times 1^h.2 &= \qquad 1\ .37 \\
\hline
\text{Cor'd Eq. t.} &= 4^m\ 51^s.17
\end{aligned}$$

$$\begin{aligned}
\text{Sun's Az.*} &= \text{S } 36°\ 48'\ \text{E} \\
\text{cot Az.} \times \text{sec Lat.} &= 1.80
\end{aligned}$$

$$\begin{aligned}
\log\sin \tfrac{1}{2}P &= 9.51998 \\
\tfrac{1}{2}P &= 19°\ 20'.2 \\
P &= 38°\ 40'.4 \\
&= 2^h\ 34^m\ 41^s.6 \\
\text{L. A. T.} &= 9^h\ 25^m\ 18^s.4 \\
\text{Eq. t.} &= \qquad 4\ \ 51\ .2 \\
\hline
\text{L. M. T.} &= 9^h\ 30^m\ 09^s.6 \\
\text{G. M. T.} &= 1\ \ \ 12\ \ \ 33 \\
\hline
\text{Long.} &= 3^h\ 42^m\ 23^s.4 \\
&= 55°\ 35'.8 \\
\text{Run} &= +60' \\
\hline
\text{Cor'd Long.} &= 56°\ 35'.8
\end{aligned}$$

$$\begin{aligned}
\text{Lat.} &= 42°\ 00' \\
\text{Run} &= \qquad 1' \\
\hline
\text{Cor'd Lat.} &= 42°\ 01'
\end{aligned}$$

---

* By table or by Equa. [30]; see Art. 93, p. 155.

Second Observation

semidiam. =	$+16'\ 17''$	Decl. at G. M. N. =	$-22^\circ\ 47'\ 22''.3$
dip =	$-\ 5\ \ 53$	$+15''.15 \times 6^h.1$ =	$+1\ \ 32\ .4$
r & p =	$-\ 2\ \ 54$		
Corr. =	$+7'\ 30''$	Decl. =	$-22^\circ\ 45'\ 49''.9$
Obs. Alt. =	$17\ \ 33\ \ 30$	p =	$112^\circ\ 45'\ 49''.9$
True Alt. =	$17^\circ\ 41'\ 00''$		

L =	$42^\circ\ 01'$	sec 0.12904	Eq. t. =	$4^m\ 49^s.80$
p =	$112\ \ 45\ .8$	csc 0.03522	$1^s.145 \times 6^h.1$ =	$6\ .98$
h =	$17\ \ 41$			
	$172\ \ 27\ .8$		Cor'd Eq. t. =	$4^m\ 56^s.78$
s =	$86^\circ\ 13\ .9$	cos 8.81771	Sun's Az.* =	S $33^\circ\ 30'$ W
$s - h$ =	$68^\circ\ 32'.9$	sin 9.96883	cot Az. $\times$ sec Lat. =	2.03
		2)8.95080		

$$
\begin{aligned}
\log \sin \tfrac{1}{2} P &= 9.47540 \\
\tfrac{1}{2} P &= 17^\circ\ 23'.2 \\
P &= 34^\circ\ 46'.4 \\
\text{L. A. T.} &= 2^h\ 19^m\ 05^s.6 \\
\text{Eq. t.} &= 4\ \ 56\ .8 \\
\hline
\text{L. M. T.} &= 2^h\ 24^m\ 02^s.4 \\
\text{G. M. T.} &= 6\ \ 05\ \ 31 \\
\hline
\text{Long.} &= 3^h\ 41^m\ 28^s.6 \\
&= 55^\circ\ 22'.1
\end{aligned}
$$

1st Long. $= 56^\circ\ 35'.8$
2d Long. $= 55\ \ 22\ .1$
Diff. $= 1^\circ\ 13'.7 = 73'.7$

$19.2 \times 1.80 = 34'.6$ Corr. to 1st Long.
$19.2 \times 2.03 = 39\ .0$ Corr. to 2d Long.

1st Long. =	$56^\circ\ 35'.8$	2d Long. =	$55^\circ\ 22'.1$
Corr. =	$34\ .6$	Corr. =	$39\ .0$
Long. =	$56^\circ\ 01'.2$	Long. =	$56^\circ\ 01'.1$

$$\frac{73'.7}{1.80 + 2.03} = 19'.2 \text{ Corr. to the Lat.}$$

$\therefore$ Lat. $= 42^\circ\ 20'$ N. $\qquad \therefore$ Long. $= 56^\circ\ 01'$ W.

* By table or by Equa. [30]; see Art. 93, p. 155.

TABLES

TABLE I. MEAN REFRACTION.

Barometer, 29.5 inches. Thermometer, 50° F.

App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.
0° 00′	33′ 51″	10° 00′	5′ 13″	20° 00′	2′ 36″	35° 00′	1′ 21″
30	28 11	30	4 59	30	2 32	36 00	1 18
1 00	23 51	11 00	4 46	21 00	2 28	37 00	1 16
30	20 33	30	4 34	30	2 24	38 00	1 13
2 00	17 55	12 00	4 22	22 00	2 20	40 00	1 08
30	15 49	30	4 12	30	2 17	42 00	1 03
3 00	14 07	13 00	4 02	23 00	2 14	44 00	0 59
30	12 42	30	3 54	30	2 11	46 00	0 55
4 00	11 31	14 00	3 45	24 00	2 08	48 00	0 51
30	10 32	30	3 37	30	2 05	50 00	0 48
5 00	9 40	15 00	3 30	25 00	2 02	52 00	0 45
30	8 56	30	3 23	26 00	1 57	54 00	0 41
6 00	8 19	16 00	3 17	27 00	1 52	56 00	0 38
30	7 45	30	3 10	28 00	1 47	58 00	0 36
7 00	7 15	17 00	3 05	29 00	1 43	60 00	0 33
30	6 49	30	2 59	30 00	1 39	65 00	0 27
8 00	6 26	18 00	2 54	31 00	1 35	70 00	0 21
30	6 05	30	2 49	32 00	1 31	75 00	0 15
9 00	5 46	19 00	2 44	33 00	1 28	80 00	0 10
30	5 29	30	2 40	34 00	1 24	85 00	0 05
10 00	5 13	20 00	2 36	35 00	1 21	90 00	0 00

TABLE II. FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

(Increase in Sun's Right Ascension in Sidereal h. m. s.)

Mean Time = Sidereal Time − C'.

Sid. Hrs.	Corr.	Sid. Min.	Corr.	Sid. Min.	Corr.	Sid. Sec.	Corr.	Sid. Sec.	Corr.
	m s		s		s		s		s
1	0 9.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 8.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 7.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 6.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

TABLE III. FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

(Increase in Sun's Right Ascension in Solar h. m. s.)

Sidereal Time = Mean Time + C.

Mean Hrs.	Corr.	Mean Min.	Corr.	Mean Min.	Corr.	Mean Sec.	Corr.	Mean Sec.	Corr.
	m s		s		s		s		s
1	0 9.856	1	0.164	31	5.093	1	0.003	31	0.085
2	0 19.713	2	0.329	32	5.257	2	0.005	32	0.088
3	0 29.569	3	0.493	33	5.421	3	0.008	33	0.090
4	0 39.426	4	0.657	34	5.585	4	0.011	34	0.093
5	0 49.282	5	0.821	35	5.750	5	0.014	35	0.096
6	0 59.139	6	0.986	36	5.914	6	0.016	36	0.099
7	1 8.995	7	1.150	37	6.078	7	0.019	37	0.101
8	1 18.852	8	1.314	38	6.242	8	0.022	38	0.104
9	1 28.708	9	1.478	39	6.407	9	0.025	39	0.107
10	1 38.565	10	1.643	40	6.571	10	0.027	40	0.110
11	1 48.421	11	1.807	41	6.735	11	0.030	41	0.112
12	1 58.278	12	1.971	42	6.900	12	0.033	42	0.115
13	2 8.134	13	2.136	43	7.064	13	0.036	43	0.118
14	2 17.991	14	2.300	44	7.228	14	0.038	44	0.120
15	2 27.847	15	2.464	45	7.392	15	0.041	45	0.123
16	2 37.704	16	2.628	46	7.557	16	0.044	46	0.126
17	2 47.560	17	2.793	47	7.721	17	0.047	47	0.129
18	2 57.417	18	2.957	48	7.885	18	0.049	48	0.131
19	3 7.273	19	3.121	49	8.049	19	0.052	49	0.134
20	3 17.129	20	3.285	50	8.214	20	0.055	50	0.137
21	3 26.986	21	3.450	51	8.378	21	0.057	51	0.140
22	3 36.842	22	3.614	52	8.542	22	0.060	52	0.142
23	3 46.699	23	3.778	53	8.707	23	0.063	53	0.145
24	3 56.555	24	3.943	54	8.871	24	0.066	54	0.148
		25	4.107	55	9.035	25	0.068	55	0.151
		26	4.271	56	9.199	26	0.071	56	0.153
		27	4.435	57	9.364	27	0.074	57	0.156
		28	4.600	58	9.528	28	0.077	58	0.160
		29	4.764	59	9.692	29	0.079	59	0.162
		30	4.928	60	9.856	30	0.082	60	0.164

TABLE IV.

PARALLAX — SEMIDIAMETER — DIP.

(A) Sun's parallax.

Sun's altitude.	Sun's parallax.
0°	9″
10	9
20	8
30	8
40	7
50	6
60	4
70	3
80	2
90	0

(B) Sun's semidiameter.

Date.	Semidiameter.
Jan. 1	16′ 18″
Feb. 1	16 16
Mar. 1	16 10
Apr. 1	16 02
May 1	15 54
June 1	15 48
July 1	15 46
Aug. 1	15 47
Sept. 1	15 53
Oct. 1	16 01
Nov. 1	16 09
Dec. 1	16 15

(C) Dip of the sea horizon.

Height of eye in feet.	Dip of sea horizon.
1	0′ 59″
2	1 23
3	1 42
4	1 58
5	2 11
6	2 24
7	2 36
8	2 46
9	2 56
10	3 06
11	3 15
12	3 24
13	3 32
14	3 40
15	3 48
16	3 55
17	4 02
18	4 09
19	4 16
20	4 23
21	4 29
22	4 36
23	4 42
24	4 48
25	4 54
26	5 00
27	5 06
28	5 11
29	5 17
30	5 22
35	5 48
40	6 12
45	6 36
50	6 56
55	7 16
60	7 35
65	7 54
70	8 12
75	8 29
80	8 46
85	9 02
90	9 18
95	9 33
100	9 48

TABLE V.

Local Mean (Astronomical) Times of Culmination and Elongation of *Polaris* for 1910, computed for Longitude 90° West of Greenwich and for Latitude 40° N.*

Date.	Upper culmination.	Western elongation.	Lower culmination.	Eastern elongation.
Jan. 1	6^h 44^m	12^h 39^m	18^h 42^m	0^h 49^m
Jan. 15	5 48	11 44	17 46	23 49
Feb. 1	4 41	10 36	16 39	22 42
Feb. 15	3 46	9 41	15 44	21 47
Mar. 1	2 51	8 46	14 49	20 52
Mar. 15	1 56	7 51	13 54	19 57
Apr. 1	0 49	6 44	12 47	18 50
Apr. 15	23 50	5 49	11 52	17 55
May 1	22 47	4 46	10 49	16 52
May 15	21 52	3 51	9 54	15 57
June 1	20 45	2 44	8 47	14 50
June 15	19 50	1 49	7 52	13 55
July 1	18 48	0 47	6 50	12 53
July 15	17 54	23 49	5 56	11 59
Aug. 1	16 47	22 42	4 48	10 51
Aug. 15	15 52	21 47	3 54	9 57
Sept. 1	14 45	20 40	2 47	8 50
Sept. 15	13 50	19 45	1 52	7 55
Oct. 1	12 47	18 42	0 49	6 52
Oct. 15	11 52	17 47	23 50	5 57
Nov. 1	10 46	16 41	22 44	4 50
Nov. 15	9 50	15 46	21 48	3 55
Dec. 1	8 47	14 42	20 45	2 52
Dec. 15	7 52	13 47	19 50	1 57

* This table may be used to find the approximate times for any year. For dates falling between those given in the table the times may be found by interpolation, the daily difference being about 4^m. For the method of converting this local time into Standard time see Art. 35.

TABLE VI. CORRECTION TO THE ALTITUDE OF POLARIS *

(Equa. [80], Art. 69.)

	Latitudes.									
H.A.	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°
10°	0″	0″	1″	1″	1″	1″	1″	1″	2″	2″
20	1	1	2	2	3	4	4	5	6	7
30	2	3	4	5	6	8	9	11	13	16
40	3	5	7	9	11	13	15	18	22	26
50	5	7	10	12	15	18	22	26	31	37
60	6	9	12	15	19	23	27	33	39	47
70	7	10	14	18	22	27	32	38	46	55
80	7	11	15	20	24	29	35	42	49	60
90	8	11	16	20	25	30	36	43	51	61
100	7	11	15	19	24	29	35	41	49	59
110	6	10	13	17	21	26	31	37	44	53
120	5	8	11	15	18	22	26	31	37	45
130	4	6	9	11	14	17	20	24	29	35
140	3	4	6	8	10	12	14	17	20	24
150	2	3	4	5	6	7	9	10	12	15
160	1	1	2	2	3	3	4	5	6	7
170	0	0	0	1	1	1	1	1	2	2

* This table is calculated for a polar distance = 1° 10′. An increase of 1′ in the polar distance produces an increase of about 3% in the tabulated term. The hour angle in the table is measured from 0° at upper culmination either to the east or to the west.

TABLE VII.

VALUES OF FACTOR 112.5 × 3600 × SIN 1″ TAN Z_e.

Z_e	Factor.	Z_e	Factor.	Z_e	Factor.
1° 00′	.0343	1° 20′	.0457	1° 40′	.0571
01	.0348	21	.0463	41	.0577
02	.0354	22	.0468	42	.0583
03	.0360	23	.0474	43	.0589
04	.0366	24	.0480	44	.0594
05	.0371	25	.0486	45	.0600
06	.0377	26	.0491	46	.0606
07	.0383	27	.0497	47	.0611
08	.0388	28	.0503	48	.0617
09	.0394	29	.0508	49	.0623
10	.0400	30	.0514	50	.0629
11	.0406	31	.0520	51	.0634
12	.0411	32	.0526	52	.0640
13	.0417	33	.0531	53	.0646
14	.0423	34	.0537	54	.0651
15	.0428	35	.0543	55	.0657
16	.0434	36	.0548	56	.0663
17	.0440	37	.0554	57	.0669
18	.0446	38	.0560	58	.0674
19	.0451	39	.0566	59	.0680

GREEK ALPHABET

Letters.	Name.	Letters.	Name.
Α, α,	Alpha	Ν, ν,	Nu
Β, β,	Beta	Ξ, ξ,	Xi
Γ, γ,	Gamma	Ο, ο,	Omicron
Δ, δ,	Delta	Π, π,	Pi
Ε, ε,	Epsilon	Ρ, ρ,	Rho
Ζ, ζ,	Zeta	Σ, σ, ς,	Sigma
Η, η,	Eta	Τ, τ,	Tau
Θ, θ, ϑ,	Theta	Υ, υ,	Upsilon
Ι, ι,	Iota	Φ, φ,	Phi
Κ, κ,	Kappa	Χ, χ,	Chi
Λ, λ,	Lambda	Ψ, ψ,	Psi
Μ, μ,	Mu	Ω, ω,	Omega

ABBREVIATIONS USED IN THIS BOOK

♈ or V	= vernal equinox.
R. A. or Rt. Asc.	= right ascension.
D or Decl.	= declination.
p	= polar distance.
h or Alt.	= altitude.
z	= zenith distance.
P or H. A.	= hour angle.
L or Lat.	= latitude.
Long.	= longitude.
Sid.	= sidereal.
Sol.	= solar.
G. M. N.	= Greenwich Mean Noon.
G. M. T.	= Greenwich Mean Time.
G. A. T.	= Greenwich Apparent Time.
G. S. T.	= Greenwich Sidereal Time.
L. M. N.	= Local Mean Noon.
L. M. T.	= Local Mean Time.
L. A. T.	= Local Apparent Time.
L. S. T.	= Local Sidereal Time.
Eq. T.	= equation of time.
Astr.	= astronomical time.
Civ.	= civil.
E. S. T.	= Eastern Standard Time.
U. C.	= upper culmination.
L. C.	= lower culmination.
$\overline{\odot}$ or U. L.	= upper limb.
$\underline{\odot}$ or L. L.	= lower limb.
RL ,LL	= right limb, left limb.
*	= star.
Corr.	= correction.
I. C.	= index correction.
r. or refr.	= refraction correction.
p	= parallax correction.
s. d.	= semidiameter.
Z or Az.	= azimuth.
N, E, S, W	= north, east, south, west.

APPENDIX

THE TIDES

The Tides.

The engineer may occasionally be called upon to determine the height of mean sea level or of mean low water as a datum for levelling or for soundings. The exact determination of these heights requires a long series of observations, but an approximate determination, sufficiently accurate for many purposes, may be made by means of a few observations. In order to make these observations in such a way as to secure the best results the engineer should understand the general theory of the tides.

Definitions.

The periodic rise and fall of the surface of the ocean, caused by the moon's and the sun's attraction, is called the **tide.** The word " tide " is sometimes applied to the horizontal movement of the water (**tidal currents**), but in the following discussion it will be used only to designate the vertical movement. When the water is rising it is called **flood tide**; when it is falling it is called **ebb tide.** The maximum height is called **high water**; the minimum is called **low water.** The difference between the two is called the **range** of tide.

Cause of the Tides.

The principal cause of the tide is the difference in the force of attraction exerted by the moon upon different parts of the earth. Since the force of attraction varies inversely as the square of the distance, the portion of the earth's surface nearest the moon is attracted with a greater force than the central portion, and the latter is attracted more powerfully than the portion farthest from the moon. If the earth and moon were at rest the surface of the water beneath the moon would be elevated as shown in Fig. 72 at A. And since the attraction at B is the least, the water surface will also be elevated at this point. The same forces which tend to elevate the surface at A and B tend to depress it at C and D. If the earth were set rotating, an observer at any point O, Fig. 72, would be carried through two high and two low tides each day, the approximate interval between the high and the low tides being about $6\frac{1}{4}$ hours. This explanation shows what would happen if the tide were developed while the two bodies were at rest; but, owing to the high velocity of the earth's rotation, the shallowness of the water, and the interference of continents, the actual

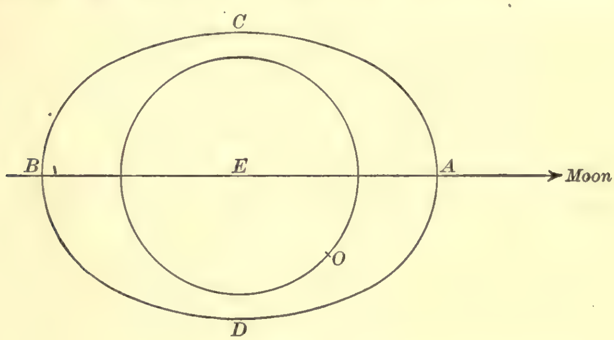

FIG. 72

tide is very complex. If the earth's surface were covered with water, and the earth were at rest, the water surface at high tide would be about two feet above the surface at low tide. The interference of continents, however, sometimes forces the tidal wave into a narrow, or shallow, channel, producing a range of tide of fifty feet or more, as in the Bay of Fundy.

The sun's attraction also produces a tide like the moon's, but considerably smaller. The sun's mass is much greater than the moon's but on account of its greater distance the ratio of the tide-producing forces is only about 2 to 5. The tide actually observed, then, is a combination of the sun's and the moon's tides.

Effect of the Moon's Phase.

When the moon and the sun are acting along the same line, at new or full moon, the tides are higher than usual and are called **spring tides.** When the moon is at quadrature (first or last quarter), the sun's and the moon's tides partially neutralize each other and the range of tide is less than usual; these are called **neap tides.**

Effect of Change in Moon's Declination.

When the moon is on the equator the two successive high tides are of nearly the same height. When the moon is north

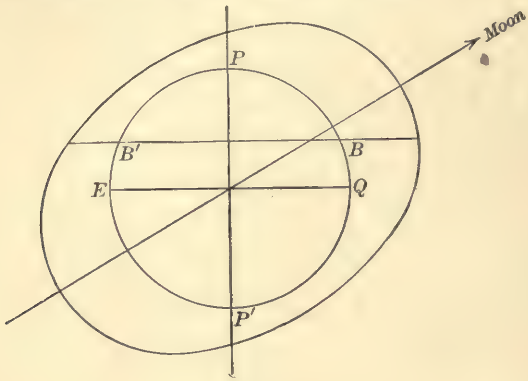

FIG. 73

or south of the equator the two differ in height, as is shown in Fig. 73. At point B under the moon it is high water, and the depth is greater than the average. At B', where it will again be high water about 12 later, the depth is less than the average. This is known as the **diurnal inequality.** At the points E and Q, on the equator, the two tides are equal.

Effect of the Moon's Change in Distance.

On account of the large eccentricity of the moon's orbit the tide-raising force varies considerably during the month. The actual distance of the moon varies about 13 per cent, and as a result the tides are about 20 per cent greater when the moon is nearest the earth, at perigee, than they are when the moon is farthest, at apogee.

Priming and Lagging of the Tides.

On the days of new and full moon the high tide at any place follows the moon's meridian passage by a certain interval of time, depending upon the place, which is called the **establishment of the port.** For a few days after new or full moon the crest of the combined tidal wave is west of the moon's tide and high water occurs earlier than usual. This is called the **priming** of the tide. For a few days before new or full moon the crest is east of the moon's tide and the time of high water is delayed. This is called **lagging** of the tide.

All of these variations are shown in Fig. 74, which was constructed by plotting the predicted times and heights from the U. S. Coast Survey Tide Tables and joining these points by straight lines. It will be seen that at the time of new and full moon the range of tide is greater than at the first and last quarters; at the points where the moon is farthest north or south of the equator (shown by *N*, *S*,) the diurnal inequality is quite marked, whereas at the points where the moon is on the equator (*E*) there is no inequality; at perigee (*P*) the range is much greater than at apogee (*A*).

Effect of Wind and Atmospheric Pressure.

The actual height and time of a high tide may differ considerably from the normal values at any place, owing to the weather conditions. If the barometric pressure is great the surface is depressed, and *vice versa.* When the wind blows steadily into a bay or harbor the water is piled up and the height of the tide is increased. The time of high water is delayed because the water continues to flow in after the true time of high water has passed; the maximum does not occur until the ebb and the effect of wind are balanced.

Observation of the Tides.

In order to determine the elevation of mean sea level, or, more properly speaking, of mean half-tide, it is only necessary to observe, by means of a graduated staff, the height of high and low water for a number of days, the number depending upon

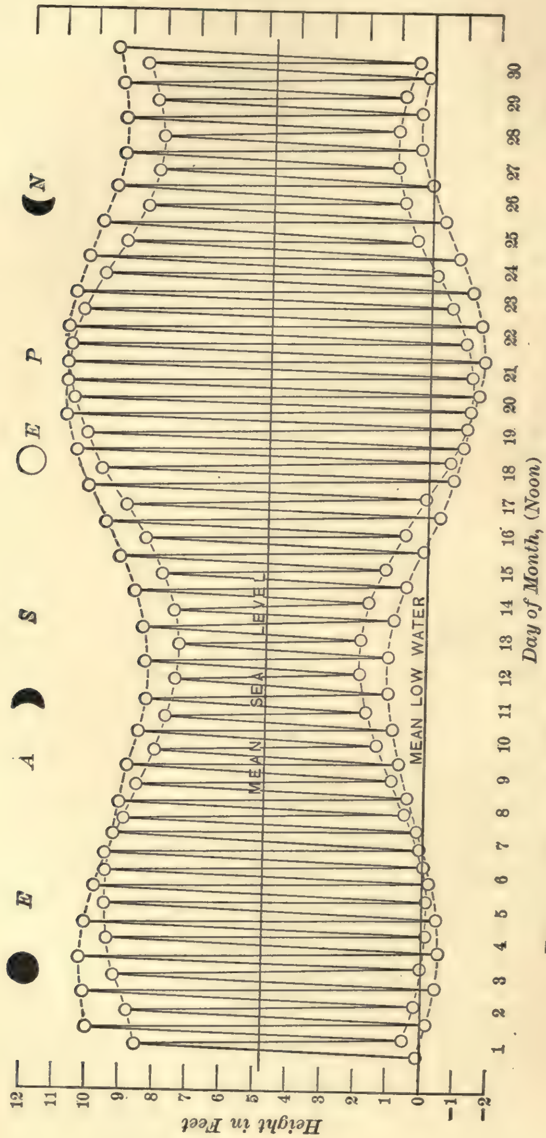

FIG. 74. DIAGRAM OF PREDICTED TIDES AT BOSTON, MASS., SEPTEMBER, 1910

the accuracy desired, and to take the mean of the gauge readings. If the height of the zero point of the scale is referred to some bench mark, by means of a line of levels, the height of the bench mark above mean sea level may be computed. In order to take into account all of the small variations in the tides it would be necessary to carry on the observations for a series of years; a very fair approximation may be obtained, however, in one lunar month, and a rough result, close enough for many purposes, may be obtained in a few days.

Tide Gauges.

If an elaborate series of observations is to be made, the self-registering tide gauge is the best one to use. This consists of a float, which is enclosed in a vertical wooden box and which rises and falls with the tide. A cord is attached to the float and is connected by means of a reducing mechanism with the pen of a recording apparatus. The record sheet is wrapped about a cylinder, which is revolved by means of clockwork. As the tide rises and falls the float rises and falls in the box and the pen traces out the tide curve on a reduced scale. The scale of heights is found by taking occasional readings on a staff gauge which is set up near the float box and referred to a permanent bench mark. The time scale is found by means of reference marks made on the sheet at known times.

When only a few observations are to be made the staff gauge is the simplest to construct and to use. It consists of a vertical graduated staff fastened securely in place, and at such a height that the elevation of the water surface may be read on the graduated scale at any time. Where the water is comparatively still the height may be read directly on the scale; but where there are currents or waves the construction must be modified. If a current is running rapidly by the gauge but the surface does not fluctuate rapidly, the ripple caused by the water striking the gauge may be avoided by fastening wooden strips on the sides so as to deflect the current at a slight angle. The horizontal cross section of such a gauge is shown in

Fig. 75. If there are waves on the surface of the water the height will vary so rapidly that accurate readings cannot be made. In order to avoid this difficulty a glass tube about $\frac{3}{4}$ inch in diameter is placed between two wooden strips (Fig. 76), one of which is used for the graduated scale. The water enters the glass tube and stands at the height of the water surface outside. In order to check sudden variations in height the water is allowed to enter this tube only through a very small tube (1^{mm} inside diameter) placed in a cork or rubber stopper at the lower end of the large tube. The water can rise in the tube rapidly enough to show the general level of the water surface, but small waves have practically no effect upon the reading. For convenience the gauge is made in sections about three feet long. These may be placed end to end and the large tubes connected by means of the smaller ones passing through the stoppers. In order to read the gauge at a distance it is convenient to have a narrow strip of red painted on the back of the tube or else blown into the glass.* Above the water surface this strip shows its true size, but below the surface, owing to the refraction of light by the water, the strip appears several times its true width, making it easy to distinguish the dividing line. Such a gauge may be read from a considerable distance by means of a transit telescope or field glasses.

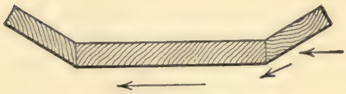

FIG. 75

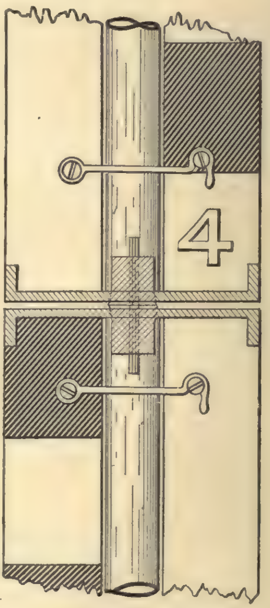

FIG. 76

* Tubes of this sort are manufactured for use in water gauges of steam boilers.

Location of Gauge.

The spot chosen for setting up the gauge should be near the open sea, where the true range of tide will be obtained. It should be somewhat sheltered, if possible, against heavy seas. The depth of the water and the position of the gauge should be such that even at extremely low or extremely high tides the water will stand at some height on the scale.

Making the Observations.

The maximum and minimum scale readings at the times of high and low tides should be observed, together with the times at which they occur. The observations of scale readings should be begun some thirty minutes before the predicted time of high or low water, and continued, at intervals of about 5^m, until a little while after the maximum or minimum is reached. The height of the water surface sometimes fluctuates at the time of high or low tide, so that the first maximum or minimum reached may not be the true time of high or low water. In order to determine whether the tides are normal the force and direction of the wind and the barometric pressure may be noted.

Reducing the Observations.

If the gauge readings vary so that it is difficult to determine by inspection where the maximum or minimum occurred, the observations may be plotted, taking the times as abscissæ and gauge readings as ordinates. A smooth curve drawn through the points so as to eliminate accidental errors will show the position of the maximum or minimum point. (Figs. 77a and 77b.) When all of the observations have been worked up in this way the mean of all of the high-water and low-water readings may be taken as the scale reading for mean half-tide. There should of course be as many high-water readings as low-water readings. If the mean half-tide must be determined from a very limited number of observations, these should be combined in pairs in such a way that the diurnal inequality does not introduce an error. In Fig. 78 it will be seen that the mean of *a* and *b*, or the mean of c and d, or e and f, will give nearly the mean half-tide; but if b and c, or d and e, are combined, the mean is in

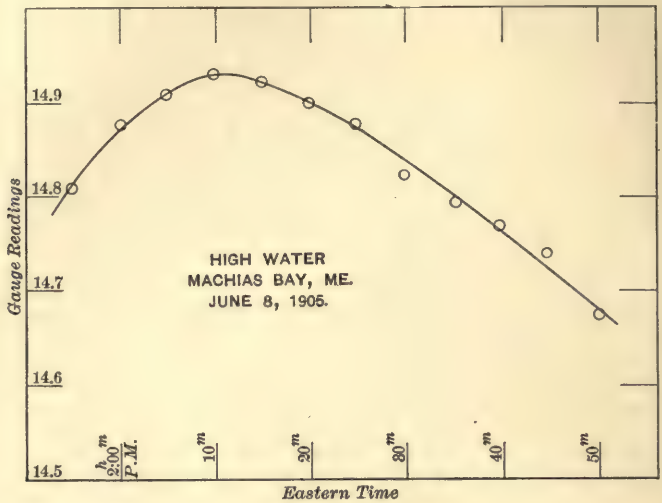

FIG. 77a

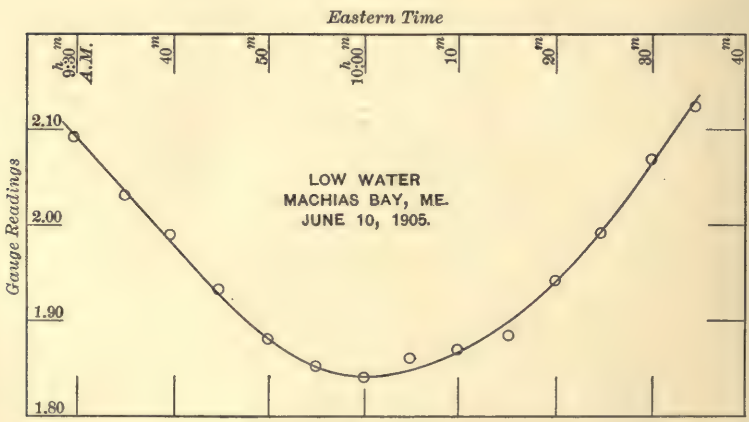

FIG. 77b

one case too small and in the other case too great. The proper selection of tides may be made by examining the predicted heights and times given in the tables issued by the U. S. Coast and Geodetic Survey. By examining the predicted heights the exact relation may be found between mean sea level and the mean half-tide as computed from the predicted heights corresponding to those tides actually observed. The difference between these two may be applied as a correction to the mean of the observed tides to obtain mean sea level. For example, suppose that the predicted heights at a port near the place of observation indicate that the mean of a, b, c, d, e, and f is 0.2 ft.

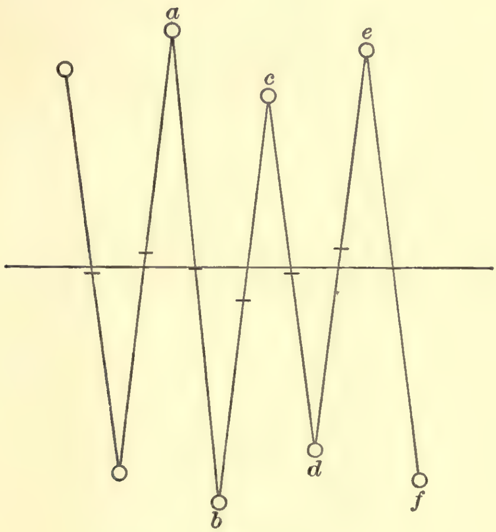

FIG. 78

below mean sea level. Then if these six tides are observed and the results averaged, a correction of 0.2 ft. should be added to the mean of the six heights in order to obtain mean sea level.

Prediction of Tides.

Since the local conditions have such a great influence in determining the tides at any one place, the prediction of the times and heights of high and low water for that place must be based upon a long series of observations made at the same point. *Tide Tables* giving predicted tides for one year are published annually by the United States Coast and Geodetic Survey; these tables give the times and heights of high and low water for the principal ports of the United States, and also for many foreign ports. The method of using these tables is explained in a note at the foot of each page. A brief statement of the theory of tides is given in the Introduction.

The approximate time of high water at any place may be computed from the time of the moon's meridian passage, provided we know the average interval between the moon's transit and the following high water, i.e., the "establishment of the port." The mean time of the moon's transit over the meridian of Greenwich is given in the Nautical Almanac for each day, together with the change per hour of longitude. The local time of transit is computed by adding to the tabular time the hourly change multiplied by the number of hours in the west longitude; this result, added to the establishment of the port, gives the approximate time of high water. The result is nearly correct at the times of new and full moon, but at other times is subject to a few minutes variation.

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