FUNDAMENTALS OF PRACTICAL MATHEMATICS

Florian Cajori.

WENTWORTH-SMITH MATHEMATICAL SERIES

FUNDAMENTALS OF PRACTICAL MATHEMATICS

BY

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PREFACE

General Plan. The development of a more practical type of education in this country has now reached such a stage as to demand a series of textbooks that shall fully meet this need as to content and that shall be prepared with the view to their usability in the classroom. In arranging to meet this demand the authors have made a careful survey of both the general and the vocational schools in the leading cities in the United States, and have come to the conclusion that the most usable type of book should be based upon the assumption that the student has had a good course in elementary arithmetic, including the simple graph, but is in need of a brief review of the fundamental operations. Upon this assumption they have proceeded to build.

The present book contains those basic principles which the student must know, whatever special vocation he is to follow. He may go into machine work of one kind or another, into electrical work, into carpentry, into cabinetwork, into the clothing industry, or into printing, but in any case he will need most of the fundamental work which is set forth in this book. This work consists of a review of such topics as the four fundamental operations with integers and fractions, the practical use of percentage, the applications of proportion, the elements of mensuration, the use of the formula and the equation, the finding of areas by plotting on squared paper, the finding of roots, and the simplest elements of trigonometry. The arrangement of the pages, with the exercises facing the blueprints, will be found especially convenient and will contribute to the general appearance of reality of the work. As to the exercises themselves, they have been carefully chosen from practical fields.

Practical Nature of the Work. This review is undertaken, however, with an entirely new set of motives on the part of the student. Instead of mere mechanical drill on abstract calculations he at once finds himself in the atmosphere of the shop, and he meets with precisely the type of problem that will confront him in his practical work. If he has to add fractions he will find the problem related to a blueprint taken from the workroom, and whenever any other operation is to be performed the student will find that the work always relates to a real situation. Arithmetic thus ceases to be merely formal work with abstract numbers, algebra takes on an aspect of genuine utility, trigonometry becomes a tool to be used, and mensuration refers to things that the student knows are worth measuring.

Schools for which the Work is Adapted. As stated above, the authors have had in mind the general high school as well as the vocational school. There are many high schools in which certain classes will receive greater benefit from the type of work herein set forth than from the more abstract mathematics commonly offered. The book has therefore been prepared to meet the needs of the junior high school and the four-year high school as well as the needs of the technical and continuation schools.

Technical Works. After completing the fundamental work laid down in this textbook the student will be ready to take up the special preparation for his chosen vocation. For this preparation he will need a textbook that relates to the technical work to be undertaken, such, for example, as the "Machine-Shop Mathematics" in this series. For all such special fields the present textbook will be found to give the necessary preparation.

The authors hope that, in preparing a work with the same care that characterizes all the books of the Wentworth-Smith Series, they have taken a forward step in general and vocational education that will meet with the same approval that teachers in this country have so generously given to this series in the past.

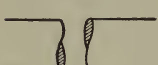
CONTENTS

SYMBOLS

Symbols. The following mathematical symbols and abbreviations are used most frequently in the shop:

$+$	plus, addition	$3''$	means 3 inches (3 in.)
$-$	minus, subtraction	$2'$	means 2 feet (2 ft.)
$\times$	times, multiplication; by (as in $2' \times 3'$)	yd.	yard or yards
$\div$	divided by, division	sq.	square (as in sq. ft.)
$\surd$	square root	$=$	equal, equals
5^2	means "5 square," or 5×5	$\angle$	angle
:	ratio (as in 2 : 3)	#	number (as in marking sizes of wires)
2/3	means $\frac{2}{3}$ or $2 \div 3$	/	per (as in 7 lb./cu. ft., read "seven pounds per cubic foot")
%	per cent, hundredths		

Conventional Signs. The following conventional signs are used frequently in the blueprints upon which the exercises are based:



This indicates that part of the rod is omitted in the drawing.

This indicates the point from which or to which we measure.

IRON STEEL BRASS WOOD

A drafting-room convention for cross sections of the different materials named.

The conventional way to show screw threads.

The conventional way to indicate an incandescent lamp.

FUNDAMENTALS OF PRACTICAL MATHEMATICS

CHAPTER I

FUNDAMENTAL OPERATIONS

Review of the Fundamentals. Before beginning this book the student is supposed to be able to add, subtract, multiply, and divide in cases involving whole numbers, decimals, or common fractions. Nevertheless, a brief review of these operations, undertaken from the strictly practical standpoint, will be found desirable unless the student has recently been doing a considerable amount of computing.

Checks. Speed is desirable, but accuracy is essential. Therefore *check* every operation.

$$\begin{array}{r} 34.75 \\ 2.864 \\ 10.096 \\ \hline 47.71 \end{array}$$

For example, in this case in addition, first add from the bottom of the column upwards and then check the result by adding from the top downwards.

$$\begin{array}{r} 4.07 \\ .638 \\ \hline 3.432 \end{array}$$

There are several methods of subtracting. Use the one that you find best, but in any case always check by adding the result to the smaller number and seeing that the sum is the larger number. In the case of the subtraction of 0.638 from 4.07, as here shown, we have $3.432 + 0.638 = 4.07$, and hence the work is correct.

Exercises. Addition and Subtraction

1. In the blueprint of the piston on page 3 the lengths are given to the nearest 0.001″ (thousandth of an inch). Find the length A; that is, find the value of $1.656'' + 0.063''$.

The expression 1.656″ means 1.656 inches. Errors are less likely to arise by writing 0.063 instead of .063, although the two have the same meaning. In this book we use the first form except in column addition or subtraction, although the second form is often used in practice.

2. In the piston find each of the lengths B, C, and D.

3. Add the results in Exs. 1 and 2, thus finding the total length of the piston, and check by adding the fourteen numbers that represent the separate lengths.

4. In the cone pulley find each of the lengths $A + B$, $A + B + C$, and $A + B + C + D$.

5. In the cone pulley find the length $D + E$ and the total length of the pulley.

6. In the cone pulley how much longer is D than E?

In every case be sure to check the result. An incorrect result in a simple operation of this kind is never excusable.

In the lathe spindle find each of the following lengths:

7. $A + B$. **9.** $C + D$. **11.** $E + F$. **13.** $A + B + C$.

8. $B + C$. **10.** $D + E$. **12.** $F + G$. **14.** $E + F + G$.

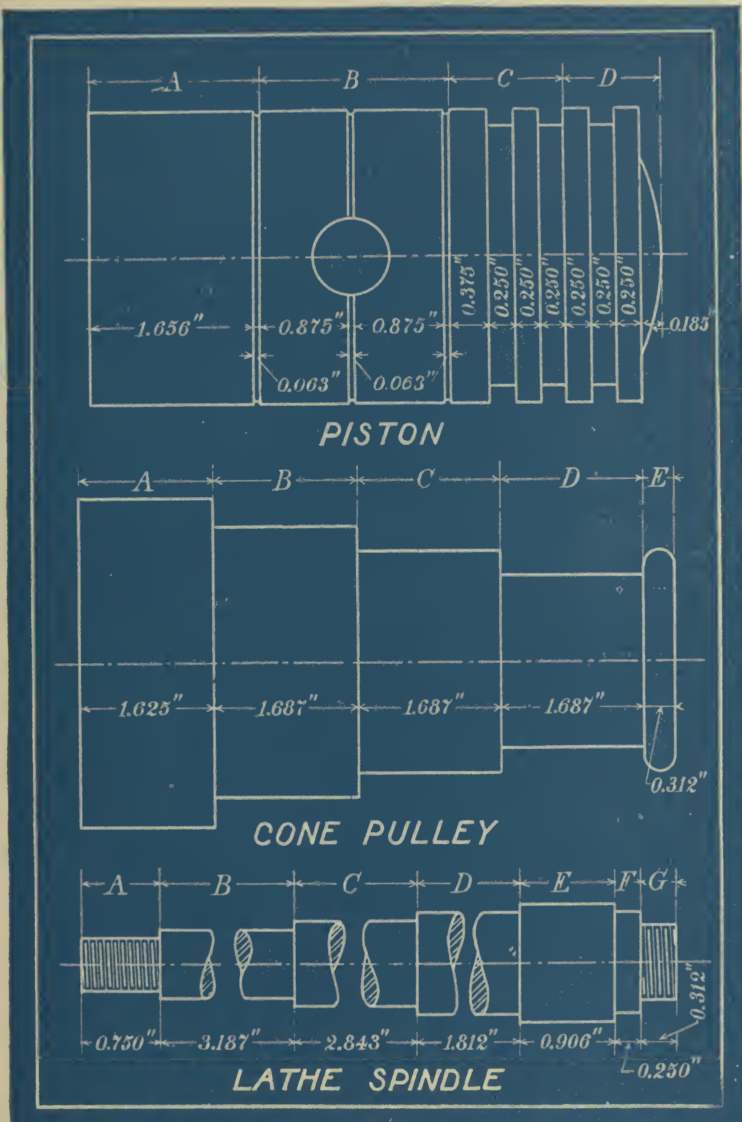
15. Find the total length of the lathe spindle.

16. In the lathe spindle how much longer is G than F?

In the lathe spindle find the difference in length of the parts in each of the following cases:

17. B and A. **19.** C and D. **21.** E and F.

18. B and C. **20.** D and E. **22.** A and G.


A
B
C
D
1.656"
0.875"
0.875"
0.375"
0.250"
0.250"
0.250"
0.250"
0.250"
0.250"
0.185"
0.063"
0.063"
PISTON
A
B
C
D
E
1.625"
1.687"
1.687"
1.687"
0.312"
CONE PULLEY
A
B
C
D
E
F
G
0.312"
0.750"
3.187"
2.843"
1.812"
0.906"
0.250"
LATHE SPINDLE

Multiplication of a Decimal by a Whole Number. If all the numbers to be added are equal, as in the case of the eight dimensions 0.375″ in the cross section of the piston on page 5, we can save time by multiplying. Thus, we can find the length A by finding the product, or result, of $8 \times 0.375''$.

$$\begin{array}{r} 0.375 \\ \underline{8} \\ 3.000 \\ =3 \quad\;\; \end{array}$$

In actual practice we omit the abbreviation for inches (″) and arrange the numbers as shown.

We then have $8 \times 5 = 40$, and we write 0 and add 4 to the next product. Then $8 \times 7 + 4 = 56 + 4 = 60$, and we write another 0 and add 6 to the next product. Finally, we have $8 \times 3 + 6 = 24 + 6 = 30$, and we write 30 in the product.

Since we multiplied thousandths by a whole number, the result is thousandths, and so we have three decimal places. Since the three decimal figures are zeros, we write 3″ as the result.

In multiplying a decimal by a whole number, point off from the right as many decimal places in the product as there are in the number multiplied.

Exercises. Multiplication of Decimals

1. Find the length of four of the parts of section A of the piston shown on page 5. Then multiply the result by 2, and thus check the multiplication shown above.

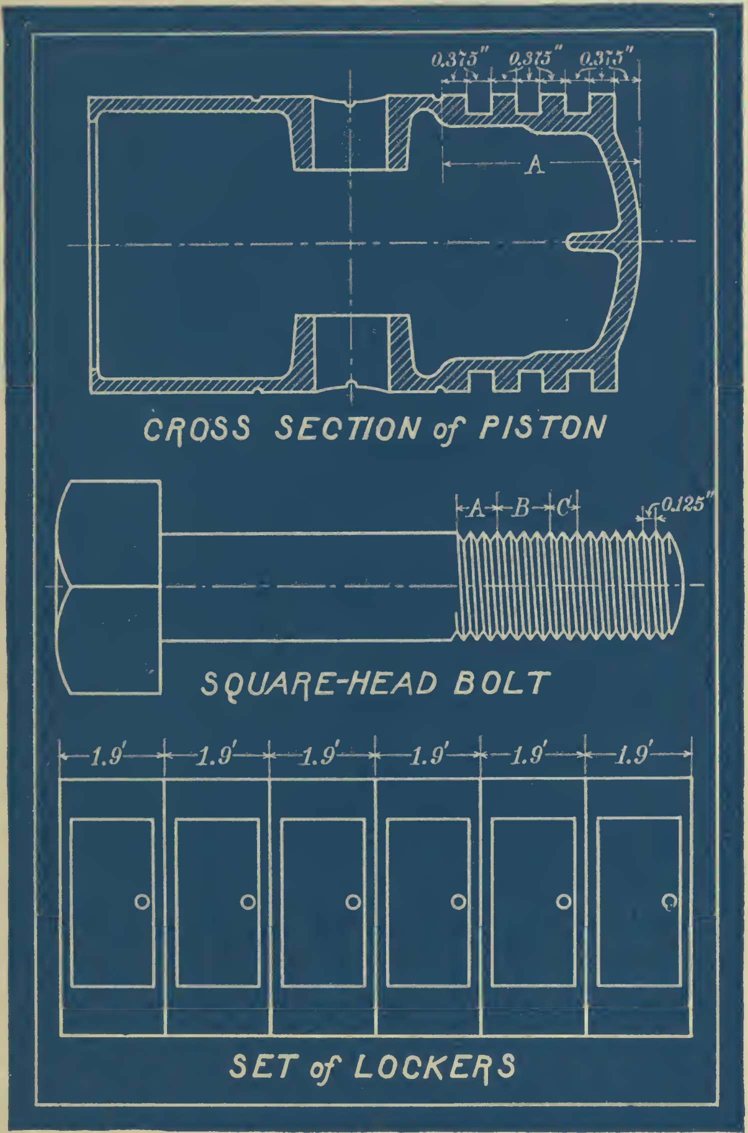
2. In Ex. 1 find the length of three of the parts.

3. In the square-head bolt the pitch, that is, the distance between successive threads, is shown. Find the distance A; the distance B; the distance C.

4. In Ex. 3 find the distance $A + B$ by one multiplication and check by adding the first two results in Ex. 3.

5. Find the width of three lockers of the set shown in the blueprint; of four lockers; of all six lockers.

6. Check the last result found in Ex. 5 by multiplying the first result by 2.


0.375″
0.375″
0.375″
A
CROSS SECTION of PISTON
A
B
C
0.125″
SQUARE-HEAD BOLT
1.9′
1.9′
1.9′
1.9′
1.9′
1.9′
SET of LOCKERS

Multiplication of a Decimal by a Decimal. If we have a lathe faceplate as shown on page 7, the diameter is easily and accurately measured by the use of calipers of some kind, but the circumference cannot be so easily measured, since it would require the use of a tape, which does not give as high a degree of accuracy as a caliper.

There is a rule, however, that the circumference is equal to π (pī) times the diameter, where π is $3\frac{1}{7}$, or 3.14, for ordinary work, and 3.1416 for cases requiring a higher degree of accuracy.

Therefore, to find the circumference of the faceplate we multiply 11.4″ by 3.14.

$$\begin{array}{r} 11.4 \\ 3.14 \\ \hline 456 \\ 114 \\ 342 \\ \hline 35.796 \\ \text{or } 35.8 \end{array}$$

We first multiply as with whole numbers, the product being 35 796. Since we multiplied tenths by hundredths, the product is thousandths; and so, beginning at the right, we point off three decimal places, the result being 35.796.

Since, in this case, the diameter was given only to 0.1″, the circumference cannot be found accurately beyond 0.1″. Therefore we write the result as 35.8″.

Beginning at the right, point off as many decimal places in the product as there are in the two numbers together.

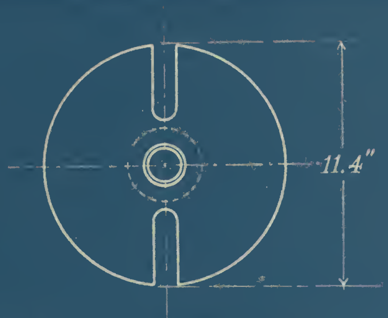
Exercises. Multiplication of Decimals

1. Find the dimensions corresponding to those given in the collar pin on page 7 for a pin 2.5 times as large.

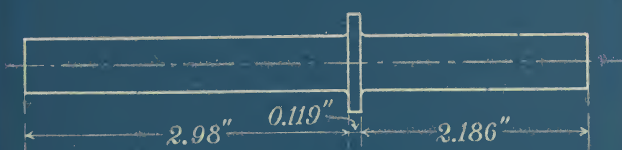
2. Find the dimensions corresponding to those given in the cartridge fuse for a fuse 1.35 times as large.

Multiply as follows:

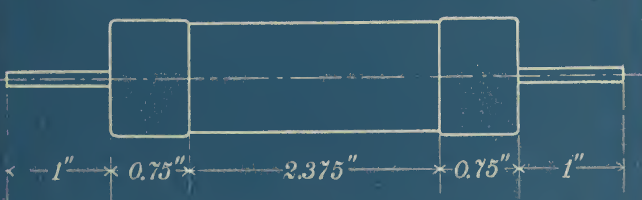
3. 2.4 × 3.96.	**6.** 4.71 × 0.683.	**9.** 4.9 × 0.087.
4. 0.57 × 0.873.	**7.** 52.8 × 37.9.	**10.** 0.72 × 0.096.
5. 2.63 × 48.72.	**8.** 6.82 × 83.5.	**11.** 3.48 × 5.793.



LATHE FACEPLATE



COLLAR PIN



CARTRIDGE FUSE

Division of a Decimal by a Whole Number. **1.** In the bridge shown on page 9 the four struts, that is, the upright pieces, divide the length into five equal parts. Find the length of each part.

$$\begin{array}{r} 5\overline{)237.5} \\ 47.5 \end{array}$$

Here we have to find the value of 237.5′ ÷ 5. Dividing as with whole numbers, we place the decimal point in the result below the decimal point in the number divided. The length of each part is found to be 47.5′.

2. Divide 237.5′ by 25.

$$\begin{array}{r} 9.5 \\ 25\overline{)237.5} \\ 225 \\ \hline 12\ 5 \\ 12\ 5 \\ \hline \end{array}$$

In long division we write the result above the number divided, placing the decimal point in the result directly above the decimal point in the number divided. The result is 9.5′.

We may check this result by observing that 25 × 9.5′ = 237.5′; that is, expressed as a general rule, the number by which we divide times the result is equal to the number divided.

Exercises. Division of Decimals

1. If the struts of a steel bridge 186.5′ long divide the length into five equal parts, how long is each part?

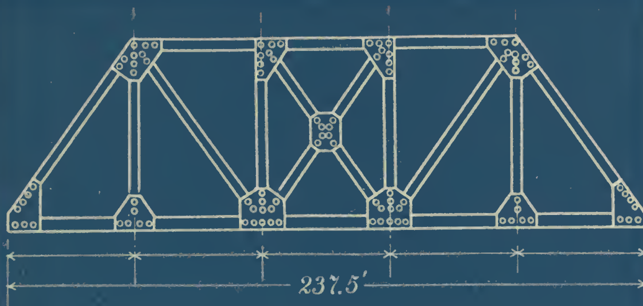
2. If the struts divide a bridge 207.2′ long into seven equal parts, how long is each part?

3. In the stairway shown in the blueprint on page 9 there are 15 steps. Find the rise R of each step.

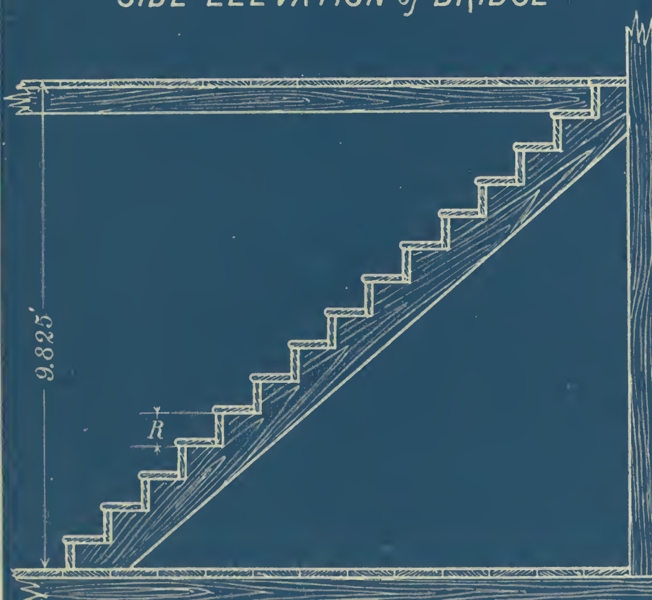
Divide, finding each result to the nearest hundredth:

4. 2.9 ÷ 4.	**7.** 0.0728 ÷ 5.	**10.** 1 ÷ 3.
5. 3.8 ÷ 52.	**8.** 0.0639 ÷ 4.	**11.** 3.1416 ÷ 4.
6. 48.1 ÷ 26.	**9.** 527.8 ÷ 649.	**12.** 100 ÷ 3183.

In finding a result to the nearest hundredth, if the figure in thousandths' place is less than 5, it is disregarded; if the figure is 5 or greater, the figure in hundredths' place is increased by 1.



SIDE ELEVATION of BRIDGE



CROSS SECTION of STAIRWAY

Division of a Decimal by a Decimal. We often need to find the diameter of a circle that shall have a given circumference.

Since $\text{circumference} = \pi \times \text{diameter},$

we have $\dfrac{\text{circumference}}{\pi} = \text{diameter},$

or $\dfrac{1}{\pi} \times \text{circumference} = \text{diameter}.$

For example, if the circumference of the spur-gear blank shown on page 11 is 22.7″, what is the diameter to the nearest 0.1″?

$$\begin{array}{r|l} & 7.2 \\ \hline 3\,14 & 22\,70. \\ & 21\,98 \\ \hline & 720 \\ & 628 \\ \hline & 92 \end{array}$$

We may divide 22.7″ by 3.1416, but it will be near enough for our purpose here if we divide by 3.14.

To make 3.14 a whole number we multiply both numbers by 100 by moving both decimal points two places to the right.

Dividing, the result is 7.2″ to the nearest 0.1″. Since 92, the difference between 720 and 628, is obviously less than $\frac{1}{2}$ of 314, it will not affect the tenths' figure.

Exercises. Division of Decimals

1. Using 3.1416 as the value of π, find the value of $1 \div \pi$ to the nearest ten-thousandth. Multiply 22.7″ by the result and show that it agrees with the 7.2″ found above.

2. The boring-mill table shown on page 11 has a circumference of 157.38″. By dividing by 3.142, find the diameter to the nearest 0.01″. Check as in Ex. 1.

3. The emery wheel has a circumference of 78.54″. Find the diameter and the radius, using 3.1416 as the value of π.

Divide, finding each result to the nearest hundredth:

4. $7 \div 3.142$.	6. $5.2 \div 3.1416$.	8. $1 \div 0.3183$.
5. $6 \div 3.142$.	7. $82.4 \div 3.1416$.	9. $5 \div 0.3183$.


SPUR-GEAR BLANK
BORING-MILL TABLE
EMERY WHEEL

Exercises. Review Drill

Add as follows:

1.	2.	3.	4.	5.
92.85	62.38	2.728	712.3	9.288
72.97	93.97	5.683	528.2	.396
61.82	75.63	3.929	887.7	.486
32.09	76.06	6.635	771.1	.567
43.58	29.37	4.004	928.6	8.06
21.23	82.05	4.8	671.8	9.
3.89	40.63	7.303	466.5	.079

Subtract as follows:

6.	7.	8.	9.	10.
90.24	70.26	79.	65.3	87.03
30.08	41.9	32.98	28.96	7.6

Multiply as follows:

11. $3 \times 7.04''$. **17.** 1.2×3.26. **23.** 6.6×4.92.

12. $8 \times 6.38'$. **18.** 56×82.2. **24.** 6.2×12.48.

13. $4 \times 5.32''$. **19.** 9.4×3.28. **25.** 0.28×2009.

14. $6 \times 3.08'$. **20.** 8.1×448. **26.** 2.73×816.2.

15. 9×8.754. **21.** 8.5×8.23. **27.** 35.5×70.04.

16. 8×0.7382. **22.** 0.34×5.814. **28.** 72.39×32.016.

Divide, finding each result to the nearest hundredth:

29. $428.3 \div 41$. **32.** $73.42 \div 0.68$. **35.** $290.9 \div 4.26$.

30. $720.6 \div 8.4$. **33.** $82.37 \div 7.49$. **36.** $4.702 \div 32.8$.

31. $293.4 \div 7.3$. **34.** $64.86 \div 63.8$. **37.** $53.09 \div 0.772$.

Divide, finding each result to the nearest thousandth:

38. $427.1 \div 0.93$. **40.** $63.85 \div 4.7$. **42.** $8.030 \div 23.7$.

39. $5.826 \div 0.67$. **41.** $52.27 \div 6.3$. **43.** $6.282 \div 42.4$.

Exercises. Miscellaneous Applications

1. The three sides of a triangle are 4.75″, 4.9″, and 4.97″. Find the perimeter; that is, the sum of the sides.

2. On four successive days an automobile ran 97.6 mi., 67.8 mi., 99.7 mi., and 82.5 mi. What was the total distance?

3. From a city lot that contained 8635.2 sq. ft. the owner sold 3526.8 sq. ft. and 2259.3 sq. ft. How much was left?

4. The perimeter of a triangle is 19.4″, and two of the sides are 6.48″ and 7.96″. Find the third side.

5. If a nine-story city building averages 10.75′ to a story, how high is the building?

6. The measure called the meter is equivalent to a length of 39.37″. How many inches are there in 6.5 meters?

7. If each side of a square building lot is 66.6′, find the perimeter of the lot.

8. A machinist made a certain steel plate 0.75 as wide as it was long. If the length was 9.2″, what was the width?

9. A certain rectangle is 3.45 times as long as it is wide. If the rectangle is 6.7′ wide, what is the length?

10. How long will it take a railway train traveling 0.9 mi. a minute to go 29.7 mi.? to go 37 mi.?

In every case where a rate of speed is involved, the rate is to be considered as uniform, unless otherwise stated.

11. How long will it take an airplane traveling at the rate of 1.2 mi. a minute to go 20.4 mi.? to go 69 mi.?

12. As already learned, $1 \div \pi = 0.3183$, approximately. Compare the results of $0.3183 \times 13.421''$ and $13.421'' \div 3.1416$, finding each result to the nearest 0.001″. Which method do you find easier? State the reason for your answer.

Reduction of a Fraction. In practical mathematics the common fractions used most often are halves, fourths, eighths, sixteenths, thirty-seconds, and sixty-fourths. By examining a ruler divided into sixteenths of an inch the student will see that $\frac{1}{2}'' = \frac{2}{4}'' = \frac{4}{8}'' = \frac{8}{16}''$. Learn the following:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64} \qquad \frac{1}{8} = \frac{2}{16} = \frac{4}{32} = \frac{8}{64}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64} \qquad \frac{3}{8} = \frac{6}{16} = \frac{12}{32} = \frac{24}{64}$$

$$\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{24}{32} = \frac{48}{64} \qquad \frac{5}{8} = \frac{10}{16} = \frac{20}{32} = \frac{40}{64}$$

From these equalities we see that the following is true:

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

Adding Fractions. For example, in the blueprint of the shaft on page 15 find the length of $A+B$.

Since B is given in thirty-seconds of an inch, we first reduce A to thirty-seconds of an inch, as shown at the right. Since both fractions are now expressed with the same denominator, we can add as shown. The length of $A + B$ is thus found to be $\frac{17}{32}''$.

$$\begin{array}{rr} \frac{7}{16} = & \frac{14}{32} \\ & \underline{\frac{3}{32}} \\ & \frac{17}{32} \end{array}$$

Exercises. Addition of Fractions

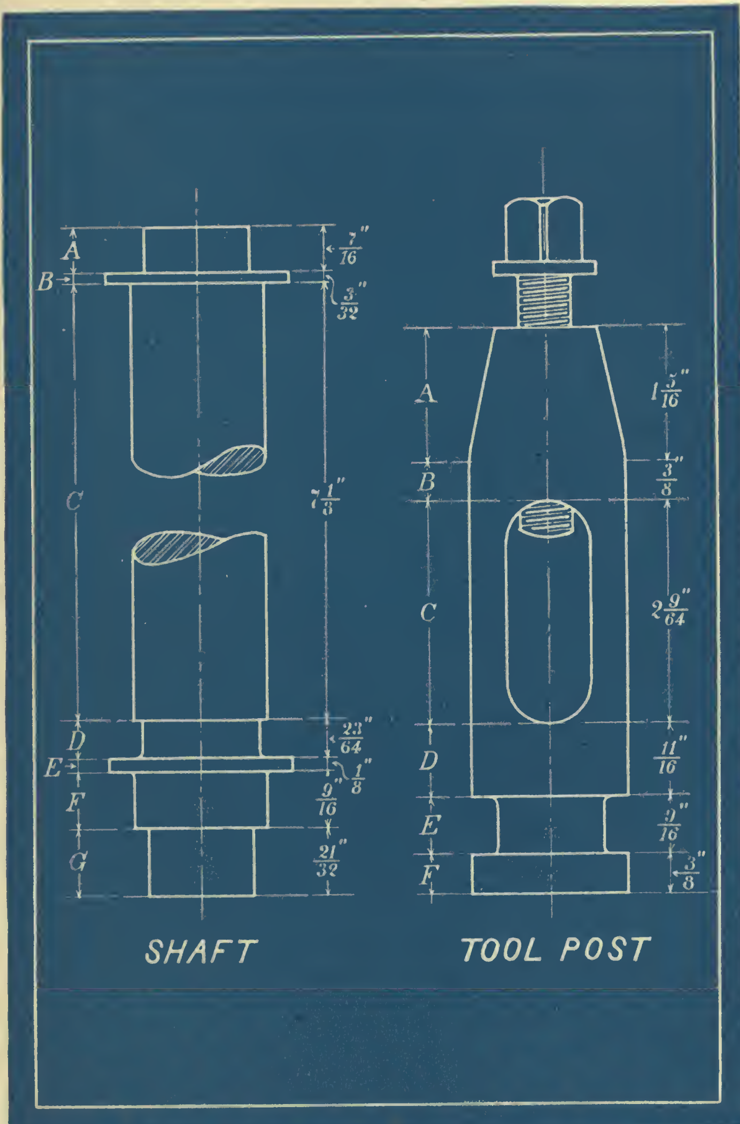
In the shaft on page 15 find each of the following lengths:

1. $B+C$.	**3.** $D+E$.	**5.** $A+B+C$.
2. $C+D$.	**4.** $E+F$.	**6.** $D+E+F+G$.

7. Find the total length of the shaft.

8. From the blueprint of the tool post find the length of $A+B$; of $E+F$; of the tool post from A to F inclusive.

9. In the tool post show that $D+F = \frac{17}{16}'' = 1\frac{1}{16}''$.


A
B
C
D
E
F
G
7/16"
3/32"
7 1/8"
23/64"
1/8"
9/16"
21/32"
SHAFT
A
B
C
D
E
F
1 5/16"
3/8"
2 9/64"
11/16"
9/16"
3/8"
TOOL POST

Subtracting Fractions. **1.** In the feed screw on page 17 find whether A or C is the longer, and also find the difference in length of the two parts.

We know that $\frac{1}{2}'' = \frac{8}{16}''$, so that A is the longer. Since both fractions are now expressed as sixteenths of an inch, we can subtract. We thus find that A is $\frac{1}{16}''$ longer than C.

$$\begin{array}{lr} A = & \frac{9}{16} \\ C = \frac{1}{2} = & \frac{8}{16} \\ \hline & \frac{1}{16} \end{array}$$

2. In the crankshaft find how much longer B is than A.

If we try to take $7\frac{15}{16}''$ from $11\frac{12}{16}''$ we find that $\frac{12}{16}''$ is smaller than $\frac{15}{16}''$. So we change $11\frac{12}{16}''$ to $10'' + 1'' + \frac{12}{16}''$, or $10'' + \frac{16}{16}'' + \frac{12}{16}''$, or $10\frac{28}{16}''$. We can then subtract, and we have $3\frac{13}{16}''$ as the result.

$$\begin{array}{lr} B = 11\frac{3}{4} = 11\frac{12}{16} = & 10\frac{28}{16} \\ A = & 7\frac{15}{16} \\ \hline & 3\frac{13}{16} \end{array}$$

Exercises. Subtraction of Fractions

1. In the blueprint of the feed screw on page 17 how much longer is E than D? How much longer is B than E?

2. In the crankshaft how much longer is B than C?

In the crankshaft find the difference in length of the parts in each of the following cases:

3. A and D. **4.** B and D. **5.** D and C.

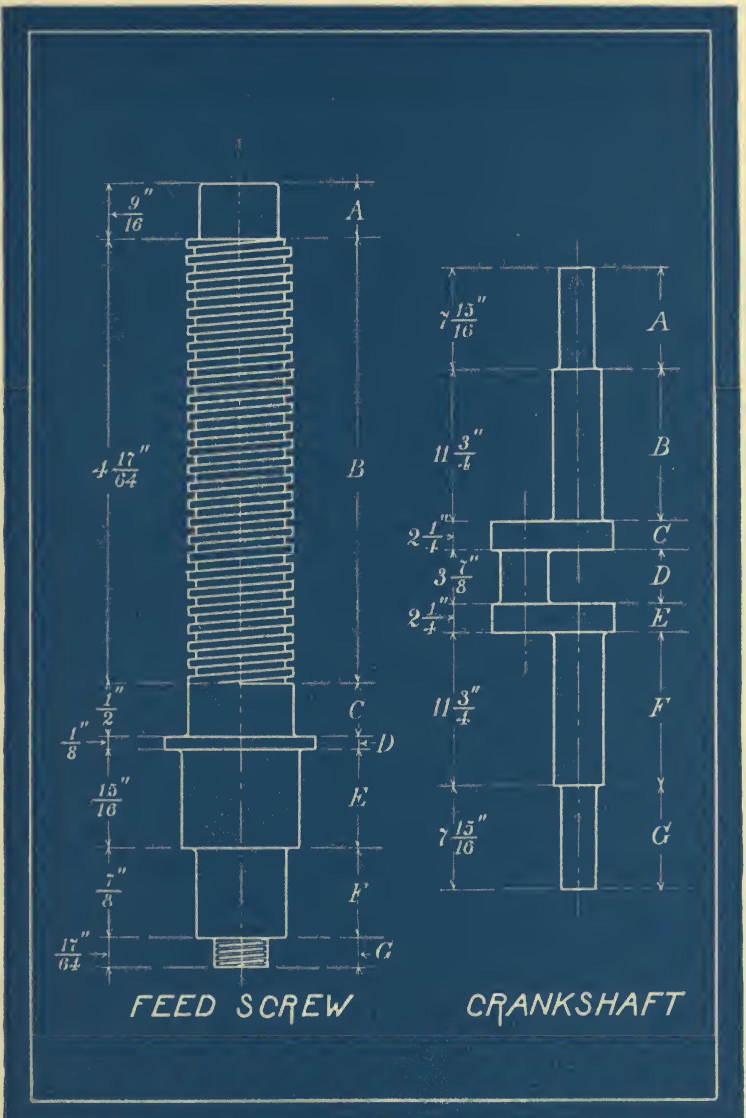
In the feed screw find the difference in length of the parts in each of the following cases:

6. B and C. **8.** E and C. **10.** E and G.

7. F and G. **9.** E and A. **11.** G and D.

In every case express the result in the simplest form. For example, a result like $\frac{6}{16}''$ should be written $\frac{3}{8}''$.

12. In the feed screw find the amount by which the length of $A+B$ exceeds that of $C+D+E+F+G$.



9/16"
A
4 17/64"
B
1/2"
C
1/8"
D
15/16"
E
7/8"
F
17/64"
G
7 15/16"
A
11 3/4"
B
2 1/4"
C
3 7/8"
D
2 1/4"
E
11 3/4"
F
7 15/16"
G
FEED SCREW
CRANKSHAFT

Multiplication of a Fraction by a Whole Number. In the stock-room bins shown on page 19 it is seen that the distances between the centers of the shelves are all the same, namely, $5\frac{1}{32}''$. To find the sum of four such distances we might add, but it is easier to multiply. Thus, if we wish to find the distance from the center of the first shelf to the center of the fifth shelf, we have to find the value of $4 \times 5\frac{1}{32}''$.

We multiply the fraction by 4, and we have $\frac{4}{32}''$, which is equal to $\frac{1}{8}''$. We then multiply the whole number by 4, and we have $20''$. Adding $\frac{1}{8}''$ and $20''$ we have $20\frac{1}{8}''$ as the result.

$$\begin{array}{l} 5\frac{1}{32} \\ \ \ 4 \\ \hline 20\frac{4}{32} = 20\frac{1}{8} \end{array}$$

Instructors will find that the detailed explanation of reducing $\frac{4}{32}$ to $\frac{1}{8}$ does not add to the student's understanding. With such simple fractions and with the aid of a ruler marked in fractions of an inch, the student's intuition will lead him safely.

If, in some other problem, we wished to find the sum of 40 distances, each $5\frac{1}{32}''$ in length, we should have

$$40 \times 5\frac{1}{32}'' = 200\frac{40}{32}'' = 200'' + 1'' + \frac{8}{32}'' = 201\frac{1}{4}''.$$

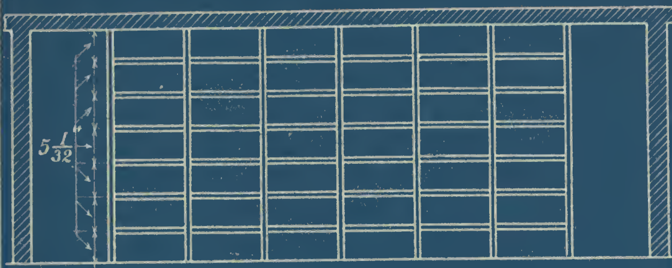
Exercises. Multiplication of Fractions

1. In the intake pipe for a 6-cylinder engine, shown on page 19, find the value of $2 \times 3\frac{15}{16}''$; find the total distance between centers for the six cylinders.

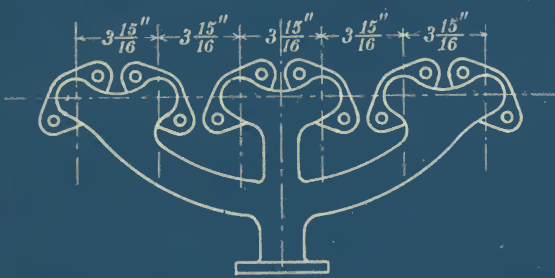
2. Find the height of that portion of the stock-room bins shown in the blueprint, disregarding the thickness of the top piece. Solve by one multiplication.

3. In the planer table, a part of which is shown in the blueprint, the distance between centers is $1\frac{7}{32}''$. Find by multiplication the distance from the first to the ninth hole; from the first to the seventeenth hole; from the first to the thirty-third hole; from the first to the forty-ninth hole.

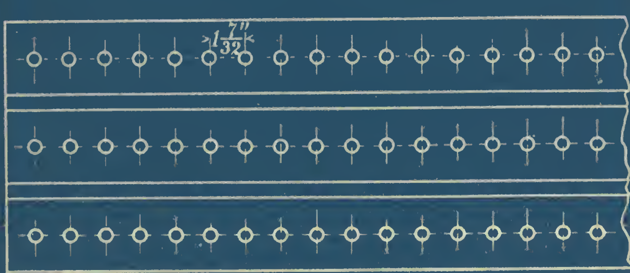
The distance between the centers of the holes is always understood.



STOCK-ROOM BINS



INTAKE PIPE for 6-CYLINDER ENGINE



PLANER TABLE

Multiplication by a Fraction. **1.** In the closet door shown on page 21 the length P of the lower panel is $\frac{3}{4}$ the width of the door. Find P.

We have to find $\frac{3}{4}$ of 27″. We multiply 27″ by 3 and divide the result by 4, as shown at the right. The length of the panel is thus found to be $20\frac{1}{4}$″.

$$\frac{3 \times 27}{4} = \frac{81}{4} = 20\tfrac{1}{4}$$

2. The height H of the I-beam shown in the blueprint is $2\frac{3}{8}$ times the width. Find H.

We have to find the value of $2\frac{3}{8} \times 15''$. We first find $\frac{3}{8}$ of 15″, as in Ex. 1, and see that it is $5\frac{5}{8}$″. Then $2 \times 15'' = 30''$. We add these results and find that the height of the beam is $35\frac{5}{8}$″.

In such a case as that of finding $\frac{3}{8}$ of 16″, we first divide 16″ by 8 and then multiply the result by 3.

$$\frac{3 \times 15}{8} = \frac{45}{8} = 5\tfrac{5}{8}$$

$$\begin{array}{r} 15 \\ 2\frac{3}{8} \\ \hline 5\frac{5}{8} \\ 30 \\ \hline 35\frac{5}{8} \end{array}$$

3. In the planer bolt the width W of the head is $\frac{3}{4}$ the diameter of the shank. Find W.

We have to find $\frac{3}{4}$ of $\frac{7}{8}$″. We first multiply the two numerators together and then multiply the two denominators together. The width of the head is thus found to be $\frac{21}{32}$″.

$$\frac{3 \times 7}{4 \times 8} = \frac{21}{32}$$

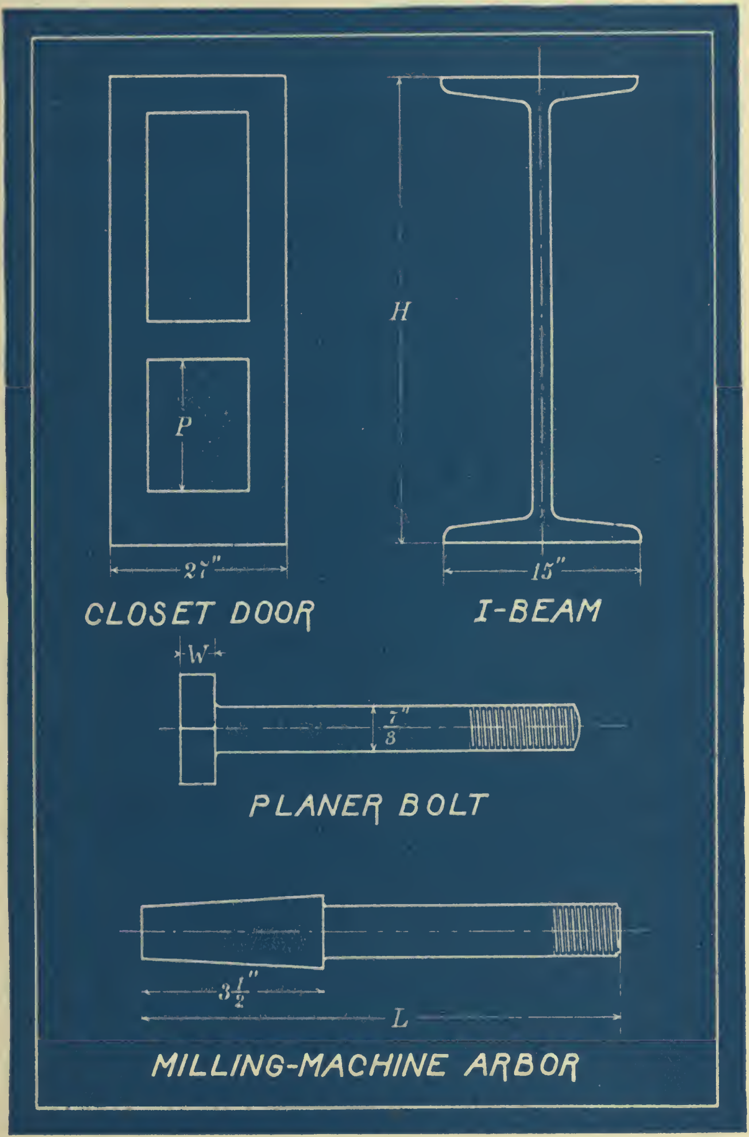
If the diameter of the shank of the planer bolt were $\frac{15}{16}$″, and the width of the head were $\frac{4}{5}$ the diameter, we should have to find $\frac{4}{5}$ of $\frac{15}{16}$″. In such a case we first indicate the multiplication as shown, cancel, and find the width of the head to be $\frac{3}{4}$″.

$$\frac{\cancel{4} \times \overset{3}{\cancel{15}}}{\cancel{5} \times \underset{4}{\cancel{16}}} = \frac{3}{4}$$

4. In the milling-machine arbor the length L is $2\frac{5}{8}$ times the length of the tapered part. Find L.

$$2\tfrac{5}{8} \times 3\tfrac{1}{2} = \frac{21 \times 7}{8 \times 2} = \frac{147}{16} = 9\tfrac{3}{16}$$

We have to find the value of $2\frac{5}{8} \times 3\frac{1}{2}$″. We change $2\frac{5}{8}$ to $\frac{21}{8}$, and $3\frac{1}{2}$″ to $\frac{7}{2}$″. Multiplying as in Ex. 3, we have $\frac{147}{16}$″, or $9\frac{3}{16}$″. Hence the length L is $9\frac{3}{16}$″.



P
27"
CLOSET DOOR
H
15"
I-BEAM
W
7/8"
PLANER BOLT
3 1/2"
L
MILLING-MACHINE ARBOR

Exercises. Multiplication by Fractions

1. The width of a door is to be $\frac{1}{3}$ the height, and it is required that the height be $8\frac{1}{4}'$. Find the width.

2. The height of an I-beam is $23\frac{1}{4}''$, and the width is one third the height. Find the width.

3. The circumference of a wheel is $3\frac{1}{7}$ times the diameter. If the diameter is $2.1'$, what is the circumference?

Find the value of each of the following:

4. $\frac{3}{8}$ of 1464.
5. $\frac{7}{8}$ of 327.2.
6. $\frac{5}{6}$ of 77.64.
7. $\frac{5}{8}$ of 9.768.
8. $\frac{1}{6}$ of 7740.6.
9. $\frac{5}{6}$ of 7776.6.
10. $2\frac{5}{8} \times 33.76$.
11. $3\frac{5}{6} \times 1.758$.
12. $7\frac{3}{8} \times 497.6$.

13. A man has a city lot that contains $\frac{16}{25}$ of an acre. If he sells $\frac{5}{8}$ of it, what part of an acre does he sell? What part of an acre has he left?

14. If the length of the paddle of a water wheel in a mill is $\frac{3}{7}$ of the diameter, and the diameter is $63''$, what is the length of the paddle?

15. If a glass jar holds $\frac{15}{16}$ qt., what part of a quart does it contain when it is $\frac{4}{5}$ full? What part of a quart does it contain when it is $\frac{2}{5}$ full? when it is $\frac{3}{5}$ full?

Multiply as follows:

16. $\frac{1}{8} \times \frac{4}{5}$.
17. $\frac{4}{5} \times \frac{5}{8}$.
18. $\frac{3}{4} \times \frac{8}{15}$.
19. $\frac{5}{6} \times \frac{3}{5}$.
20. $\frac{2}{3} \times \frac{9}{10}$.
21. $\frac{2}{3} \times \frac{15}{16}$.
22. $\frac{7}{16} \times \frac{5}{7}$.
23. $\frac{7}{16} \times \frac{8}{21}$.

24. Find the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$; the value of $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{5}{12}$.

Multiply as follows:

25. $\frac{2}{3} \times \frac{5}{8} \times \frac{3}{15}$.
26. $\frac{3}{7} \times \frac{21}{25} \times \frac{5}{9}$.
27. $\frac{3}{8} \times \frac{4}{15} \times \frac{5}{6}$.
28. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{15}$.
29. $\frac{3}{5} \times \frac{8}{9} \times \frac{15}{16}$.
30. $\frac{5}{8} \times \frac{3}{5} \times 12$.

Division of Fractions. The several cases involving the division of a fraction or the division of a number by a fraction, which the student will meet in practical work, may best be understood by first considering the following examples:

1. Divide $\frac{7}{8}$ by 3.

Since $\frac{7}{8} \div 3$ is the same as $\frac{1}{3}$ of $\frac{7}{8}$, we may use multiplication, as on page 20, instead of division. We then have

$$\frac{7}{8} \div 3 = \frac{1}{3} \times \frac{7}{8} = \frac{7}{24}.$$

2. Divide $\frac{15}{16}$ by 3.

As in Ex. 1, $\frac{15}{16} \div 3 = \frac{1}{3} \times \frac{15}{16} = \frac{5}{16}.$

We might have canceled the 3 into the 15, but in such simple cases as are here given this can best be done mentally.

3. Divide $7\frac{1}{2}$ by 2.

Since $7\frac{1}{2}$ may be written as $\frac{15}{2}$, we have

$$7\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{15}{2} = \frac{15}{4} = 3\frac{3}{4}.$$

4. Divide $7\frac{1}{2}$ by 3.

As in Ex. 3, $7\frac{1}{2} \div 3 = \frac{1}{3} \times \frac{15}{2} = \frac{5}{2} = 2\frac{1}{2}.$

The method given in Exs. 3 and 4 can often be simplified by using the method shown in Exs. 9 and 10 on page 24.

5. Divide 8 by $\frac{1}{2}$.

On a ruler we see that $\frac{1}{2}''$ is contained twice in $1''$, and therefore $\frac{1}{2}''$ is contained 8×2 times, or 16 times, in $8''$.

Hence $8 \div \frac{1}{2} = 2 \times 8 = 16.$

6. Divide 15 by $\frac{3}{4}$.

As in Ex. 5, we see on a ruler that $\frac{3}{4}''$ is contained 4 times in $3''$.

That is, $3'' \div \frac{3}{4}'' = 4.$

Now 15 is equal to 5×3, and therefore

$$15 \div \frac{3}{4} = 5 \times 4 = 20.$$

We also see that we could get the same result by multiplying 15 by $\frac{4}{3}$.

From Exs. 5 and 6 we have the following rule:

To divide by a fraction multiply by the fraction inverted.

7. Using the rule at the foot of page 23, divide $\frac{3}{8}$ by $\frac{2}{3}$.

We have $$\tfrac{3}{8} \div \tfrac{2}{3} = \tfrac{3}{2} \times \tfrac{3}{8} = \tfrac{9}{16}.$$

8. Divide $3\frac{1}{8}$ by $2\frac{1}{2}$.

Expressing $3\frac{1}{8}$ as $\frac{25}{8}$ and $2\frac{1}{2}$ as $\frac{5}{2}$, and canceling mentally, we have

$$3\tfrac{1}{8} \div 2\tfrac{1}{2} = \tfrac{2}{5} \times \tfrac{25}{8} = \tfrac{5}{4} = 1\tfrac{1}{4}.$$

9. In the line of water pipe shown on page 25 it is found that B is $\frac{1}{2}$ of A. Find the length of B.

$$\boxed{\begin{array}{r} 2\,\underline{)\,12\frac{1}{2}} \\ 6\frac{1}{4} \end{array}}$$

We have to divide $12\frac{1}{2}''$ by 2. Without expressing $12\frac{1}{2}''$ as $\frac{25}{2}''$, we can readily see that $12'' \div 2 = 6''$, that $\frac{1}{2}'' \div 2 = \frac{1}{4}''$, and hence that B is $6\frac{1}{4}''$ long.

10. If it is also found in Ex. 9 that E is $\frac{1}{4}$ of C, find the length of E.

$$\boxed{\begin{array}{r} 4\,\underline{)\,17\frac{7}{8}} = 4\,\underline{)\,16\frac{15}{8}} \\ 4\frac{15}{32} \end{array}}$$

We have to divide $17\frac{7}{8}''$ by 4. If we write $16'' + 1\frac{7}{8}''$ for $17\frac{7}{8}''$, the division is easier as we then have a whole number divisible by 4. Expressing $1\frac{7}{8}''$ as $\frac{15}{8}''$, we see that the length of B is $4\frac{15}{32}''$.

Exercises. Division of Fractions

1. In the line of water pipe shown on page 25 it is found that E is $\frac{1}{2}$ of F. Find the length of E, thus verifying the result found in Ex. 10 above.

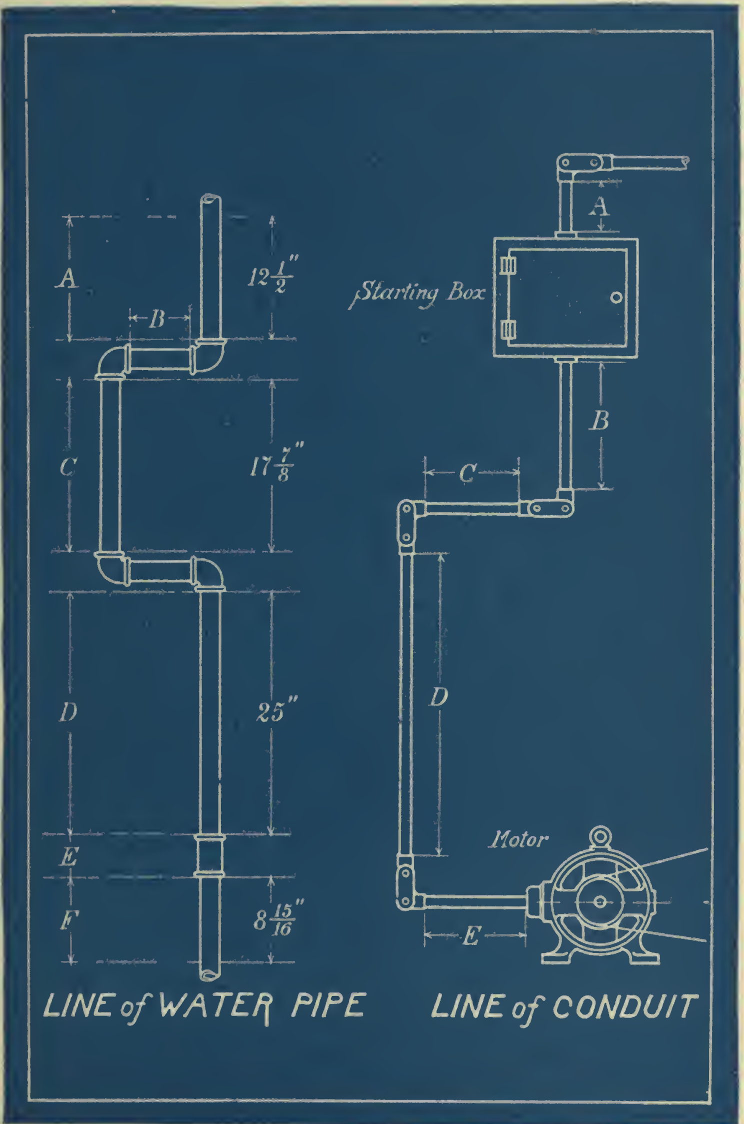
2. In the line of conduit it is found that E is $\frac{1}{3}$ of D. If D is $4'\ 9\frac{3}{4}''$ long, what is the length of E?

Express $4'\ 9\frac{3}{4}''$ as $3'\ 21\frac{3}{4}''$ or as $57\frac{3}{4}''$.

3. Using the result of Ex. 2, find how many pieces of conduit the length of E can be cut from a piece of conduit $8'$ long. How long a piece is left over?

4. If E in the line of conduit is $\frac{7}{8}$ the length of B, what is the length of B?

5. From Exs. 2 and 4 find what part B is of D.


A
$12\frac{1}{2}''$
B
C
$17\frac{7}{8}''$
D
25''
E
F
$8\frac{15}{16}''$
LINE of WATER PIPE
A
Starting Box
B
C
D
Motor
E
LINE of CONDUIT

Fractions to Decimals. Sometimes the dimensions in a blueprint are given in feet and inches, sometimes they include a common fraction, and sometimes they include a decimal fraction. It is necessary to be able to express any one of these forms in either of the other two forms. We shall now see how to change from a common fraction to a decimal fraction, or, as usually said, from a fraction to a decimal.

1. One of the dimensions of the binding post on page 27 is $\frac{3}{4}''$. Express this as a decimal.

$$\begin{array}{r} 4\,)\,3.00 \\ \hline 0.75 \end{array}$$

Since $\frac{3}{4}$ means $3 \div 4$, we simply divide 3 by 4, as shown. We may annex as many zeros as we wish after a decimal point, for $3 = 3.0 = 3.00$, and so on, just as $\$3 = \$3.00$. We see that $\frac{3}{4}''$ is equivalent to $0.75''$.

2. Express as a decimal the dimension $\frac{21}{64}''$, which is used in the blueprint of the binding post.

$$\begin{array}{r} 0.328\frac{1}{8} \\ 64\,\overline{)\,21.000} \\ \underline{19\;2} \\ 1\;80 \\ \underline{1\;28} \\ 520 \\ \underline{512} \\ 8 \end{array}$$

We have $21 \div 64 = 0.328$, with 8 left over from the thousandths' place. Since this 8 is still to be divided by 64, we may write $\frac{8}{64}$, or $\frac{1}{8}$, after the thousandths' place in the result.

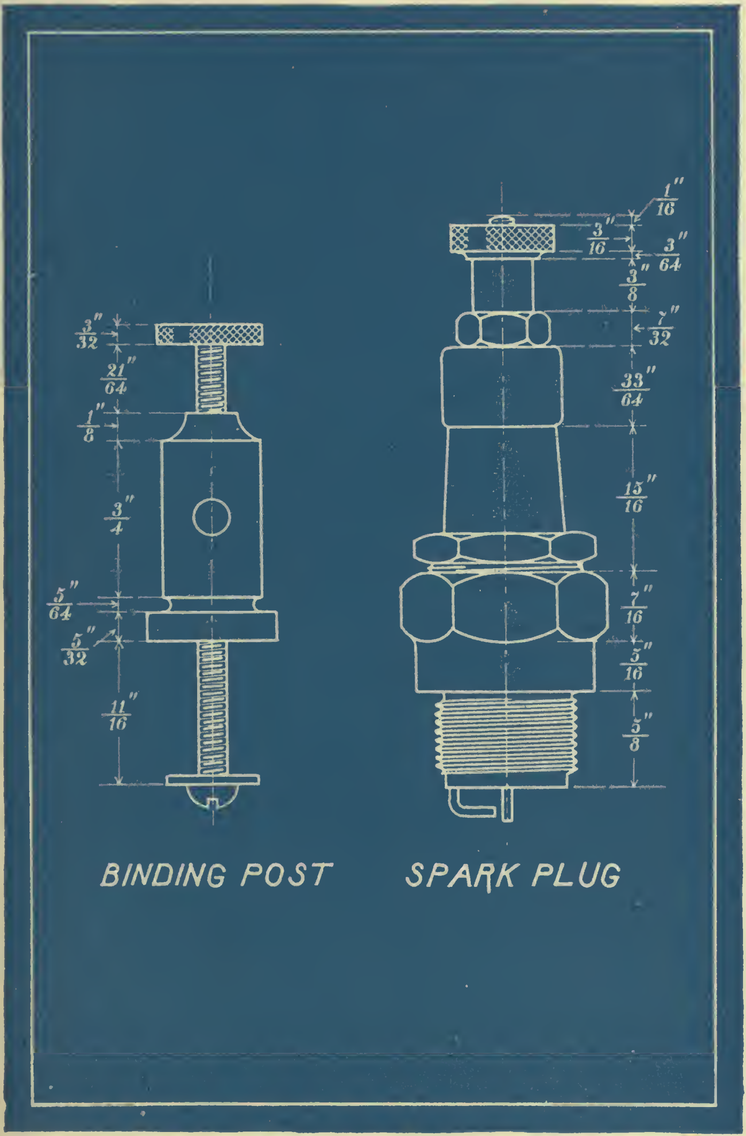
In practical measurements of this kind we rarely need to carry such a result beyond $0.001''$. Since $\frac{1}{8}$ is less than $\frac{1}{2}$, we express $\frac{21}{64}''$ as a decimal to the nearest $0.001''$ as $0.328''$.

If the remainder in this case had been $\frac{1}{2}$ or greater, we should have given the result to the nearest $0.001''$ as $0.329''$ instead of $0.328''$.

Exercises. Fractions to Decimals

1. Omitting the two dimensions used in the examples above, express each of the other dimensions of the binding post on page 27 as a decimal to the nearest $0.001''$.

2. Express each of the dimensions of the spark plug as a decimal to the nearest $0.001''$.


3/32"
21/64"
1/8"
3/4"
5/64"
5/32"
11/16"
1/16"
3/16"
3/64"
3/8"
7/32"
33/64"
15/16"
7/16"
5/16"
5/8"
BINDING POST
SPARK PLUG

Decimals to Fractions. Sometimes a blueprint gives a dimension as a decimal of an inch, and the workman finds it more convenient to use a common fraction with a denominator which is either 2, 4, 8, 16, 32, or 64. If the accuracy of the work requires that dimensions be given to $\frac{1}{64}''$, he reduces the decimal to a common fraction to the nearest $\frac{1}{64}''$.

For example, in the taper spindle on page 29 the threaded part at the top is $0.437''$ long. Express this dimension as a common fraction to the nearest $\frac{1}{64}''$.

$$\begin{array}{r l} 0.437 & \\ 64 & \\ \hline 1748 & \\ 2622 & \\ \hline 27.968 & (64\text{ths}) \end{array}$$

$$\tfrac{28}{64} = \tfrac{7}{16}$$

Since $0.437'' = \frac{0.437''}{1}$, we can express the dimension in sixty-fourths of an inch by multiplying both terms by 64, and we have 27.968 sixty-fourths, which is almost $\frac{28}{64}''$, or $\frac{7}{16}''$. To prove this we find that $7 \div 16 = 0.4375$; that is, $0.437''$ is too small by $0.0005''$ to be expressed to an exact $\frac{1}{64}''$.

Exercises. Decimals to Fractions

1. In the taper spindle on page 29 express $0.734''$ as a common fraction to the nearest $\frac{1}{64}''$; to the nearest $\frac{1}{32}''$.

2. In the same figure express each of the other dimensions as a common fraction or a mixed number to the nearest $\frac{1}{32}''$.

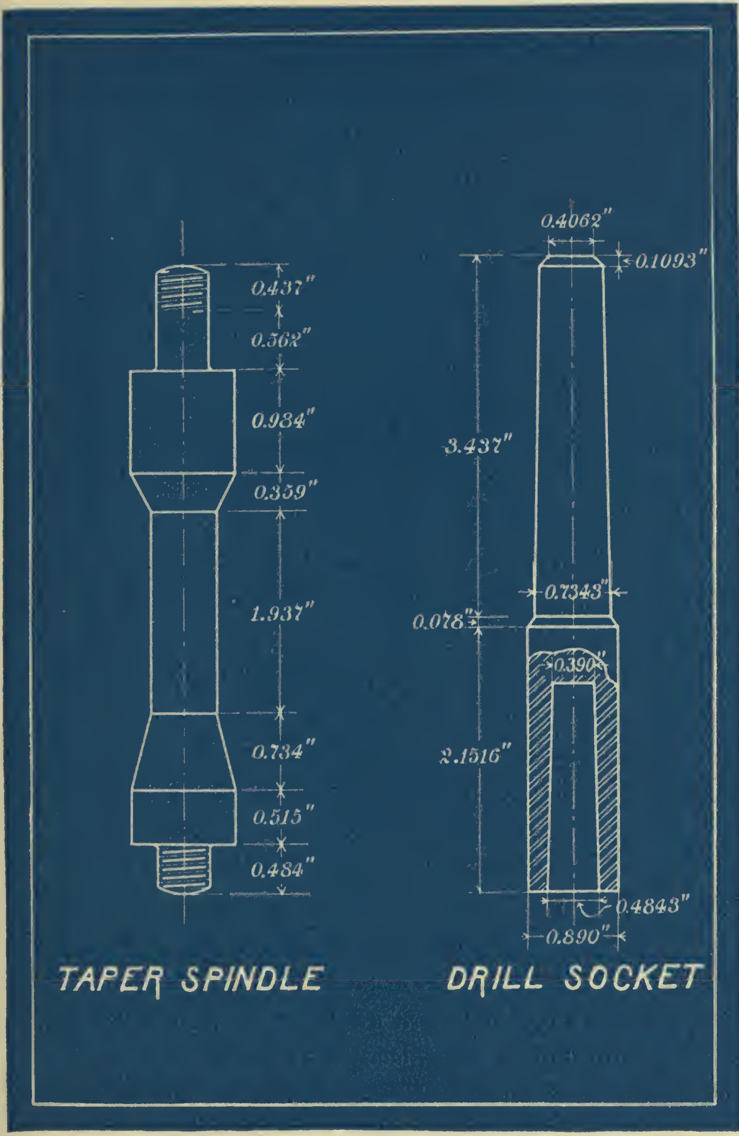
3. In the drill socket express each of the dimensions as a common fraction or a mixed number to the nearest $\frac{1}{64}''$.

4. The diameters of certain numbered drills are as follows: #15, $0.1800''$; #19, $0.1660''$; #24, $0.1520''$; #31, $0.1200''$. Express each size as a common fraction in lowest terms.

In the first case reduce $\frac{1800}{10000}''$ to lowest terms by first dividing by 100 and then by 2. The symbol # is commonly used for "number."

Reduce to common fractions or to mixed numbers:

5. 0.8. **6.** 0.35. **7.** 1.375. **8.** 3.75. **9.** 0.625.


0.437"
0.562"
0.984"
0.359"
1.937"
0.734"
0.515"
0.484"
TAPER SPINDLE
0.4062"
0.1093"
3.437"
0.7343"
0.078"
0.390"
2.1516"
0.4843"
0.890"
DRILL SOCKET

Exercises. Review

1. Express as a decimal each of the dimensions given in the blueprint of the dining chair shown on page 31. Give each result to the nearest $0.001''$.

2. Express each dimension of the lead-pipe stop as a decimal to the nearest $0.001''$.

3. In the figure of the chair find the dimension lettered A.

4. In the same figure how much greater is the distance from the top of the front leg to the top of the back than the distance from the floor to the top of the front leg?

5. In the same figure find the height H of the back.

6. How long a piece of wood is needed to make four pieces, each $17\frac{3}{4}''$ long? to make six pieces, each $18\frac{1}{4}''$ long? to make nine pieces, each $3\frac{3}{4}''$ long?

7. If a child's chair is made half as large as the dining chair shown in the blueprint, how long is the piece corresponding to the one that is $17\frac{3}{4}''$ long?

8. In the pipe stop find the total length of $A + B + C + D$.

9. In the same figure find the height E.

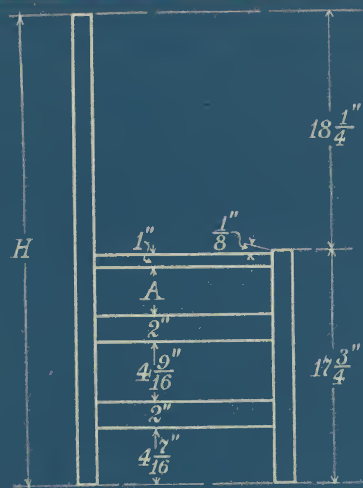
10. If the dimensions of a pipe stop were twice as large as those given in the blueprint, what would be the length of D?

11. If the dimensions of a pipe stop were half as large as those given, what would be the length of C?

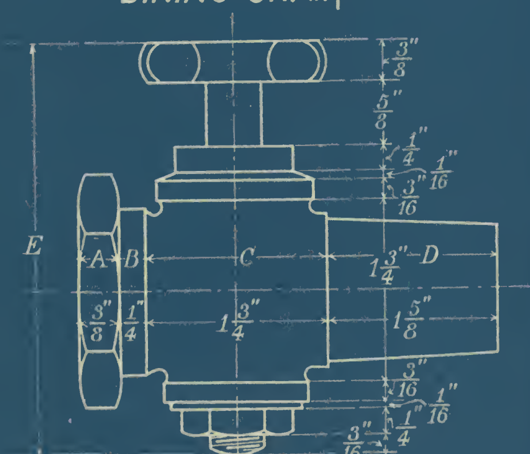
12. Lead is $11\frac{7}{20}$ times as heavy as water, but this fact is usually stated by using a decimal. Express the number $11\frac{7}{20}$ with a decimal instead of a common fraction.

13. From a piece of board $16\frac{1}{8}''$ long a piece $13\frac{3}{4}''$ long is cut off. How long is the piece that is left?

14. From a steel rod $26\frac{1}{2}''$ long a piece $14\frac{3}{4}''$ long is cut off. How long is the piece that is left?



DINING CHAIR



LEAD-PIPE STOP

Exercises. Review Drill Work

Add as follows:

1. $\frac{1}{2}+\frac{1}{3}$.	**3.** $7\frac{1}{2}+3\frac{1}{3}$.	**5.** $2\frac{1}{4}+3\frac{1}{6}$.	**7.** $9\frac{3}{4}+7\frac{1}{5}$.
2. $\frac{1}{5}+\frac{1}{3}$.	**4.** $2\frac{1}{5}+5\frac{1}{3}$.	**6.** $5\frac{4}{5}+6\frac{2}{3}$.	**8.** $7\frac{5}{6}+9\frac{2}{3}$.

Subtract as follows:

9. $\frac{4}{5}-\frac{1}{2}$.	**11.** $9\frac{4}{5}-2\frac{1}{2}$.	**13.** $8\frac{2}{3}-7\frac{1}{2}$.	**15.** $8\frac{3}{4}-3\frac{7}{8}$.
10. $\frac{3}{4}-\frac{2}{5}$.	**12.** $9\frac{3}{4}-7\frac{2}{5}$.	**14.** $3\frac{2}{3}-1\frac{7}{8}$.	**16.** $8\frac{1}{2}-6\frac{7}{8}$.

Multiply as follows:

17. $5\times 2\frac{2}{5}$.	**19.** $1\frac{3}{4}\times 9$.	**21.** $6\frac{3}{8}\times 4\frac{1}{3}$.	**23.** $2\frac{7}{8}\times 5\frac{1}{2}$.
18. $8\times 9\frac{7}{8}$.	**20.** $6\frac{1}{3}\times 7\frac{1}{5}$.	**22.** $5\frac{1}{3}\times 3\frac{1}{2}$.	**24.** $7\frac{1}{5}\times 4\frac{1}{4}$.

Divide as follows:

25. $32\div\frac{1}{8}$.	**30.** $4\frac{3}{4}\div\frac{1}{4}$.	**35.** $3\frac{2}{3}\div\frac{1}{24}$.	**40.** $7\frac{1}{3}\div\frac{2}{3}$.
26. $24\div\frac{1}{16}$.	**31.** $2\frac{2}{5}\div\frac{1}{5}$.	**36.** $5\frac{7}{8}\div\frac{1}{32}$.	**41.** $5\frac{2}{3}\div 4\frac{2}{3}$.
27. $15\div\frac{1}{12}$.	**32.** $3\frac{5}{8}\div\frac{1}{8}$.	**37.** $\frac{5}{8}\div 2\frac{1}{5}$.	**42.** $\frac{4}{5}\div 13\frac{1}{2}$.
28. $\frac{5}{6}\div\frac{1}{12}$.	**33.** $4\frac{3}{4}\div\frac{1}{12}$.	**38.** $\frac{2}{3}\div 3\frac{1}{3}$.	**43.** $7\frac{3}{8}\div 1\frac{3}{4}$.
29. $3\frac{1}{2}\div\frac{1}{2}$.	**34.** $2\frac{3}{4}\div\frac{1}{24}$.	**39.** $3\frac{5}{8}\div\frac{2}{3}$.	**44.** $5\frac{5}{8}\div 1\frac{1}{8}$.

Find the value of each of the following:

45. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft.	**48.** $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.
46. $\frac{1}{2}$ of 5×86 sq. ft.	**49.** $\frac{3}{5}$ of 15×485 sq. ft.
47. $\frac{7}{8}$ of 4×164 sq. in.	**50.** $\frac{3}{7}$ of 13×147 sq. in.

Divide as follows:

51. $21\div 4\frac{2}{3}$.	**55.** $16\frac{4}{5}\div 4\frac{1}{5}$.	**59.** $20\frac{1}{8}\div 15\frac{1}{3}$.
52. $48\div 3\frac{5}{6}$.	**56.** $15\frac{1}{3}\div 3\frac{5}{6}$.	**60.** $30\frac{1}{4}\div 15\frac{1}{8}$.
53. $36\div 1\frac{1}{5}$.	**57.** $17\frac{1}{2}\div 2\frac{1}{2}$.	**61.** $27\frac{3}{4}\div 9\frac{1}{4}$.
54. $24\div 4\frac{4}{5}$.	**58.** $42\frac{2}{5}\div 10\frac{2}{3}$.	**62.** $45\frac{3}{5}\div 15\frac{1}{5}$.

Exercises. Miscellaneous Applications

1. If a sheet of cardboard is $\frac{1}{16}''$ thick, how many sheets pressed together will have a thickness of $1\frac{3}{8}''$?

2. If a sheet of veneer is $\frac{1}{16}''$ thick, how many sheets pressed together will have a thickness of $2\frac{1}{4}''$?

3. If a sheet of blotting paper is $\frac{1}{32}''$ thick, how many sheets are there in a pile that is $12\frac{7}{8}''$ high?

4. If a wagon wheel makes $\frac{1}{12}$ of a revolution while the wagon is going $1'$, how many feet will the wagon go while the wheel is making 144 revolutions?

5. How many books, each $\frac{7}{8}''$ thick, will it take to make a pile $23\frac{5}{8}''$ high?

6. How many strips of wood, each $\frac{9}{16}''$ thick, will it take to make a pile $19\frac{11}{16}''$ high?

7. How many sheets of bookbinding board, each $\frac{3}{32}''$ thick, will it take to make a pile $9\frac{3}{8}''$ high?

8. To divide a sheet of paper $7\frac{7}{8}''$ wide into four equal columns, what width must be spaced off for each column?

9. Making no allowance for doors and windows, how much picture molding will be needed to go round a room $22\frac{3}{4}'$ long and $18\frac{1}{2}'$ wide?

10. If you plan to make a box to go under a shelf in a space $21\frac{1}{4}''$ high and allow a space of $3\frac{1}{8}''$ between the cover and the shelf, what must be the inside depth of the box if you use lumber $\frac{5}{8}''$ thick?

11. If you are working from a blueprint that gives the dimensions in decimals, and your instruments are graduated in thirty-seconds of an inch, what common fractions will you use in place of the following: $0.125''$, $0.375''$, $0.0625''$, $0.1875''$, $0.625''$, $0.5625''$, $0.875''$, $0.9375''$?

Feet and Inches. 1. In the border of lamps on page 35 find the length of the border between the centers of the first and fourth lamps.

′	″
2	4
3	9
1	5
7	6

Since we have to add three dimensions which are given in feet and inches, we arrange them in columns as shown. Adding the inches we have 18″, or 1′ 6″, and we write 6 under the inches column. Adding the feet, including the 1′ already found, we have 7′. Therefore the required length is 7′ 6″.

2. The studs in that part of the partition shown in the blueprint are equally spaced. Find to the nearest $\frac{1}{16}''$ the distance between the centers of the studs.

$$10\overline{)21'\,6''}$$
$$2' + \tfrac{1}{10} \text{ of } 1'\,6'' = 2'\,1.8''$$
$$0.8'' = 0.8 \text{ of } \tfrac{16}{16}'' = \tfrac{12.8}{16}'', \text{ or } \tfrac{13}{16}''$$
$$2'\,1\tfrac{13}{16}''. \quad \textit{Ans.}$$

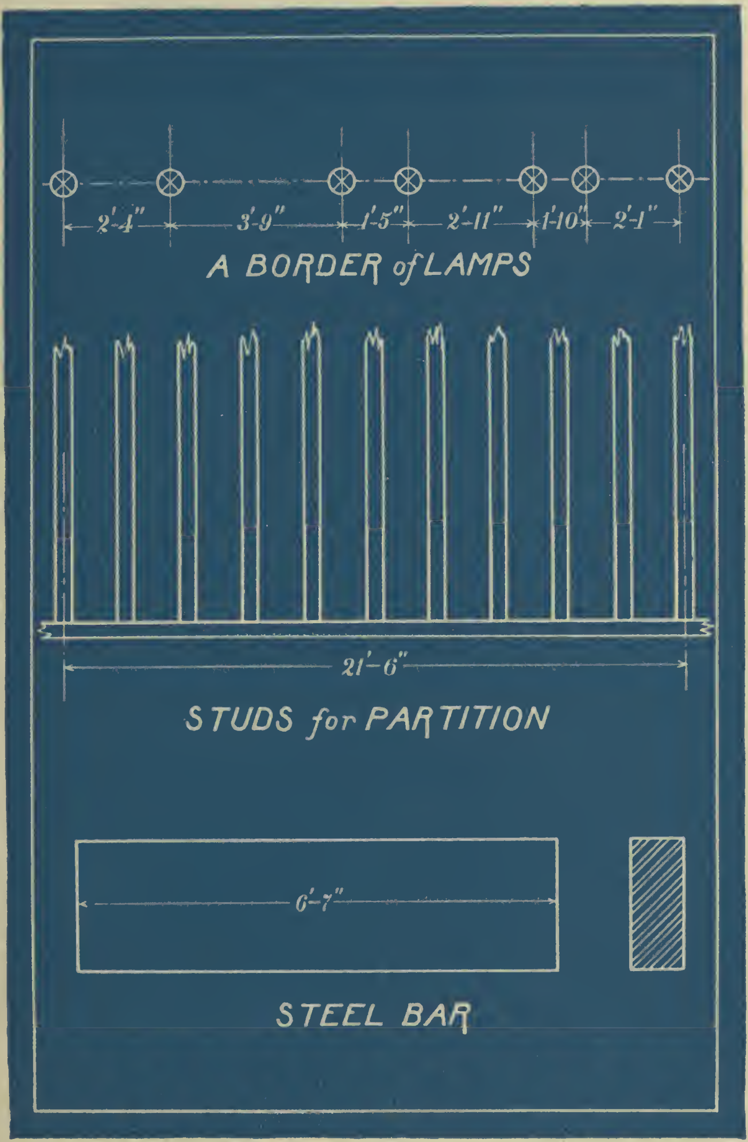
We divide 21′ 6″ by 10, since there are ten spaces for the eleven studs. We see that $21'\,6'' \div 10 = 2'$, with 1′ 6″, or 18″, left over, which is still to be divided by 10. Then $18'' \div 10 = 1.8''$. Expressing 0.8″ as a common fraction to the nearest $\frac{1}{16}''$, we have $\frac{13}{16}''$, and therefore the distance between the centers of the studs in the partition is $2'\,1\frac{13}{16}''$.

Exercises. Feet and Inches

1. If the lamps on page 35 were equally spaced, find to the nearest $\frac{1}{8}''$ the distance between the centers of the lamps.

2. Find to the nearest $\frac{1}{16}''$ the distance between the centers of the studs in a partition when there are 14 equally spaced studs in a length of 32′; 7 studs in a length of 18′; 18 studs in a length of 33′; 15 studs in a length of 28′ 7″; 12 studs in a length of 16′ 3″; 50 studs in a length of 75′ 10″.

3. If the steel bar in the blueprint is cut into five equal parts, find to the nearest $\frac{1}{32}''$ the length of each part.


2'-4"
3'-9"
1'-5"
2'-11"
1'-10"
2'-1"
A BORDER of LAMPS
21'-6"
STUDS for PARTITION
6'-7"
STEEL BAR

Exercises. Review

1. The rivet holes in the structural steel girder, a part of which is shown on page 37, are evenly spaced. If the first and the last of the holes shown in each row are $3'\ 11\frac{1}{4}''$ apart, find the distance between successive holes.

In such cases distances are always measured from center to center.

2. What would be the distance in Ex. 1 if there were 26 holes in the given length?

3. A machinist is laying out the cover for the transmission case shown in the blueprint for drilling. What distance between centers does he use for the holes in the longitudinal rows, the holes being spaced equally? What distance does he use for the holes in the transverse rows?

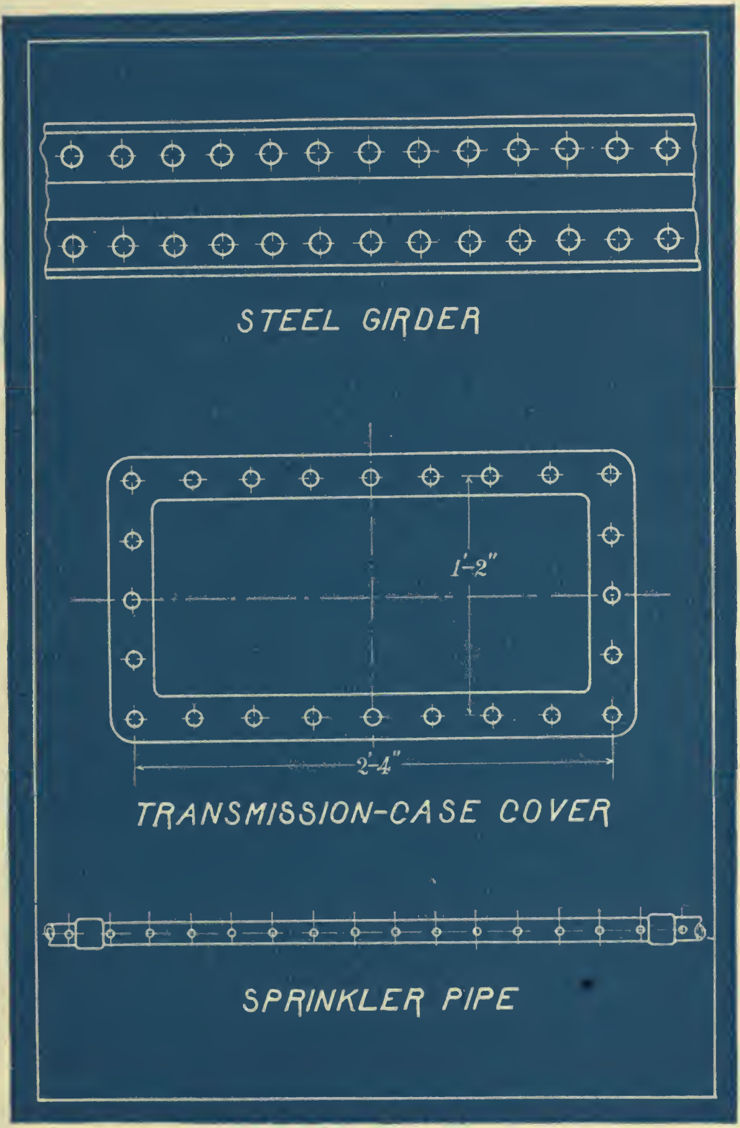
4. The sprinkler pipe, a portion of which is shown in the blueprint, is $21'\ 10''$ long, and there are 52 equally spaced holes, the first and last holes being each $\frac{5}{16}''$ from the ends of the pipe. Find the distance between successive holes.

5. If the blueprint of the sprinkler pipe in Ex. 4 had called for 53 holes with distances between centers of $4\frac{3}{8}''$, what would be the length of the pipe?

6. A certain blueprint for a steel girder calls for a distance between the centers of the rivet holes of $4.375''$. Express this distance with a common fraction instead of the decimal.

7. If in a girder the distance between the first rivet hole and the second is $3\frac{15}{16}''$ and that between the second and the third is $4\frac{3}{4}''$, find the distance from the first hole to the third.

8. If in a steel girder the distance between the first rivet hole and the second is $3\frac{3}{8}''$, that between the second and the third is $4\frac{1}{16}''$, and that between the third and the fourth is $3\frac{15}{16}''$, find the distance from the first hole to the third; from the second to the fourth; from the first to the fourth.


STEEL GIRDER
1'-2"
2'-4"
TRANSMISSION-CASE COVER
SPRINKLER PIPE

Exercises. Miscellaneous Applications

1. In laying the floor of a corridor 72′ long, a carpenter uses boards 16′ long. There are 15 boards in the width of the corridor. Allowing for the loss of one board in matching, how many boards does the carpenter use?

2. If the circumference of a wagon wheel is 12′, how many times will the wheel turn while the wagon is going 14 mi.? while the wagon is going $7\frac{3}{4}$ mi.?

3. The diameter of an iron rod is $\frac{14}{16}''$. Express this in eighths of an inch. Is the diameter less than $\frac{1}{2}''$? If so, how much less? Is it more than $\frac{3}{4}''$? If so, how much more?

4. A piece of plate glass is $\frac{8}{32}''$ thick. Express this fraction in lowest terms. Express the thickness of the glass in eighths of an inch; in sixteenths of an inch.

5. A foreman ordered some iron strips that were $\frac{1}{8}''$ thick and some that were $\frac{5}{32}''$ thick. Which strips were the thicker? How much thicker?

6. Which wire has the greater diameter, a wire which is $\frac{16}{64}''$ across or one which is $\frac{3}{8}''$ across? How much greater?

7. A workman has several drills which are respectively $\frac{1}{2}''$, $\frac{15}{32}''$, $\frac{29}{64}''$, $\frac{7}{16}''$, $\frac{31}{64}''$, and $\frac{27}{64}''$ in diameter. Reduce these fractions to sixty-fourths and arrange them in order of size, beginning with the smallest.

8. If plaster $\frac{3}{8}''$ thick is coated with a finer plaster $\frac{3}{16}''$ thick, how thick is the plaster then?

9. A plate of brass $\frac{1}{32}''$ thick is laid on a plate of iron $\frac{3}{16}''$ thick. What is then the total thickness of the plates?

10. An iron rod $\frac{15}{16}''$ in diameter is covered with thin rolled brass $\frac{1}{32}''$ thick. What is then the diameter of the rod?

11. A piece of cardboard $\frac{1}{64}''$ thick is laid on a book $\frac{5}{8}''$ thick. How thick are the two together?

12. How many pieces of molding 8″ long can be cut from a strip 5′ long, and how much molding will be left over?

13. Making no allowance for doors and windows, how much picture molding will be needed to go round a room 38′ 6″ long and 24′ 8″ wide?

14. From a board 16′ long a workman saws off a piece 3′ $5\frac{3}{4}$″ long and another piece 4′ $8\frac{3}{8}$″ long. How long is the piece of the board that is left?

15. A plate of glass $18\frac{3}{4}$″ by $23\frac{5}{8}$″ was set in a picture frame that covered it $\frac{3}{8}$″ from each edge. What are the inside dimensions of the frame?

16. A gas fitter, in running a pipe into a room, has four pieces of pipe respectively 7′ $4\frac{1}{2}$″, 8′ $2\frac{5}{8}$″, 9′ $5\frac{1}{4}$″, and 8′ 6″ long, and finds that he has 4′ 9″ more than he needs. What is the length of the pipe required?

17. What is the cost of 16′ 4″ of iron rod, weighing $4\frac{1}{2}$ lb. to the foot, at $3\frac{3}{8}$¢ a pound?

18. What is the weight of a steel girder which is 18′ 10″ long and weighs $46\frac{1}{2}$ lb. to the running foot?

19. A pile of $\frac{3}{8}$-inch boards is 5′ $3\frac{7}{8}$″ high. Allowing 5″ as the total of the spaces left for ventilation between the boards in piling, how many boards are there in the pile?

20. The distance between two railroad stations is given in a time-table as 16.32 mi. Express this distance in miles and a common fraction.

21. The sides of a triangular flower bed are 11′ 8″, 16′ 9″, and 14′ 7″ respectively. Find the perimeter of the bed.

22. The sides of a triangular plate of steel are $8\frac{3}{16}$″, $6\frac{15}{32}$″, and $8\frac{3}{16}$″ respectively. Find the perimeter of the plate.

23. If the perimeter of a triangular plate is $23\frac{7}{8}$″ and if each of two sides is $7\frac{3}{16}$″, what is the length of the third side?

Per Cent. Another name for "hundredths" is *per cent.* For example, instead of saying "ten hundredths" we may say "ten per cent." The two expressions mean the same.

There is a special symbol for per cent, %. Thus we write **40%** for forty per cent, and it has the same meaning as 0.40.

Because hundredths and per cents are the same, any fraction with denominator 100 may be written in the form of a per cent; thus:

$$\frac{1}{100} = 1\% \qquad \frac{3}{100} = 3\% \qquad \frac{21}{100} = 21\% \qquad \frac{200}{100} = 200\%$$

The per cents commonly needed may be easily written in fractional form as follows:

$$50\% = \frac{50}{100} = \frac{1}{2} \qquad 12\tfrac{1}{2}\% = \frac{1}{8} \qquad 37\tfrac{1}{2}\% = \frac{3}{8}$$

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 16\tfrac{2}{3}\% = \frac{1}{6} \qquad 62\tfrac{1}{2}\% = \frac{5}{8}$$

$$75\% = \frac{75}{100} = \frac{3}{4} \qquad 33\tfrac{1}{3}\% = \frac{1}{3} \qquad 87\tfrac{1}{2}\% = \frac{7}{8}$$

That part of arithmetic which treats of per cents is called *percentage.*

First Problem in Percentage. The most common type of problem in percentage is finding some per cent of a number.

For example, a certain water tank holds 85,000 gal., and a large pump at the water works pumps 4% of this amount at a single stroke. How many gallons is this?

$$\begin{array}{r} 85000 \\ 0.04 \\ \hline 3400.00 \\ = 3400 \end{array}$$

We have to find 4% of 85,000 gal., and, as is shown above, this is the same as 0.04 of 85,000 gal. Then $4 \times 85{,}000 = 340{,}000$, and $0.04 \times 85{,}000$ is $\frac{1}{100}$ as much, or 3400. Therefore 3400 gal. are pumped at a single stroke.

In multiplying by any per cent, multiply as by a whole number and divide the result by 100 by moving the decimal point.

When it is easier, use the fractional form of a per cent.

Exercises. Finding Per Cents

1. A certain company manufactures electric engines of 18,600 H.P. (horse power) and other engines of $66\frac{2}{3}\%$ as much power. Find the power of the latter engines.

Since $66\frac{2}{3}\% = \frac{2}{3}$, simply take $\frac{2}{3}$ of 18,600 H.P.

Most types of engines or motors are rated by the horse power that they develop. A horse power is the force necessary to lift 33,000 lb. a distance of 1′ in 1 min.

2. How much does $37\frac{1}{2}\%$ of a cubic foot of steel weigh if 1 cu. ft. of steel weighs 490 lb.?

Use the fractional form $\frac{3}{8}$ instead of $37\frac{1}{2}\%$ or $0.37\frac{1}{2}$.

3. If a steel car when full carries 96,000 lb. of coal, how many tons (2000 lb.) does it carry when loaded to 75% of its capacity?

4. If a locomotive weighing 124 T. (tons) can exert a pull equal to $22\frac{1}{2}\%$ of its weight, how great a pull can it exert?

Multiply by $22\frac{1}{2}$ and insert the decimal point two places to the left.

5. If a manufacturer sells shoes at a profit of 15% and it costs him $3.45 a pair to make and sell them, how much is his profit on 1000 pairs?

6. If a shop manufactures 276 locomotives and sells 75% of them for $16,125 each and the rest for $12,825 each, how much is received for all?

7. The wooden pattern from which an iron casting is made weighs $6\frac{3}{4}\%$ as much as the iron. If the casting weighs 1500 lb., how much does the pattern weigh?

8. An iron tire expands $1\frac{9}{16}\%$ on being heated for shrinking on a wheel. A certain wooden wheel needs a tire 16′ 8″ in circumference. How much longer is the tire when heated?

Express 16′ 8″ as inches and then multiply.

Second Problem in Percentage. The second type of problem in percentage is to find what per cent one number is of another.

For example, the purity of gold is measured in carats, or twenty-fourths, 18 carats, or 18 carats fine, meaning that the article is $\frac{18}{24}$ pure gold. What is the per cent of pure gold in a 14-carat ring?

$$\begin{array}{r} 0.58\frac{1}{3} \\ 12\overline{)7.} \\ \underline{60} \\ 100 \\ \underline{96} \\ 4 \end{array}$$

In a 14-carat ring the pure gold is $\frac{14}{24}$, or $\frac{7}{12}$, of the metal. To express $\frac{7}{12}$ as a decimal we divide 7 by 12, as shown at the right, the result being 0.58 with a remainder of 4 (hundredths). Dividing 4 by 12, we have $\frac{4}{12}$, or $\frac{1}{3}$.

Hence the per cent of pure gold is $58\frac{1}{3}$; that is, the answer is $58\frac{1}{3}\%$.

Exercises. Second Problem in Percentage

1. What is the per cent of pure gold in a watch case that is 18 carats fine? in a chain that is 10 carats fine?

2. A carpenter needs a plank $30'\ 8''$ in length when finished. To allow for ending he orders it $31'$ long in the rough. The waste is what per cent of the length in the rough?

We see that $31' - 30'\ 8'' = 4''$ and that $31' = 372''$. We therefore have to express $\frac{4}{372}$ as per cent.

In all problems, unless otherwise directed, if the remainder (hundredths) does not reduce to one of the simple common fractions, such as thirds, fourths, eighths, and so on, find the decimal to the nearest 0.001 and give the answer as a per cent to the nearest 0.1%. In practice it might be sufficient to give the answer to Ex. 2 to the nearest 1%.

3. After a rough casting weighing 282 lb. is turned in a lathe, it is found to weigh 271 lb. The loss in weight of the casting is what per cent of the weight in the rough? of the weight when finished?

4. If $6\frac{1}{4}$ tons of iron are obtained from $117\frac{1}{2}$ tons of ore, what per cent of the ore is iron?

5. An agent bought an automobile for $600 and sold it at a profit of $120. His gain was what per cent of the cost? of the selling price?

6. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 800 cu. ft. of water 6250 lb. of water are drawn off. What per cent of the water is drawn off?

7. If a baker uses 639 lb. of flour in making a certain amount of bread, and adds to the flour 213 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture? The weight of the flour is what per cent of the weight of the mixture?

8. In preparing a solution for spraying, 1 oz. of Paris green is added to $6\frac{1}{4}$ gal. of water. Taking 8.4 lb. as the weight of 1 gal. of water, the weight of the Paris green is what per cent of the weight of the water?

Express the result in Ex. 8 to the nearest 0.01%.

9. A boiler that supplies steam for an engine has a gage attached to it that shows how many pounds of pressure the steam exerts against every square inch of the boiler surface. If the steam pressure increases from 120 lb. to 150 lb. per square inch, it is then what per cent greater than before?

10. A plumber who has been receiving $45.60 a week for a 48-hour week has his pay raised to $52.80 a week for a 44-hour week. Find the per cent of increase per hour.

11. An electrical-appliance dealer buys 45 box bells for $16.20 and sells them at 55¢ apiece. Find his per cent of gain on the cost; on the selling price.

12. A rod that is $3\frac{7}{8}'$ long is what per cent as long as a rod that has a length of $7\frac{3}{4}'$? The length of the longer rod is what per cent of the length of the shorter one? It is what per cent longer than the shorter rod?

Third Problem in Percentage. It often happens that we need to know what number it is that, when a certain per cent is taken, gives a certain other number.

For example, in shipping an order of wet cells it was found that 1050 cells were lost through breakage, this being 15% of the total shipment. How many cells were shipped?

$$\begin{array}{r} 7000 \\ 15\overline{)105000} \\ \underline{105} \end{array}$$

Since	$1050 =$	15% of the total,
we see that	$1050 \div 15 =$	1% of the total,
and	$100 \times 1050 \div 15 =$	100% of the total.

Therefore we may simply multiply 1050 by 100 by annexing two zeros, and then divide by 15, which gives us 7000.

Hence there were 7000 cells shipped.

Of course we might just as well divide 1050 by 0.15, but the above method is somewhat clearer for purposes of explanation.

The problem frequently appears in such a form as this: In a shipment of wet cells, after losing 15% in breakage, a dealer in electrical supplies could use only 5950 cells. How many cells were shipped to him?

$$\begin{array}{r} 7000 \\ 85\overline{)595000} \\ \underline{595} \end{array}$$

Since he lost 15% of the cells, he lost $\frac{15}{100}$, so there were left $\frac{100}{100} - \frac{15}{100}$, or $\frac{85}{100}$.

Since $\frac{85}{100}$ of the cells is 5950,

$\frac{1}{100}$ of them is $5950 \div 85$,

and $\frac{100}{100}$ of them is $100 \times 5950 \div 85$.

Therefore we may simply multiply 5950 by 100 by annexing two zeros, and then divide by 85, which gives us 7000.

Hence 7000 cells were shipped to the dealer.

As in the preceding case, we might divide 5950 by 0.85, but the method used here avoids the use of a decimal.

Rules could be given for this case, but the student who clearly understands the above solutions will be able to make up his own rules if necessary.

Exercises. Third Problem in Percentage

1. Find the cost of a D.C. (direct current) generator which, when sold at a loss of 8% on the cost, brought $575.

2. A cabinetmaker sold a tool chest at a profit of $7, which was 20% of the cost. Find the cost of the tool chest and also the selling price.

3. A clothier sold a suit of clothes at a profit of 20% on the selling price, his gain being $8.40. Find the selling price and the cost.

4. 12′ 5.6″ is 11% of what length? 5% of what length?

5. 126 lb. 14 oz. is 35% of what weight?

6. A hardware dealer had a bargain sale on a certain lot of hammers, but failed to sell $12\frac{1}{2}$% of them. If he had 10 hammers left, how many were there in the lot?

7. A man sells some lumber for $360, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

8. If a cabinetmaker sold a desk so as to gain 20% on the cost of manufacture, and received $84, how much did it cost to make the desk?

9. A man saved $1350.40 last year, which was 32% of his income. How much was his income?

10. A contractor figures that of the lumber needed to finish a certain job he has $11\frac{1}{2}$% on hand. If he has on hand 23 M bd. ft. (23,000 board feet), how much does he need to finish the job? How much more must he buy?

11. A manufacturer sold a suit of clothes to a dealer at a profit of $12\frac{1}{2}$% on the cost of manufacture. The dealer sold the suit to a customer for $48 and made a profit of $33\frac{1}{3}$% on what it cost him. How much did the suit cost the dealer? What was the cost of manufacture?

Exercises. Miscellaneous Applications

1. An agent who buys for an automobile-supply company receives from the company a commission of $\frac{7}{8}\%$ on the amount of his purchases. If he buys 600 bbl. of oil at $7.60 per barrel, the total freight amounting to $57.50, find to the next higher 25¢ the price per barrel at which the company must sell the oil in order to gain 25% on the total cost.

2. A hardware dealer bought 15 gross of screw drivers at $1.85 a dozen. He sold three fourths of the lot at a profit of $33\frac{1}{3}\%$ on the cost and the rest at a loss of 5% on the cost. Find his per cent of gain on the cost for the whole transaction.

3. A line shaft is driven by a 15 H.P. motor. There is, however, a loss of $2\frac{1}{2}\%$ due to the slipping of the belt connecting the motor and the line shaft. Find the actual power delivered to the line shaft.

4. A maintenance engineer found that in a certain year the per cent of breakage of incandescent lamps in a plant was 15% of the number used. By more careful placing of the lights the breakage the second year was cut in half. If in the first year the plant used 4840 lamps, how many did it use the second year? What was the per cent of breakage the second year?

5. A manufacturer of electric supplies shipped 300 doz. incandescent bulbs to a jobber. Owing to an accident in transportation, 15% of the bulbs were broken. How many bulbs were broken? What per cent were not broken?

6. The profits on a business this year are $12,688, and are 22% more than last year. What were the profits last year?

7. Water in freezing expands 9% of its volume. How many cubic feet of water are needed to make 1199 cu. ft. of ice? How many gallons (231 cu. in.) of water are needed?

Discount. Merchants who sell at wholesale, that is, in large quantities to dealers, often sell at a certain per cent off the *list price*, as the price stated in their catalogues is called.

A reduction from a price or amount is called a *discount.*

The per cent of discount, or the common fraction to which this per cent is equivalent, is called the *rate of discount.*

The amount of a price after the discount has been taken off is called the *net price* or *selling price*, and, similarly, the amount of a bill after the discount has been taken off is called the *net amount* of the bill.

For example, find the discount and the net price when the list price of an adding machine is \$245 and a discount of 12% is allowed the purchaser.

We first take 12% of \$245 and find that the discount is \$29.40.

We then subtract the discount from the list price and find that the net price is \$215.60.

$$\begin{array}{r} \$245 \\ 0.12 \\ \hline \$29.40 \end{array} \qquad \begin{array}{r} \$245. \\ 29.40 \\ \hline \$215.60 \end{array}$$

In cases where the rate of discount can be expressed easily as a simple common fraction, it is better to use that fraction.

For example, find the discount and the net amount of a bill of goods for \$1248, the rate of discount being $16\frac{2}{3}\%$.

We first express the rate of discount as a fraction, $16\frac{2}{3}\%$ being equivalent to the fraction $\frac{1}{6}$.

We then find the discount by taking $\frac{1}{6}$ of \$1248 and obtain \$208 as the result.

We then subtract this discount from the amount of the bill and find that the net amount is \$1040.

$$16\tfrac{2}{3}\% = \tfrac{1}{6}$$

$$\begin{array}{r} 6\overline{)\$1248} \\ \hline \$208 \end{array} \qquad \begin{array}{r} \$1248 \\ 208 \\ \hline \$1040 \end{array}$$

Although there is no fixed custom in business, in solving problems in this book the student should discard any fraction less than $\frac{1}{2}$¢ in a discount and call $\frac{1}{2}$¢ or more a full cent.

Discounted Bill. A bill sent by a wholesale dealer usually shows the discount allowed. A sample receipted bill follows:

Chicago, Ill., Dec. 17, 1924

Mr. B. S. Cole
Oklahoma City, Okla.

Bought of HILL, SMITH & CO.
MANUFACTURING JEWELERS
8378 Burlington Ave.

Terms: 2/10, net 30

Dec.	17	3 doz. silver forks @ $32	96			
		1/6 doz. salad forks @ $33	5	50		
			101	.50		
		Less 20%	20	30		
					81	20
		Less 2%			*1*	*62*
					79	*58*
		RECEIVED PAYMENT DEC. 22, 1924 HILL, SMITH & CO. *Per* *C. T. M.*				

A bill sent by a manufacturer or a jobber is often called an *invoice*. The above bill might be called an invoice.

The expression "Terms: 2/10, net 30" in the above bill means that Mr. Cole may take off an additional 2% discount if he pays the bill within ten days or that he may have thirty days in which to settle the account for the net amount, \$81.20. When Mr. Cole received the bill it showed the net amount after the 20% discount, or *trade discount* as it is often called, had been taken off. When Mr. Cole paid the bill he deducted the 2%, as shown in the script type.

Exercises. Bills and Discounts

Make out bills for each of the following, take off the discounts given, and write the receipt on each bill as on page 48:

1. 2 carloads coal, 23,800 lb., 25,200 lb., @ \$9.80 per ton; 3 carloads coal, 24,800 lb., 23,700 lb., 22,900 lb., @ \$9.95 per ton. Terms: 3/10, net 90.

In all these problems insert the dates and the names of dealer and purchaser, and assume that the bill is paid within the time allowed for the *cash discount*, as the 3% in 10 da. is called.

In all problems consider the ton as 2000 lb. unless the long ton of 2240 lb. is specifically mentioned.

2. 7 rocking chairs @ \$8.30; 9 kitchen tables @ \$2.75; 3 bedroom sets @ \$54.30; 18 extension dining tables @ \$16.50; 9 sideboards @ \$24.60. Terms: 2/10, net 30.

3. 1 T. fence wire @ 23¢ per pound. Trade discount: 35%. Terms: 2/10, net 60.

4. 1500 lb. of 3d. nails @ \$3.67 per 100 lb. Trade discount: 15%. Terms: 3/10, net 30.

5. 45 bu. of hair for plastering, 8 lb. to the bushel, @ 9¢ per pound. Trade discount: 12%. Terms: 3/10, net 60.

6. 2 automobile cylinders complete with head cover, valves, and plugs @ \$69.70. Trade discount: 8%.

7. $20\frac{1}{2}$ doz. $\frac{5}{16}$-inch cylinder-head cover-cap screws @ 90¢ per dozen. Trade discount: 12%. Terms: 2/10, net 60.

8. 5892′ #18 annunciator wire, 150′ to the pound, @ 53¢ a pound. Trade discount: $7\frac{1}{2}$%.

9. 12 lb. black rubber tape, $\frac{1}{4}$-pound rolls, @ 15¢ a roll. Trade discount: $\frac{1}{6}$. Terms: 2/10, net 30.

10. 11,646′ #14 galvanized telephone wire, 96 lb. to the mile, @ $7\frac{1}{4}$¢ a pound. Trade discount: $\frac{1}{8}$.

Several Discounts. In some lines of wholesale business two or more discounts are occasionally allowed on the same bill.

For example, a dealer may buy \$400 worth of hardware with discounts of 20% and 10%, often written "20%, 10%," or simply "20, 10." This means that 20% is first deducted from the amount of the bill and then 10% is deducted from the remainder. What is the net amount of the bill?

The amount of the bill is \$400.
The amount less 20% is \$320.
The \$320 less 10% is \$288, the net amount.

This explains the general nature of the problem. Practically we should take short cuts. If we take away 20% of an amount we have 80% left, and if we take away 10% of this result we have 90% of it left. The problem becomes a case of finding 90% of 80% of \$400, and we have

$$0.90 \times 0.80 \times \$400 = 0.72 \times \$400 = \$288.$$

Proceeding in the same way we can solve a problem of this type when it is easier to use the fractional form of the per cents. If the above discounts were $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, we should have to find $\frac{5}{6}$ of $\frac{7}{8}$ of \$400, and we should have

$$\frac{5 \times 7 \times \$\overset{\overset{25}{\cancel{50}}}{\cancel{400}}}{\underset{3}{\cancel{6}} \times \cancel{8}} = \frac{\$875}{3} = \$291.66\tfrac{2}{3}.$$

Therefore the net amount would be \$291.67.

Follow the rule given at the bottom of page 47 in connection with fractions of a cent in any discount.

The bill for the above goods sent by the wholesaler would follow the form of the discounted bill shown on page 48. If any cash discount is allowed, it is deducted, when the account is settled, from the net amount shown on the wholesaler's bill.

Exercises. Several Discounts

1. What is the difference between a discount of 50% and the two discounts of 25%, 25% on $1000?

2. Is there any difference between a discount of 5%, 4% and one of 4%, 5% on $900? on $600?

3. A manufacturer lists a desk at $52 less 25%, and a rival manufacturer offers a similar desk for $57 less $\frac{1}{3}$. Which is the lower net price? How much lower? If the first dealer increases his discount to 25, 3, which will then be the lower net price? How much lower?

4. A wholesale dealer allows a trade discount of 20, 5, and a 10-day cash discount of 2% from his list prices. Find the amount paid on Jan. 18 for the following items bought on Jan. 10: 9 dining tables @ $34.50; 68 dining chairs @ $7.85; 7 buffets @ $47.25; 5 serving tables @ $29.75.

5. Make out an invoice for the following goods bought Sept. 20 and paid for Oct. 5, terms 2/20, N 90: 5 #264 plows @ $42.50 less 20%; 3 #178 self-dumping hayrakes @ $18.60 less 15%; 9 #325 hay stackers @ $46.50 less 15%.

The symbol 2/20, N 90 means the same as 2/20, net 90, the explanation of which was given on page 48.

6. Find the amount paid for 2469′ of 100-conductor interior cable, $1\frac{1}{2}'$ to the pound, @ 54¢ a pound, net.

Following the model on page 48, make out a receipted bill for each of the following shipments:

7. 36 doz. files @ $8.30. Discount: 30, 20.

8. 9 doz. pairs hinges @ $6.75; 48 doz. table knives @ $12.60; 36 doz. table forks @ $8.40. Discount: 20, 10.

9. 36 doz. locks @ $6.30; 19 doz. mortise locks @ $6.75; 28 doz. pairs hinges @ $8.85. Discount: 25, 8, 4.

Exercises. Pay Rolls

1. Fill out each space that is marked with an asterisk (*) in the following form, which is part of the pay roll of a small manufacturing concern for the week ending July 26, 1924:

PAY ROLL OF J. R. MOLLER & CO.

For the week ending July 26, *1924*

No.	Name	M.	T.	W.	T.	F.	S.	Total Time	Wages per Hour	Total Wages	
1	J. P. Drew	8	8	8	6	8	4	42	80¢	$33	60
2	R. L. Bond	8	$7\frac{1}{2}$	8	8	8	4	*	85¢	*	*
3	P. F. Cram	7	$7\frac{1}{4}$	8	$7\frac{1}{2}$	8	4	*	80¢	*	*
4	B. J. Mead	8	7	$6\frac{1}{2}$	8	$7\frac{1}{2}$	3	*	75¢	*	*
5	R. K. King	8	8	7	$7\frac{3}{4}$	8	$3\frac{1}{2}$	*	72¢	*	*
6	M. L. Drake	$7\frac{3}{4}$	6	8	0	7	$3\frac{3}{4}$	*	$67\frac{1}{2}$¢	*	*
		*	*	*	*	*	*	*		*	*

Make out pay rolls (inserting names) when the men's numbers, the hours per day, and the wages per hour are as follows:

2. No. 1: $7\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, 8, 8, $3\frac{1}{2}$, 90¢; No. 2: 8, 8, 8, 8, 8, 4, 85¢; No. 3: 8, 8, 7, 7, 8, 4, 72¢; No. 4: 8, $7\frac{1}{2}$, $6\frac{3}{4}$, 8, 8, 4, 68¢; No. 5: 8, 8, 8, 0, $6\frac{1}{2}$, 4, 65¢.

3. No. 1: 7, 8, 8, 8, $7\frac{1}{2}$, 4, $92\frac{1}{2}$¢; No. 2: 8, 7, 8, 7, 6, 4, 80¢; No. 3: 8, $7\frac{1}{2}$, 8, 8, 8, 4, $78\frac{1}{4}$¢; No. 4: 8, 8, 8, 8, $7\frac{1}{2}$, 4, 75¢; No. 5: 8, 7, 8, 8, 8, 4, 72¢.

4. No. 1: 8, 7, 6, 8, 8, 4, $92\frac{1}{2}$¢; No. 2: 8, 7, 6, $6\frac{3}{4}$, $7\frac{1}{2}$, 4, 84¢; No. 3: 8, 6, 6, 8, 8, 4, 82¢; No. 4: 7, 8, $7\frac{3}{4}$, $6\frac{1}{2}$, 8, 4, 77¢; No. 5: 8, 6, 7, $7\frac{1}{2}$, $6\frac{1}{4}$, 4, 68¢.

5. No. 1: 7, 7, 8, 8, 8, 4, 96¢; No. 2: 8, 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{1}{4}$, 4, 95¢; No. 3: 8, 8, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, 4, 88¢; No. 4: $7\frac{1}{2}$, $7\frac{1}{4}$, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $3\frac{3}{4}$, 82¢; No. 5: 8, 8, 8, 8, 8, 4, $72\frac{1}{2}$¢.

6. Fill each space marked with an asterisk in the following pay roll, allowing double pay for overtime as explained below:

PAY ROLL OF R. E. THURSTON & CO.											
For the week ending Jan. 19, *1924*											
No.	Name	M.	T.	W.	T.	F.	S.	Total Time	Wages per Hour	Total Wages	
1	R. S. Jones	$\surd$	$\overset{2}{\surd}$	$\surd$	$\overset{1\frac{1}{2}}{\surd}$	$\overset{1\frac{1}{2}}{\surd}$	$\surd$	54	90¢	*	*
2	M. L. Downs	$\overset{1}{\surd}$	$\surd$	$\overset{1}{\surd}$	$\overset{1\frac{1}{2}}{\surd}$	$6\frac{1}{2}$	$\overset{1\frac{1}{2}}{\surd}$	*	85¢	*	*
3	J. M. Reed	$\surd$	$\overset{1}{\surd}$	$\overset{2}{\surd}$	$\overset{\frac{1}{2}}{\surd}$	—	$\overset{\frac{1}{2}}{\surd}$	*	$82\frac{1}{2}$¢	*	*
		*	*	*	*	*	*	*		*	*

Before assigning Ex. 6 the instructor should explain that from one and a half to two times the regular hourly wage is usually paid for overtime, and that the check ($\surd$) in the above pay roll means full time for the day. In this pay roll the full time is 8 hr. except on Saturday, when it is 4 hr. The symbol $\overset{2}{\surd}$ means 8 hr. + 2 hr. overtime. A dash (—) indicates absence. Part time, like $6\frac{1}{2}$ hr., is indicated as above on Friday for Downs. Since the allowance for overtime is double that for regular work, Jones's time is $8 + 8 + 8 + 8 + 8 + 4$ (regular time) and $4 + 3 + 3$ (overtime), or 54 hr. in all.

Make out pay rolls (inserting names) when the men's numbers, the hours per day, and the wages per hour are as follows, a full day being 8 hr. except on Saturday, when it is 4 hr., and double pay being given for overtime:

7. No. 1: 8, 9, 8, 9, 8, 5, $97\frac{1}{2}$¢; No. 2: $8\frac{1}{2}$, 9, $9\frac{1}{2}$, 8, 8, 4, 95¢; No. 3: 8, 8, 8, 10, 8, 6, 92¢; No. 4: 8, 9, 9, 9, 7, 4, 90¢; No. 5: $8\frac{1}{2}$, $8\frac{1}{2}$, 9, 8, 8, 5, 80¢.

8. No. 1: 8, 10, 8, 10, 8, 6, 90¢; No. 2: $9\frac{1}{2}$, 8, 6, 9, 8, 4, 88¢; No. 3: 10, 10, 10, 10, 8, 5, $82\frac{1}{2}$¢; No. 4: 8, 0, 8, 8, 10, 9, 80¢; No. 5: 8, 8, 8, 9, $8\frac{1}{2}$, $6\frac{1}{2}$, 75¢.

Exercises. Review

1. A casting is 14′ 3″ long, and the metal expands $\frac{3}{16}''$ to the foot when heated to a red heat. Compute the length of this casting when red hot.

2. In casting brass hinges an allowance of $\frac{1}{64}$ in length has to be made for shrinkage when the brass cools. Find to the nearest 0.001″ the length of the mold for a hinge that is to be $3\frac{7}{8}''$ long; for a hinge that is to be $5\frac{3}{4}''$ long.

3. Find the weight of a piece of round steel shafting $18\frac{3}{4}'$ long, weighing 24.05 lb. to the running foot.

4. Find the cost of 780′ of #000 stranded rubber-covered wire @ $375 per 1000′, discounts of 12%, 5% being allowed.

5. Find the cost of 967′ of $\frac{3}{4}$-inch rigid conduit @ $16.15 per 100′, discounts of 8%, 3% being allowed.

6. A manufacturer determines his selling price by adding approximately $22\frac{1}{2}\%$ to the manufacturing cost. If it costs $72.50 to manufacture a certain machine, find to the next higher dollar the price at which he should sell it.

7. A furniture dealer buys some tables at a prime cost of $16.75 each. His buying and selling expenses and overhead total 24% of the prime cost. If he sells the tables at $25 each, what per cent of profit does he make on the prime cost? on the total cost?

The first cost, or wholesale net price, of an article is called the *prime cost.* Such expenses as heat, light, insurance, and so on make up the *overhead charges,* which are often called simply *overhead.*

8. A dealer bought some glassware for $578.50, but through his fault some of the glassware was broken. He sold the rest for $600, and estimated his cost of doing business at 22% of the selling price. Did he gain or lose on the transaction? What per cent of the total cost did he gain or lose?

CHAPTER II

RATIO AND PROPORTION

Ratio. In practical mathematics we make much use of the idea of ratio, as of the ratio of one length to another length. The relation of one number to another of the same kind, as expressed by the division of the first number by the second, is called the *ratio* of the first number to the second.

Thus the ratio of \$3 to \$6 is $\frac{3}{6}$, or, in its simplest form, $\frac{1}{2}$; the ratio of 1 yd. to 1 ft. is the same as the ratio of 3 ft. to 1 ft., or 3; the ratio of 5 to 2 is $\frac{5}{2}$, or $2\frac{1}{2}$; and the ratio of any number to itself is 1.

The ratio of 2 to 3 may be written in the fraction forms $\frac{2}{3}$ or 2/3, or in the form 2 : 3.

The ratio of 12′ to 4′, for example, may be written $\frac{12'}{4'}$, $\frac{12}{4}$, 12/4, 12 : 4, or simply 3. The word "ratio" is used for each of these forms. The expression 12 : 4 is read "the ratio of 12 to 4" or "as 12 is to 4," 12 and 4 being called the *terms* of the ratio.

Illustrative Problem. If the driving wheel of a locomotive has a diameter of 6′ 9″ and a circumference of 21′ 2.47″, find the ratio of the diameter to the circumference.

To find the ratio we have to divide the diameter by the circumference, and in this case we have

$$\frac{6'\,9''}{21'\,2.47''} = \frac{81''}{254.47''} = 0.3183.$$

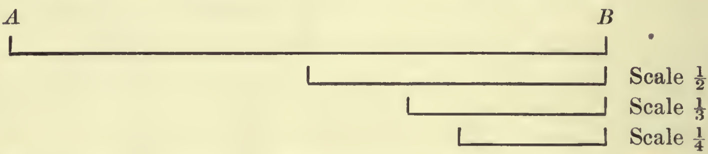
Hence the ratio of the diameter to the circumference is 0.3183.

The above value is the same as that obtained on page 10 by dividing 1 by 3.1416; that is, in any circle the ratio of the diameter to the circumference is $1 \div \pi$.

Ratios and Scales. One of the simplest uses of ratio is found in the blueprints commonly seen in shops.

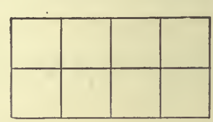
For example, if a room is 30′ long and 20′ wide, and we make a floor plan 3″ long and 2″ wide, we draw the plan to scale, 1″ representing 10′. We indicate this by writing "Scale, 1″=10′." We may also write this "Scale, 1″=120″," or "Scale $\frac{1}{120}$," which simply means that the ratio of the length of a line in the drawing to the length of the line on the floor which it represents is 1 : 120.

The following shows a line *AB* drawn to different scales:



The figures shown below illustrate the drawing of a rectangle to scale. In this case the lower rectangle is a drawing of the upper rectangle to the scale $\frac{1}{2}$, or 1 to 2, or 1″ to 2″.

Notice that the area of the lower rectangle is only $\frac{1}{4}$ that of the upper one; that is, the ratio of the areas is 1 : 4. When we draw to the scale $\frac{1}{2}$ we mean that the length of every line in the drawing is $\frac{1}{2}$ the length of the corresponding line in the original. Whatever the shape of the figure, the area of the drawing is $\frac{1}{4}$ the area of the original figure.

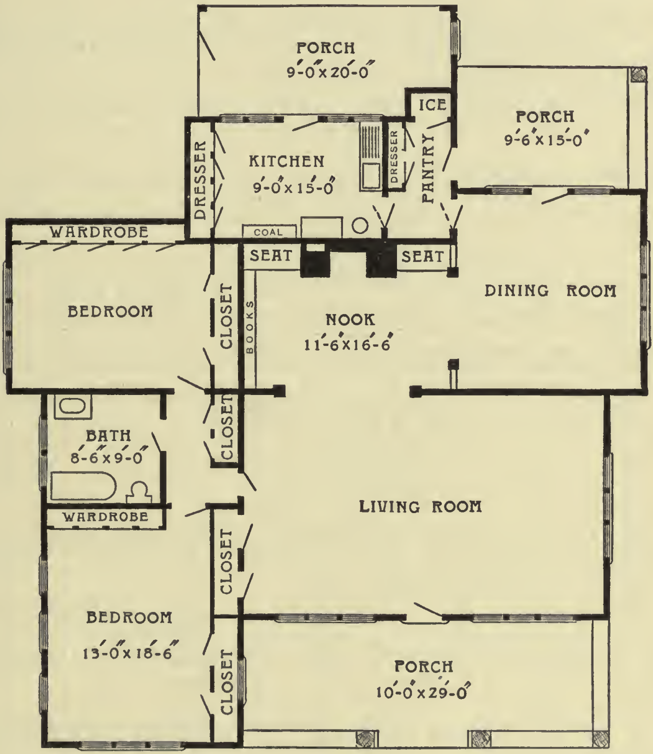


Maps are figures drawn to scale. The scale is usually stated on the map, as you will see in any geography. The scale used in drawing a map is often expressed by means of a line divided to represent miles, and sometimes by such a statement as "1″=100 mi."

Exercises. Drawing to Scale

1. A room is 30′ long and 25′ wide. Draw a plan of the floor, using the scale $\frac{1}{120}$.

2. Draw a plan of a floor 24′ long and 16′ wide, using the scale of $\frac{1}{8}''$ to 1′; using the scale of 1″ to 8′.



3. The drawing above is the floor plan for a concrete bungalow. Find the scale used in drawing the plan.

4. Find the dimensions of the living room, dining room, and smaller bedroom, including the wardrobe but not the closet.

Exercises. Miscellaneous Applications

1. The specific gravity of copper is 8.9 and the weight of 1 cu. ft. of water is $62\frac{1}{2}$ lb. Find the weight of 1 cu. ft. of copper; of 7.53 cu. ft. of copper.

The *specific gravity* of a metal is the ratio of the weight of a piece of the metal to the weight of an equal volume of water.

2. The specific gravity of tin is 7.29. Find to the nearest 0.1 oz. the weight of 1 cu. in. of tin.

First find the weight of 1 cu. ft. of tin, using the weight of 1 cu. ft. of water as given in Ex. 1. Then 1 cu. in. = $\frac{1}{1728}$ cu. ft.

3. The weight of 1 cu. in. of water is 0.58 oz., and the specific gravity of steel is 7.83. Find the weight of 1 cu. in. of steel; of 1 cu. ft. of steel.

4. Using the specific gravity of copper given in Ex. 1, find the weight of a bar of copper 8″ long, 2″ wide, and 1″ thick.

5. In Babbitt metal, also known as white metal, there are, by weight, 4 parts of copper, 8 parts of antimony, and 96 parts of tin; that is, the ratio of the copper to the total is 4 : 108. In 54 lb. of Babbitt metal what is the weight of the copper? of the antimony? of the tin?

6. From the data of Ex. 5 find the weight of the copper in 500 lb. of Babbitt metal.

7. From the data of Ex. 5 find the weight of each of the three component metals in 800 lb. of Babbitt metal; in 900 lb. of Babbitt metal; in 1 T. of Babbitt metal.

8. A certain kind of gunpowder is made by taking 15 parts by weight of saltpeter to 3 parts of charcoal and 2 parts of sulphur. Find the number of pounds of saltpeter used in making 1 T. of gunpowder.

9. In Ex. 8 find the number of pounds of charcoal used; the number of pounds of sulphur used.

Proportion. An expression of equality between two ratios is called a *proportion.*

For example, the ratio \$2 : \$3 is equal to the ratio $10' : 15'$. Therefore \$2 : \$3 $= 10' : 15'$ is a proportion. This proportion is read "\$2 is to \$3 as $10'$ is to $15'$." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

Extremes and Means. The first and last terms of a proportion are called the *extremes*; the second and third terms are called the *means.* In the proportion $3 : 7 = 15 : 35$, or

$$\frac{15}{35} = \frac{3}{7},$$

if we multiply both fractions by 7×35 we see that

$$7 \times 15 = 3 \times 35.$$

Therefore, it follows that in any proportion

The product of the means equals the product of the extremes.

If we let x stand for some number that we are to find, and if

$$x : 29 = 26 : 105,$$

then

$$x = \frac{29 \times 26}{105}.$$

In any proportion the product of the means divided by either extreme equals the other extreme, and the product of the extremes divided by either mean equals the other mean.

Illustrative Problems. 1. If $x : 7 = 13 : 21$, find the value of x.

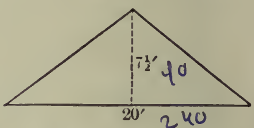
As above, we see that $$x = \frac{\cancel{7} \times 13}{\underset{3}{\cancel{21}}} = \frac{13}{3} = 4\frac{1}{3}.$$

2. If $19.17 : 21.3 = x : 3$, find the value of x.

Here we see that $$x = \frac{\cancel{3} \times 19.17}{\underset{7.1}{\cancel{21.3}}} = \frac{191.7}{71} = 2.7.$$

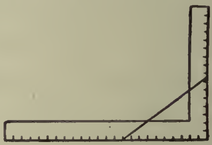
Exercises. Proportion

1. A student draws a plan of the gable end of a roof as shown below, using 8″ to represent 20′. What length in the plan represents the $7\frac{1}{2}$-foot "rise"? How can the student find the length of the slope of the roof from the plan?



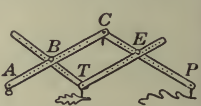
2. The instructor says that the plan in Ex. 1 is not as convenient as a plan showing half of the gable in which the 10-foot "run" is represented by 10″. Draw such a plan and measure the slope. How long is the slope of the roof?

3. In solving Ex. 1 a carpenter would use his square. On the tongue, or short arm, he would take a point $7\frac{1}{2}$″ from the corner; then on the blade, or long arm, he would take a point 10″ from the corner. He would then measure the distance between these points, and say that the number of inches in this distance is the same as the number of feet in the slope of the roof. Draw the figure to scale and write out the reason involved.



4. By means of a pantograph a student enlarges the floor plan for a house in the ratio of 7 : 4. If the dining room in the original plan measures $2\frac{1}{2}$″ by 3″, what are the dimensions in the enlarged plan?

The pantograph is extensively used in enlarging or reducing plans or designs. The bars of the instrument here shown are pivoted at *B*, *C*, *E*, and *T*, and the point *A* is fixed. As the tracing point *T* is moved over the outline of the design which is being enlarged, the pencil at *P* draws the enlargement similar to the design.



5. How far is it around a piece of land represented by a rectangle 14″ by 18″ on a map to the scale of 1″ = 0.8 mi.?

6. If 8 T. of coal cost \$70.96, how much will $48\frac{1}{2}$ T. cost?

Since the ratio of the costs is equal to the ratio of the number of tons in each case, $x : 70.96 = 48\frac{1}{2} : 8$.

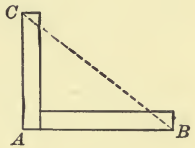
7. If 18 hammers cost \$24, what is the cost of 28 hammers?

8. If 1000′ of double-braided stranded wire cost \$50.67, how much will 850′ cost?

9. In Ex. 8 find to the nearest foot the amount of wire which you could buy for \$69.50.

10. An iron casting weighing 425 lb. costs \$18.75. At this rate what is the cost of a similar casting weighing 379 lb.?

11. Two joists 6″ wide are fitted together at right angles, as here shown. The distance from A to B is 8′, that from A to C is 6′, and that from B to C is 10′. In fitting another joist along the dotted line BC the carpenter has to saw off the ends of the first joists on the slant. Find the length of the slanting cut across the upright piece; across the horizontal piece.



Express the results in each case in inches.

12. If a certain type of engine requires $\frac{7}{8}$ pt. of kerosene per horse power per hour, how much kerosene will be required per hour for a 10 H.P. engine of this type? How much kerosene will be required for this engine for 6 hr.?

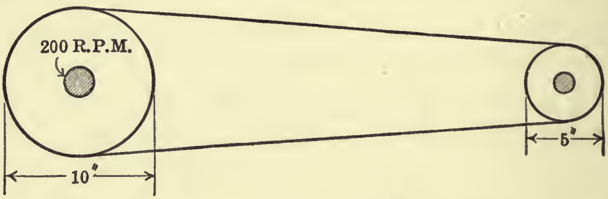
13. If a boiler evaporates 9 lb. of water per hour per pound of coal used, how many pounds of water will it evaporate in 8 hr. if 7 T. of coal are used?

Such a problem may be solved by two proportions, or it may be solved without using any proportion whatever.

14. If the weight of a certain quality of sheet steel is 487.7 lb. per cubic foot, what is the weight of a plate of this steel which is $\frac{1}{16}$″ thick and has 3 sq. ft. of surface?

Inversely Proportional. Sometimes the ratio of the elements of one figure is not equal to the ratio of the corresponding elements of another figure, but is equal to the inverse of that ratio. The elements are then said to be *inversely proportional.*

For example, the figure below represents a large wheel on a driving shaft connected by a belt with a smaller wheel.



Since the two wheels are connected by the belt, the ratio of their speeds must be equal to some ratio of their diameters. But since the belt travels at the same rate as a point on the circumference of the larger wheel, the number of revolutions per minute (R.P.M.) of the smaller wheel must be greater than that of the larger wheel. Therefore, instead of having a proportion in which the speed of the smaller wheel is to the speed of the larger wheel as the smaller diameter is to the larger diameter, the latter ratio is inverted.

Technically such wheels for transmitting power are called *pulleys.*

Illustrative Problem. Using the data given in the above figure, find the speed of the 5-inch driven pulley.

Since the speeds of the pulleys are inversely proportional to the diameters, we have the proportion

$$x : 200 = 10 : 5,$$

from which $$x = \frac{200 \times \overset{2}{\cancel{10}}}{\cancel{5}} = 400.$$

The speed of the driven pulley is therefore 400 R.P.M.

In problems of this type, if the result is not a whole number, the nearest whole number should be taken.

Exercise. Inverse Proportion

1. A 24-inch pulley fixed to a line shaft which makes 500 R.P.M. is belted to an 8-inch pulley. Find the number of R.P.M. of the smaller pulley.

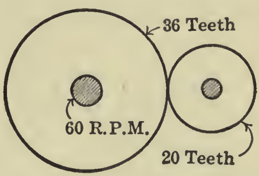
2. A driving pulley has a diameter of 15″ and its speed is 180 R.P.M. Find the speed of a 6-inch driven pulley.

3. A circular saw with a 6-inch pulley is to be driven at 1000 R.P.M. from a line shaft which makes 240 R.P.M. What should be the diameter of the driving pulley on the line shaft in order to obtain this speed?

4. The diameters of the steps of a cone pulley are $4\frac{1}{4}''$, $6\frac{1}{2}''$, $7\frac{3}{4}''$, and $9\frac{1}{2}''$ respectively, and the pulley is driven by a similar cone pulley with the steps arranged in reverse order on a shaft making 210 R.P.M. Beginning with the belt on the largest step of the driving pulley, find the speed of the driven pulley for each position of the belt.

A cone pulley is illustrated in the blueprint on page 3.

5. A driving gear with 36 teeth meshes with a driven gear with 20 teeth, as shown in the figure. The driving gear makes 60 R.P.M. Find the number of R.P.M. made by the driven gear.

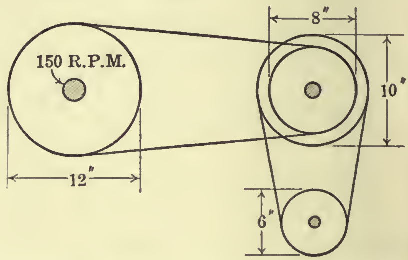


A *gear* is a wheel with teeth cut on the rim to prevent slipping in transmitting power. In the figure the individual teeth are not shown, the words "36 Teeth" and "20 Teeth" indicating that the wheels are gears. The rule for gears is similar to that for pulleys, and, expressed as a proportion, is as follows:

$$\frac{\text{number of R.P.M. of driven gear}}{\text{number of R.P.M. of driving gear}} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}.$$

6. A gear having 76 teeth meshes with one having 30 teeth. At what speed should the smaller gear be driven so that the larger gear will make 115 R.P.M.?

Pulley Train. A series of pulleys connected by belting, as here shown, the power coming from one of the pulleys, is called a *pulley train.*



In the figure the 10-inch pulley and the 8-inch pulley are fixed to the same shaft and consequently they revolve at the same speed.

There is a simple rule which covers the relation of the diameters and the R.P.M. of the pulleys and which we express in the form of a proportion as follows:

$$\frac{\text{R.P.M. of last driven pulley}}{\text{R.P.M. of first driving pulley}} = \frac{\text{product of diameters of all driving pulleys}}{\text{product of diameters of all driven pulleys}}.$$

For example, in the pulley train shown above find the speed of the 6-inch pulley.

$$\frac{x}{150} = \frac{12 \times 10}{8 \times 6}; \quad \text{whence} \quad x = \frac{\overset{25}{\cancel{150}} \times \overset{3}{\cancel{12}} \times \overset{5}{\cancel{10}}}{\underset{2}{\cancel{8}} \times \cancel{6}} = 375.$$

Hence the speed of the 6-inch pulley is 375 R.P.M.

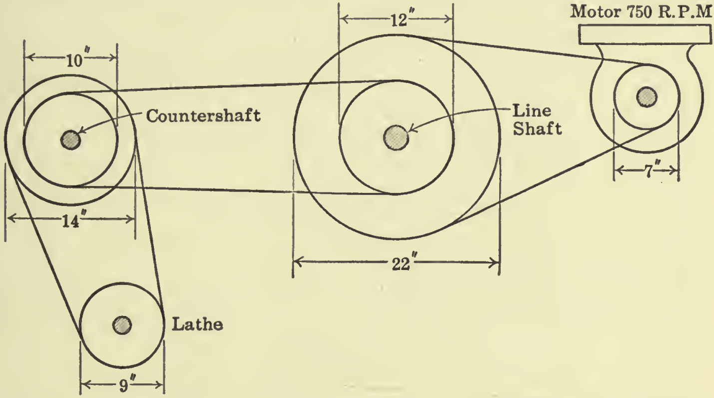
Gear Train. A series of gears running together, the power coming from one of the gears, is called a *gear train.*

The rule for a gear train is similar to the rule for a pulley train, and, expressed as a proportion, is as follows:

$$\frac{\text{R.P.M. of last driven gear}}{\text{R.P.M. of first driving gear}} = \frac{\text{product of number of teeth of driving gears}}{\text{product of number of teeth of driven gears}}.$$

Exercises. Pulley Trains and Gear Trains

1. The figure below shows the belt connections of a lathe, the power from the motor being transmitted through the pulley train as shown. Find the number of R.P.M. of the lathe.

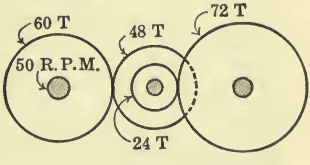


2. In Ex. 1, if a new motor having a 6-inch pulley and a speed of 325 R.P.M. were put in, what size of driven pulley to the nearest $\frac{1}{8}''$ would be needed on the line shaft in order to run the lathe at the same speed?

3. In Ex. 2 what size of pulley would be needed in order to run the lathe at a speed of 350 R.P.M.?

4. In the gear train here shown the driving shaft has a speed of 50 R.P.M. Find the speed of the last driven gear.

The symbol 60 T means that the gear has 60 teeth. This conventional symbol will hereafter be used in connection with gears and gear trains.



5. In Ex. 4 suppose that the 72-T gear were the driving gear and that its shaft made 100 R.P.M., what would then be the speed of the 60-T gear?

6. The change gears in the blueprint on page 67 are used on a lathe when cutting screw threads. From the data on the drawing find the speed of the lead screw.

7. In Ex. 6 find the number of teeth that the driven gear on the compound should have in order that the speed of the lead screw may be doubled.

Express the result in such a problem as the nearest whole number.

8. In the gear train shown in the blueprint find the speed of the 52-T gear.

9. If the 52-T gear in the gear train were the driving gear, and if its speed were 125 R.P.M., what would be the speed of the 28-T gear?

10. In the lathe gear box shown in the blueprint, gears *A* and *B* can be brought into engagement with any of the gears on the lead screw. Gear *B* is an idler, or an intermediate gear, and has no effect upon the speed of the driven gear on the lead screw. Find the speed of the lead screw when gears *A* and *B* engage, in turn, each of the gears on the lead screw, beginning with the smallest.

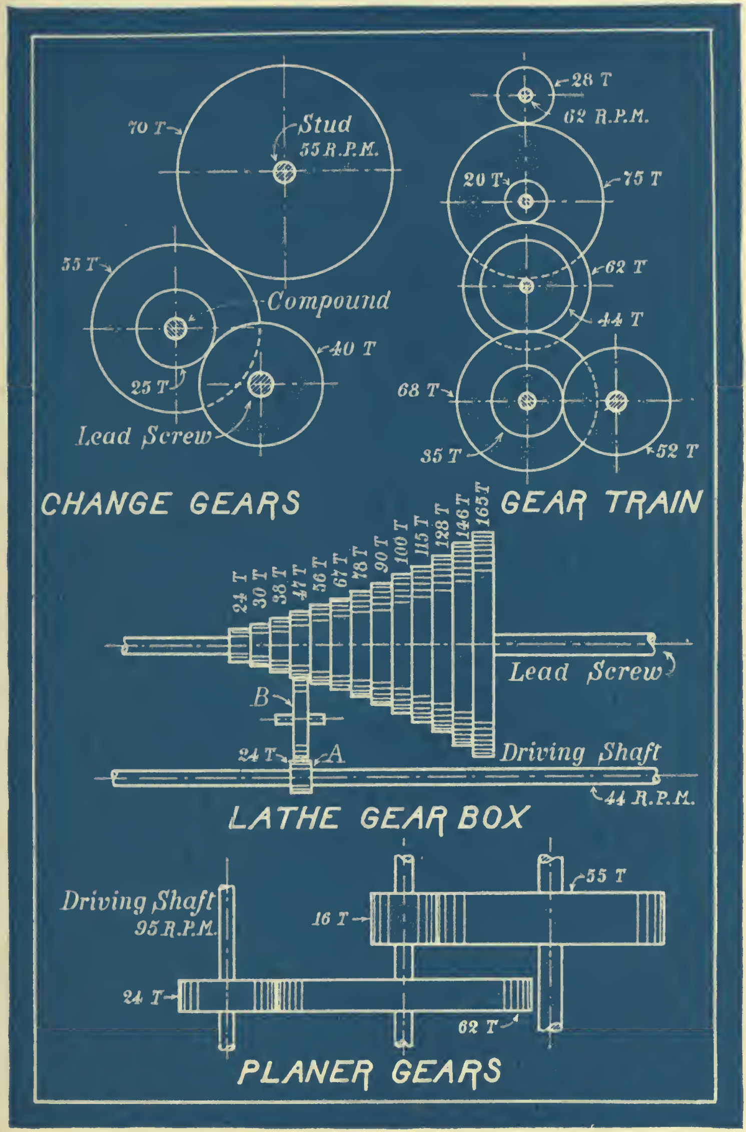
11. In the planer gears shown in the blueprint find the speed of the last driven gear.

12. What would be the result in Ex. 11 if the driving shaft made 110 R.P.M.?

13. In Ex. 11 what would be the result if the 55-T gear were changed to a 60-T gear?

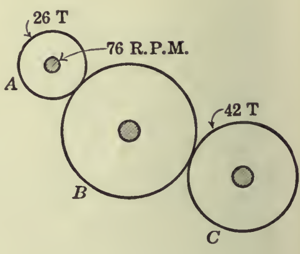
14. In Ex. 11 what would be the result if we should replace the 62-T gear with a 50-T gear?

15. The line shaft in a machine shop runs at 200 R.P.M. A grinder with a pulley 6″ in diameter should run at 1800 R.P.M. In order to obtain this speed, what should be the diameter of the pulley placed on the line shaft?

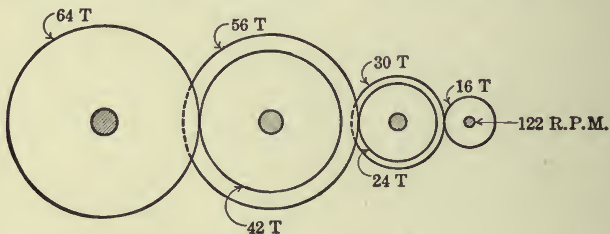

70 T
Stud
55 R.P.M.
55 T
Compound
40 T
25 T
Lead Screw
28 T
62 R.P.M.
20 T
75 T
62 T
44 T
68 T
35 T
52 T
CHANGE GEARS
GEAR TRAIN
24 T
30 T
38 T
47 T
56 T
67 T
78 T
90 T
100 T
115 T
128 T
146 T
165 T
Lead Screw
B
24 T
A
Driving Shaft
44 R.P.M.
LATHE GEAR BOX
Driving Shaft
95 R.P.M.
16 T
55 T
24 T
62 T
PLANER GEARS

16. In this gear train find the speed of the gear marked C.

It should be noticed that gear B in this train is an idler, or intermediate gear. Such a gear was shown in the lathe gear box on page 67 and is frequently used where it is desired to transmit power between two points, such as A and C, which are too far apart to use only two gears of convenient size. An intermediate gear does not affect the speed of the driven gear.



17. In the gear train below find the speed of the 64-T gear.



18. In Ex. 17, if the 64-T gear were the driving gear and made 100 R.P.M., what would then be the number of R.P.M. of the 16-T gear?

19. In the blueprint on page 69, which shows how power is transmitted by belts, find the speed of the surfacer.

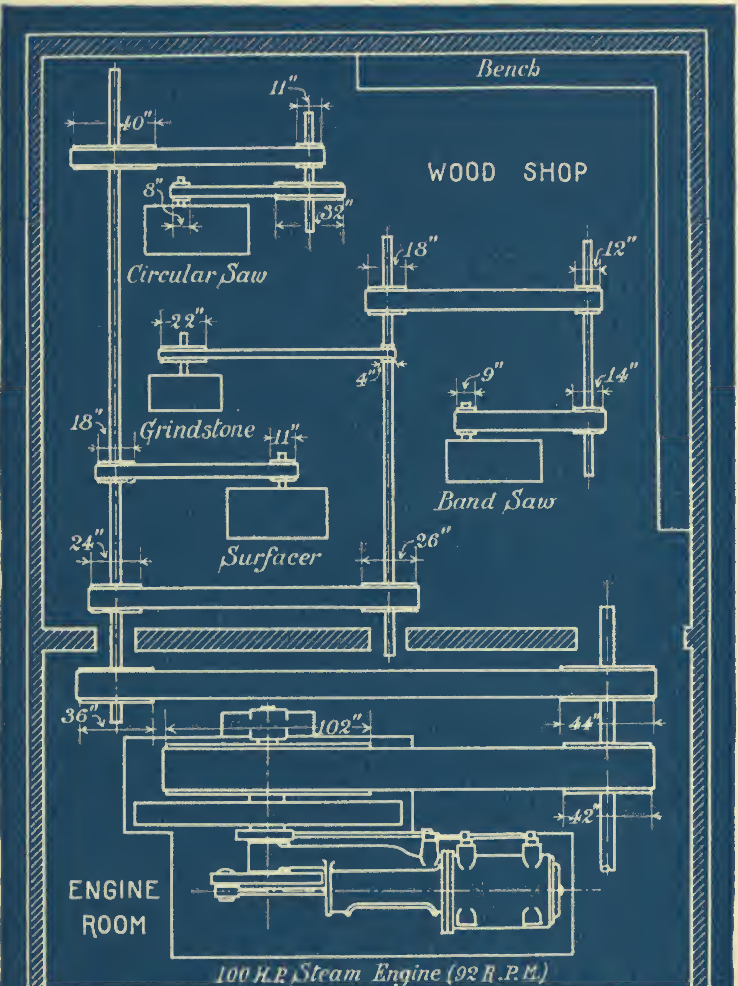
20. Find the number of R.P.M. of the grindstone.

21. Find the speed of the circular saw.

22. Find the number of R.P.M. of the band saw.

23. What size of driven pulley to the nearest $\frac{1}{8}''$ should be placed on the grindstone to increase its speed to 60 R.P.M.? to increase its speed to 75 R.P.M.?

24. What would be the speed of the band saw if the 18-inch pulley on the shaft which drives the grindstone were placed so as to drive the band saw direct?



FLOOR PLAN of WOOD SHOP

Exercises. Review

1. If $\frac{3}{4}''$ on a map represents 375 mi., what is the distance between two places which are $2\frac{1}{8}''$ apart on the map?

2. If 2.8 bbl. of lime are required for 75 sq. yd. of plastering, how many barrels are needed for 675 sq. yd.?

3. A stretch of railroad track runs 685′, with a uniform grade of $8\frac{1}{2}''$ per 100′. What is the difference in level between the bottom and the top of the grade?

4. Gun metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper by weight. How many pounds of tin must be added to 420 lb. 12 oz. of copper to make gun metal?

5. How many pounds of tin are there in 464 lb. 12 oz. of gun metal such as that described in Ex. 4?

6. If 35 men in 16 da. can complete half of an excavation, how long would it take to complete the other half if five more men were added to the working force?

7. A 14-inch pulley fixed to a line shaft, which runs at 150 R.P.M., is belted to a 12-inch pulley on a countershaft. Find the number of R.P.M. of the countershaft.

8. A grindstone with a 28-inch pulley is to be driven at 50 R.P.M. from the countershaft in Ex. 7. What size of driving pulley should be placed on the countershaft to obtain this speed?

9. A 6-inch pulley on a countershaft drives a 22-inch pulley on a hacksaw. What should be the speed of the countershaft to drive the hacksaw at 33 R.P.M.?

10. What size of pulley should be placed on the countershaft in Ex. 9 to increase the speed of the hacksaw $33\frac{1}{3}\%$?

11. A grinder which should have a speed of 480 R.P.M. is to be run by a 19-inch pulley on a shaft making 180 R.P.M. What size of pulley should be placed on the grinder?

CHAPTER III

MENSURATION

Common Measures. Industry in general makes use of a relatively small number of the measures which are usually taught in the schools.

In measuring lengths the inch, foot, and yard are the most common units. While the abbreviations *ft.* and *in.* are used for feet and inches respectively, the symbols ′ and ″ are more common in the shop.

In practical measuring, fractional parts of an inch are usually expressed with denominators 2, 4, 8, 16, 32, or 64; less often with denominators 3, 6, 12, 24, or 48; and with growing frequency as decimals.

The common units used in measuring areas are given on page 72, and those used in measuring solids on page 84.

Liquid and dry measures are used in industry, but many substances that were formerly measured by the quart, gallon, or bushel are now measured by weight.

In measuring weight, the ounce, pound, and ton of 2000 lb. are used, although the long ton of 2240 lb. is still found.

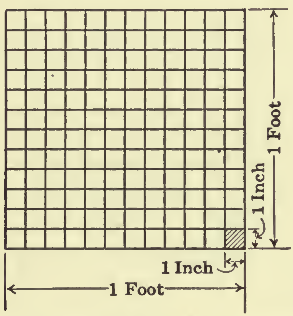
The metric measures are considered on pages 122–130.

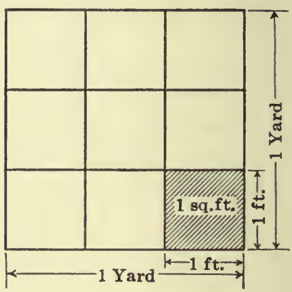
The tables on pages 193–194 should be consulted if necessary.

In finished machine work, results are usually carried to the nearest 0.001″; in plumbing, carpentry, and sheet-metal work, to the nearest $\frac{1}{64}$″; in weights, to the nearest hundredth of the unit employed; and in speeds of pulleys, gears, and machines, to the nearest unit.

Square Measure. In measuring areas the square inch, square foot, and square yard, with their decimal subdivisions or with the simplest common fractions, are used.

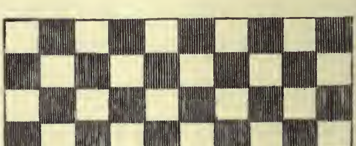
A square inch (sq. in.) is the area of a square that is one inch ($1''$) on a side; a square foot (sq. ft.) is the area of a square that is one foot ($1'$) on a side; and a square yard (sq. yd.) is the area of a square that is one yard (1 yd.) on a side.





The left-hand figure is drawn to the scale $\frac{1}{10}$, and consequently the square which represents 1 sq. ft. is $0.1'$, or $1.2''$, on a side. It will be seen that there are 144 sq. in. in 1 sq. ft. The right-hand figure is drawn to the scale $\frac{1}{30}$, and the square which represents 1 sq. yd. is $\frac{1}{30}$ of a yard, or $1.2''$, on a side. It will be seen that there are 9 sq. ft. in 1 sq. yd.

Area of a Rectangle. For example, if the floor of a rectangular hall, which is $10'$ long and $4'$ wide, is made of marble squares, $1'$ on a side, as here shown, there are 10 squares in each row, and there are 4 rows of squares. Since there are 4×10 squares, we have



$$\text{Area} = 4 \times 10 \text{ sq. ft.}$$
$$= 40 \text{ sq. ft.}$$

To be more precise we might write $4 \times 10 = 40$, the number of square feet, but for brevity the abbreviation *sq. ft.* is commonly used as shown.

The area of a rectangle is the product of the base and height.

Exercises. Square Measure

1. Find the floor area of a room 28′ 6″ long and 18′ wide.

2. Find the cross-section area of a square beam $3\frac{1}{4}''$ on a side. Verify the result by drawing a $3\frac{1}{4}$-inch square and ruling it off into $\frac{1}{4}$-inch squares.

3. A room 16′ by 19′ is $9\frac{1}{2}'$ high. Find the total area of the four walls, the floor, and the ceiling.

No allowance for doors and windows is to be made unless specified.

4. A garden 48′ by 66′ contains a 3-foot walk laid inside the garden along the four sides. The mid points of the long sides are joined by a 2-foot path. Find the area left for cultivation and draw a plan to any convenient scale.

5. On the floor of a room 36′ long and 28′ wide a border 2′ wide is to be painted. Find the cost of painting the border at 60¢ per square yard.

6. At 26¢ per square foot find the cost of a concrete walk 6′ wide round the outside of a garden 66′ by 84′.

7. Draw a plan to any convenient scale of the basement of a house, the walls of which are laid out as follows: Starting at the southeast corner the wall runs north 22′, then west 12′, north 14′, west 16′, south 36′, and east to the starting point 28′, all dimensions being inside measurements. Find the cost of cementing the floor at 25¢ per square foot.

8. A rectangle is $6\frac{7}{8}''$ long and $4\frac{1}{4}''$ wide. Find the area correct to the nearest $\frac{1}{8}$ sq. in.

9. A workman measures the side of a square and finds it to be 5.24″, but the last figure is uncertain, being possibly either 3 or 5. If he uses 5.24″ for finding the area of the square, what is the greatest possible error that can arise?

First find the values of 5.24×5.24, 5.23×5.23, and 5.25×5.25.

Formula. In all kinds of applied mathematics there is need for symbols. Thus we use + for "plus," − for "minus," × for "times," ÷ for "divided by," and √ for "the square root of." This form of mathematical shorthand is still further extended when we come to the rules for measuring. Instead, for example, of writing "The area of a rectangle is the product of the base and height," we simply write the *formula*

$$A = bh,$$

using the initial letters for "area," "base," and "height," and understanding that when two letters are written side by side their product is to be taken.

In practical mathematics we have no time for long rules when we can more easily express these rules by formulas.

Either capital letters or small letters may be used in a formula.

Use of the Formula. For example, if the base of a rectangle is 9″ and the height is $7\frac{1}{2}''$, find the area.

$$A = bh = 9 \times 7\tfrac{1}{2} = 67\tfrac{1}{2}.$$

Since the dimensions were given in inches, the area is $67\frac{1}{2}$ sq. in.

Evaluating. In using such a formula as $A = bh$ the student will often be asked to find the value of A when b and h have certain given values. It is convenient to have a single word to use for the expression "find the value of," and we use the word *evaluate* for this purpose.

For example, to evaluate bh for the values $b = 6$ and $h = 4$ we have $bh = 6 \times 4 = 24$. That is, $bh = 24$ for these values of b and h.

Similarly, we evaluate a^2 for $a = 7$ by writing 7 for a, whence $a^2 = 7 \times 7 = 49$.

The expression a^2, read "a square," means aa, or $a \times a$. Similarly, a^3, read "a cube," means aaa, or $a \times a \times a$.

Exercises. Formulas

1. The area of a square of side s is s^2. The formula for the area may therefore be written $A = s^2$. Evaluate this formula for A when $s = 2\frac{1}{2}''$.

2. From a steel beam weighing b pounds there hangs a block and tackle weighing t pounds, and by this a load of w pounds is being lifted. Write a formula for L, the total resulting load on the beam, including the weight of the beam.

3. Evaluate the formula found in Ex. 2 for L, given that $b = 750$, $t = 325$, and $w = 1675$.

4. The area of a rectangle is shown on page 74 to be expressed by the formula $A = bh$. Evaluate this formula for A when $b = 7.5$ and $h = 3.4$.

5. Evaluate the formula of Ex. 4 for A when $b = 10.2$ and $h = 7.3$. If b and h are dimensions in inches, what is the area of the rectangle?

6. It will be shown on page 76 that the area of a triangle of base b and height h is expressed by the formula $A = \frac{1}{2}bh$. Evaluate this formula for A when $b = 3\frac{3}{4}$ and $h = 1\frac{1}{2}$.

7. The area of a circle is expressed by the formula $A = \frac{22}{7}r^2$. Find the value of A when r, the radius, is 35 units.

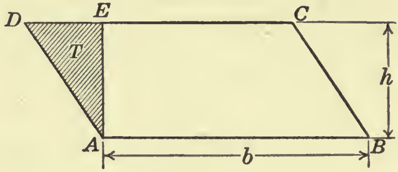
If the unit is 1″, the area will be in square inches; if the unit is 1′, the area will be in square feet.

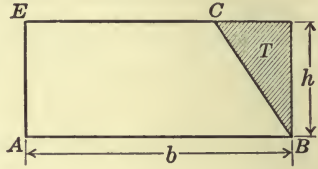
8. An odd number is indicated by the expression $2n + 1$, where n is 0 or any whole number. In this expression give to n all the various values from 0 to 10 and evaluate the expression for each of these values, thus obtaining all the odd numbers from 1 to 21.

While not a practical problem, this shows the general nature of a formula and gives an idea of the meaning of an algebraic expression.

P

Area of a Parallelogram. If from any parallelogram, like $ABCD$ in the left-hand figure below, we cut off the shaded triangle T by a line perpendicular to DC and place this triangle at the other end of the parallelogram, as shown in the figure at the right, the resulting figure is a rectangle.





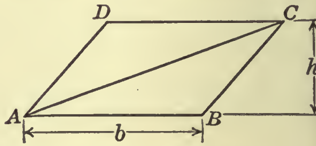
That is, the area of a parallelogram is equal to the area of a rectangle of the same base and the same height. Since the formula for the area of a rectangle is $A = bh$,

The area of a parallelogram is the product of the base and height.

This rule may be conveniently expressed by the formula

$$A = bh.$$

Area of a Triangle. If we draw the diagonal AC in the parallelogram $ABCD$ below, we divide the parallelogram into two equal obtuse triangles. The diagonal DB would give two equal acute triangles, and the diagonal of a rectangle would give two equal right triangles.



Therefore, since any triangle is half of a parallelogram of the same base and the same height, we see that

The area of a triangle is half the product of the base and height.

This rule may be conveniently expressed by the formula

$$A = \frac{1}{2}bh.$$

Exercises. Parallelograms and Triangles

Draw the following parallelograms to scale and find the area of the original parallelogram and of the drawing in each case:

1. Base, $40''$; other side, $30''$; height, $16''$; scale $\frac{1}{8}$.

2. Base, $16''$; other side, $9''$; height, $8''$; scale $\frac{1}{4}$.

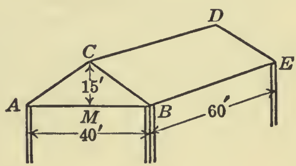
3. Base, $4.5''$; other side, $9.6''$; height, $3.75''$; scale $\frac{1}{3}$.

4. The base of a triangle is $5.7''$ and the height of the triangle is $3.4''$. Find the area of the triangle correct to the nearest 0.1 sq. in.

5. Find the cost of plastering the walls and ceiling of a room $28'\ 9''$ by $36'\ 6''$ and $11'$ high at 68¢ per square yard, deducting 20% of the wall area for doors and windows.

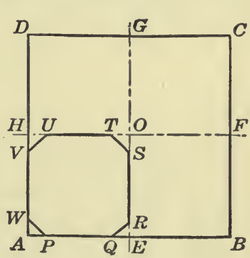
6. What fraction of a square yard of bunting is there in a triangular pennant which has a width of $27''$ measured along the flagstaff and a length of 2 yd.?

7. The span AB of a roof is $40'$, the rise MC is $15'$, the slope BC is $25'$, and the length BE is $60'$. Find the area of each gable end and also the area of the roof.



In this problem do not consider any overhang of the eaves.

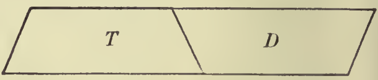
8. In this figure $ABCD$ represents a 6-inch square, E, F, G, and H being the mid points of the sides. In the square $AEOH$, $AP = QE = ER = SO = \cdots = WA = \frac{1}{6}AE$. Find the area of each of the small triangles such as APW and also of the octagon $PQRSTUVW$.



The dots $(\cdots)$ mean "and so on" and in this particular case they take the place of $OT = UH = HV$.

An *octagon* is a figure of eight sides.

Area of a Trapezoid. If a trapezoid T has its duplicate cut from paper and turned over and fitted to it, as D in this figure, the two together form a parallelogram. How does the area of the whole parallelogram compare with the area of the trapezoid T? How does the base of the parallelogram compare with the sum of the upper and lower bases of the trapezoid? How do you find the area of the parallelogram? Then how do you find the area of the trapezoid?



If from the trapezoid $ABCD$, shown in this figure, the shaded portion is cut off and is fitted into the space marked by the dotted lines, what kind of figure is formed? How is the area of the resulting figure found?

From these illustrations we infer the following:

The area of a trapezoid is the product of one half the height and the sum of the parallel sides.

This rule may be conveniently expressed by the formula

$$A = \frac{1}{2}h(B + b),$$

where A stands for the area, h for the height, B for the lower base, and b for the upper base.

The parentheses show that B and b are to be added before the sum is multiplied by $\frac{1}{2}h$. For example, if $h = 4$, $B = 7$, and $b = 5$, we have

$$\begin{aligned} A &= \tfrac{1}{2}h(B + b) \\ &= \tfrac{1}{2} \times 4 \times (7 + 5) \\ &= 2 \times 12 = 24. \end{aligned}$$

Since all ordinary rectilinear figures, that is, figures formed by straight lines, are made up of parallelograms of some kind (including squares and rectangles), triangles, or trapezoids, we have now found how to measure the area of a rectilinear figure of any shape.

Exercises. Areas

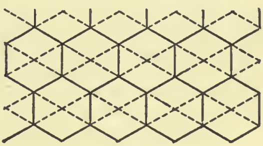
1. Find the area of a piece of ground in the form of a trapezoid, the parallel sides being $63\frac{1}{4}'$ and $37\frac{3}{4}'$ and the perpendicular distance between these parallel sides being $24'$.

2. If the area of a trapezoid is 396 sq. in. and the bases are $19''$ and $21''$ respectively, what is the height?

3. The bases of a trapezoid are $8\frac{1}{4}''$ and $6\frac{3}{8}''$ respectively and the height is $5\frac{1}{16}''$. Find the area of the trapezoid correct to the nearest $\frac{1}{4}$ sq. in.

4. Draw to any convenient scale a plane rectilinear figure with seven sides (a heptagon) and show that its area can be found by cutting the figure into smaller figures, the areas of which can be found by the formulas already given.

5. A floor is paved with six-sided tiles, as here shown. In the picture the tiles have been divided by dotted lines to suggest a method of measuring them. What measurements would you take to find the area of each tile? What other divisions of the tiles can you suggest for convenience in finding this area?

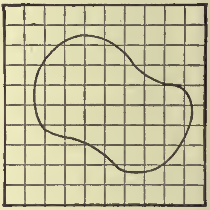


6. The sides of a rectangle are given as $3''$ and $3.8''$, where the width is exact and the length is correct to the nearest $0.1''$. The length therefore lies between $3.75''$ and $3.85''$. Draw the figure as accurately as you can for each of these lengths and show that the greatest possible error arising from taking $3.8''$ as the length is 0.15 sq. in.

7. The base of a triangle is given as $5.6''$ exactly, and the height is measured as $3.2''$ correct to the nearest $0.1''$. Between what limits does the area of the triangle lie?

8. How many sheets of tin $18''$ by $24''$ are needed to cover a roof $75'$ by $120'$, allowing 20 sheets for waste?

Estimates of Area. If a figure is drawn on squared paper, the area inclosed may be estimated approximately by counting the squares. In counting the squares cut by the outline of the figure exclude any square which does not lie at least half within the figure, count as a half square one that is practically evenly divided, and count as a full square one that lies more than half within the outline.



For example, in this figure if each square represents 1 sq. in., the area inclosed by the curve is approximately 33 sq. in., as there are approximately 33 squares inclosed.

In practical work this method is commonly used even with rectilinear figures, the areas of which can be computed by the methods already given.

Exercises. Estimates of Areas

1. The squared paper shown on page 81 is ruled with ten lines to the inch, every tenth line being heavy. The figures are drawn to the scale $\frac{1}{10}$. Find the approximate area of figure I by counting the squares.

2. Find the area of figure I by dividing it into two trapezoids and finding the area of each by the formula on page 78.

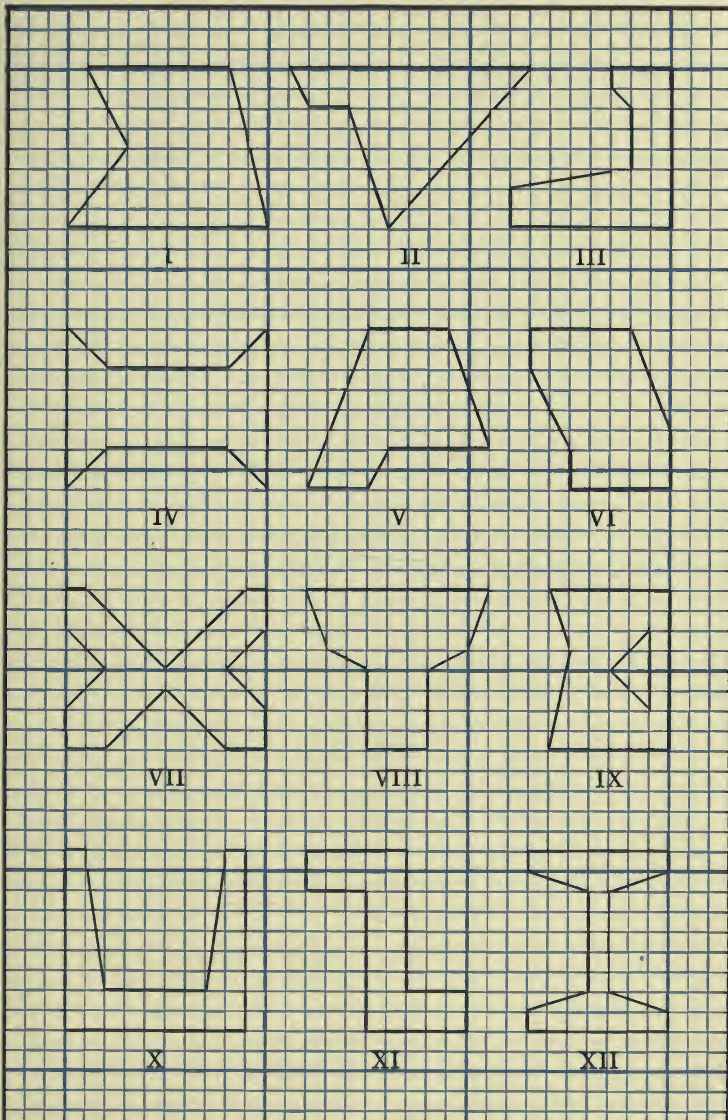
By counting the squares and by dividing the figures into parts, the areas of which can be computed by the formulas, find the area of each of the following:

3. Figure II. **5.** Figure IV. **7.** Figure VI. **9.** Figure VIII.

4. Figure III. **6.** Figure V. **8.** Figure VII. **10.** Figure XII.

11. As in Exs. 3–10, find the area of figure IX, deducting the area of the triangle from the area of the whole figure.

12. By counting squares and by dividing into parts in any convenient way, find the area of figure X; of figure XI.


I
II
III
IV
V
VI
VII
VIII
IX
X
XI
XII

Application to Working Drawings. The blueprints on page 83 represent the kind of working drawings from which the workman has to compute areas. In computing the areas in the exercises below, the student may lay out the figures on squared paper, or he may apply the formulas for mensuration.

When, in computing such areas from large blueprints, a dimension not given in the drawing is needed, the measurement should be taken with a pair of dividers and then, from the scale on the drawing or by proportion, the required dimension should be found.

Owing to the small size to which these blueprints have of necessity been reduced, such measurements would be too inaccurate, and the problems given can be solved from the dimensions in the blueprints.

Exercises. Computing Areas

1. The gusset plate on page 83 is made of steel weighing 18.77 lb. per square foot. Find the weight of the plate.

It will be desirable in this case to lay out the figure to the scale $\frac{1}{10}$ or to the scale $\frac{1}{5}$ on squared paper with ten lines to the inch.

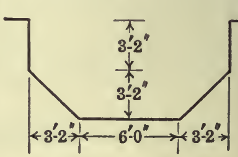
2. If a bridge builder has to chip all round the gusset plate in order to true it, what length does he chip?

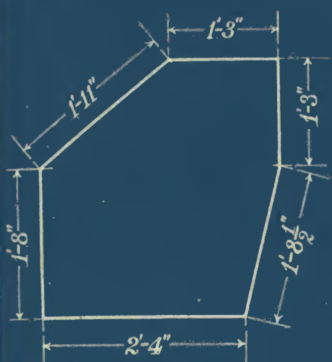
3. The angle clamp is made of brass 0.032″ thick, such brass weighing 1.367 lb. per square foot. Find to the nearest 0.01 oz. the weight of the clamp.

4. Find the area of the cross section of the metal part of the bearing support, as shown by the shaded portion.

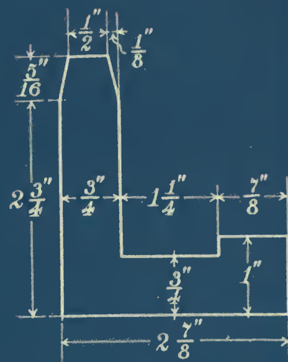
5. Find the area of the side of the desk.

6. How much floor space is added to a room if the bay window here shown is built out on one side of the room? How much floor space would be added if the dimensions of the bay window were each made $1\frac{1}{4}$ times as great?

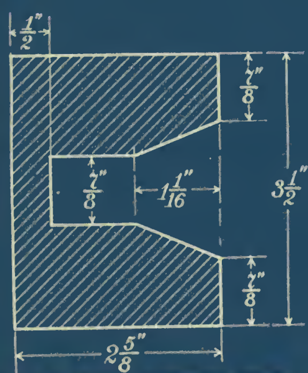




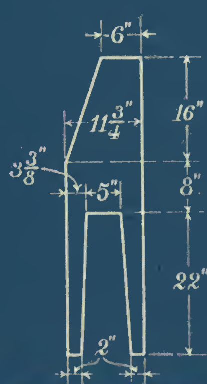
GUSSET PLATE



ANGLE CLAMP



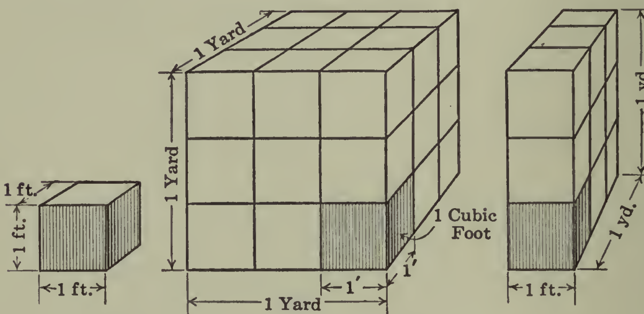
BEARING SUPPORT



SIDE of DESK

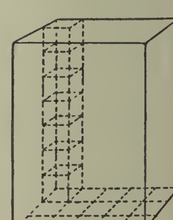
Cubic Measure. In measuring volumes the cubic inch, cubic foot, and cubic yard, with their decimal subdivisions or with the simplest common fractions, are used.

A cubic inch (cu. in.) is the volume of a cube that is one inch ($1''$) on an edge; a cubic foot (cu. ft.) is the volume of a cube that is one foot ($1'$) on an edge; and a cubic yard (cu. yd.) is the volume of a cube that is one yard (1 yd.) on an edge.



The above figures, drawn to the scale $\frac{1}{36}$, represent respectively from left to right 1 cu. ft., 1 cu. yd., and a rectangular solid 1 ft. thick, 1 yd. long, and 1 yd. high. We see that the third solid contains 9 cu. ft. and that 1 cu. yd. is made up of three such solids. Hence 1 cu. yd. contains 27 cu. ft. Similarly, it may be shown that 1 cu. ft. contains 1728 cu. in.

Volume of a Rectangular Solid. If this figure represents a rectangular solid $5''$ long, $3''$ wide, and $7''$ high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3×5 such columns. Hence the volume is $3 \times 5 \times 7$ cu. in., or 105 cu. in. Therefore



The volume of a rectangular solid is the product of the three dimensions.

This may be expressed by the formula

$$V = lwh.$$

The letters l, w, h are the initials of "length," "width," and "height."

Exercises. Volumes

Find the volumes of solids of the following dimensions:

1. $23''$, $24''$, $26''$.	**4.** $27'$, $12'$, $13\frac{1}{2}'$.	**7.** $36''$, $28\frac{1}{2}''$, $8\frac{1}{2}''$.
2. $40''$, $34''$, $25''$.	**5.** $28'$, $36'$, $14'$.	**8.** $16\frac{1}{2}''$, $8\frac{1}{2}''$, $4\frac{1}{2}''$.
3. $63''$, $48''$, $30''$.	**6.** $32''$, $23''$, $12\frac{3}{8}''$.	**9.** $5\frac{3}{4}'$, $2\frac{1}{4}'$, $1\frac{3}{8}'$.

10. A storeroom is $66'$ long, $32'$ wide, and $16\frac{1}{2}'$ high. Find the number of cubic feet of space in the room.

11. A cellar $36' \times 32' \times 6'$ is to be excavated. How much will the excavation cost at 95¢ a cubic yard?

Dimensions are often given in this form, and $36' \times 32' \times 6'$ means that the excavation is to be $36'$ long, $32'$ wide, and $6'$ deep.

12. At 92¢ a cubic yard, how much will it cost to dig a ditch $160'$ long, $3'$ wide, and $5'$ deep?

13. The box of an ordinary farm wagon is $3' \times 10'$, and the depth is usually $24''$ or $26''$. Find the volume in cubic feet and also in cubic inches for each of these depths.

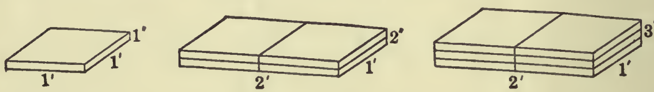
14. The interior of a certain freight car is $36'$ long, $8'\,4''$ wide, and $7'\,6''$ high. How many cubic feet does it contain? If the car is filled with grain to a depth of $4'\,9''$, what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

15. To what depth must a rectangular tank $6' \times 9'\,8''$ be filled with water to contain 160 cu. ft. of water?

16. If $2\frac{2}{5}$ cu. ft. of corn in the ear produce 1 bu. of shelled corn, how many bushels of shelled corn can be obtained from a crib $16' \times 24' \times 8'$ filled with corn in the ear?

17. A cellar $24' \times 36' \times 7'$ is to be dug for a house on a level lot $62' \times 140'$. The dirt taken from the cellar is to be used to raise the level of the whole lot. How much will the level of the lot be raised if the dirt is evenly distributed?

Measuring Lumber. A board foot (1 bd. ft.) of lumber means a piece that is 1 sq. ft. on one surface and 1″ or less in thickness. A board foot is often called simply a foot.



From left to right the figures shown above represent pieces of lumber that contain 1 bd. ft., 4 bd. ft., and 6 bd. ft. respectively.

A board 24′ long, 10″ wide, and 1″ or less thick contains $24 \times \frac{10}{12}$ bd. ft., or 20 bd. ft. A beam 16′ long, 6″ wide, and 8″ thick contains $16 \times \frac{6}{12} \times 8$ bd. ft., or 64 bd. ft. A fraction of a board foot is counted as 1 bd. ft.

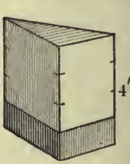
We may express the number of board feet as follows:

$$B = \frac{lwt}{12},$$

where B is the number of board feet, l the length in feet, w the width in inches, and t the thickness in inches.

In large quantities lumber is usually referred to as so many thousand feet. Thus 9 M bd. ft. means 9000 board feet.

Volume of a Prism. This figure represents a prism. On each square foot of the base there can be placed 1 cu. ft., reaching to the height of 1′ in the prism. Therefore the shaded part of the bottom of the prism will contain as many cubic feet as there are square feet in the base. Therefore the whole prism, which is 4′ high, will contain four times as much, so that if the base contains 3 sq. ft., the volume is 4×3 cu. ft., or 12 cu. ft. That is,



The volume of a prism is the product of the base and height.

This rule may be conveniently expressed by the formula

$$V = bh.$$

Exercises. Volumes

1. A box is $6\frac{1}{2}''$ long, $4\frac{1}{2}''$ wide, and $2\frac{1}{4}''$ deep. Write the formula for the volume and find the volume.

In the case of boxes and other receptacles all dimensions are inside measurements except when otherwise stated. In such problems estimate the results in advance as a check on the accuracy of computation.

2. An excavation is $69'$ long, $45'$ wide, and $6'\,4''$ deep. Write a formula for the number of loads of earth removed and then find this number.

In excavation work a load is equivalent to a cubic yard.

3. A prism has a base of $37\frac{1}{2}$ sq. in. and a height of $16''$. Find the volume.

4. A bin $16' \times 28'$ is filled with coal to a depth of $6'$. Allowing 35 cu. ft. to a ton of coal, write a formula for the number of tons of coal in the bin and then find this number.

5. A machine part made of steel weighing 490 lb. per cubic foot is cast in the form of a prism which has a length of $4'\,8''$ and a cross-section area of 26 sq. in. Find the volume and the weight of the part.

6. To floor a barn 136 planks, each $12'$ long, $1'$ wide, and $2''$ thick, were used. From the formula for board measure find the amount of lumber used.

7. The sill of a barn is $18'$ long, $8''$ wide, and $10''$ thick. Find how many feet of lumber the sill contains.

Find the number of board feet in each of the following lots:

8. 12 boards, each $16'$ long, $10''$ wide, and $1''$ thick.

9. 42 planks, each $14'$ long, $1'$ wide, and $2''$ thick.

10. 46 boards, each $16'$ long, $8''$ wide, and $\frac{7}{8}''$ thick.

A thickness of less than $1''$ is always considered as $1''$.

11. 68 joists, each $14'$ long, $8''$ wide, and $3''$ thick.

Exercises. Areas and Volumes

1. Find the area of the elevation of the angle bracket shown in the blueprint on page 89.

The term "elevation" is used extensively in connection with working drawings and building plans. Here it means the surface shown in outline in the blueprint, that is, one face of the angle bracket. In a set of plans for a house the front elevation is a drawing, not in perspective, showing the house as seen from the front; the side elevation is a similar drawing of the side; and so on.

2. If the angle bracket is made of cast steel such that a bar $\frac{7}{8}''$ square and $1'$ long weighs 2.587 lb., how much does the bracket weigh?

3. In planing the angle bracket a machinist removes $\frac{1}{8}''$ of metal from the side shown in the drawing. How many cubic inches of metal does he remove?

4. The cross section of an iron joist is shown in the blueprint. Find the area of this cross section.

The rounding of the corners should be neglected entirely.

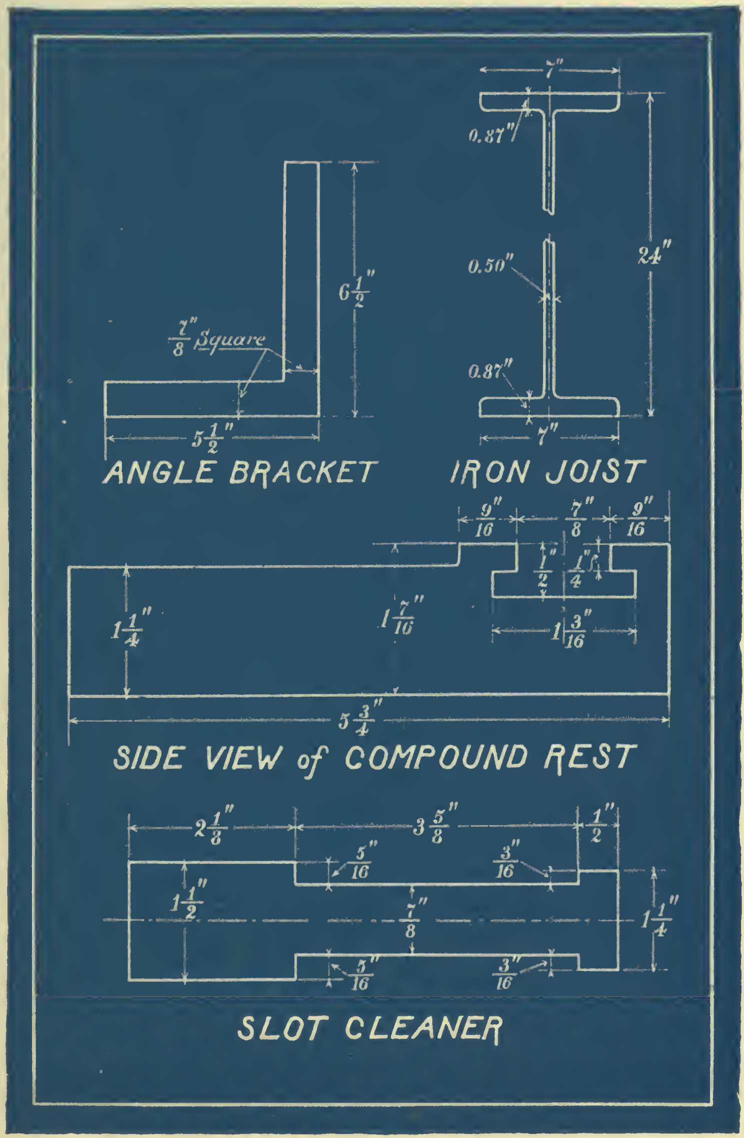
5. In Ex. 4 what would be the area of the web (the part $0.50''$ thick) if the thickness were $0.875''$ instead of $0.50''$?

6. How many square inches of metal would be removed in planing the side of the compound rest?

7. The slot cleaner shown in the blueprint is made of #16 B. & S. gage copper weighing 2.302 lb. per square foot of surface. Find the weight of the cleaner. At 38¢ per pound, what is the value of the metal in 250 cleaners?

8. What would be the area of the elevation of the angle bracket shown in the blueprint if the base were $6\frac{7}{8}''$, the height $7\frac{3}{8}''$, and the thickness $\frac{15}{16}''$?

9. In Ex. 8 what would be the number of cubic inches in the bracket if the length of the base were $14''$?



7"
0.87"
24"
0.50"
0.87"
7"
$6\frac{1}{2}$"
$\frac{7}{8}$" Square
$5\frac{1}{2}$"
ANGLE BRACKET
IRON JOIST
$\frac{9}{16}$"
$\frac{7}{8}$"
$\frac{9}{16}$"
$\frac{1}{2}$"
$\frac{1}{4}$"
$1\frac{1}{4}$"
$1\frac{7}{16}$"
$1\frac{3}{16}$"
$5\frac{3}{4}$"
SIDE VIEW of COMPOUND REST
$2\frac{1}{8}$"
$3\frac{5}{8}$"
$\frac{1}{2}$"
$\frac{5}{16}$"
$\frac{3}{16}$"
$1\frac{1}{2}$"
$\frac{7}{8}$"
$1\frac{1}{4}$"
$\frac{5}{16}$"
$\frac{3}{16}$"
SLOT CLEANER

Exercises. Volumes

1. How many cubic feet of lumber are there in 15 planks, each $14' \, 6''$ long, $6''$ wide, and $2''$ thick? How many board feet are there?

2. If a bar of iron $1''$ square and $1'$ long weighs 3.333 lb., what is the weight of a bar $2'' \times \frac{1}{4}'' \times 1'$?

3. Using the data given in Ex. 2, find the weight of a bar of iron if the dimensions are $2\frac{1}{8}'' \times \frac{7}{8}'' \times 3'$.

4. If cast iron weighs 0.260 lb. per cubic inch, what is the weight of a bar of cast iron $12' \, 4'' \times 6\frac{1}{2}'' \times 2''$?

5. Find the number of cubic inches of metal in the angle plate shown on page 91. If the angle plate is made of cast iron weighing 450 lb. per cubic foot, what is the cost of 150 plates of this type at $7\frac{1}{2}$¢ per pound?

6. If the angle plates in Ex. 5 are finished all over by machining off $\frac{3}{64}''$, that is, by removing $\frac{3}{64}''$ of metal from each surface, what is then the weight of 150 plates?

7. The step block is made of steel weighing 490 lb./cu. ft. Find the weight of the block.

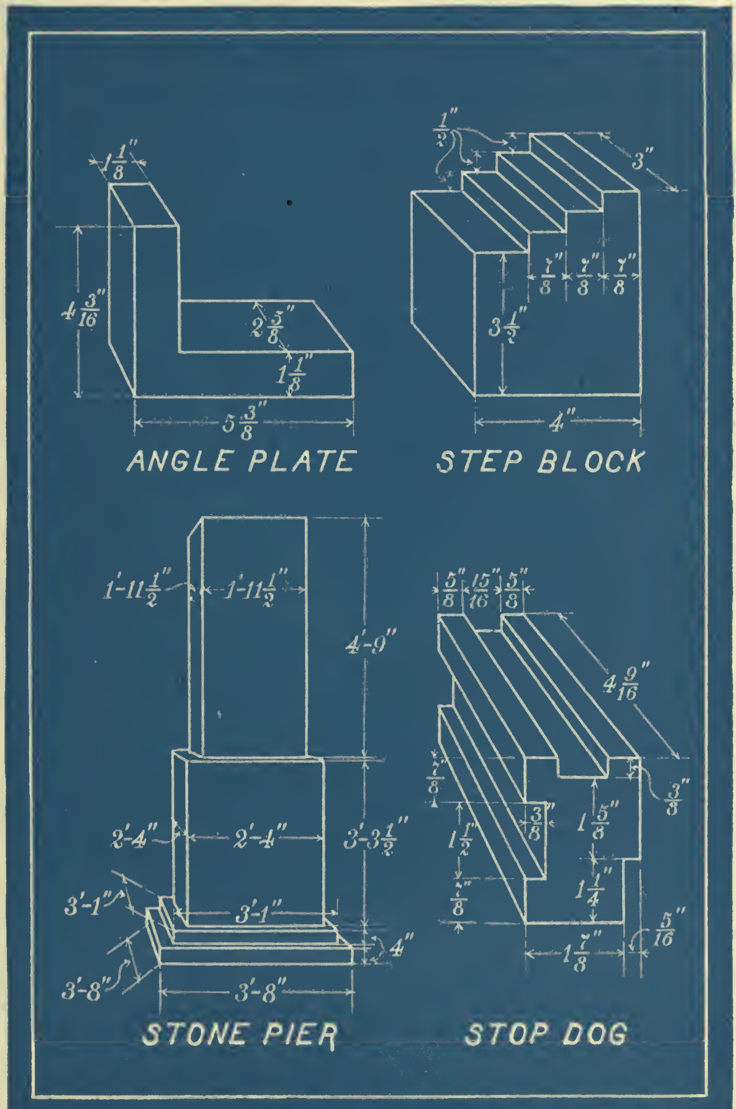
The expression "490 lb./cu. ft." means "490 lb. per cubic foot," and in general the symbol / is read "per" in all cases of this kind.

8. At $1.75 per perch of $24\frac{3}{4}$ cu. ft., find the cost of the stone for the pier shown in the blueprint.

The perch used in stone work is generally 1 rd. ($16\frac{1}{2}'$) long, $1'$ wide, and $1\frac{1}{2}'$ thick, but varies locally, being often taken as 25 cu. ft.

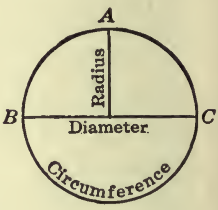
9. Allowing 575 bricks per cubic yard, and 5% above this for waste and breakage, how many bricks would be required if the pier in Ex. 8 were to be made of brick?

10. Taking the weight of copper as 542 lb./cu. ft., and the cost as 38¢ a pound, find the cost of 125 stop dogs of the type shown in the blueprint.


ANGLE PLATE
STEP BLOCK
STONE PIER
STOP DOG

Circumference of a Circle. If we measure the diameter and the circumference of a circle, and then divide the circumference by the diameter, we shall find that

$$\frac{\text{circumference}}{\text{diameter}} = 3\tfrac{1}{7}, \text{ approximately.}$$



The number 3.1416 is a closer approximation, and 3.14159 is still closer, but for practical work $3\frac{1}{7}$, $\frac{22}{7}$, or 3.14 is used unless a higher degree of accuracy is necessary.

A special name is given to the ratio $3\frac{1}{7}$; it is called pī (written π, a Greek letter). That is, $c : d = \pi$, or

$$c = \pi d.$$

If we divide these two equal expressions by π, we have

$$d = \frac{c}{\pi}.$$

Since the diameter of a circle is twice the radius, that is, since $d = 2r$, we have

$$c = 2\pi r.$$

If we divide these two equal expressions by 2π, we have

$$r = \frac{c}{2\pi}.$$

In this book use $\frac{22}{7}$, or 3.14, for π, and $\frac{7}{22}$, or 0.32, for $\frac{1}{\pi}$, in all cases, unless otherwise directed.

Illustrative Problems. **1.** Find the circumference of a fly-wheel that is 28″ in diameter.

$$c = \pi d = \tfrac{22}{7} \times 28'' = 88''.$$

2. What radius should be used in drawing a circle for the pattern of a pulley that is to be 48.4″ in circumference?

$$r = \frac{c}{2\pi} = \frac{1}{\cancel{2}} \times \frac{7}{\cancel{22}} \times \overset{\overset{1.1}{\cancel{2.2}}}{\cancel{48.4}}'' = 7.7''.$$

Exercises. Circles

1. A pulley has a diameter of 8″. Find the circumference to the nearest 0.1″.

2. An automobile wheel has a diameter of 36″. Find the radius and the circumference.

3. A locomotive has a driving wheel 6′ 10″ in diameter. Not allowing for slipping, how far will the locomotive travel during one revolution of the driving wheel?

4. In Ex. 3 how many R.P.M. does the driving wheel make when the train is traveling 45 mi./hr.?

5. When a 14-inch steel casting is being turned in a lathe, how long a chip is cut off at each turn of the casting?

Consider that the casting is a cylinder with a diameter of 14″ and that the length of the chip is equal to the circumference.

6. Some *BX* cable is coiled up in 18 turns, the average diameter of the coil being 2′ 4″. Find the length of the cable.

7. The flywheel of an engine is 14″ in diameter and runs at a speed of 150 R.P.M. Find the rim speed, that is, the speed of a point on the rim, in F.P.M. (feet per minute).

8. An automobile tire is 34″ in diameter. At what speed should the wheel revolve in order that the car may have a speed of 18 mi./hr.?

9. A boiler is 6′ 8″ in diameter. How many rivets will there be in a circumferential seam if the rivet holes are spaced approximately 3″ apart?

10. The circumference of a boiler is 14′. Find the diameter to the nearest $\frac{1}{8}$″.

11. In making a drawing of a wheel that is to have a circumference of 110″, the scale of the drawing being $\frac{1}{10}$, what length of radius should the draftsman use?

12. Find the circumference of the pulley shown at the top of the blueprint on page 95.

13. If the diameter of the pulley were half as large as shown, what would be the circumference?

14. If the circumference of the pulley were $1' \, 10''$, what would be the diameter?

15. If a pattern maker were drawing the circle for a pulley $2' \, 9''$ in circumference, what radius should he use?

16. The circular saw shown in the blueprint has 35 teeth. Using 3.142 as the value of π, find to the nearest $0.001''$ the distance between the points of adjacent teeth.

17. If in Ex. 16 the distance between the points of adjacent teeth were $1.571''$, how many teeth would there be? Use 3.1416 as the value of π.

18. How many revolutions will the automobile wheel make while the car is going 1.5 mi.? 3.9 mi.? 11.2 mi.?

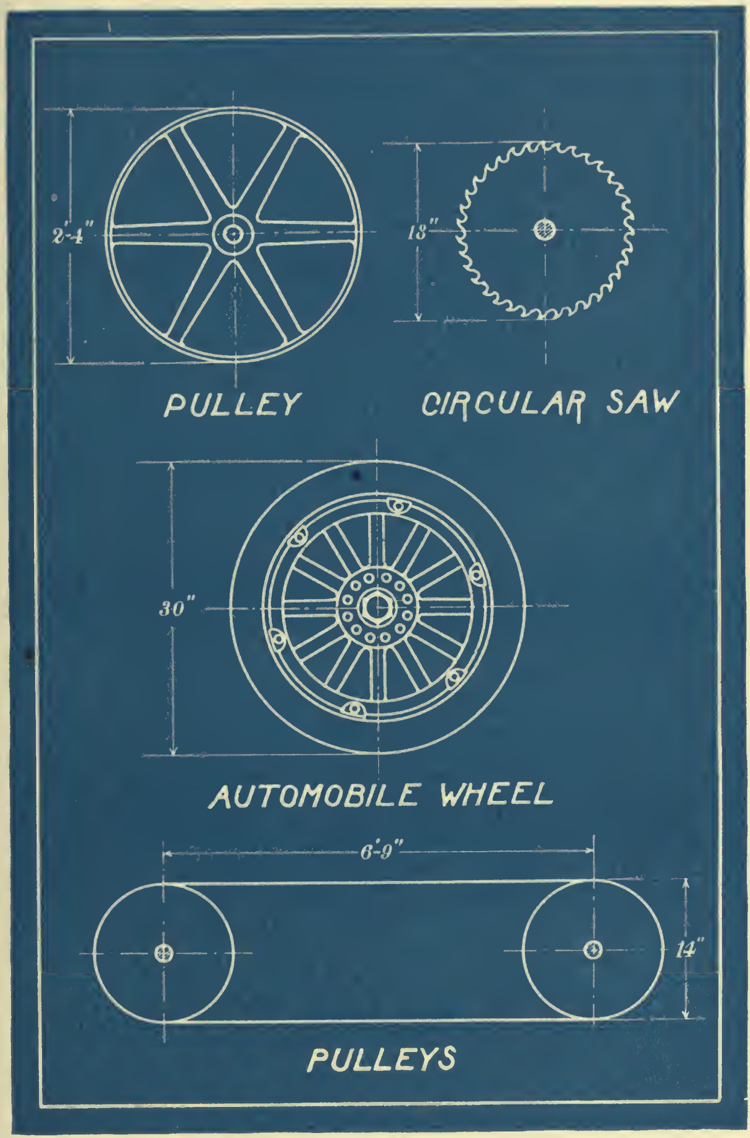
19. In Ex. 18 how many revolutions would there be in each case if the diameter of the wheel were $36''$? With which size of wheel would there be the less wear on the tire? Write the reason for your answer.

20. Find the number of R.P.M. made by a wheel $34''$ in diameter on a car that is going at the rate of 25 mi./hr.

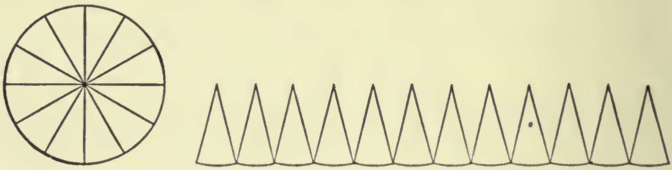
21. On a certain locomotive the drive wheels are $6'$ in diameter and the truck wheels are $2'$ in diameter. Find the number of R.P.M. of each type of wheel when the locomotive is going at the rate of 40 mi./hr.

22. In the two pulleys connected by belting, as shown in the blueprint, what is the length of the belting?

23. In Ex. 22 what would be the length of the belting if the diameter of each pulley were twice as great?


2'-4"
PULLEY
13"
CIRCULAR SAW
30"
AUTOMOBILE WHEEL
6'-9"
14"
PULLEYS

Area of a Circle. A circle may be separated into figures which are nearly triangles, the height of each triangle being the radius, and the sum of the bases being the circumference. If these figures were exact triangles, the area of the circle



would be $\frac{1}{2}\times$ height $\times$ sum of bases; that is, the area would be $\frac{1}{2}\times$ radius $\times$ circumference. It is proved in geometry that this is the true area of a circle.

We may now express this as a formula, thus:

$$A=\frac{1}{2}rc.$$

Since $c=2\pi r$, $\quad A=\frac{1}{2}r\times 2\pi r,$

or $\quad A=\pi r^2.$

Since $\frac{d}{2}=r$, the formula $A=\pi r^2$ may be written

$$A=\frac{\pi d^2}{4}.$$

Illustrative Problems. **1.** Find the area of a circle which is drawn with a radius of 5″.

$$A=\pi r^2=3.14\times 5\times 5=78.5.$$

Hence the area of the circle is 78.5 sq. in.

2. Find the cross-section area of a shaft 14″ in diameter.

$$A=\frac{\pi d^2}{4}=\frac{22\times\overset{2}{\cancel{14}}\times\overset{7}{\cancel{14}}}{\cancel{7}\times\underset{2}{\cancel{4}}}=154.$$

Hence the cross-section area of the shaft is 154 sq. in.

Exercises. Circles

Find the areas of circles, given the radii as follows:

1. 7″. **2.** 14″. **3.** 1.4″. **4.** 4.9″. **5.** 56″.

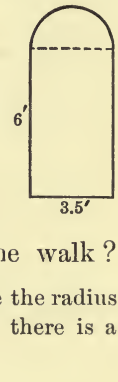
6. If the radius of one circle is twice as long as the radius of another circle, how do the circumferences of the circles compare? How do the areas compare?

7. If one circle has a radius three times as long as the radius of another circle, how do the circumferences compare? How do the areas compare?

8. A circular mirror is 2′ 3″ in diameter. Find the cost of resilvering the mirror at 68¢ a square foot.

9. What is the area of the cross section of a water pipe that is 6″ in diameter?

10. Find the entire area of a window the lower part of which is a rectangle and the upper part a semicircle, or half circle, as shown in the figure.



11. In a park there is a circular lake 240′ in diameter. Find the number of square yards in a walk 6′ wide around the lake. If the width of the walk is doubled, what is then the area of the walk?

This is a case of finding the area of a ring. If we let R be the radius of the outer circle and r be the radius of the inner circle, there is a convenient formula for the area, which is

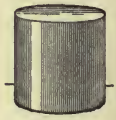
$$A = \pi(R + r)(R - r).$$

The parentheses indicate that we must first add R and r, then subtract r from R, and then multiply the product of these two results by π.

12. A tree the cross section of which may be assumed to be a circle has a circumference of 12′ 3″ at a certain height. What is the area of the top of the stump that is formed by sawing horizontally through the tree at this point?

Measure of a Cylinder. We frequently need to find the area of the curve surface and also the volume of a cylinder.

The area of the curve surface of a cylinder is the product of π times the diameter and the height.



Since π times the diameter is the circumference, if we think of the curve surface of the cylinder as being unrolled, we see that the product of the circumference and the height is the area of the curve surface.

This rule is too long for practical use, and so we write it as a formula, thus:

$$S = \pi dh.$$

For example, suppose that we wish to find the area of the curve surface of a pipe 15′ long and 3″ in diameter.

Expressing the diameter 3″ as 0.25′, we have

$$S = \pi dh = 3.14 \times 0.25 \times 15 = 11.7750.$$

Hence the area of the curve surface is approximately 11.78 sq. ft.

The volume of a cylinder is the product of the area of the base and the height.

We write this rule as a formula, thus:

$$V = bh.$$

Since the area of the base is πr^2 or $\frac{1}{4}\pi d^2$, we may write the formula for V in two different forms, as follows:

$$V = \pi r^2 h,$$
$$V = \frac{1}{4}\pi d^2 h.$$

For example, suppose that we wish to find the volume of a steel shaft 3′ 6″ long and 4″ in diameter.

Expressing the length 3′ 6″ as 42″, we have

$$V = \frac{1}{4}\pi d^2 h = \frac{1}{\cancel{4}} \times \frac{22}{\cancel{7}} \times \cancel{4} \times 4 \times \overset{6}{\cancel{42}} = 528.$$

Hence the volume of the shaft is 528 cu. in.

Exercises. Cylinders

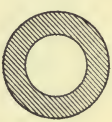
1. A water tank is 20′ in diameter and 16′ high, inside measurements. Find the area of the interior curve surface.

2. Find the capacity of the tank in Ex. 1.

3. A piece of water pipe is 16′ long and the internal diameter is 4″. Find the number of cubic inches of water required to fill the pipe.

4. A cylindric iron pillar supporting a ceiling is 14′ high and has a diameter of 5″. If the pillar is solid and weighs 441 lb./cu. ft., what is its weight?

5. The cross section of a hollow cylindric iron pillar is as here shown. The external diameter is 6″, the internal diameter 4″, and the length 12′. If the grade of iron used weighs 441 lb./cu. ft., what is the weight of the pillar?



The formula for the area of a ring was given on page 97. From this formula it is evident that if we are dealing with a hollow cylinder the formula for the volume is

$$V = \pi(R + r)(R - r)h,$$

where R is the external radius, r the internal radius, and h the height, or length, of the cylinder.

6. In finding the strength of an iron rod the cross-section area is required. Find this area for a rod $3\frac{1}{2}''$ in diameter.

7. A tinsmith cuts out 100 circular pieces of tin for the bottoms of cups, using a radius of 2″. How many square inches of tin are there in all the pieces?

8. The number of pounds which can be supported at the center of the distance between the supports of a cylindric cast-iron shaft l feet long and d inches in diameter is $500\ d^3/l$. Find the weight which can be thus supported by a shaft which is 3″ in diameter and has a length of 16′ between the supports.

9. The blind collar shown on page 101 is made of cast iron weighing 450 lb./cu. ft. Find the weight of the collar, the hole running only part of the length, as shown.

10. In Ex. 9 what would be the weight of the collar if the hole ran all the way through it?

11. Before the bronze bushing was turned to its present dimensions there was $\frac{1}{8}''$ left on the entire outside surface, including the ends, for finishing. An automobile factory uses 2500 of these bushings per day. If the scrap is worth 35¢ per pound and bronze weighs 529 lb./cu. ft., how much is the value of the scrap for a day?

12. The jig bushing shown in the blueprint is turned and drilled from $1\frac{15}{16}$-inch round stock. How many pounds of steel weighing 490 lb./cu. ft. are required in manufacturing 75 bushings of this type?

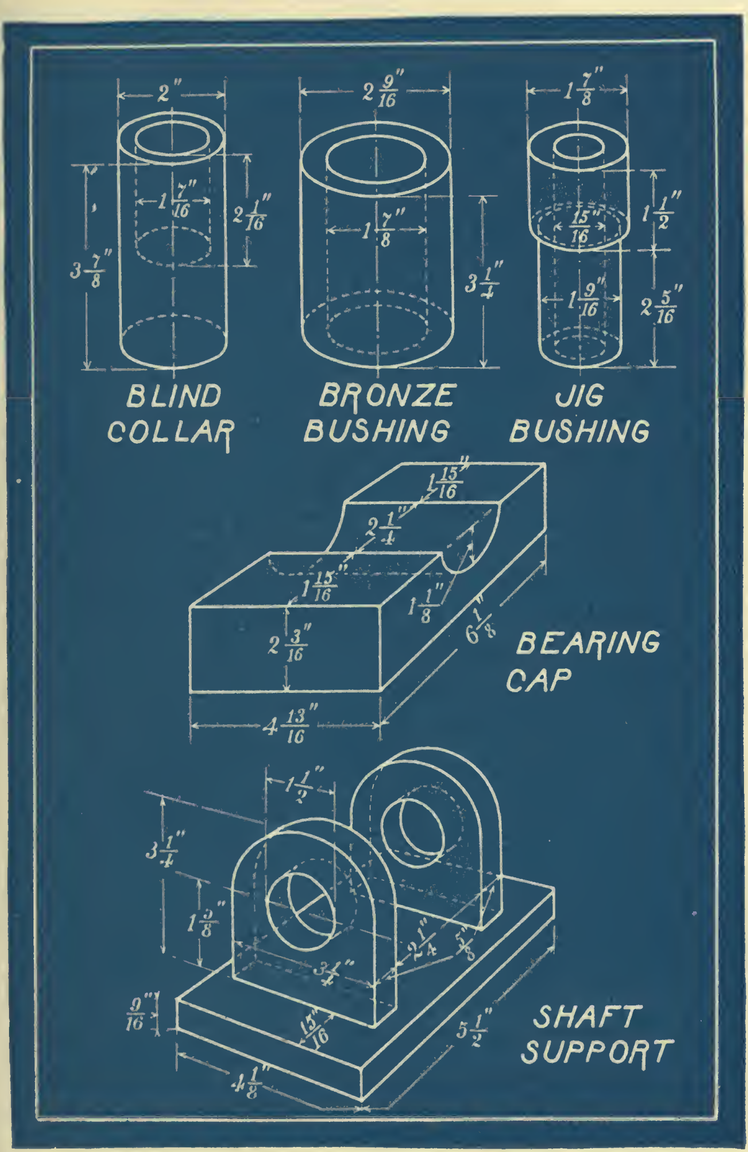
13. Find the weight of the steel which is wasted in turning and drilling the lot mentioned in Ex. 12.

14. The pattern for the bearing cap is made of mahogany weighing 53 lb./cu. ft. If the bearing cap is made of cast iron weighing 450 lb./cu. ft., the cap is how many pounds heavier than the pattern?

The slight shrinkage in casting may be neglected.

15. A factory orders 125 cast-iron shaft supports and 25 bronze supports of the pattern shown in the blueprint. The cost of the cast-iron supports is 8¢ a pound, and that of the bronze supports is 38¢ a pound. Taking the weights of the metals as given in Exs. 9 and 11, find the entire cost.

16. How many cubic inches of metal are there in the bronze bushing shown in the blueprint? How many would there be if all the dimensions were doubled? if they were multiplied by three? if they were half the size given?


BLIND
COLLAR
BRONZE
BUSHING
JIG
BUSHING
BEARING
CAP
SHAFT
SUPPORT

Meaning of Roots. In engineering and in various lines of industry it is often necessary to use the roots of numbers, particularly the square root.

If a number is the product of two equal factors, either factor is called the *square root* of the number. For example, because $25 = 5 \times 5$, we say that 5 is the square root of 25, and we indicate this by the symbol $\sqrt{\ }$. Thus $5 = \sqrt{25}$.

According to this definition 3 would have no square root, because 3 has not two equal factors. We therefore extend the idea of root to include approximate factors, and say that $\sqrt{3} = 1.732$ to four *significant figures* or to three decimal places, because $1.732^2 = 3$, approximately.

Approximate roots being allowed as in the case of the square root, if a number is the product of three equal factors, any one of these factors is called the *cube root* of the number; if of four equal factors, the *fourth root*; and so on. Thus the cube root of 8, written $\sqrt[3]{8}$, is 2, the *index of the root* being 3; and $\sqrt[4]{81}$ is 3, the index of the root being 4.

In practice all roots are found by means of tables, and hence we shall give but little attention to the older methods. We shall pay some attention to finding square roots because they are the ones that the student will most often need.

Square of a Sum. If we wish to find the square of 25 we may, if desired, multiply $20 + 5$ by itself, as here shown.

	$20 + 5$
	$20 + 5$
Product by 5	$20 \times 5 + 5^2$
Product by 20	$20^2 + 20 \times 5$
Total product	$20^2 + 2 \times 20 \times 5 + 5^2$

Thus, if we take 20 as the first part and 5 as the second part of 25, the square of 25 consists of the following three parts, as shown above: (1) the square of 20, (2) twice the product of 20 and 5, and (3) the square of 5.

Square Root. In the preceding work, if we put t for the tens and u for the units, we see that

$$(t+u)^2 = t^2 + 2\,tu + u^2.$$

The square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, the square root of any number between 1 and 100 lies between 1 and 10, and the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, the integral part of the square root of any whole number of *one* or *two* figures is a number of *one* figure; that of any whole number of *three* or *four* figures is a number of *two* figures; and so on.

If, therefore, a whole number is separated into periods of two figures each, the number of figures in the integral part of the square root is equal to the number of periods. Since we mark off the periods from right to left, the period at the left may have one or two figures; for example, 22 09 and 7 89 04.

Illustrative Problem. Find the square root of 3481.

Separate the figures of the number into periods of two figures each, beginning at the right.

The first period contains the square of the tens' number of the root. Since the greatest square in 34 is 25, then 5, the square root of 25, is the tens' figure of the root.

$$\begin{array}{r|l} & 34\ 81\,(59 \\ & 25 \\ \hline 100 & 9\ 81 \\ 109 & 9\ 81 \\ \hline \end{array}$$

Subtracting the square of the tens, the remainder contains twice the tens times the units, plus the square of the units. Dividing by twice the tens (that is, by 100, which is 2×5 tens), we find, approximately, the units' figure. Dividing 981 by 100, we have 9 as the units' figure.

Since twice the tens times the units, plus the square of the units, is equal to (twice the tens plus the units) times the units, that is, since $2 \times 50 \times 9 + 9^2 = (2 \times 50 + 9) \times 9$, we add 9 to 100 and multiply the sum by 9. The product is 981, and there is no remainder. Hence the square root is 59. Checking the work, $59^2 = 3481$.

Square Root with Decimals. In finding the square root of a number which contains a decimal point we proceed in the same way as before, except that we separate the number into periods of two figures each, beginning at the decimal point.

For example, find the square root of 151.29.

Separate the figures into periods, beginning at the decimal point.

The greatest square of the tens in 151.29 is 100, and the square root of 100 is 10.

Then 51.29 contains 2×10 times the units' number of the root, plus the square of the units' number.

Dividing 51 by 2×10, or 20, we find that the next figure of the root is 2.

We have now found 12, the square being $100 + 40 + 4$, or 144.

Then 7.29 contains 2×12 times the tenths' number of the root, plus the square of the tenths' number, because we have subtracted 144, which is the square of 12.

$$
\begin{array}{r|r}
 & 1\ 51.29\,(12.3 \\
 & 1 \\
\hline
20 & 51 \\
22 & 44 \\
\hline
240 & 7\ 29 \\
243 & 7\ 29 \\
\hline
\end{array}
$$

Dividing by 240, we find that the tenths' figure of the root is 3.

There is no remainder, and hence the square root of 151.29 is 12.3.

If the number is not a perfect square we may annex pairs of zeros at the right and find the root to as many decimal places as we choose.

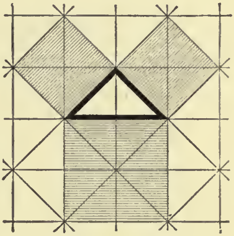
Exercises. Square Root

Find the square root of each of the following:

1. 625.
2. 6.25.
3. 0.0625.
4. 324.
5. 3.24.
6. 0.0324.
7. 484.
8. 0.0484.
9. 729.
10. 7.29.
11. 0.0729.
12. 2025.
13. 20.25.
14. 0.2025.
15. 0.1296.
16. 0.1089.
17. 1225.
18. 12.25.
19. 0.1225.
20. 4096.
21. 40.96.
22. 4.096.
23. 14,641.
24. 146.41.

Square on the Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.

If a floor is paved with triangular tiles as in this figure, it is easy to mark out a right triangle, as shown by the heavy lines. It is seen that the square on the hypotenuse contains eight small triangles, while each square on a side contains four such triangles. Hence

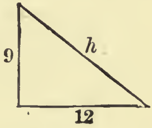


The square on the hypotenuse is the sum of the squares on the other two sides.

Letting the hypotenuse be h and the other sides be a and b, we have the following formulas:

$$h^2 = a^2 + b^2, \quad h = \sqrt{a^2 + b^2}, \quad a = \sqrt{h^2 - b^2}, \quad b = \sqrt{h^2 - a^2}.$$

Illustrative Problems. **1.** Given that $a = 9''$ and $b = 12''$ in a right triangle, find the value of h.



$$h = \sqrt{a^2 + b^2} = \sqrt{9^2 + 12^2} = \sqrt{225} = 15.$$

Hence the length of the hypotenuse is $15''$.

2. In a right triangle it is given that $h = 20'$ and $b = 12'$. Find the value of a.

$$a = \sqrt{h^2 - b^2} = \sqrt{20^2 - 12^2} = \sqrt{256} = 16.$$

Hence the length of side a is $16'$.

Exercises. Square Root

Find the sides of squares that have the following areas:

1. 3136 sq. ft.	**3.** 6561 sq. ft.	**5.** 8281 sq. yd.
2. 3844 sq. ft.	**4.** 6889 sq. ft.	**6.** 9409 sq. yd.

7. If the hypotenuse of a right triangle is $60''$ and one side is $36''$, what is the length of the other side?

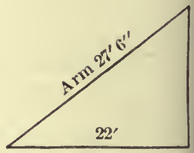
8. How long is the diagonal of a floor 48′ by 64′?

In those examples on this page which involve numbers that are not perfect squares find each square root to the nearest hundredth.

9. Find the length of the diagonal of a square that contains 13 sq. ft.

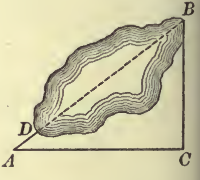
10. Find the length, between the points where it is attached, of a wire drawn taut from the top of a 70-foot flagpole to a stake set in the ground 30′ from the foot of the pole.

11. A wire is to be fastened to a telegraph pole at a point 20′ above the ground and is to be stretched taut to a stake in the ground 15′ 6″ from the foot, so as to hold the pole perpendicular. If 5′ is allowed for fastening the wire at each end, find the length of wire required.



12. The arm of a derrick for hoisting coal is 27′ 6″ long and swings over an opening 22′ from the base of the derrick. How far is the top of the arm above the opening?

13. The foot of a 48-foot ladder is 16′ from the wall of a building against which the top rests. How high does the ladder reach on the wall?



14. To find the length of this pond a group of students laid off the right triangle *ACB*, as shown. They found by measuring that $AC = 428'$, $BC = 321'$, and $AD = 75'$. Calculate the length of DB.

15. How far from the wall of a house must the foot of a 36-foot ladder be placed so that the top of the ladder may touch a window sill 32′ above the ground?

16. A square building lot has an area of 17,500 sq. ft. Find the perimeter of the lot to the nearest 0.1′. Find the perimeter of a lot with four times this area.

Table of Square Roots. People can no longer afford the time to find square roots by the method on pages 103 and 104. They use such a table as the one on pages 108–111.

In that table the first two figures of the number whose root we seek are given at the left in the column marked **N**, and the third figure is given at the top. For example, on page 108 the square root of 1.00 is 1.000, this being opposite the number 1.0 and under 0; the square root of 1.01 is 1.005, this being opposite the number 1.0 and under 1; the square root of 1.02 is 1.010; and so on.

Furthermore, since $\sqrt{100}=10$, multiplying a number by 100 multiplies the square root by 10. Thus:

$\sqrt{1.76}=1.327.$ $\sqrt{8.38}=2.895.$

$\sqrt{176}=13.27.$ $\sqrt{838}=28.95.$

$\sqrt{17600}=132.7.$ $\sqrt{83800}=289.5.$

On each page of the table the right-hand columns headed 1, 2, 3, ..., 9 show the numbers to be added to the root when the number whose square root is desired contains a fourth figure. For example, on page 108 we find that $\sqrt{2.68}=1.637$; but if we wish to find $\sqrt{2.687}$ we look along the line from 2.6 and find 2 in the right-hand column under 7. This means that we should add 0.002 to 1.637, giving $\sqrt{2.687}=1.639$. Consider also the following:

$$\sqrt{934.9}=30.56+0.01=30.57.$$

$$\sqrt{81.55}=9.028+0.003=9.031.$$

$$\sqrt{8155}=90.28+0.03=90.31.$$

The last two roots above are found on page 111.

Thus, the table gives the square roots of all numbers from 1 to 9999. The table on page 112 gives on one page, for easy reference, the squares, cubes, square roots, and cube roots of all whole numbers from 1 to 100.

SQUARE ROOTS OF NUMBERS FROM 1 TO 9.999

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	0	1	1	2	2	3	3	4	4
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0	1	1	2	2	3	3	4	4
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136	0	1	1	2	2	3	3	4	4
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179	0	1	1	2	2	3	3	3	4
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0	1	1	2	2	2	3	3	4
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	0	1	1	2	2	2	3	3	4
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	0	1	1	2	2	2	3	3	4
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338	0	1	1	2	2	2	3	3	3
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375	0	1	1	1	2	2	3	3	3
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0	1	1	1	2	2	3	3	3
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	0	1	1	1	2	2	2	3	3
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	0	1	1	1	2	2	2	3	3
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0	1	1	1	2	2	2	3	3
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0	1	1	1	2	2	2	3	3
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	0	1	1	1	2	2	2	3	3
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0	1	1	1	2	2	2	3	3
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	0	1	1	1	2	2	2	2	3
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	0	1	1	1	2	2	2	2	3
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	0	1	1	1	1	2	2	2	3
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	0	1	1	1	1	2	2	2	3
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	0	1	1	1	1	2	2	2	3
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786	0	1	1	1	1	2	2	2	3
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0	1	1	1	1	2	2	2	2
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841	0	1	1	1	1	2	2	2	2
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868	0	1	1	1	1	2	2	2	2
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895	0	1	1	1	1	2	2	2	2
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921	0	1	1	1	1	2	2	2	2
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	0	1	1	1	1	2	2	2	2
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	0	1	1	1	1	2	2	2	2
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997	0	1	1	1	1	2	2	2	2
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022	0	0	1	1	1	1	2	2	2
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	0	0	1	1	1	1	2	2	2
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0	0	1	1	1	1	2	2	2
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095	0	0	1	1	1	1	2	2	2
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	0	0	1	1	1	1	2	2	2
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	0	0	1	1	1	1	2	2	2
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0	0	1	1	1	1	2	2	2
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	0	0	1	1	1	1	2	2	2
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	0	0	1	1	1	1	2	2	2
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	0	0	1	1	1	1	2	2	2
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	0	0	1	1	1	1	2	2	2
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	0	0	1	1	1	1	2	2	2
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300	0	0	1	1	1	1	2	2	2
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322	0	0	1	1	1	1	2	2	2
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	0	0	1	1	1	1	1	2	2

SQUARE ROOTS OF NUMBERS FROM 1 TO 9.999

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0	0	1	1	1	1	1	2	2
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	0	0	1	1	1	1	1	2	2
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0	0	1	1	1	1	1	2	2
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	0	0	1	1	1	1	1	2	2
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	0	0	1	1	1	1	1	2	2
6.0	2.449	2.452	2.454	2.056	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1	1	1	2	2
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1	1	1	2	2
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1	1	1	2	2
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1	1	1	2	2
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1	1	1	2	2
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	0	0	1	1	1	1	1	2	2
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	0	0	1	1	1	1	1	2	2
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1	1	1	2	2
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	0	0	1	1	1	1	1	2	2
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1	1	1	2	2
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	0	0	1	1	1	1	1	2	2
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1	1	1	1	2
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1	1	1	1	2
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718	0	0	1	1	1	1	1	1	2
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	0	0	1	1	1	1	1	1	2
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	0	0	1	1	1	1	1	1	2
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1	1	1	1	2
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1	1	1	1	2
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	0	0	1	1	1	1	1	1	2
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	0	0	1	1	1	1	1	1	2
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1	1	1	1	2
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	0	0	1	1	1	1	1	1	2
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1	1	1	1	2
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	0	0	1	1	1	1	1	1	2
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1	1	1	1	2
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1	1	1	1	2
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1	1	1	1	2
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	0	0	1	1	1	1	1	1	2
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	0	0	1	1	1	1	1	1	2
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	0	0	1	1	1	1	1	1	2
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	0	0	0	1	1	1	1	1	1
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	0	0	0	1	1	1	1	1	1
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	0	0	0	1	1	1	1	1	1
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1	1	1	1	1
9.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	0	0	0	1	1	1	1	1	1
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	0	0	0	1	1	1	1	1	1
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1	1	1	1	1
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	0	0	0	1	1	1	1	1	1
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145	0	0	0	1	1	1	1	1	1
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1	1	1	1	1

SQUARE ROOTS OF NUMBERS FROM 10 TO 99.99

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8	9	11	12	14
11	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450	1	3	4	6	7	9	10	12	13
12	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	1	3	4	6	7	8	10	11	13
13	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	1	3	4	5	7	8	10	11	12
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7	8	9	11	12
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	1	3	4	5	6	8	9	10	11
16	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6	7	9	10	11
17	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	1	2	4	5	6	7	8	10	11
18	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	1	2	3	5	6	7	8	9	10
19	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	1	2	3	5	6	7	8	9	10
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	1	2	3	4	6	7	8	9	10
21	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5	6	8	9	10
22	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	1	2	3	4	5	6	7	8	9
23	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5	6	7	8	9
24	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	1	2	3	4	5	6	7	8	9
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	1	2	3	4	5	6	7	8	9
26	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5	6	7	8	9
27	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5	6	7	8	9
28	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5	6	7	7	8
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	1	2	3	4	5	5	6	7	8
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	1	2	3	4	4	5	6	7	8
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	3	4	5	6	7	8
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	3	4	5	6	7	8
33	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	1	2	3	3	4	5	6	7	8
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4	5	6	7	8
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	1	2	2	3	4	5	6	7	8
36	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	1	2	2	3	4	5	6	7	7
37	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4	5	6	7	7
38	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237	1	2	2	3	4	5	6	6	7
39	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	1	2	2	3	4	5	6	6	7
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	1	2	2	3	4	5	6	6	7
41	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	1	2	2	3	4	5	5	6	7
42	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	1	2	2	3	4	5	5	6	7
43	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	1	2	2	3	4	5	5	6	7
44	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	2	2	3	4	5	5	6	7
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	1	1	2	3	4	4	5	6	7
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4	4	5	6	7
47	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4	4	5	6	7
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4	4	5	6	6
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	1	1	2	3	4	4	5	6	6
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4	4	5	6	6
51	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	4	4	5	6	6
52	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3	4	5	6	6
53	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342	1	1	2	3	3	4	5	5	6
54	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3	4	5	5	6

SQUARE ROOTS OF NUMBERS FROM 10 TO 99.99

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477	1	1	2	3	3	4	5	5	6
56	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	1	1	2	3	3	4	5	5	6
57	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	1	1	2	3	3	4	5	5	6
58	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675	1	1	2	3	3	4	5	5	6
59	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	1	1	2	3	3	4	4	5	6
60	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804	1	1	2	3	3	4	4	5	6
61	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868	1	1	2	3	3	4	4	5	6
62	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931	1	1	2	3	3	4	4	5	6
63	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994	1	1	2	3	3	4	4	5	6
64	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056	1	1	2	2	3	4	4	5	6
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118	1	1	2	2	3	4	4	5	5
66	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179	1	1	2	2	3	4	4	5	5
67	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	1	1	2	2	3	4	4	5	5
68	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301	1	1	2	2	3	4	4	5	5
69	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361	1	1	2	2	3	4	4	5	5
70	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420	1	1	2	2	3	4	4	5	5
71	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479	1	1	2	2	3	4	4	5	5
72	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538	1	1	2	2	3	3	4	5	5
73	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597	1	1	2	2	3	3	4	5	5
74	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	1	1	2	2	3	3	4	5	5
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712	1	1	2	2	3	3	4	5	5
76	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769	1	1	2	2	3	3	4	5	5
77	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826	1	1	2	2	3	3	4	4	5
78	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	1	1	2	2	3	3	4	4	5
79	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939	1	1	2	2	3	3	4	4	5
80	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994	1	1	2	2	3	3	4	4	5
81	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050	1	1	2	2	3	3	4	4	5
82	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105	1	1	2	2	3	3	4	4	5
83	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	1	1	2	2	3	3	4	4	5
84	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	1	1	2	2	3	3	4	4	5
85	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	1	1	2	2	3	3	4	4	5
86	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322	1	1	2	2	3	3	4	4	5
87	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375	1	1	2	2	3	3	4	4	5
88	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	1	1	2	2	3	3	4	4	5
89	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482	1	1	2	2	3	3	4	4	5
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	1	1	2	2	3	3	4	4	5
91	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586	1	1	2	2	3	3	4	4	5
92	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	1	1	2	2	3	3	4	4	5
93	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690	1	1	2	2	3	3	4	4	5
94	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	1	1	2	2	3	3	4	4	5
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	1	1	2	2	3	3	4	4	5
96	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	1	1	2	2	3	3	4	4	5
97	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	1	1	2	2	3	3	4	4	5
98	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945	0	1	1	2	2	3	3	4	4
99	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995	0	1	1	2	2	3	3	4	4

POWERS AND ROOTS

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	**51**	2 601	132 651	7.141	3.708
2	4	8	1.414	1.260	52	2 704	140 608	7.211	3.733
3	9	27	1.732	1.442	53	2 809	148 877	7.280	3.756
4	16	64	2.000	1.587	54	2 916	157 464	7.348	3.780
5	25	125	2.236	1.710	55	3 025	166 375	7.416	3.803
6	36	216	2.449	1.817	56	3 136	175 616	7.483	3.826
7	49	343	2.646	1.913	57	3 249	185 193	7.550	3.849
8	64	512	2.828	2.000	58	3 364	195 112	7.616	3.871
9	81	729	3.000	2.080	59	3 481	205 379	7.681	3.893
10	100	1 000	3.162	2.154	**60**	3 600	216 000	7.746	3.915
11	121	1 331	3.317	2.224	61	3 721	226 981	7.810	3.936
12	144	1 728	3.464	2.289	62	3 844	238 328	7.874	3.958
13	169	2 197	3.606	2.351	63	3 969	250 047	7.937	3.979
14	196	2 744	3.742	2.410	64	4 096	262 144	8.000	4.000
15	225	3 375	3.873	2.466	65	4 225	274 625	8.062	4.021
16	256	4 096	4.000	2.520	66	4 356	287 496	8.124	4.041
17	289	4 913	4.123	2.571	67	4 489	300 763	8.185	4.062
18	324	5 832	4.243	2.621	68	4 624	314 432	8.246	4.082
19	361	6 859	4.359	2.668	69	4 761	328 509	8.307	4.102
20	400	8 000	4.472	2.714	**70**	4 900	343 000	8.367	4.121
21	441	9 261	4.583	2.759	71	5 041	357 911	8.426	4.141
22	484	10 648	4.690	2.802	72	5 184	373 248	8.485	4.160
23	529	12 167	4.796	2.844	73	5 329	389 017	8.544	4.179
24	576	13 824	4.899	2.884	74	5 476	405 224	8.602	4.198
25	625	15 625	5.000	2.924	75	5 625	421 875	8.660	4.217
26	676	17 576	5.099	2.962	76	5 776	438 976	8.718	4.236
27	729	19 683	5.196	3.000	77	5 929	456 533	8.775	4.254
28	784	21 952	5.292	3.037	78	6 084	474 552	8.832	4.273
29	841	24 389	5.385	3.072	79	6 241	493 039	8.888	4.291
30	900	27 000	5.477	3.107	**80**	6 400	512 000	8.944	4.309
31	961	29 791	5.568	3.141	81	6 561	531 441	9.000	4.327
32	1 024	32 768	5.657	3.175	82	6 724	551 368	9.055	4.344
33	1 089	35 937	5.745	3.208	83	6 889	571 787	9.110	4.362
34	1 156	39 304	5.831	3.240	84	7 056	592 704	9.165	4.380
35	1 225	42 875	5.916	3.271	85	7 225	614 125	9.220	4.397
36	1 296	46 656	6.000	3.302	86	7 396	636 056	9.274	4.414
37	1 369	50 653	6.083	3.332	87	7 569	658 503	9.327	4.431
38	1 444	54 872	6.164	3.362	88	7 744	681 472	9.381	4.448
39	1 521	59 319	6.245	3.391	89	7 921	704 969	9.434	4.465
40	1 600	64 000	6.325	3.420	**90**	8 100	729 000	9.487	4.481
41	1 681	68 921	6.403	3.448	91	8 281	753 571	9.539	4.498
42	1 764	74 088	6.481	3.476	92	8 464	778 688	9.592	4.514
43	1 849	79 507	6.557	3.503	93	8 649	804 357	9.644	4.531
44	1 936	85 184	6.633	3.530	94	8 836	830 584	9.695	4.547
45	2 025	91 125	6.708	3.557	95	9 025	857 375	9.747	4.563
46	2 116	97 336	6.782	3.583	96	9 216	884 736	9.798	4.579
47	2 209	103 823	6.856	3.609	97	9 409	912 673	9.849	4.595
48	2 304	110 592	6.928	3.634	98	9 604	941 192	9.899	4.610
49	2 401	117 649	7.000	3.659	99	9 801	970 299	9.950	4.626
50	2 500	125 000	7.071	3.684	**100**	10 000	1 000 000	10.000	4.642

Square Root by Trial. If we have no square-root tables at hand or have forgotten the method given on pages 103 and 104, the square root of a number may be found by trial. To understand this method it must be remembered that if any number is divided by its square root, the result is equal to the square root; for example:

$$\sqrt{4} = 2, \quad \text{and} \quad 4 \div 2 = 2.$$
$$\sqrt{25} = 5, \quad \text{and} \quad 25 \div 5 = 5.$$

```
        2.8
 2 5)7 0.
     5 0
     2 00
     2 00
```

Illustrative Problem. Find the square root of 7 by the trial method.

Since 7 is about halfway between 4, which is the square of 2, and 9, which is the square of 3, we first try 2.5 as the square root of 7. We then divide 7 by this trial root and obtain 2.8 as the result, as shown. Since the result is greater than the number by which we divided, 2.5 is evidently too small.

We next try 2.65, the number halfway between 2.5 and 2.8. Dividing 7 by 2.65 we obtain 2.642 as the result to the nearest thousandth, as shown. Since the result is now less than the number by which we divided, 2.65 is too large.

```
           2.641(2)
 2 65)7 00.
      5 30
      1 700
      1 590
        1100
        1060
          400
          265
          135
```

We then take 2.646, the number halfway between 2.65 and 2.642. Dividing 7 by 2.646 we find the result to be 2.646. Hence the square root of 7 is 2.646.

By referring to the table it will be seen that 2.646 is there given as the square root of 7 to the nearest thousandth.

The student will soon learn to judge from the difference between the trial root and the result in any division what number is the most sensible to use as the next trial root.

In the subsequent exercises the student may use any of the methods of finding square root that he may wish, but he should endeavor to become familiar with all the methods given.

Application of Square Root to the Circle. From the formula for the area of a circle, $A = \pi r^2$, it is seen that in finding the radius r we have to find a square root, since

$$r = \sqrt{\frac{A}{\pi}}.$$

The formula may be written $r = \sqrt{\frac{7}{22} A}$, $r = \sqrt{0.32\,A}$, $r = \sqrt{0.3183\,A}$, according to the degree of accuracy required.

For example, a draftsman who is to draw a circle representing the cross section of an iron column of cross-section area 30 sq. in. needs to find the radius of the circle. If the drawing is to be full size, what radius should he use?

$$r = \sqrt{0.3183\,A} = \sqrt{0.3183 \times 30} = \sqrt{9.5490} = 3.090.$$

Hence to the nearest 0.01″ the radius he should use is 3.09″.

Exercises. Square Root

1. Find the radius that a tinsmith should use in laying out a circular hole for a pipe, the cross-section area of which is 167 sq. in.

In such a case find the radius to the nearest 0.01″ and then express the result to the nearest $\frac{1}{64}$″.

2. What must be the diameter of a water pipe in order that the area of the cross section shall be 5 sq. in.?

3. Find the diameter of a water main of which the area of the cross section is 5.6 sq. ft.

4. Find the diameter of a cylinder head whose area is 136 sq. in., and of one whose area is 154 sq. in.

5. A tinsmith is required to make some cylindric cans to hold a gallon (231 cu. in.) each and to be 8″ high. What radius should he use in laying out the base?

Exercises. Review

Using the table, find the square root of each of the following:

1. 8.5.	**3.** 235.	**5.** 3.345.	**7.** 74.8.
2. 3.8.	**4.** 4.45.	**6.** 2.548.	**8.** 75.76.

By trial, find the square root of each of the following:

9. 24.	**11.** 989.	**13.** 687.9.	**15.** 98.41.
10. 38.	**12.** 787.	**14.** 332.4.	**16.** 8873.

17. The foot of a 42-foot ladder is 15′ from the wall of a building against which the top rests. How high does the ladder reach on the wall?

18. In order to have an iron pillar capable of supporting a certain weight, the cross-section area must be 72 sq. in. What radius should a pattern maker use in drawing the circle for the pattern?

19. What must be the diameter of a water pipe in order that the area of the cross section shall be 7 sq. in.?

20. What must be the diameter of the piston of an engine in order that the area of the cross section may be 146 sq. in.?

21. A tinsmith wishes to make some cylindric gallon cans which are to be 9″ high. What must be the area of the base? What radius must he use for the base?

22. A cylindric water tank is 28′ high and has a capacity of 38,000 cu. ft. What is the diameter of the tank? What would be the diameter if the capacity of the tank were doubled?

23. A water main has a diameter of 18″. What is the area of its cross section? What is the area of the cross section of a water main that will carry twice the amount of water under the same conditions? What is the diameter of this larger main?

Exercises. Estimates of Areas

1. The squared paper upon which the figures on page 117 are drawn is ruled with ten lines to the inch, and the scale of the figures is $\frac{1}{10}$. By counting the squares find the approximate area of figure I. Verify the result by finding the more nearly exact area from the proper formula.

The general rule to be followed in counting or rejecting parts of a square is given on page 80. In each exercise given below, the result found by counting the squares should be verified by finding the area from the proper formula.

2. Find the area of figure II, deducting the area of the small circle.

3. Find the area of figure III, which represents seven eighths of the area of a circle.

4. Find the area of figure IV, the length of each curve line being a fourth of a circumference.

5. Find the area of figure V, the length of each curve line being either a fourth or a half of a circumference.

6. Find the area of figure VI, the length of the curve line being a semicircumference.

7. Find the area of figure VII, deducting the area of the small circle from that of the large one.

8. Find the area of figure VIII, the length of the curve line being a fourth of a circumference.

Using your judgment as to how the curve lines are drawn, find the area of each of the following:

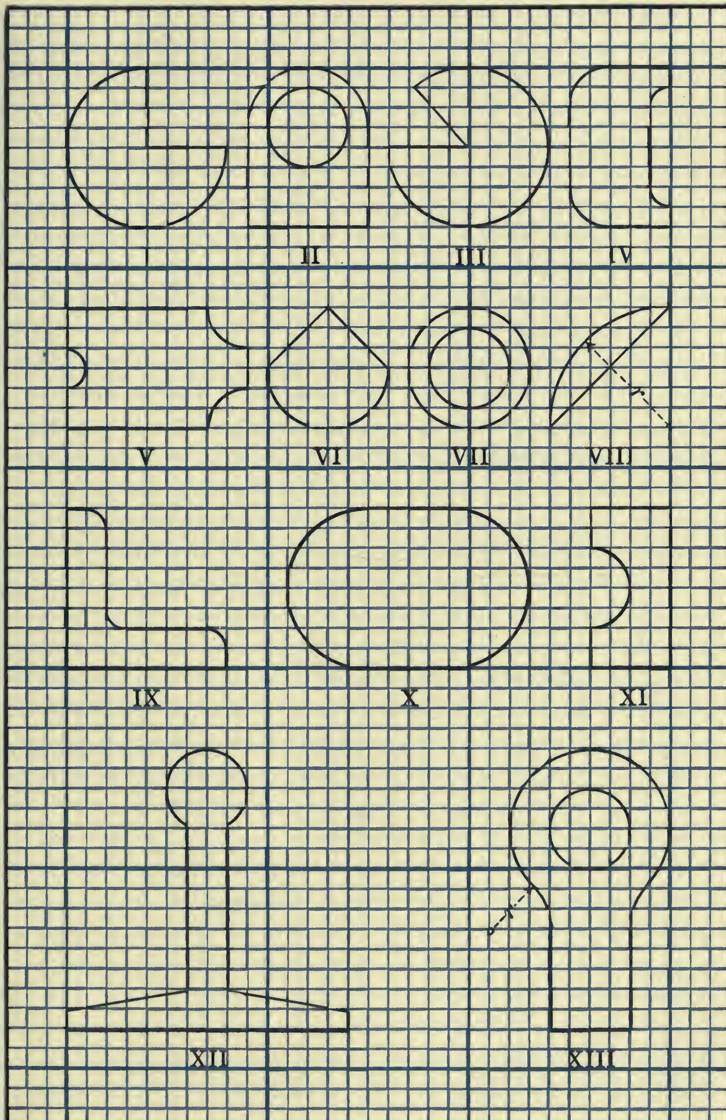
9. Figure IX.

10. Figure X.

11. Figure XI.

12. Figure XII.

13. Find the area of figure XIII, deducting the area of the small circle.



II
III
IV
V
VI
VII
VIII
IX
X
XI
XII
XIII

Useful Formulas. The formulas shown in the blueprint on page 119 are useful for various cases in mensuration, and most of them have already been given. Some of the formulas already given are here printed in a slightly different form. Those not already given may be used with the others in solving the exercises which follow. They represent the kind of aids often given on blueprints in workshops, and the student should be familiar with their use.

In the following exercises use the formulas given on page 119.

Exercises. Useful Formulas

1. The side of a certain square is given as 6.8″. Find the length of the diagonal.

The side being given to the nearest 0.1″, the result should not be given to any higher degree of accuracy, and similarly in all such cases.

2. Find to the nearest 0.001 sq. in. the area of a rectangle 13.725″ long and 6.375″ wide.

3. Find to the nearest 0.1 sq. in. the area of a triangle of base 14.9″ and height 7.3″.

4. Find the height of an equilateral triangle of side 7.2″.

5. Find the base of an equilateral triangle of height 7.2″; of an equilateral triangle of height 14.4″.

6. Find the hypotenuse of a right triangle of sides $a = 7.2''$, $b = 9.6''$; of a right triangle of sides $a = 14.4''$, $b = 19.2''$.

7. Find the base of a right triangle of hypotenuse 9.8″ and height 7.2″.

8. Find the height of a right triangle of hypotenuse 17.2″ and base 6.8″.

9. Find the areas of circles of diameters 2.2″ and 4.4″ respectively; of radii 8.8″ and 17.6″ respectively.

SQUARE

$d = 1.414\,b$

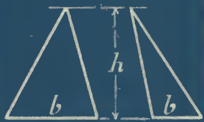
$b = \frac{d}{1.414} = 0.707\,d$

$A = b^2$

RECTANGLE

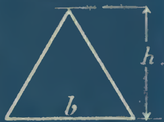
$A = bh$

TRIANGLE



$A = \frac{bh}{2}$

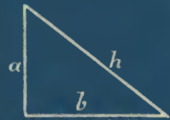
EQUILATERAL TRIANGLE



$h = 0.866\,b$

$b = \frac{h}{0.866} = 1.155\,h$

RIGHT TRIANGLE



$h = \sqrt{a^2 + b^2}$

$a = \sqrt{h^2 - b^2}$

$b = \sqrt{h^2 - a^2}$

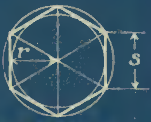
CIRCLE

$d = \frac{cir}{\pi} = \frac{cir}{3.1416}$

$cir = \pi d = 3.1416\,d$

$A = 0.7854\,d^2$

HEXAGON



$r = 0.866\,s$

$A = 2.598\,s^2$

$= 3.464\,r^2$

OCTAGON

$r = 1.207\,s$

$A = 4.828\,s^2$

$= 3.314\,r^2$

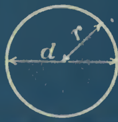
CIRCULAR RING

$A = \pi(R^2 - r^2)$

$A = 3.1416(R^2 - r^2)$

$= 3.1416(R+r)(R-r)$

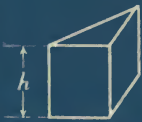
SPHERE



$S = \pi d^2 = 4\pi r^2$

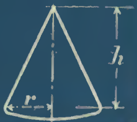
$V = \frac{\pi d^3}{6} = \frac{4\pi r^3}{3}$

PRISM



$V = \text{Area of base} \times h$

CONE



$V = \text{Area of base} \times \frac{h}{3}$

$= \frac{\pi r^2 h}{3}$

10. Find the circumferences of circles of diameters 3.3″, 6.6″, and 9.9″ respectively.

11. Find the radii of circles that have areas of 10 sq. in. and 20 sq. in. respectively.

12. Find the diameters of circles of circumferences 21″ and 42″ respectively.

13. The radius of the circle inscribed in the hexagonal head of a bolt is $\frac{7}{16}''$. Find the area of the head of the bolt.

In the hexagon on page 119 the inside circle, of which r is the radius, is called an *inscribed circle*; the outside circle is called a *circumscribed circle,* the radius of which is equal to the side of the hexagon.

14. What radius must be used to circumscribe a circle about a hexagonal nut which is $\frac{3}{8}''$ on a side? to inscribe a circle within the nut?

15. A side of the octagonal head of a bolt is $\frac{1}{4}''$. Find the area of the head of the bolt.

16. The internal and external radii of a water pipe are respectively $9\frac{7}{8}''$ and $10\frac{1}{8}''$. Find the area of the metal in the cross section of the pipe.

17. At 26¢ per square inch what is the cost of gilding a ball on the top of the dome of a building, the ball being 4′ 6″ in diameter and no allowance being made for the base on which the ball is supported.

18. What is the weight of a spherical shell which is $1\frac{7}{8}''$ thick, which has an external diameter of $14\frac{3}{8}''$, and is made of cast iron weighing 450 lb./cu. ft.?

19. Find the volume of a prism 8″ long, the cross section of which is a triangle 1″ on each side.

20. Find the volume of a cone $7\frac{3}{4}''$ high, the base being a circle of diameter $3\frac{1}{2}''$. What would be the volume of the cone if each of these dimensions were doubled?

Exercises. Review

1. The base of the support of a certain shaft is a square 16.8″ on a side. Find to the nearest 0.1 sq. in. the area covered by the base of the support.

2. Find the area of the cross section of a cylindric shaft 7.2″ in diameter.

In solving these problems and those which occur on subsequent pages continue to use $3\frac{1}{7}$, or 3.14, for π, unless otherwise directed. The formulas on page 119, which give 3.1416, may be used in such practical work as may require a very high degree of accuracy.

3. Find the radius of a cylindric shaft of cross-section area 26.8 sq. in.

4. By the aid of calipers measuring to $\frac{1}{64}''$ the diameter of a solid shaft was found to be $4\frac{7}{64}''$. Find the circumference of the shaft and the area of the cross section.

5. Find the circumference of a shaft of cross-section area 21 sq. in., and one of cross-section area 63 sq. in.

6. A steel plate is in the form of a trapezoid, the two parallel sides being respectively $23\frac{1}{2}''$ and $17\frac{3}{4}''$. The perpendicular distance between the parallel sides is $16\frac{3}{8}''$. Find to the nearest $\frac{1}{8}$ sq. in. the area of the plate.

Such a statement always refers to the area of one face.

7. If the steel plate in Ex. 6 is $\frac{5}{8}''$ thick, how many cubic inches does it contain?

8. A solid cast-iron pillar is 12′ 6″ high and has a diameter of 8″. Taking the weight of cast iron as 450 lb./cu. ft., find the weight of the pillar.

9. If the curve surface of the pillar in Ex. 8 is to be painted, how many square feet must be covered?

10. The area of the curve surface of a cylinder 14″ in height is 176 sq. in. Find the circumference and the diameter.

Metric System. The measures that were formerly used by the world proved to be so unsystematic and inconvenient, varying so much in different countries, that most of the civilized nations adopted a new system in the nineteenth century. This set of measures was devised in France about the year 1800 and is known as the *metric system* and, like our system of money, is based upon the scale of ten.

The use of the metric system is now obligatory in more than thirty countries, and the system is in partial use in most of the others. Before the World War we had little need for the metric system except in scientific work, where it is used almost universally. Our recent great expansion of foreign trade, however, necessitates the use of this system in describing goods intended for export and in making machinery of various kinds. Furthermore, in practical mathematics we need to know the principal units of the system since we often find them used in technical journals and handbooks. We read of the diameters of automobile cylinders in millimeters, of 75-millimeter guns, and of such distances as 28 kilometers and 600 meters, and it is convenient to know what these various metric measures mean.

It is not for the schools to decide whether the metric system will probably replace our common system, but it is important that they should give to students some knowledge of the nature of the system.

It is easy to understand the metric system, for there are only six important prefixes and but few measures to learn. The prefixes and their meanings are as follows:

milli-	**0.001**	**deka-**	**10**
centi-	**0.01**	**hekto-**	**100**
deci-	**0.1**	**kilo-**	**1000**

The work is so arranged that pages 122–130 may be omitted if desired.

Exercises. Metric System

1. A mill is what part of \$1? A millimeter is what part of a meter? If a meter is about 40″, a millimeter is about what part of an inch? Draw a line about a millimeter long.

The meter is equivalent to 39.37″, but in the problems on this page, where the purpose is to help visualize the metric measures, the meter may be taken as approximately equivalent to 40″.

2. A cent is what part of \$1? A centimeter is what part of a meter? A centimeter is about what part of an inch? Draw a line about a centimeter long.

3. What is the meaning of deci-? A decimeter is what part of a meter? A decimeter is about how many inches long? Draw a line about a decimeter long.

4. A French "75" is a French gun that fires a projectile 75 millimeters in diameter. To what size of gun in use in the American army does this correspond?

5. What is the meaning of kilo-? A kilometer is how many meters? Taking a meter as equivalent to $3\frac{1}{4}'$, how many feet are there in a kilometer? how many tenths of a mile?

6. A gram is approximately equivalent to $\frac{1}{500}$ lb., and so a kilogram is equivalent to how many pounds?

A kilogram is more nearly equivalent to 2.2 lb.

7. A regiment protected by a battery of 125-millimeter guns advanced 3 kilometers, captured a hill 160 meters high, and bombed the enemy's trenches with bombs weighing a kilogram apiece. Write the sentence, replacing the metric measures with their equivalents in our common measures.

8. A liter is equivalent to a quart, and so a hektoliter is equivalent to how many quarts? how many gallons?

The word *liter* is pronounced "lee-ter."

By answering the questions in these exercises the student will learn some of the more important measures of the metric system.

Metric Length. The table of metric length is as follows:

A **kilometer (km.)** = 1000 meters
A hektometer = 100 meters
A dekameter = 10 meters
Meter (m.)
A decimeter (dm.) = 0.1 of a meter
A **centimeter (cm.)** = 0.01 of a meter
A **millimeter (mm.)** = 0.001 of a meter

The meter is equivalent to 39.37″, or about $3\frac{1}{4}'$, or a little over a yard; the kilometer is equivalent to 0.62 mi. When the metric system was invented, the meter was intended to be one ten-millionth of the distance on the surface of the earth from the equator to the pole, but it varies slightly from this standard.

The figure at the right is a 10-centimeter, or 1-decimeter, rule. The small graduations between 0 and 1 represent millimeters.

In the metric system only those measures which are printed in black letters in the tables are in common use.

Any one of these measures may be expressed in terms of any other measure by *simply moving the decimal point to the right or left.*

Thus, as 245¢ = 24.5 dimes = \$2.45,

so 2475 mm. = 247.5 cm. = 24.75 dm. = 2.475 m.

All the units of the system are derived from the meter. Every compound name is accented on the first syllable, as, for example, *mil′limeter*.

The school should be supplied with a meter stick, a liter, and a cubic centimeter, and these can easily be made if necessary.

The abbreviations in this book are in common use. Some, however, use Km., Dm., and dm. for kilometer, dekameter, and decimeter.

Exercises. Metric Length

1. The distance from Chicago to New York by one route is about 1500 km. Using 0.62 mi. as the equivalent of the kilometer, express this distance in miles.

2. The distance from New York to Albany is 229 km. Express this distance in miles as in Ex. 1.

3. In a gymnasium where scientific records are kept it is found that a certain boy is 144 cm. tall. Using 39.37″ as the equivalent of the meter, express this height in inches; in feet and inches; in feet and a decimal.

Practically we work in the metric system or in our common system, rarely having any occasion to transfer from one to the other. As in Exs. 1 and 2, the purpose here is merely to visualize the measures.

4. Measure the length of this page to the nearest 0.1 cm.; to the nearest 0.01 dm.; to the nearest millimeter. Measure the width of the printed portion in the same way.

If not provided with a ruler marked in millimeters, transfer the length to a strip of paper, and use the rule shown on page 124.

5. Measure the diameter of a 5-cent piece and the diameter of a dime, giving the results to the nearest millimeter.

6. Draw a line AX and from one end A mark off AC equal to 17 mm.; then CD equal to 9 mm.; then DE equal to 26 mm. Measure AE, write the result, and check by adding the separate lengths.

7. Make a fine dot with ink on the rim of a silver "quarter." Before the ink dries roll the coin along a piece of paper until the ink has marked the paper twice, and draw a line connecting these two points. Measure in millimeters this line, which is equal to the circumference of the coin, and also measure the diameter of the coin. Check the length of the circumference by the formula given on page 92.

Metric Area. The table of metric area is as follows:

A square kilometer (sq. km.) = 1,000,000 square meters
A square hektometer = 10,000 square meters
A square dekameter = 100 square meters
Square meter (sq. m.)
A square decimeter = 0.01 of a square meter
A **square centimeter (sq. cm.)** = 0.0001 of a square meter
A **square millimeter (sq. mm.)** = 0.000001 of a square meter

There is no generally recognized set of abbreviations for square measure. Instead of using sq. m., scientific writers often use m^2., and similar abbreviations for the other measures.

In measuring land a square dekameter is called an *are* (pronounced är); a square hektometer is called a *hektare* (ha.). The hektare is equivalent to 2.47 A. (acres), or nearly $2\frac{1}{2}$ A. The student will have no present use for these measures, and so they need not be learned.

Metric Volume. The table of metric volume is as follows:

Cubic meter (cu. m.)
A **cubic decimeter (cu. dm.)** = 0.001 of a cubic meter
A **cubic centimeter (cu. cm.)** = 0.000001 of a cubic meter
A cubic millimeter (cu. mm.) = 0.000000001 of a cubic meter

There is no generally recognized set of abbreviations for cubic measure. Instead of using cu. m., scientific writers often use m^3., and similar abbreviations for the other measures.

We also have cubic kilometers, cubic hektometers, and cubic dekameters. These terms, however, are seldom used.

In measuring wood a cubic meter is called a *stere* (st.). The word *stere* is pronounced "stair." The student will have no real use for this measure, however, and so it will not be considered further.

It is evident that the above tables can be obtained at once from the table of length by merely squaring or cubing. They need not be specially learned, since they can always be obtained in this way, as is evident from Exs. 2 and 3 on page 127.

Exercises. Metric Area and Volume

1. How many feet in 1 yd.? How many square feet in 1 sq. yd.? How many cubic feet in 1 cu. yd.? How many cubic feet in a cube that is $1\frac{1}{2}$ yd. on an edge?

2. How many centimeters in 1 m.? How many square centimeters in 1 sq. m.? How many cubic centimeters in 1 cu. m.? How many square centimeters in a square that is 1.2 m. on a side?

3. How many inches in 1 yd.? How many square inches in 1 sq. yd.? How many square millimeters in 1 sq. cm.? How many cubic millimeters in 1 cu. cm.?

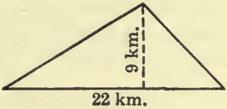
4. Find the area of a rectangle 17 mm. by 28 mm.; of a parallelogram of base 19.2 cm. and height 9.7 cm.

5. Find the area of a triangle, the base of which is 5 cm. and the height 7 cm. Express the result first as square centimeters and then as square millimeters.

6. Find the cross-section area of the bore of a French "75." This means the area of the cross section of the bore of a gun that has an internal diameter of 75 mm.

7. In Ex. 6 find the circumference of the bore.

8. During the war an army advanced on a straight front of 22 km. and occupied a triangular piece of territory 9 km. in depth, as shown by the figure. How many square kilometers of territory did the army gain?



9. In Ex. 8, in order to grasp the situation in terms of our common units, express the lengths in miles and find the area in square miles.

10. The specifications for a piece of machinery require a cylinder 0.48 m. long and 0.2 m. in diameter. Find the volume of the cylinder to the nearest 0.1 cu. cm.

Metric Capacity. The table of metric capacity is as follows:

A **hektoliter (hl.)** = 100 liters
A dekaliter = 10 liters
Liter (l.)
A deciliter (dl.) = 0.1 of a liter

A liter is the volume of a cube 1 dm. on an edge, and so it is possible to express this table in terms of cubic measures. For this reason the centiliter and milliliter are not often used.

The liter is practically equivalent to our quart.

A liter contains about 61.024 cu. in. and is equivalent to 1.0567 liquid quarts or 0.908 of a dry quart. These details need not be memorized.

Metric Weight. The table of metric weight is as follows:

A **metric ton (t.)** = 1000 kilograms
A **kilogram (kg.)** = 1000 grams
Gram (g.)

The *quintal*, which is equal to 100 kg., is also used; but, like "dekagram" and "hektogram," the term is not employed frequently enough to demand our attention. The meaning of the words "decigram," "centigram," and "milligram" is evident, but the student will commonly find these units expressed as decimals of a gram.

A kilogram is equivalent to 2.2 lb., and a metric ton to 2204.6 lb. A kilogram is usually called a *kilo* (kē-lo).

A gram is equivalent to 15.432 grains, or about 0.035 avoirdupois ounces. A pound is equivalent to about 0.4536 kg. Our common ton is equivalent to 0.907 of a metric ton.

Using the metric system, it is easy to calculate the weight of a given volume of a substance if we know its specific gravity, since, for all practical purposes,

1 cu. cm. of water weighs 1 g.,
1 l. of water weighs 1 kg.,
and 1 cu. m. of water weighs 1 t.

Exercises. Metric Capacity and Weight

1. A manufacturer has a demand from a South American country for cylinders which shall contain 125 l. He may roughly estimate this as how many gallons per cylinder?

2. An exporting house receives an order for 25 metric tons of copper plates. How many pounds are ordered?

3. A tank 3 m. long, 1.8 m. wide, and 1.2 m. deep is filled with water. What is the weight of the water?

4. A cylindric gas tank is 1.8 m. long and has a diameter of 0.3 m. Find the number of liters in the cylinder.

5. An exporting house receives an order for a shipment of liter measures, each measure to be a cylinder 0.1 m. in diameter. Find the height of each measure.

6. A liter measure in the form of a cylinder is 0.2 m. high. Find the diameter of the cylinder.

7. A manufacturer is required to supply a tank that shall be 3.4 m. long, 2.8 m. wide, and that shall have a capacity of 19,000 l. Find the height of the tank to the nearest 0.1 m.

8. An order is received by an exporting concern for 225 metric tons of cotton. Find to the nearest bale the number of bales of 500 lb. each that will fill the order.

9. Find in kilograms the weight of the water that a tank 3 m. $\times$ 4 m. $\times$ 6 m. will hold. Also find in pounds the weight of the water that a tank $3' \times 4' \times 6'$ will hold, taking $62\frac{1}{2}$ lb. as the weight of 1 cu. ft. of water.

10. The specific gravity of copper is 8.9. Find the weight of a cube of copper 24 cm. on an edge. Using the weight of 1 cu. ft. of water as given in Ex. 9, find the weight of a cube of copper $9\frac{1}{2}''$ on an edge. Compare this result with the result obtained by converting into pounds the weight as found in kilograms in the first part of the problem.

Exercises. Review

1. A square is to be made in which the diagonal cannot exceed 13.2 cm. Find the greatest length of side that can be allowed in making the square.

2. The side of one square is half as long as the side of another square. The area of the first square is what part of the area of the second? The perimeter of the first square is what part of the perimeter of the second?

3. The side of one square is equal to the diagonal of another square. The area of the first square is how many times the area of the second?

4. The diameter of one water pipe is 0.4 m. and that of another is 0.3 m. What must be the diameter of a third pipe that shall have the same carrying capacity as the two smaller pipes together?

5. It is desired to replace two steam pipes that have the same length and the same diameter by one pipe which shall have the same length as each of the old pipes and a radiating surface equal to that of the two pipes combined. If the external diameter of each of the old pipes was 50 mm., what should be the external diameter of the new pipe?

6. Find the weight of the water that it will take to fill a cylindric tank 2.5 m. in diameter to a depth of 2.1 m.

7. The entire surface area of a cube is 1014 sq. cm. Find the volume of the cube.

8. The volume of a cube is 13,824 cu. cm. Find the entire surface area of the cube.

9. A rectangular water tank 8 m. long, 6 m. wide, and 5 m. deep is to be replaced by a cylindric tank of the same depth and the same capacity. Find the diameter of the new tank.

10. In Ex. 9 find the circumference and the area of the base.

CHAPTER IV

TRIGONOMETRY

Nature of Trigonometry. In practical measurements one of the simplest and most valuable methods is *trigonometry.* The word comes from the Greek and means "tri-angle-measure." Since we can easily cut all plane surfaces actually or approximately into triangles, we can measure any plane surface, at least approximately, if we can measure a triangle.

As an illustration of the use of the triangle in measuring, suppose that a 4-foot post, PB in the figure below, casts a shadow $6'$ long at the same time that the shadow of the tree TY is $60'$ long. Find the height of the tree.

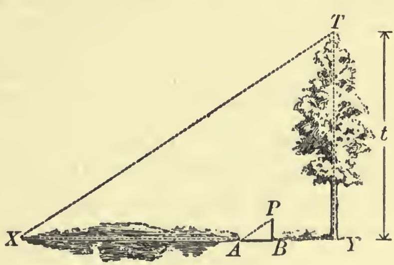
We have two right triangles ABP and XYT of the same shape. We might consider triangle ABP as a small-scale plan of the large triangle XYT. We then have a proportion between the sides, in which

$$\frac{\text{height of tree}}{\text{shadow of tree}}=\frac{\text{height of post}}{\text{shadow of post}}.$$

That is, in this case we have

$$\frac{t}{60}=\frac{4}{6}, \quad \text{or} \quad t=\frac{4\times\overset{10}{\cancel{60}}}{\cancel{6}}=40.$$

Hence the tree is $40'$ high.



We have here found the height of the tree by indirect measurement; that is, we do not need to climb the tree and measure the height directly by a tape line; we find the height indirectly by taking certain other measures.

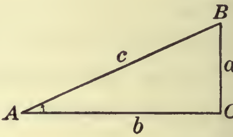
In such cases the object is assumed to stand on a horizontal plane.

Tangent of an Angle. In finding the height of the tree on page 131 we multiplied the length of the shadow of the tree by the ratio of the height of the post to its shadow; that is, by the result of height divided by shadow. The height of the post is immaterial, for the ratio of height to shadow is the same whatever the height of the post which we take.

That is, in the right triangle ACB below, if angle A remains the same, the ratio of BC to AC does not change, whatever the length BC may be.

This ratio of BC to AC is called the *tangent* of the angle A. It is customary to write **tan** A for "tangent of angle A."

Using this symbol and designating BC in the figure by a, and AC by b, we have



$$\tan A = \frac{a}{b};$$

whence
$$a = b \tan A.$$

If we know $\tan A$ and can measure b, we can compute the value of a. If, therefore, we can find the tangents of the various angles that we are likely to use, we can find any height by this method. Our first problem, therefore, is to find the tangents of angles.

Because of the variety of its applications, the tangent is the most natural trigonometric function with which to introduce the subject.

In the above triangle if we measure the angle A with a protractor, we find it to be about 25°, and if we measure AC we find it to be 1″. We have just seen that

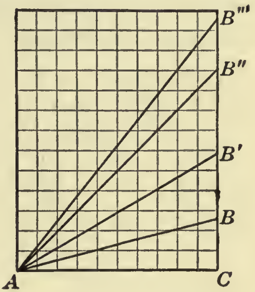
$$a = b \tan A,$$

or
$$a = 1 \times \tan 25°,$$

so that if we can find the value of tan 25° we can find the length of a without measuring it directly.

We shall presently see that tan 25° = 0.47, approximately, so that the length of side a is approximately 0.47″.

Finding Tangents. The method of finding the tangents of various angles depends upon higher algebra, but we can determine approximately the tangent of any angle by the aid of squared paper.



In this figure angle $CAB = 14°$, and since the tangent is $BC \div AC$ we find, by counting the spaces, that

$$\tan 14° = \frac{2\frac{1}{2}}{10} = 0.25.$$

A closer approximation found by higher algebra is 0.2493.

Similarly, $\tan CAB' = \tan 30° = \dfrac{5.8}{10} = 0.58,$

$$\tan CAB'' = \tan 45° = \frac{10}{10} = 1,$$

and $\tan CAB''' = \tan 51°\ 20' = \dfrac{12.5}{10} = 1.25.$

B' is read "B prime"; B'' is read "B second"; B''' is read "B third."

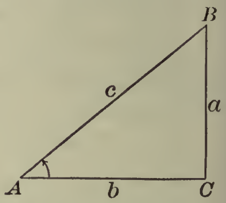
Table of Tangents. The following short table of tangents of angles may be used with the exercises on page 134:

Angle	tan	Angle	tan	Angle	tan
10°	0.1763	40°	0.8391	70°	2.7475
20°	.3640	50°	1.1918	80°	5.6713
30°	.5774	60°	1.7321	90°	∞

The symbol ∞ means "infinity"; that is, the tangent of 90° is infinitely long. This can easily be seen by drawing an angle of 90°.

Students should be reminded that the sum of the angles in any triangle is 180°, that the square on the hypotenuse of a right triangle is equal to the sum of the squares on the two sides, that each angle of an equilateral triangle is 60°, and that the angles at the base of an isosceles triangle are equal.

Practical Use of the Tangent. An observer at A in the figure below determines the angle A, made by sighting at the top B of a tower, as 40°. With a tape with which he can measure accurately to 0.01′, he measures the distance from A to the base C of the tower and finds it to be 150′. Find the height of the tower.



Since $a = b \tan A$,

we have $a = 150 \tan 40°$

$= 150 \times 0.8391$

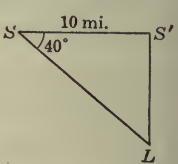
$= 125.865.$

That is, to the nearest 0.01′, the height of the tower is 125.87′.

If we had no instruments for measuring the distance b more closely than to the nearest foot, we should give the height of the tower as 126′, since *no result can be more nearly accurate than the measurements upon which it is based.*

Exercises. Tangents

1. The captain of a ship at S observes a lighthouse L to lie 40° south of an east-and-west line. After the ship has sailed to the position S', which is 10 mi. east of S, the lighthouse is seen to be directly south of the ship. Find how far the lighthouse is from S'.



Referring to the triangle at the top of this page, find in each case the value of a to four figures, given that:

2. $b = 38'$, $A = 10°$.

3. $b = 42'$, $A = 20°$.

4. $b = 84'$, $A = 30°$.

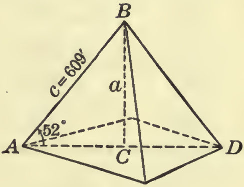
5. $b = 105.4'$, $A = 50°$.

6. $b = 116.4'$, $A = 70°$.

7. $b = 42.57'$, $A = 80°$.

8. If each angle of a triangle is 60° and the height of the triangle is 8″, find the base.

Sine of an Angle. Suppose that we know that the edge AB of the Great Pyramid is 609′ to the nearest foot and that the angle DAB is 52°, measures that are easily taken. If we knew the ratio $a \div c$, which is shown more clearly in the second figure below, we could easily find the value of a, the height. That is, because



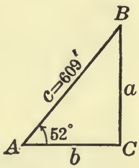
$$a = c \times \frac{a}{c},$$

we have $$a = 609 \times \frac{a}{c}.$$

The ratio $\frac{a}{c}$ is called the *sine* of A, which is written $\sin A$.

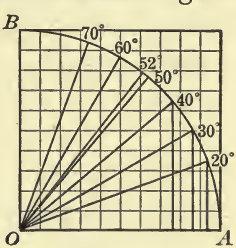
In this figure, $$\sin A = \frac{a}{c};$$

whence $$a = c \sin A.$$



In any right triangle the sine of either acute angle is the ratio of the side opposite the angle to the hypotenuse.

Finding Sines. We can determine approximately the sine of any angle by the aid of squared paper. In this figure the arc AB is drawn with center O and radius 10. Then sin 20° is equal to the length of the perpendicular drawn to OA from the point marked 20° in the figure, divided by the radius. The perpendicular is about 3.4 squares long, and hence sin 20° is approximately $3.4 \div 10$, or 0.34.



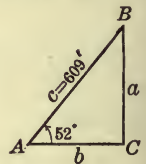
Proceeding as in the above case, we find the following table of sines for the remaining angles shown in the figure:

Angle	sin	Angle	sin	Angle	sin
30°	0.50	50°	0.77	60°	0.87
40°	.64	52°	.79	70°	.94

Table of Sines. The following short table of sines may be used in solving the exercises which are given on page 137:

Angle	sin	Angle	sin	Angle	sin
10°	0.1736	40°	0.6428	70°	0.9397
20°	.3420	50°	.7660	75°	.9659
30°	.5000	52°	.7880	80°	.9848
38°	.6157	60°	.8660	90°	1.0000

Practical Use of the Sine. **1.** Find a, the height of the Great Pyramid referred to on page 135.



Since $a = c \sin A$,

we have $a = 609 \sin 52°$

$= 609 \times 0.7880$

$= 479.892.$

Since c was given to the nearest foot, we say that the height of the Great Pyramid is 480′ to the nearest foot.

2. Given that angle B is 38°, as is really the case in the Great Pyramid, find b, which is half the diagonal of the base.

Since $b = c \sin B$,

we have $b = 609 \sin 38°$

$= 609 \times 0.6157$

$= 374.9613.$

Hence the length of b is 375′ to the nearest foot.

Check. We can check the approximate results for a and b, above, from the formula for the right triangle $h^2 = a^2 + b^2$, which in this case we may write $c^2 = a^2 + b^2$.

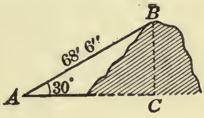
While we shall at first use as illustrations such problems of general interest as the one given above, it is evident that the methods of trigonometry may be used in industrial problems as well, and we shall soon apply them in this way.

Exercises. Sines

1. If a kite string 360′ long makes an angle of 40° with the ground, how high is the kite?

The kite string would not be perfectly straight, but we can obtain a very good approximation by this method.

2. In order to find the height of a mound a string is stretched from A to B, as shown in the figure, and AB is found to be 68′ 6″ long. If $\angle A$ is found to be 30°, what is the height of the mound?

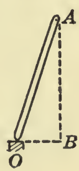


The symbol $\angle A$ is used to indicate the angle at A, or the angle CAB.

3. In planning for a pontoon bridge across the head of a lake to save marching through swamp land, a squad of engineer troops sighted from a point C due west to A, as shown in the figure. They then sighted from C directly north, thus laying out the line CD. On this line they took a point B, measured AB, and found it to be 978′. They found $\angle A$ to be 20°. Find the distance BC.

4. When the 36-foot arm OA of a steel crane makes an angle of 70° with the horizontal line OB, what is the vertical distance AB?

5. In Ex. 4 find the horizontal distance OB, the angle with the vertical line AB being 20°. Verify the lengths of AB and OB by the right-triangle formula.

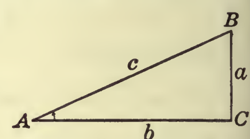


Given an acute angle and the hypotenuse of a right triangle as follows, find the side opposite the acute angle in each case:

6. 40°, 42.3′.	**9.** 60°, 12 mi.	**12.** 70°, 16 yd.
7. 50°, 38.7′.	**10.** 30°, 28 mi.	**13.** 80°, 350 yd.
8. 38°, 72.1″.	**11.** 40°, 36 mi.	**14.** 75°, 400 yd.

Cosine of an Angle. In any triangle the sum of all the angles is 180°, so that in a right triangle the sum of the acute angles is 90°. Angle B in the figure below is the *complement* of angle A; that is, $\angle B$ is equal to 90° minus $\angle A$, which is what we mean when we speak of the complement of an angle.

The sine of the complement of an angle is called the *cosine* of the angle, the syllable "co-" standing for "complement." We write **cos *A*** for "cosine of angle A." By definition, cos A is the same as sin B, and in this figure



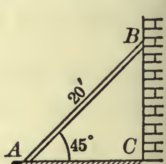
$$\cos A = \frac{b}{c};$$

whence $$b = c \cos A.$$

Table of Cosines. The following short table of cosines of angles may be used with the exercises given on page 139:

ANGLE	COS	ANGLE	COS	ANGLE	COS
10°	0.9848	40°	0.7660	70°	0.3420
15°	.9659	45°	.7071	75°	.2588
20°	.9397	50°	.6428	80°	.1736
30°	.8660	60°	.5000	90°	.0000

Practical Use of the Cosine. In shoring up the wall BC it is desired to use 20-foot timbers and to have the shores make an angle of 45° with the horizontal. How far from the wall should the foot A of each shore be placed?



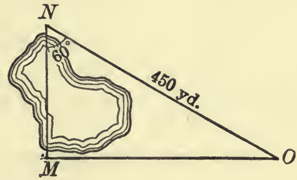
Since $$b = c \cos A,$$

we have
$$\begin{aligned} AC &= 20 \cos 45° \\ &= 20 \times 0.7071 \\ &= 14.142. \end{aligned}$$

Since to the nearest foot AC is 14′, the foot of each shore should be placed approximately 14′ from the base of the wall.

Exercises. Cosines

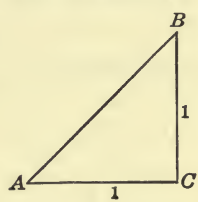
1. To find the distance from M to N across a pond, as here shown, some students sighted from M to N and then ran a line MO at right angles to MN. They measured ON, found it to be 450 yd., and found $\angle N$ to be $60°$. Find the distance MN.



2. A boy walking along a straight road leaves it at a point P and goes along a straight oblique path $500'$ to a spring at S. He then takes a path that is perpendicular to the road and reaches the road at a point Q, which is $433'$ from P. Find the angle QPS which the oblique path makes with the road and find the angle at S which it makes with the other path. Find also the length of the path SQ.

First draw the figure freehand while reading the problem. What ratio relating to angle P can be found? What is its value? What angle has this value for this ratio? How may the angle S be found?

3. In this right triangle suppose that $AC = CB = 1$ and then find the length of AB. From this find the value of $\sin 45°$; $\cos 45°$; $\tan 45°$.



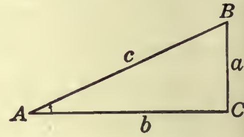
4. A ship starts from a port and sails N. 20° W. a distance of 32 mi. Find the distance due north and the distance due west that the ship has sailed from the port.

The expression "sails N. 20° W." means that the ship sails in a direction 20° west of north.

5. A ship sails N. 10° E. a distance of 48 mi. Find how far due north and also how far due east the ship has sailed.

6. Draw an equilateral triangle $1''$ on a side, draw a perpendicular from any vertex to the opposite side, and find the value of $\sin 60°$; $\cos 60°$; $\tan 60°$; $\sin 30°$; $\cos 30°$; $\tan 30°$.

Cotangent of an Angle. The tangent of the complement of an angle is called the *cotangent* of the angle. In this figure, therefore, the cotangent of A, written **cot A**, is the same as tan B, or, expressed as a formula,



$$\cot A = \frac{b}{a};$$

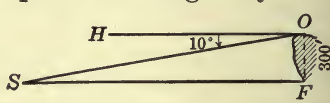
whence $$b = a \cot A.$$

Table of Cotangents. The following short table of cotangents of angles may be used with the exercises on page 141:

ANGLE	COT	ANGLE	COT	ANGLE	COT
10°	5.6713	40°	1.1918	70°	0.3640
20°	2.7475	50°	0.8391	80°	.1763
30°	1.7321	60°	.5774	90°	.0000

The student should compare this table of cotangents with the table of tangents given on page 133.

Practical Use of the Cotangent. An observer at O, on the top of a cliff 300′ high, sees the top of a floating buoy at S at an angle of depression of 10°. How far is the buoy from F, the foot of the cliff?



The *angle of depression* is the angle which the line of sight OS in this figure makes with the horizontal line OH. The angle of depression, or the angle HOS, is equal to the angle S.

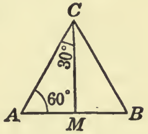
Since $$b = a \cot A,$$

we have
$$SF = 300 \cot 10°$$
$$= 300 \times 5.6713$$
$$= 1701.39.$$

Hence the buoy is approximately 1701′ from the foot of the cliff.

Exercises. Cotangents

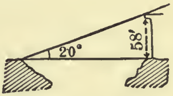
1. A steel truss was made up of sections like ABC in this figure. If the sections were equilateral triangles 48′ on a side, find the height CM of each section.



Each angle of an equilateral triangle is 60°. Hence $\angle A = 60°$ and $\angle ACM = 30°$. We also have $AM = 24'$. We may now find CM in several ways, as by using $\tan 60°$ or by using $\cot 30°$.

Since $CM \div AM = \cot 30°$, to what is CM equal?

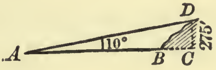
2. A building known to be 58′ high is observed from across a ravine, the angle of elevation of the top being 20°. How wide is the ravine?



In practice the angle of elevation is usually taken from a point above the base of the building, and allowance has to be made for this height.

3. How far from the foot of a tree 60′ high must an observer lie in order that he may see the top of the tree at an angle of 50°? at an angle of 40°?

4. The top D of a hill is known to be 275′ above the level of a lake AB. An observer at A finds the angle of elevation of D to be 10°. Find the distance AC shown in the diagram.



Referring to the triangle on page 140, find b, given that:

5. $a = 34'$, $A = 10°$.

6. $a = 27'$, $A = 20°$.

7. $a = 65'$, $A = 30°$.

8. $a = 130'$, $A = 40°$.

9. $a = 350'$, $A = 50°$.

10. $a = 14.7''$, $A = 60°$.

11. $a = 32.6'$, $A = 70°$.

12. $a = 32'\,4''$, $A = 80°$.

13. $a = 26'\,4''$, $A = 50°$.

14. $a = 32'\,5''$, $A = 30°$.

15. $a = 28'\,6''$, $A = 60°$.

16. $a = 48'\,9''$, $A = 70°$.

17. $a = 52'\,8''$, $A = 80°$.

18. $a = 42'\,7''$, $A = 80°$.

Tables of Natural Functions. We speak of $\sin A$, $\cos A$, $\tan A$, and $\cot A$ as *functions* of the angle A. The functions of the angles thus far used have been given in brief tables as occasion required. We shall now show how to find these functions from complete tables. These tables are given on pages 144–151, the four functions being given for every $6'$, that is, for every $0.1°$, from $0°$ to $90°$.

In this case $6'$ means 6 minutes, or $\frac{6}{60}$ of $1°$. We therefore have the same symbol ($'$) used to mean two different things; that is, feet and minutes. In case any uncertainty is likely to arise we shall hereafter use the abbreviation *ft.* for feet, but in general the context will make clear the meaning of the symbol.

The tables are called tables of natural functions to distinguish them from tables of logarithmic functions, which are used in more extended courses in trigonometry.

For example, to find $\sin 28° 36'$, look for $28°$ in the column at the left on page 144, and then to the right in that line under $36'$ we find 0.4787, which is the sine of $28° 36'$.

To find $\sin 28° 40'$, however, we shall have to use the *columns of differences* at the right-hand side of the table.

Since $28° 40'$ is $4'$ more than $28° 36'$, which we found above, we look in the columns of differences under $4'$ and in line with $28°$ and find the number 10. We then have

	$\sin 28° 36' =$	0.4787, as above.
Difference for	$4' =$	.0010
Adding,	$\sin 28° 40' =$	0.4797

The above addition should be made mentally as we look at $\sin 28° 36'$; we write only 0.4797.

There is another way of finding $\sin 28° 40'$. We notice that $40'$ is $\frac{2}{3}$ of the way from $36'$ to $42'$ and that the difference between $\sin 28° 36'$ and $\sin 28° 42'$ is 0.0015, so we simply add $\frac{2}{3}$ of 0.0015 to $\sin 28° 36'$. The result is the same as the one given above, but this process, which is ordinarily known as *interpolation*, is longer.

Illustrative Problems. **1.** Find $\sin 62° 19'$.

From page 145	$\sin 62° 18' = 0.8854$
Difference for	$1' = 1$
Adding,	$\sin 62° 19' = 0.8855$

As already stated, in such cases we write only the result.

2. Find $\cos 39° 50'$.

In the case of the cosine and cotangent we must bear in mind that these functions decrease as the angle increases. To call attention to this fact the columns of differences are marked "— Differences."

From page 146	$\cos 39° 48' = 0.7683$
Difference for	$2' = 4$
Subtracting,	$\cos 39° 50' = 0.7679$

3. Find $\tan 77° 39'$.

Here the tangent is changing so rapidly that the column of differences ceases to be accurate enough. We therefore use the plan of interpolation suggested at the foot of page 142.

From page 149	$\tan 77° 42' = 4.5864$
	$\tan 77° 36' = 4.5483$
Subtracting, the difference for	$6' = 0.0381$

Taking half of this, the difference for $3' = 0.0191$.
Adding to $\tan 77° 36'$, $\tan 77° 39' = 4.5674$.

4. Find the angle of which the sine is 0.9673.

We look in the table for sines beginning with the figures 96, and find on page 145 that 0.9673 is in the line for 75° and under the column 18′.

Therefore the angle of which the sine is 0.9673 is 75° 18′.

5. Find the angle of which the cotangent is 0.3512.

As in Ex. 4, $0.3522 = \cot 70° 36'$.

But 0.3512 is 0.0010 less than this, and from the columns of differences 0.0010 is to be subtracted for an increase of 3′. Therefore we have

$$0.3512 = \cot 70° 39'.$$

That is, the angle of which the cotangent is 0.3512 is 70° 39′.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	3	6	9	12	15
1	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	3	6	9	12	15
2	.0349	.0366	.0384	.0401	.0419	.0436	.0454	.0471	.0488	.0506	3	6	9	12	15
3	.0523	.0541	.0558	.0576	.0593	.0610	.0628	.0645	.0663	.0680	3	6	9	12	15
4	.0698	.0715	.0732	.0750	.0767	.0785	.0802	.0819	.0837	.0854	3	6	9	12	14
5	.0872	.0889	.0906	.0924	.0941	.0958	.0976	.0993	.1011	.1028	3	6	9	12	14
6	.1045	.1063	.1080	.1097	.1115	.1132	.1149	.1167	.1184	.1201	3	6	9	12	14
7	.1219	.1236	.1253	.1271	.1288	.1305	.1323	.1340	.1357	.1374	3	6	9	12	14
8	.1392	.1409	.1426	.1444	.1461	.1478	.1495	.1513	.1530	.1547	3	6	9	12	14
9	.1564	.1582	.1599	.1616	.1633	.1650	.1668	.1685	.1702	.1719	3	6	9	12	14
10	.1736	.1754	.1771	.1788	.1805	.1822	.1840	.1857	.1874	.1891	3	6	9	11	14
11	.1908	.1925	.1942	.1959	.1977	.1994	.2011	.2028	.2045	.2062	3	6	9	11	14
12	.2079	.2096	.2113	.2130	.2147	.2164	.2181	.2198	.2215	.2233	3	6	9	11	14
13	.2250	.2267	.2284	.2300	.2317	.2334	.2351	.2368	.2385	.2402	3	6	8	11	14
14	.2419	.2436	.2453	.2470	.2487	.2504	.2521	.2538	.2554	.2571	3	6	8	11	14
15	.2588	.2605	.2622	.2639	.2656	.2672	.2689	.2706	.2723	.2740	3	6	8	11	14
16	.2756	.2773	.2790	.2807	.2823	.2840	.2857	.2874	.2890	.2907	3	6	8	11	14
17	.2924	.2940	.2957	.2974	.2990	.3007	.3024	.3040	.3057	.3074	3	6	8	11	14
18	.3090	.3107	.3123	.3140	.3156	.3173	.3190	.3206	.3223	.3239	3	6	8	11	14
19	.3256	.3272	.3289	.3305	.3322	.3338	.3355	.3371	.3387	.3404	3	5	8	11	14
20	.3420	.3437	.3453	.3469	.3486	.3502	.3518	.3535	.3551	.3567	3	5	8	11	14
21	.3584	.3600	.3616	.3633	.3649	.3665	.3681	.3697	.3714	.3730	3	5	8	11	14
22	.3746	.3762	.3778	.3795	.3811	.3827	.3843	.3859	.3875	.3891	3	5	8	11	14
23	.3907	.3923	.3939	.3955	.3971	.3987	.4003	.4019	.4035	.4051	3	5	8	11	14
24	.4067	.4083	.4099	.4115	.4131	.4147	.4163	.4179	.4195	.4210	3	5	8	11	13
25	.4226	.4242	.4258	.4274	.4289	.4305	.4321	.4337	.4352	.4368	3	5	8	11	13
26	.4384	.4399	.4415	.4431	.4446	.4462	.4478	.4493	.4509	.4524	3	5	8	10	13
27	.4540	.4555	.4571	.4586	.4602	.4617	.4633	.4648	.4664	.4679	3	5	8	10	13
28	.4695	.4710	.4726	.4741	.4756	.4772	.4787	.4802	.4818	.4833	3	5	8	10	13
29	.4848	.4863	.4879	.4894	.4909	.4924	.4939	.4955	.4970	.4985	3	5	8	10	13
30	.5000	.5015	.5030	.5045	.5060	.5075	.5090	.5105	.5120	.5135	3	5	8	10	13
31	.5150	.5165	.5180	.5195	.5210	.5225	.5240	.5255	.5270	.5284	2	5	7	10	12
32	.5299	.5314	.5329	.5344	.5358	.5373	.5388	.5402	.5417	.5432	2	5	7	10	12
33	.5446	.5461	.5476	.5490	.5505	.5519	.5534	.5548	.5563	.5577	2	5	7	10	12
34	.5592	.5606	.5621	.5635	.5650	.5664	.5678	.5693	.5707	.5721	2	5	7	10	12
35	.5736	.5750	.5764	.5779	.5793	.5807	.5821	.5835	.5850	.5864	2	5	7	9	12
36	.5878	.5892	.5906	.5920	.5934	.5948	.5962	.5976	.5990	.6004	2	5	7	9	12
37	.6018	.6032	.6046	.6060	.6074	.6088	.6101	.6115	.6129	.6143	2	5	7	9	12
38	.6157	.6170	.6184	.6198	.6211	.6225	.6239	.6252	.6266	.6280	2	5	7	9	11
39	.6293	.6307	.6320	.6334	.6347	.6361	.6374	.6388	.6401	.6414	2	4	7	9	11
40	.6428	.6441	.6455	.6468	.6481	.6494	.6508	.6521	.6534	.6547	2	4	7	9	11
41	.6561	.6574	.6587	.6600	.6613	.6626	.6639	.6652	.6665	.6678	2	4	7	9	11
42	.6691	.6704	.6717	.6730	.6743	.6756	.6769	.6782	.6794	.6807	2	4	6	9	11
43	.6820	.6833	.6845	.6858	.6871	.6884	.6896	.6909	.6921	.6934	2	4	6	8	11
44	.6947	.6959	.6972	.6984	.6997	.7009	.7022	.7034	.7046	.7059	2	4	6	8	10

All the above sines are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	.7071	.7083	.7096	.7108	.7120	.7133	.7145	.7157	.7169	.7181	2	4	6	8	10
46	.7193	.7206	.7218	.7230	.7242	.7254	.7266	.7278	.7290	.7302	2	4	6	8	10
47	.7314	.7325	.7337	.7349	.7361	.7373	.7385	.7396	.7408	.7420	2	4	6	8	10
48	.7431	.7443	.7455	.7466	.7478	.7490	.7501	.7513	.7524	.7536	2	4	6	8	10
49	.7547	.7559	.7570	.7581	.7593	.7604	.7615	.7627	.7638	.7649	2	4	6	8	9
50	.7660	.7672	.7683	.7694	.7705	.7716	.7727	.7738	.7749	.7760	2	4	6	7	9
51	.7771	.7782	.7793	.7804	.7815	.7826	.7837	.7848	.7859	.7869	2	4	5	7	9
52	.7880	.7891	.7902	.7912	.7923	.7934	.7944	.7955	.7965	.7976	2	4	5	7	9
53	.7986	.7997	.8007	.8018	.8028	.8039	.8049	.8059	.8070	.8080	2	3	5	7	9
54	.8090	.8100	.8111	.8121	.8131	.8141	.8151	.8161	.8171	.8181	2	3	5	7	8
55	.8192	.8202	.8211	.8221	.8231	.8241	.8251	.8261	.8271	.8281	2	3	5	7	8
56	.8290	.8300	.8310	.8320	.8329	.8339	.8348	.8358	.8368	.8377	2	3	5	6	8
57	.8387	.8396	.8406	.8415	.8425	.8434	.8443	.8453	.8462	.8471	2	3	5	6	8
58	.8480	.8490	.8499	.8508	.8517	.8526	.8536	.8545	.8554	.8563	2	3	5	6	8
59	.8572	.8581	.8590	.8599	.8607	.8616	.8625	.8634	.8643	.8652	1	3	4	6	7
60	.8660	.8669	.8678	.8686	.8695	.8704	.8712	.8721	.8729	.8738	1	3	4	6	7
61	.8746	.8755	.8763	.8771	.8780	.8788	.8796	.8805	.8813	.8821	1	3	4	6	7
62	.8829	.8838	.8846	.8854	.8862	.8870	.8878	.8886	.8894	.8902	1	3	4	5	7
63	.8910	.8918	.8926	.8934	.8942	.8949	.8957	.8965	.8973	.8980	1	3	4	5	6
64	.8988	.8996	.9003	.9011	.9018	.9026	.9033	.9041	.9048	.9056	1	3	4	5	6
65	.9063	.9070	.9078	.9085	.9092	.9100	.9107	.9114	.9121	.9128	1	2	4	5	6
66	.9135	.9143	.9150	.9157	.9164	.9171	.9178	.9184	.9191	.9198	1	2	3	5	6
67	.9205	.9212	.9219	.9225	.9232	.9239	.9245	.9252	.9259	.9265	1	2	3	4	6
68	.9272	.9278	.9285	.9291	.9298	.9304	.9311	.9317	.9323	.9330	1	2	3	4	5
69	.9336	.9342	.9348	.9354	.9361	.9367	.9373	.9379	.9385	.9391	1	2	3	4	5
70	.9397	.9403	.9409	.9415	.9421	.9426	.9432	.9438	.9444	.9449	1	2	3	4	5
71	.9455	.9461	.9466	.9472	.9478	.9483	.9489	.9494	.9500	.9505	1	2	3	4	5
72	.9511	.9516	.9521	.9527	.9532	.9537	.9542	.9548	.9553	.9558	1	2	3	4	4
73	.9563	.9568	.9573	.9578	.9583	.9588	.9593	.9598	.9603	.9608	1	2	2	3	4
74	.9613	.9617	.9622	.9627	.9632	.9636	.9641	.9646	.9650	.9655	1	2	2	3	4
75	.9659	.9664	.9668	.9673	.9677	.9681	.9686	.9690	.9694	.9699	1	1	2	3	4
76	.9703	.9707	.9711	.9715	.9720	.9724	.9728	.9732	.9736	.9740	1	1	2	3	3
77	.9744	.9748	.9751	.9755	.9759	.9763	.9767	.9770	.9774	.9778	1	1	2	3	3
78	.9781	.9785	.9789	.9792	.9796	.9799	.9803	.9806	.9810	.9813	1	1	2	2	3
79	.9816	.9820	.9823	.9826	.9829	.9833	.9836	.9839	.9842	.9845	1	1	2	2	3
80	.9848	.9851	.9854	.9857	.9860	.9863	.9866	.9869	.9871	.9874	0	1	1	2	2
81	.9877	.9880	.9882	.9885	.9888	.9890	.9893	.9895	.9898	.9900	0	1	1	2	2
82	.9903	.9905	.9907	.9910	.9912	.9914	.9917	.9919	.9921	.9923	0	1	1	2	2
83	.9925	.9928	.9930	.9932	.9934	.9936	.9938	.9940	.9942	.9943	0	1	1	1	2
84	.9945	.9947	.9949	.9951	.9952	.9954	.9956	.9957	.9959	.9960	0	1	1	1	1
85	.9962	.9963	.9965	.9966	.9968	.9969	.9971	.9972	.9973	.9974	0	0	1	1	1
86	.9976	.9977	.9978	.9979	.9980	.9981	.9982	.9983	.9984	.9985	0	0	1	1	1
87	.9986	.9987	.9988	.9989	.9990	.9990	.9991	.9992	.9993	.9993	0	0	0	1	1
88	.9994	.9995	.9995	.9996	.9996	.9997	.9997	.9997	.9998	.9998	0	0	0	0	0
89	.9998	.9999	.9999	.9999	.9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

The precise value of all sines except sin 90° is less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9999	.9999	.9999	0	0	0	0	0
1	.9998	.9998	.9998	.9997	.9997	.9997	.9996	.9996	.9995	.9995	0	0	0	0	0
2	.9994	.9993	.9993	.9992	.9991	.9990	.9990	.9989	.9988	.9987	0	0	0	0	0
3	.9986	.9985	.9984	.9983	.9982	.9981	.9980	.9979	.9978	.9977	0	0	1	1	1
4	.9976	.9974	.9973	.9972	.9971	.9969	.9968	.9966	.9965	.9963	0	0	1	1	1
5	.9962	.9960	.9959	.9957	.9956	.9954	.9952	.9951	.9949	.9947	0	1	1	1	1
6	.9945	.9943	.9942	.9940	.9938	.9936	.9934	.9932	.9930	.9928	0	1	1	1	2
7	.9925	.9923	.9921	.9919	.9917	.9914	.9912	.9910	.9907	.9905	0	1	1	2	2
8	.9903	.9900	.9898	.9895	.9893	.9890	.9888	.9885	.9882	.9880	0	1	1	2	2
9	.9877	.9874	.9871	.9869	.9866	.9863	.9860	.9857	.9854	.9851	0	1	1	2	2
10	.9848	.9845	.9842	.9839	.9836	.9833	.9829	.9826	.9823	.9820	1	1	2	2	3
11	.9816	.9813	.9810	.9806	.9803	.9799	.9796	.9792	.9789	.9785	1	1	2	2	3
12	.9781	.9778	.9774	.9770	.9767	.9763	.9759	.9755	.9751	.9748	1	1	2	3	3
13	.9744	.9740	.9736	.9732	.9728	.9724	.9720	.9715	.9711	.9707	1	1	2	3	3
14	.9703	.9699	.9694	.9690	.9686	.9681	.9677	.9673	.9668	.9664	1	1	2	3	4
15	.9659	.9655	.9650	.9646	.9641	.9636	.9632	.9627	.9622	.9617	1	2	2	3	4
16	.9613	.9608	.9603	.9598	.9593	.9588	.9583	.9578	.9573	.9568	1	2	2	3	4
17	.9563	.9558	.9553	.9548	.9542	.9537	.9532	.9527	.9521	.9516	1	2	3	4	4
18	.9511	.9505	.9500	.9494	.9489	.9483	.9478	.9472	.9466	.9461	1	2	3	4	5
19	.9455	.9449	.9444	.9438	.9432	.9426	.9421	.9415	.9409	.9403	1	2	3	4	5
20	.9397	.9391	.9385	.9379	.9373	.9367	.9361	.9354	.9348	.9342	1	2	3	4	5
21	.9336	.9330	.9323	.9317	.9311	.9304	.9298	.9291	.9285	.9278	1	2	3	4	5
22	.9272	.9265	.9259	.9252	.9245	.9239	.9232	.9225	.9219	.9212	1	2	3	4	6
23	.9205	.9198	.9191	.9184	.9178	.9171	.9164	.9157	.9150	.9143	1	2	3	5	6
24	.9135	.9128	.9121	.9114	.9107	.9100	.9092	.9085	.9078	.9070	1	2	4	5	6
25	.9063	.9056	.9048	.9041	.9033	.9026	.9018	.9011	.9003	.8996	1	3	4	5	6
26	.8988	.8980	.8973	.8965	.8957	.8949	.8942	.8934	.8926	.8918	1	3	4	5	6
27	.8910	.8902	.8894	.8886	.8878	.8870	.8862	.8854	.8846	.8838	1	3	4	5	7
28	.8829	.8821	.8813	.8805	.8796	.8788	.8780	.8771	.8763	.8755	1	3	4	6	7
29	.8746	.8738	.8729	.8721	.8712	.8704	.8695	.8686	.8678	.8669	1	3	4	6	7
30	.8660	.8652	.8643	.8634	.8625	.8616	.8607	.8599	.8590	.8581	1	3	4	6	7
31	.8572	.8563	.8554	.8545	.8536	.8526	.8517	.8508	.8499	.8490	2	3	5	6	8
32	.8480	.8471	.8462	.8453	.8443	.8434	.8425	.8415	.8406	.8396	2	3	5	6	8
33	.8387	.8377	.8368	.8358	.8348	.8339	.8329	.8320	.8310	.8300	2	3	5	6	8
34	.8290	.8281	.8271	.8261	.8251	.8241	.8231	.8221	.8211	.8202	2	3	5	7	8
35	.8192	.8181	.8171	.8161	.8151	.8141	.8131	.8121	.8111	.8100	2	3	5	7	8
36	.8090	.8080	.8070	.8059	.8049	.8039	.8028	.8018	.8007	.7997	2	3	5	7	9
37	.7986	.7976	.7965	.7955	.7944	.7934	.7923	.7912	.7902	.7891	2	4	5	7	9
38	.7880	.7869	.7859	.7848	.7837	.7826	.7815	.7804	.7793	.7782	2	4	5	7	9
39	.7771	.7760	.7749	.7738	.7727	.7716	.7705	.7694	.7683	.7672	2	4	6	7	9
40	.7660	.7649	.7638	.7627	.7615	.7604	.7593	.7581	.7570	.7559	2	4	6	8	9
41	.7547	.7536	.7524	.7513	.7501	.7490	.7478	.7466	.7455	.7443	2	4	6	8	10
42	.7431	.7420	.7408	.7396	.7385	.7373	.7361	.7349	.7337	.7325	2	4	6	8	10
43	.7314	.7302	.7290	.7278	.7266	.7254	.7242	.7230	.7218	.7206	2	4	6	8	10
44	.7193	.7181	.7169	.7157	.7145	.7133	.7120	.7108	.7096	.7083	2	4	6	8	10

The precise value of all cosines except cos 0° is less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	.7071	.7059	.7046	.7034	.7022	.7009	.6997	.6984	.6972	.6959	2	4	6	8	10
46	.6947	.6934	.6921	.6909	.6896	.6884	.6871	.6858	.6845	.6833	2	4	6	8	11
47	.6820	.6807	.6794	.6782	.6769	.6756	.6743	.6730	.6717	.6704	2	4	6	9	11
48	.6691	.6678	.6665	.6652	.6639	.6626	.6613	.6600	.6587	.6574	2	4	7	9	11
49	.6561	.6547	.6534	.6521	.6508	.6494	.6481	.6468	.6455	.6441	2	4	7	9	11
50	.6428	.6414	.6401	.6388	.6374	.6361	.6347	.6334	.6320	.6307	2	4	7	9	11
51	.6293	.6280	.6266	.6252	.6239	.6225	.6211	.6198	.6184	.6170	2	5	7	9	11
52	.6157	.6143	.6129	.6115	.6101	.6088	.6074	.6060	.6046	.6032	2	5	7	9	12
53	.6018	.6004	.5990	.5976	.5962	.5948	.5934	.5920	.5906	.5892	2	5	7	9	12
54	.5878	.5864	.5850	.5835	.5821	.5807	.5793	.5779	.5764	.5750	2	5	7	9	12
55	.5736	.5721	.5707	.5693	.5678	.5664	.5650	.5635	.5621	.5606	2	5	7	10	12
56	.5592	.5577	.5563	.5548	.5534	.5519	.5505	.5490	.5476	.5461	2	5	7	10	12
57	.5446	.5432	.5417	.5402	.5388	.5373	.5358	.5344	.5329	.5314	2	5	7	10	12
58	.5299	.5284	.5270	.5255	.5240	.5225	.5210	.5195	.5180	.5165	2	5	7	10	12
59	.5150	.5135	.5120	.5105	.5090	.5075	.5060	.5045	.5030	.5015	3	5	8	10	13
60	.5000	.4985	.4970	.4955	.4939	.4924	.4909	.4894	.4879	.4863	3	5	8	10	13
61	.4848	.4833	.4818	.4802	.4787	.4772	.4756	.4741	.4726	.4710	3	5	8	10	13
62	.4695	.4679	.4664	.4648	.4633	.4617	.4602	.4586	.4571	.4555	3	5	8	10	13
63	.4540	.4524	.4509	.4493	.4478	.4462	.4446	.4431	.4415	.4399	3	5	8	10	13
64	.4384	.4368	.4352	.4337	.4321	.4305	.4289	.4274	.4258	.4242	3	5	8	11	13
65	.4226	.4210	.4195	.4179	.4163	.4147	.4131	.4115	.4099	.4083	3	5	8	11	13
66	.4067	.4051	.4035	.4019	.4003	.3987	.3971	.3955	.3939	.3923	3	5	8	11	13
67	.3907	.3891	.3875	.3859	.3843	.3827	.3811	.3795	.3778	.3762	3	5	8	11	13
68	.3746	.3730	.3714	.3697	.3681	.3665	.3649	.3633	.3616	.3600	3	5	8	11	14
69	.3584	.3567	.3551	.3535	.3518	.3502	.3486	.3469	.3453	.3437	3	5	8	11	14
70	.3420	.3404	.3387	.3371	.3355	.3338	.3322	.3305	.3289	.3272	3	5	8	11	14
71	.3256	.3239	.3223	.3206	.3190	.3173	.3156	.3140	.3123	.3107	3	6	8	11	14
72	.3090	.3074	.3057	.3040	.3024	.3007	.2990	.2974	.2957	.2940	3	6	8	11	14
73	.2924	.2907	.2890	.2874	.2857	.2840	.2823	.2807	.2790	.2773	3	6	8	11	14
74	.2756	.2740	.2723	.2706	.2689	.2672	.2656	.2639	.2622	.2605	3	6	8	11	14
75	.2588	.2571	.2554	.2538	.2521	.2504	.2487	.2470	.2453	.2436	3	6	8	11	14
76	.2419	.2402	.2385	.2368	.2351	.2334	.2317	.2300	.2284	.2267	3	6	8	11	14
77	.2250	.2233	.2215	.2198	.2181	.2164	.2147	.2130	.2113	.2096	3	6	9	11	14
78	.2079	.2062	.2045	.2028	.2011	.1994	.1977	.1959	.1942	.1925	3	6	9	11	14
79	.1908	.1891	.1874	.1857	.1840	.1822	.1805	.1788	.1771	.1754	3	6	9	11	14
80	.1736	.1719	.1702	.1685	.1668	.1650	.1633	.1616	.1599	.1582	3	6	9	11	14
81	.1564	.1547	.1530	.1513	.1495	.1478	.1461	.1444	.1426	.1409	3	6	9	12	14
82	.1392	.1374	.1357	.1340	.1323	.1305	.1288	.1271	.1253	.1236	3	6	9	12	14
83	.1219	.1201	.1184	.1167	.1149	.1132	.1115	.1097	.1080	.1063	3	6	9	12	14
84	.1045	.1028	.1011	.0993	.0976	.0958	.0941	.0924	.0906	.0889	3	6	9	12	14
85	.0872	.0854	.0837	.0819	.0802	.0785	.0767	.0750	.0732	.0715	3	6	9	12	14
86	.0698	.0680	.0663	.0645	.0628	.0610	.0593	.0576	.0558	.0541	3	6	9	12	15
87	.0523	.0506	.0488	.0471	.0454	.0436	.0419	.0401	.0384	.0366	3	6	9	12	15
88	.0349	.0332	.0314	.0297	.0279	.0262	.0244	.0227	.0209	.0192	3	6	9	12	15
89	.0175	.0157	.0140	.0122	.0105	.0087	.0070	.0052	.0035	.0017	3	6	9	12	15

All the above cosines are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	0.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	3	6	9	12	15
1	0.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	3	6	9	12	15
2	0.0349	.0367	.0384	.0402	.0419	.0437	.0454	.0472	.0489	.0507	3	6	9	12	15
3	0.0524	.0542	.0559	.0577	.0594	.0612	.0629	.0647	.0664	.0682	3	6	9	12	15
4	0.0699	.0717	.0734	.0752	.0769	.0787	.0805	.0822	.0840	.0857	3	6	9	12	15
5	0.0875	.0892	.0910	.0928	.0945	.0963	.0981	.0998	.1016	.1033	3	6	9	12	15
6	0.1051	.1069	.1086	.1104	.1122	.1139	.1157	.1175	.1192	.1210	3	6	9	12	15
7	0.1228	.1246	.1263	.1281	.1299	.1317	.1334	.1352	.1370	.1388	3	6	9	12	15
8	0.1405	.1423	.1441	.1459	.1477	.1495	.1512	.1530	.1548	.1566	3	6	9	12	15
9	0.1584	.1602	.1620	.1638	.1655	.1673	.1691	.1709	.1727	.1745	3	6	9	12	15
10	0.1763	.1781	.1799	.1817	.1835	.1853	.1871	.1890	.1908	.1926	3	6	9	12	15
11	0.1944	.1962	.1980	.1998	.2016	.2035	.2053	.2071	.2089	.2107	3	6	9	12	15
12	0.2126	.2144	.2162	.2180	.2199	.2217	.2235	.2254	.2272	.2290	3	6	9	12	15
13	0.2309	.2327	.2345	.2364	.2382	.2401	.2419	.2438	.2456	.2475	3	6	9	12	15
14	0.2493	.2512	.2530	.2549	.2568	.2586	.2605	.2623	.2642	.2661	3	6	9	12	16
15	0.2679	.2698	.2717	.2736	.2754	.2773	.2792	.2811	.2830	.2849	3	6	9	13	16
16	0.2867	.2886	.2905	.2924	.2943	.2962	.2981	.3000	.3019	.3038	3	6	9	13	16
17	0.3057	.3076	.3096	.3115	.3134	.3153	.3172	.3191	.3211	.3230	3	6	10	13	16
18	0.3249	.3269	.3288	.3307	.3327	.3346	.3365	.3385	.3404	.3424	3	6	10	13	16
19	0.3443	.3463	.3482	.3502	.3522	.3541	.3561	.3581	.3600	.3620	3	7	10	13	16
20	0.3640	.3659	.3679	.3699	.3719	.3739	.3759	.3779	.3799	.3819	3	7	10	13	17
21	0.3839	.3859	.3879	.3899	.3919	.3939	.3959	.3979	.4000	.4020	3	7	10	13	17
22	0.4040	.4061	.4081	.4101	.4122	.4142	.4163	.4183	.4204	.4224	3	7	10	14	17
23	0.4245	.4265	.4286	.4307	.4327	.4348	.4369	.4390	.4411	.4431	3	7	10	14	17
24	0.4452	.4473	.4494	.4515	.4536	.4557	.4578	.4599	.4621	.4642	4	7	11	14	18
25	0.4663	.4684	.4706	.4727	.4748	.4770	.4791	.4813	.4834	.4856	4	7	11	14	18
26	0.4877	.4899	.4921	.4942	.4964	.4986	.5008	.5029	.5051	.5073	4	7	11	15	18
27	0.5095	.5117	.5139	.5161	.5184	.5206	.5228	.5250	.5272	.5295	4	7	11	15	18
28	0.5317	.5340	.5362	.5384	.5407	.5430	.5452	.5475	.5498	.5520	4	8	11	15	19
29	0.5543	.5566	.5589	.5612	.5635	.5658	.5681	.5704	.5727	.5750	4	8	12	15	19
30	0.5774	.5797	.5820	.5844	.5867	.5890	.5914	.5938	.5961	.5985	4	8	12	16	20
31	0.6009	.6032	.6056	.6080	.6104	.6128	.6152	.6176	.6200	.6224	4	8	12	16	20
32	0.6249	.6273	.6297	.6322	.6346	.6371	.6395	.6420	.6445	.6469	4	8	12	16	20
33	0.6494	.6519	.6544	.6569	.6594	.6619	.6644	.6669	.6694	.6720	4	8	13	17	21
34	0.6745	.6771	.6796	.6822	.6847	.6873	.6899	.6924	.6950	.6976	4	9	13	17	21
35	0.7002	.7028	.7054	.7080	.7107	.7133	.7159	.7186	.7212	.7239	4	9	13	18	22
36	0.7265	.7292	.7319	.7346	.7373	.7400	.7427	.7454	.7481	.7508	5	9	14	18	23
37	0.7536	.7563	.7590	.7618	.7646	.7673	.7701	.7729	.7757	.7785	5	9	14	18	23
38	0.7813	.7841	.7869	.7898	.7926	.7954	.7983	.8012	.8040	.8069	5	9	14	19	24
39	0.8098	.8127	.8156	.8185	.8214	.8243	.8273	.8302	.8332	.8361	5	10	15	20	24
40	0.8391	.8421	.8451	.8481	.8511	.8541	.8571	.8601	.8632	.8662	5	10	15	20	25
41	0.8693	.8724	.8754	.8785	.8816	.8847	.8878	.8910	.8941	.8972	5	10	16	21	26
42	0.9004	.9036	.9067	.9099	.9131	.9163	.9195	.9228	.9260	.9293	5	11	16	21	27
43	0.9325	.9358	.9391	.9424	9457	.9490	.9523	.9556	.9590	.9623	6	11	17	22	28
44	0.9657	.9691	.9725	.9759	.9793	.9827	.9861	.9896	.9930	.9965	6	11	17	23	29

All tangents less than tan 45° are less than 1.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	+ Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	1.0000	.0035	.0070	.0105	.0141	.0176	.0212	.0247	.0283	.0319	6	12	18	24	30
46	1.0355	.0392	.0428	.0464	.0501	.0538	.0575	.0612	.0649	.0686	6	12	18	25	31
47	1.0724	.0761	.0799	.0837	.0875	.0913	.0951	.0990	.1028	.1067	6	13	19	25	32
48	1.1106	.1145	.1184	.1224	.1263	.1303	.1343	.1383	.1423	.1463	7	13	20	26	33
49	1.1504	.1544	.1585	.1626	.1667	.1708	.1750	.1792	.1833	.1875	7	14	21	28	34
50	1.1918	.1960	.2002	.2045	.2088	.2131	.2174	.2218	.2261	.2305	7	14	22	29	36
51	1.2349	.2393	.2437	.2482	.2527	.2572	.2617	.2662	.2708	.2753	8	15	23	30	38
52	1.2799	.2846	.2892	.2938	.2985	.3032	.3079	.3127	.3175	.3222	8	16	24	31	39
53	1.3270	.3319	.3367	.3416	.3465	.3514	.3564	.3613	.3663	.3713	8	16	25	33	41
54	1.3764	.3814	.3865	.3916	.3968	.4019	.4071	.4124	.4176	.4229	9	17	26	34	43
55	1.4281	.4335	.4388	.4442	.4496	.4550	.4605	.4659	.4715	.4770	9	18	27	36	45
56	1.4826	.4882	.4938	.4994	.5051	.5108	.5166	.5224	.5282	.5340	10	19	29	38	48
57	1.5399	.5458	.5517	.5577	.5637	.5697	.5757	.5818	.5880	.5941	10	20	30	40	50
58	1.6003	.6066	.6128	.6191	.6255	.6319	.6383	.6447	.6512	.6577	11	21	32	43	53
59	1.6643	.6709	.6775	.6842	.6909	.6977	.7045	.7113	.7182	.7251	11	23	34	45	56
60	1.7321	.7391	.7461	.7532	.7603	.7675	.7747	.7820	.7893	.7966	12	24	36	48	60
61	1.8040	.8115	.8190	.8265	.8341	.8418	.8495	.8572	.8650	.8728	13	26	38	51	64
62	1.8807	.8887	.8967	.9047	.9128	.9210	.9292	.9375	.9458	.9542	14	27	41	55	68
63	1.9626	.9711	.9797	.9883	.9970	**.0057**	.0145	.0233	.0323	.0413	15	29	44	58	73
64	2.0503	.0594	.0686	.0778	.0872	.0965	.1060	.1155	.1251	.1348	16	31	47	63	78
65	2.1445	.1543	.1642	.1742	.1842	.1943	.2045	.2148	.2251	.2355	17	34	51	68	85
66	2.2460	.2566	.2673	.2781	.2889	.2998	.3109	.3220	.3332	.3445	18	37	55	73	92
67	2.3559	.3673	.3789	.3906	.4023	.4142	.4262	.4383	.4504	.4627	20	40	60	79	99
68	2.4751	.4876	.5002	.5129	.5257	.5386	.5517	.5649	.5782	.5916	22	43	65	87	108
69	2.6051	.6187	.6325	.6464	.6605	.6746	.6889	.7034	.7179	.7326	24	47	71	95	119
70	2.7475	.7625	.7776	.7929	.8083	.8239	.8397	.8556	.8716	.8878	26	52	78	104	130
71	2.9042	.9208	.9375	.9544	.9714	.9887	**.0061**	.0237	.0415	.0595	29	58	87	116	144
72	3.0777	.0961	.1146	.1334	.1524	.1716	.1910	.2106	.2305	.2506	32	64	96	129	161
73	3.2709	.2914	.3122	.3332	.3544	.3759	.3977	.4197	.4420	.4646	36	72	108	144	180
74	3.4874	.5105	.5339	.5576	.5816	.6059	.6305	.6554	.6806	.7062	41	81	122	163	204
75	3.7321	.7583	.7848	.8118	.8391	.8667	.8947	.9232	.9520	.9812					
76	4.0108	.0408	.0713	.1022	.1335	.1653	.1976	.2303	.2635	.2972					
77	4.3315	.3662	.4015	.4373	.4737	.5107	.5483	.5864	.6252	.6646					
78	4.7046	.7453	.7867	.8288	.8716	.9152	.9594	**.0045**	.0504	.0970	Use ordinary interpolation.				
79	5.1446	.1929	.2422	.2924	.3435	.3955	.4486	.5026	.5578	.6140					
80	5.6713	.7297	.7894	.8502	.9124	.9758	**.0405**	.1066	.1742	.2432					
81	6.3138	.3859	.4596	.5350	.6122	.6912	.7720	.8548	.9395	**.0264**					
82	7.1154	.2066	.3002	.3962	.4947	.5958	.6996	.8062	.9158	**.0285**					
83	8.1443	.2636	.3863	.5126	.6427	.7769	.9152	**.0579**	.2052	.3572					
84	9.5144	.6768	.8448	**.0187**	.1988	.3854	.5789	.7797	.9882	**.2048**					
85	11.430	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.301	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.081	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.636	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.290	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					

The integral part of tangents in heavy-face type is 1 greater than preceding part.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	— Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	∞	573.0	286.5	191.0	143.2	114.6	95.49	81.85	71.62	63.66					
1	57.290	52.08	47.74	44.07	40.92	38.19	35.80	33.69	31.82	30.14					
2	28.636	27.27	26.03	24.90	23.86	22.90	22.02	21.20	20.45	19.74					
3	19.081	18.46	17.89	17.34	16.83	16.35	15.89	15.46	15.06	14.67					
4	14.301	13.95	13.62	13.30	13.00	12.71	12.43	12.16	11.91	11.66	Use ordinary interpolation.				
5	11.430	.2048	**.9882**	.7797	.5789	.3854	.1988	.0187	**.8448**	.6768					
6	9.5144	.3572	.2052	.0579	**.9152**	.7769	.6427	.5126	.3863	.2636					
7	8.1443	.0285	**.9158**	.8062	.6996	.5958	.4947	.3962	.3002	.2066					
8	7.1154	.0264	**.9395**	.8548	.7720	.6912	.6122	.5350	.4596	.3859					
9	6.3138	.2432	.1742	.1066	.0405	**.9758**	.9124	.8502	.7894	.7297					
10	5.6713	.6140	.5578	.5026	.4486	.3955	.3435	.2924	.2422	.1929					
11	5.1446	.0970	.0504	.0045	**.9594**	.9152	.8716	.8288	.7867	.7453					
12	4.7046	.6646	.6252	.5864	.5483	.5107	.4737	.4373	.4015	.3662					
13	4.3315	.2972	.2635	.2303	.1976	.1653	.1335	.1022	.0713	.0408					
14	4.0108	**.9812**	.9520	.9232	.8947	.8667	.8391	.8118	.7848	.7583					
15	3.7321	.7062	.6806	.6554	.6305	.6059	.5816	.5576	.5339	.5105	41	81	122	163	203
16	3.4874	.4646	.4420	.4197	.3977	.3759	.3544	.3332	.3122	.2914	36	72	108	144	180
17	3.2709	.2506	.2305	.2106	.1910	.1716	.1524	.1334	.1146	.0961	32	64	96	129	161
18	3.0777	.0595	.0415	.0237	.0061	**.9887**	.9714	.9544	.9375	.9208	29	58	87	116	144
19	2.9042	.8878	.8716	.8556	.8397	.8239	.8083	.7929	.7776	.7625	26	52	78	104	130
20	2.7475	.7326	.7179	.7034	.6889	.6746	.6605	.6464	.6325	.6187	24	47	71	95	119
21	2.6051	.5916	.5782	.5649	.5517	.5386	.5257	.5129	.5002	.4876	22	43	65	87	108
22	2.4751	.4627	.4504	.4383	.4262	.4142	.4023	.3906	.3789	.3673	20	40	60	79	99
23	2.3559	.3445	.3332	.3220	.3109	.2998	.2889	.2781	.2673	.2566	18	37	55	73	92
24	2.2460	.2355	.2251	.2148	.2045	.1943	.1842	.1742	.1642	.1543	17	34	51	68	85
25	2.1445	.1348	.1251	.1155	.1060	.0965	.0872	.0778	.0686	.0594	16	31	47	63	78
26	2.0503	.0413	.0323	.0233	.0145	.0057	**.9970**	.9883	.9797	.9711	15	29	44	58	73
27	1.9626	.9542	.9458	.9375	.9292	.9210	.9128	.9047	.8967	.8887	14	27	41	55	68
28	1.8807	.8728	.8650	.8572	.8495	.8418	.8341	.8265	.8190	.8115	13	26	38	51	64
29	1.8040	.7966	.7893	.7820	.7747	.7675	.7603	.7532	.7461	.7391	12	24	36	48	60
30	1.7321	.7251	.7182	.7113	.7045	.6977	.6909	.6842	.6775	.6709	11	23	34	45	56
31	1.6643	.6577	.6512	.6447	.6383	.6319	.6255	.6191	.6128	.6066	11	21	32	43	53
32	1.6003	.5941	.5880	.5818	.5757	.5697	.5637	.5577	.5517	.5458	10	20	30	40	50
33	1.5399	.5340	.5282	.5224	.5166	.5108	.5051	.4994	.4938	.4882	10	19	29	38	48
34	1.4826	.4770	.4715	.4659	.4605	.4550	.4496	.4442	.4388	.4335	9	18	27	36	45
35	1.4281	.4229	.4176	.4124	.4071	.4019	.3968	.3916	.3865	.3814	9	17	26	34	43
36	1.3764	.3713	.3663	.3613	.3564	.3514	.3465	.3416	.3367	.3319	8	16	25	33	41
37	1.3270	.3222	.3175	.3127	.3079	.3032	.2985	.2938	.2892	.2846	8	16	24	31	39
38	1.2799	.2753	.2708	.2662	.2617	.2572	.2527	.2482	.2437	.2393	8	15	23	30	38
39	1.2349	.2305	.2261	.2218	.2174	.2131	.2088	.2045	.2002	.1960	7	14	22	29	36
40	1.1918	.1875	.1833	.1792	.1750	.1708	.1667	.1626	.1585	.1544	7	14	21	28	34
41	1.1504	.1463	.1423	.1383	.1343	.1303	.1263	.1224	.1184	.1145	7	13	20	26	33
42	1.1106	.1067	.1028	.0990	.0951	.0913	.0875	.0837	.0799	.0761	6	13	19	25	32
43	1.0724	.0686	.0649	.0612	.0575	.0538	.0501	.0464	.0428	.0392	6	12	18	25	31
44	1.0355	.0319	.0283	.0247	.0212	.0176	.0141	.0105	.0070	.0035	6	12	18	24	30

The integral part of cotangents in heavy-face type is 1 less than preceding part.

°	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	– Differences				
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	1.0000	**.9965**	.9930	.9896	.9861	.9827	.9793	.9759	.9725	.9691	6	11	17	23	29
46	0.9657	.9623	.9590	.9556	.9523	.9490	.9457	.9424	.9391	.9358	6	11	17	22	28
47	0.9325	.9293	.9260	.9228	.9195	.9163	.9131	.9099	.9067	.9036	5	11	16	21	27
48	0.9004	.8972	.8941	.8910	.8878	.8847	.8816	.8785	.8754	.8724	5	10	16	21	26
49	0.8693	.8662	.8632	.8601	.8571	.8541	.8511	.8481	.8451	.8421	5	10	15	20	25
50	0.8391	.8361	.8332	.8302	.8273	.8243	.8214	.8185	.8156	.8127	5	10	15	20	24
51	0.8098	.8069	.8040	.8012	.7983	.7954	.7926	.7898	.7869	.7841	5	9	14	19	24
52	0.7813	.7785	.7757	.7729	.7701	.7673	.7646	.7618	.7590	.7563	5	9	14	18	23
53	0.7536	.7508	.7481	.7454	.7427	.7400	.7373	.7346	.7319	.7292	5	9	14	18	23
54	0.7265	.7239	.7212	.7186	.7159	.7133	.7107	.7080	.7054	.7028	4	9	13	18	22
55	0.7002	.6976	.6950	.6924	.6899	.6873	.6847	.6822	.6796	.6771	4	9	13	17	21
56	0.6745	.6720	.6694	.6669	.6644	.6619	.6594	.6569	.6544	.6519	4	8	13	17	21
57	0.6494	.6469	.6445	.6420	.6395	.6371	.6346	.6322	.6297	.6273	4	8	12	16	20
58	0.6249	.6224	.6200	.6176	.6152	.6128	.6104	.6080	.6056	.6032	4	8	12	16	20
59	0.6009	.5985	.5961	.5938	.5914	.5890	.5867	.5844	.5820	.5797	4	8	12	16	20
60	0.5774	.5750	.5727	.5704	.5681	.5658	.5635	.5612	.5589	.5566	4	8	12	15	19
61	0.5543	.5520	.5498	.5475	.5452	.5430	.5407	.5384	.5362	.5340	4	8	11	15	19
62	0.5317	.5295	.5272	.5250	.5228	.5206	.5184	.5161	.5139	.5117	4	7	11	15	18
63	0.5095	.5073	.5051	.5029	.5008	.4986	.4964	.4942	.4921	.4899	4	7	11	15	18
64	0.4877	.4856	.4834	.4813	.4791	.4770	.4748	.4727	.4706	.4684	4	7	11	14	18
65	0.4663	.4642	.4621	.4599	.4578	.4557	.4536	.4515	.4494	.4473	4	7	11	14	18
66	0.4452	.4431	.4411	.4390	.4369	.4348	.4327	.4307	.4286	.4265	3	7	10	14	17
67	0.4245	.4224	.4204	.4183	.4163	.4142	.4122	.4101	.4081	.4061	3	7	10	14	17
68	0.4040	.4020	.4000	.3979	.3959	.3939	.3919	.3899	.3879	.3859	3	7	10	13	17
69	0.3839	.3819	.3799	.3779	.3759	.3739	.3719	.3699	.3679	.3659	3	7	10	13	17
70	0.3640	.3620	.3600	.3581	.3561	.3541	.3522	.3502	.3482	.3463	3	7	10	13	16
71	0.3443	.3424	.3404	.3385	.3365	.3346	.3327	.3307	.3288	.3269	3	6	10	13	16
72	0.3249	.3230	.3211	.3191	.3172	.3153	.3134	.3115	.3096	.3076	3	6	10	13	16
73	0.3057	.3038	.3019	.3000	.2981	.2962	.2943	.2924	.2905	.2886	3	6	9	13	16
74	0.2867	.2849	.2830	.2811	.2792	.2773	.2754	.2736	.2717	.2698	3	6	9	13	16
75	0.2679	.2661	.2642	.2623	.2605	.2586	.2568	.2549	.2530	.2512	3	6	9	12	16
76	0.2493	.2475	.2456	.2438	.2419	.2401	.2382	.2364	.2345	.2327	3	6	9	12	15
77	0.2309	.2290	.2272	.2254	.2235	.2217	.2199	.2180	.2162	.2144	3	6	9	12	15
78	0.2126	.2107	.2089	.2071	.2053	.2035	.2016	.1998	.1980	.1962	3	6	9	12	15
79	0.1944	.1926	.1908	.1890	.1871	.1853	.1835	.1817	.1799	.1781	3	6	9	12	15
80	0.1763	.1745	.1727	.1709	.1691	.1673	.1655	.1638	.1620	.1602	3	6	9	12	15
81	0.1584	.1566	.1548	.1530	.1512	.1495	.1477	.1459	.1441	.1423	3	6	9	12	15
82	0.1405	.1388	.1370	.1352	.1334	.1317	.1299	.1281	.1263	.1246	3	6	9	12	15
83	0.1228	.1210	.1192	.1175	.1157	.1139	.1122	.1104	.1086	.1069	3	6	9	12	15
84	0.1051	.1033	.1016	.0998	.0981	.0963	.0945	.0928	.0910	.0892	3	6	9	12	15
85	0.0875	.0857	.0840	.0822	.0805	.0787	.0769	.0752	.0734	.0717	3	6	9	12	15
86	0.0699	.0682	.0664	.0647	.0629	.0612	.0594	.0577	.0559	.0542	3	6	9	12	15
87	0.0524	.0507	.0489	.0472	.0454	.0437	.0419	.0402	.0384	.0367	3	6	9	12	15
88	0.0349	.0332	.0314	.0297	.0279	.0262	.0244	.0227	.0209	.0192	3	6	9	12	15
89	0.0175	.0157	.0140	.0122	.0105	.0087	.0070	.0052	.0035	.0017	3	6	9	12	15

All cotangents greater than cot 45° are less than 1.

Exercises. Review

1. Find the side of the greatest square that can be milled on the round shaft shown on page 153.

After solving by using sin 45°, the student should compare the solution with the solution by the formula given in the blueprint on page 119.

2. Find the side of the square which has a diagonal of 0.675″; of 3.062″; of 7.245″.

In workshop practice the diagonal is often called the *long diameter*.

3. Find to the nearest $\frac{1}{64}''$ the side of the greatest square that can be milled on a round shaft $1\frac{15}{16}''$ in diameter.

4. In the drawing of the bay window find the angles which c makes with the vertical and with the horizontal.

In referring to the angle which a line makes with another line we always mean the acute angle unless otherwise specified. In this case first find either tangent, and from this find one of the angles.

5. Find the angle of slope of the gable roof, that is, the angle which the rafter makes with the horizontal.

6. After solving Ex. 5 find the length of the rafter.

Since $\sin A = a \div c$, it is evident that $c \sin A = a$ and $c = a \div \sin A$.

7. Find the depth of the sharp V-thread, given the pitch, or distance between two successive threads, as shown.

Find the depth of a sharp V*-thread, given the pitch as follows:*

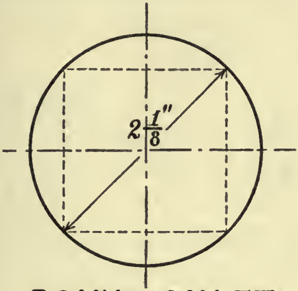
8. $\frac{1}{9}''$. **9.** $\frac{1}{16}''$. **10.** $\frac{3}{32}''$. **11.** $\frac{9}{16}''$. **12.** $\frac{7}{8}''$.

13. The drill jig has six holes evenly spaced on a circle of diameter $15\frac{1}{2}''$. Find the distance between the successive holes.

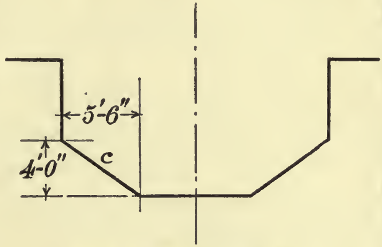
This means the straight-line distance between the centers.

In Ex. 13 find the distance if the number of holes is as follows:

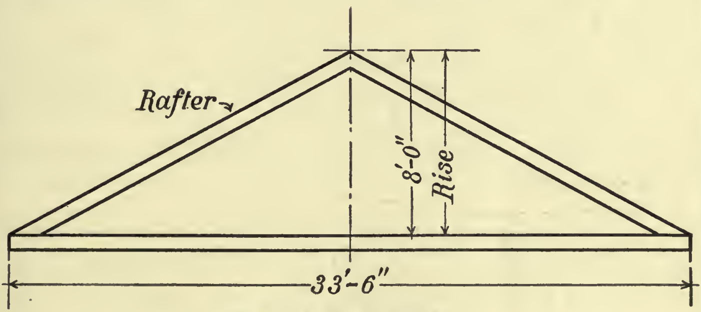
14. 5. **15.** 7. **16.** 8. **17.** 9. **18.** 12.



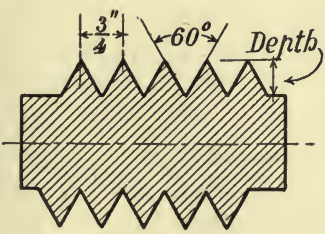
ROUND SHAFT



BAY WINDOW



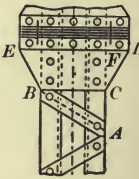
GABLE ROOF



CROSS SECTION of SHARP V-THREAD

DRILL JIG

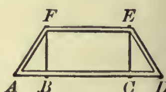
19. In this piece of construction work $BC = 14''$ and makes an angle of $30°$ with AB. Find the length of the brace AB and also of AC, the distance that the brace is set off from BC.



20. In the figure of Ex. 19 it is known that $BE = CD = 10\frac{1}{2}''$ and that each makes an angle of $60°$ with DE. Find the length of the line CF.

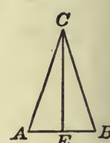
21. From Exs. 19 and 20 find the area of the trapezoid $BCDE$.

22. A steel bridge has a truss $ADEF$, as here shown. It is known that $AD = 60'$, $FE = 36'$, and $BF = 19'$. Find to the nearest degree the angle of slope which AF makes with AD.



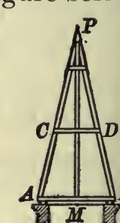
Find the value of some function of the angle A and then find the angle which comes nearest to having that value for the function used.

23. The principle of a range finder is that of an isosceles triangle. The eye is at E, and an object C is reflected at both A and B to a prism at E. The instrument is arranged so that it can be adjusted to focus the lines AC and BC on the object C. If $AE = EB = 10''$, find EC in yards when $\angle A = \angle B = 89° \, 54'$.



Practically, the distances are computed in this way when the instrument is made, and are read off in yards by the observer on a scale which is mounted inside the range finder.

24. In constructing the spire represented in the figure below it is planned to have $AB = 42'$ and $PM = 92'$. What angle of slope must the builder give to AP?



25. In Ex. 24 find the length of AP and find the angle APB.

26. In the figure of Ex. 24 the brace CD is put in $38'$ above AB. What is its length?

CHAPTER V

THE SLIDE RULE

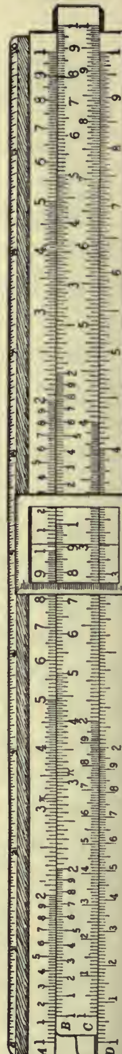
Nature of the Slide Rule. The slide rule is an instrument which consists of two rulers, one of which slides along the other. These rulers are so marked that by adding the numbers corresponding to two marks, which is done by simply sliding one of the rulers along the other, we can readily find the product of two numbers. The slide rule can also be used for division and for finding powers and roots.

We shall not attempt to explain the principle on which the slide rule is constructed, but we shall give general directions for its use. Unless the student has a slide rule of his own, however, and can actually work with it as he reads these directions, he cannot acquire the necessary facility.

Accuracy of the Work. Since all measurements are only approximately accurate, practically we need to have the results of computation relating to measurements only approximately accurate. That is, if we are able to make a certain measurement accurately to 0.01″, no computation based upon it need be carried beyond 0.01″, but the computation must be accurate to that point. The slide rule gives only approximate results, but results that are accurate within certain definite limits, depending upon the size of the rule.

The slide rule is used very extensively by mechanics and engineers and also affords one of the most convenient checks on various mathematical operations. Even if the slide rule used is so small as to give only three figures accurately, this may suffice for the purpose of checking.

Slide Rule. The illustration below shows a common and convenient form of the slide rule.



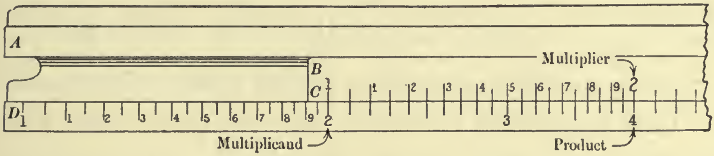
The rule in the middle slides along the two outer rules. The glass plate in the middle, known as the *runner* or *cursor*, slides either way, a hair line being ruled upon it to facilitate reading the results. Sometimes a magnifying glass is attached to the runner for greater ease in reading and to allow a higher degree of accuracy in setting the slide.

Scales A and B are alike. Each scale is duplicated, one half running from 1 to 10, and the other half also running from 1 to 10, for reasons explained later. Scales A and B are used when, owing to the different graduation of C and D, the latter are not convenient.

Scales C and D are also alike, but they are not the same as A and B. If you measure the distance from 1 to 2 (not the tenths) on C you will see that it is twice the distance from 1 to 2 on the A or B scale; in fact, the 2 on C is exactly below the 4 on B. Hence, on C and D it is easy to find the mark corresponding to 125, to judge the position of 1255 very closely, and even to judge with a fair degree of accuracy the position of such a number as 1257.

It is needless to say that no explanation of a slide rule given in a book is at all complete. No one can learn to use a slide rule without having one to work with, and no written explanation is ever as satisfactory as one given by an instructor with an instrument at hand. Large slide rules that can be read across the room can be obtained for purposes of instruction, but they are not necessary if each member of a class has a rule of his own.

Multiplication. Suppose that we wish to multiply one number by another, taking for purposes of illustration the simple case of 2×2. We place 1 on *C* exactly over 2 on *D*. We then look for 2 on *C* and find that it is exactly over 4 on *D*, and hence we see that 4 is the product of 2 and 2.



Similarly, in the same figure, the product of 2×1.5 is found just below the 5 which is between 1 and 2; that is, just below 1.5. Hence we see that $2 \times 1.5 = 3$. We have here added mechanically the length marked 2 and the length marked 1.5, the resulting length being marked 3.

If necessary, such terms as *multiplier*, *multiplicand*, *divisor*, *dividend*, and *quotient* should be informally explained.

The scheme of multiplication is seen from the following:

C	Set 1	Under 1.2	*C*	Set 1	Under 2.8
D	Over 1.5	Read 1.8	*D*	Over 2.1	Read 5.88
$1.2 \times 1.5 = 1.8$			$2.8 \times 2.1 = 5.88$		

That is, *place the 1 on C over the multiplicand on D and read the product on D below the multiplier on C. Determine the position of the decimal point by considering the numbers.*

In setting the left-hand index, the figure 1, on *C* over the multiplicand, if we find the multiplier off the rule we merely move the slide to the left, setting the right-hand index over the multiplicand. We then read the product under the multiplier. This has the same effect as repeating the *D* scale under the *C* scale when it protrudes to the right.

Exercises. Multiplication

1. A cubic foot of water weighs 62.5 lb. and the specific gravity of a certain grade of steel is 8. Find the weight of 1 cu. ft. of this grade of steel.

The problem requires the multiplication of 62.5 by 8. Place the right-hand 1 on *C* over 6.25 on *D* and read the result 500 on *D* just below the 8 on *C*, making due allowance for the decimal point.

2. The specific gravity of a certain grade of cast iron is 7.2. Find the weight of 1 cu. ft. of this grade of cast iron.

Using the slide rule, perform each of these multiplications:

3. 6×9.	**8.** 3.1×4.7.	**13.** 1.75×48.
4. 6×90.	**9.** 4.5×3.6.	**14.** 1.75×4.8.
5. 6×98.	**10.** 3.9×7.2.	**15.** 2.15×3.9.
6. 6×9.8.	**11.** 2.7×8.7.	**16.** 3.35×5.68.
7. 0.6×0.98.	**12.** 4.4×7.8.	**17.** 5.25×9.76.

18. Find the circumference of a circle of diameter $9''$.

We have to find $\pi \times 9$. Most slide rules have a mark on the *A* and *B* scales for π; that is, for $3\frac{1}{7}$. If the student's rule is not marked for π, he should use 3.14 as the value of the ratio.

19. Find the circumference of a circle of radius $6\frac{1}{2}''$.

20. An iron pillar has a diameter of $7.2''$. Find the circumference of the pillar.

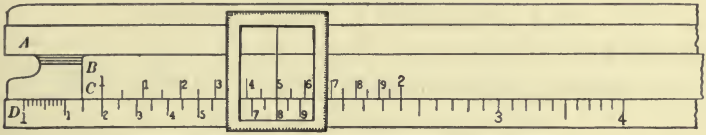
21. A box is $12.7''$ long and $7.9''$ wide. Find the area of the bottom of the box.

22. How many inches of wire will be needed to make 100 rings, each of which is $2.5''$ in diameter?

23. By the aid of the slide rule find whether 27×43 or $15\frac{1}{2} \times 84$ has the larger product.

24. As in Ex. 23, compare 23.8×64.8 and 32.4×47.6.

Continued Multiplication. By using the runner we are able to perform continued multiplication without having to read the intermediate products. For example, suppose that we wish to find the product in the case of $12 \times 15 \times 20$.



In the illustration the minor subdivisions are omitted. We set the 1 on C above 12 on D and place the runner so that the hair line crosses 15 on C. Now, instead of reading the result, bring 1 on C exactly under the hair line, and under 2 on C read the result 3600 on D.

The student must use his judgment as to how accurately the result may be given, depending upon the size of the slide rule used.

Exercises. Continued Multiplication

1. Find the volume of a box $8'' \times 12'' \times 18''$.

2. At $6.50 a day, how much will 16 men earn in 5 da.?

3. A bill of goods amounting to $2500 is allowed discounts of 8 and 10. Find the net amount.

Remember that discounts of 8 and 10 mean discounts of 8%, 10%, and that in this case we have to find the value of $0.9 \times 0.92 \times \$2500$.

4. Find the cost of 7 doz. cylinder priming cups for an automobile at 65¢ each.

We have to find the value of $7 \times 12 \times \$0.65$. The result will somewhat exceed half of 7×12, and hence will have two integral places.

5. Find the cost of 6 doz. exhaust-pipe flange gaskets for an automobile at 19¢ each.

Find the value of each of the following:

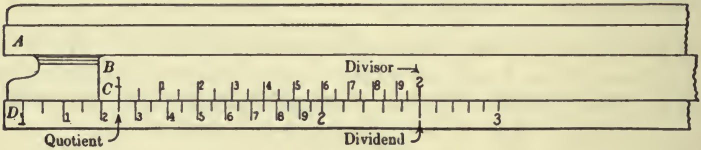
6. $4 \times 5 \times 6$. **8.** $22 \times 30 \times 60$. **10.** $37 \times 26 \times 75$.

7. $3 \times 6 \times 9$. **9.** $17 \times 13 \times 19$. **11.** $14 \times 19 \times 225$.

Division. We perform division on the slide rule by simply reversing multiplication. Expressed graphically we have:

C	Set divisor	Under 1 (right or left)
D	Over dividend	Read quotient

For example, suppose that we wish to divide 25 by 2.



As shown above, we place the divisor 2 on *C* over the dividend 25 on *D* and read 125 on *D* under the left-hand 1 on *C*. Since it is evident where the decimal point belongs, we write 12.5 as the result.

For example, suppose that we wish to divide 350 by 56. We place 56 on *C* over 350 on *D*, and under the right-hand 1 on *C* read 625 on *D*. Since we have 350 divided by a number a little over 50, the result must be a little less than 7. Hence we should place the decimal point after the first 6; that is, the result is 6.25.

There are special rules for determining where to place the decimal point in the quotient, but for these rules the student should consult the manuals which come with most slide rules.

Exercises. Division

Perform each of the following divisions:

1. $9 \div 3$.

2. $3 \div 9$.

3. $46 \div 23$.

4. $31 \div 93$.

5. $75 \div 2.5$.

6. $12.5 \div 2.5$.

7. $17.28 \div 12$.

8. $1.728 \div 1.2$.

9. $6250 \div 25$.

10. $312.5 \div 2.5$.

11. $1.584 \div 12$.

12. $15.84 \div 1.2$.

13. $3885 \div 35$.

14. $38.85 \div 3.5$.

15. $6970 \div 41$.

16. $70.11 \div 4.1$.

17. $9300 \div 6.2$.

18. $5.329 \div 73$.

Continued Multiplication and Division. In practical work we sometimes need to make calculations involving continued multiplication and division, such as the following:

$$\frac{4.25 \times 67.3 \times 300}{773 \times 0.07}$$

Using the runner, we first perform the continued multiplication of the numerator, placing the hair line over the final product. We then divide this result by 773, and divide the result found by this division by 0.07. The figures of the final result are 1, 5, 8, 6.

To determine the position of the decimal point we simply notice that the numerator is about 300 × 300, or 90,000. The denominator is about 0.07 × 800, or 56. Then 90,000 ÷ 56 is about 9000 ÷ 6, or about 1500. Hence we see that the result must be 1586.

Exercises. Multiplication and Division

1. The pull P of a locomotive is given by the formula $P = d^2ps/D$, where d is the diameter of each cylinder in inches, p the pressure of steam in pounds per square inch, s the length of the stroke in inches, and D the diameter of the drive wheels in inches. Find the value of P, given that $d = 10.5$, $p = 140$, $s = 18$, and $D = 36$.

2. For a train going at the rate of s feet per second round a curve of radius r feet the outer rail should be raised h inches above the inner, where $h = s^2G/386\,r$, in which G is the number of feet in the gage of the track. Taking $s = 60$, $r = 2700$, and $G = 4.7$, find the value of h.

Find the value of each of the following:

3. $\dfrac{3 \times 50}{5 \times 6}$.

4. $\dfrac{2 \times 7}{3 \times 11}$.

5. $\dfrac{14 \times 5 \times 81}{5 \times 7 \times 9}$.

6. $\dfrac{4 \times 40 \times 59}{6 \times 8 \times 7}$.

7. $\dfrac{130 \times 65 \times 7}{300 \times 24}$.

8. $\dfrac{65 \times 92.5}{22.5 \times 47.5}$.

Proportion. One of the features of the slide rule is that, however the slide is placed, all numbers on the slide are proportional to the numbers on the rule that coincide with them. That is, when 2 is over 4, 3 is over 6, 4 is over 8, 2.5 is over 5, 1.2 is over 2.4, and so on; in other words,

$$2:4=3:6=4:8=2.5:5=1.2:2.4=\cdots.$$

We can therefore use the slide rule in solving proportions. For example, given $1.5:3.7=4.1:x$, find the value of x.

We set 1.5 over 3.7, and read 10.11 under 4.1. In this case, since the result exceeds 10, the limit of the D scale, we use the A and B scales.

We may express the rule diagrammatically as follows:

C	Set first term	Then under third term
D	Over second term	Read fourth term

Exercises. Proportion

1. Given that 1 kg. is equivalent to 2.2 lb., set the slide so as to read the pounds corresponding to kilograms.

2. A certain map is drawn to the scale $1'' = 80$ mi. Set the slide so as to read the miles corresponding to inches.

3. Given that 1 m. is equivalent to $39.37''$, set the slide so as to read the inches corresponding to meters.

4. If the wages of 7 men for 1 da. are $31.50, find the wages of 25 men for 1 da. at the same rate.

5. If a factory can turn out 6480 pairs of shoes in 6 da., how long will it take to fill an order for 36,000 pairs?

Find the value of x in each of the following proportions:

6. $7:3=5:x$.
7. $x:7=2.3:4$.
8. $x:9=17:6.3$.
9. $5:8=7.2:x$.
10. $0.5:7=3:x$.
11. $x:0.7=2.3:4$.

Squares and Square Roots. By examining the slide rule we see that the numbers on A are the squares of the numbers just below them on D. Hence, to find the square of any number we place the hair line of the runner over the number on D and read the square under the hair line on A.

Conversely, to find the square root of any number we place the hair line of the runner over the number on A and read the square root under the hair line on D.

The slide rule may also be conveniently used for the purpose of evaluating such expressions as a^2b. For example, to find the value of $8^2 \times 5$ we have the following arrangement:

A		Read 320. *Ans.*
B	Set 1 (right)	Over 5
C		
D	Over 8	

That is, we set 1 on B over 8 on D, and over 5 on B read the answer, 320, on A. In this way we can find the value of πr^2, the area of a circle.

We can also find the value of an expression in the form of $\sqrt{a/b}$. Thus, to find the value of $\sqrt{\frac{3}{4}}$ we have the following:

A	Under 3	
B	Set 4	Under 1 (right)
C		
D		Read 0.866. *Ans.*

That is, we set 4 on B under 3 on A, and under 1 on B read the answer, 0.866, on D. In this way we can easily find the value of $\sqrt{a/\pi}$; that is, given the area of a circle we can find the radius.

Exercises. Review

Using the slide rule, find the square of each of the following:

1. 4. **3.** 7. **5.** 16.7. **7.** 38.5. **9.** 61.7.
2. 8. **4.** 12. **6.** 17.8. **8.** 5.29. **10.** 3.14.

Find the square root of each of the following:

11. 4. **13.** 16. **15.** 64. **17.** 144. **19.** 14.4.
12. 9. **14.** 49. **16.** 81. **18.** 1.44. **20.** 3.81.

21. Find the value of πr^2 when $r = 28.2''$.

22. Find the area of a circle which has a radius of $2.7''$.

23. Find the radius that should be used in drawing a circle which shall have an area of 29 sq. in.; an area of 42 sq. in.

Find the value of each of the following:

24. 3.1×16.1^2. **26.** 32×6.5^2. **28.** 9×2.75^2.
25. 3.34×2.9^2. **27.** 7.2×23^2. **29.** 0.7×2.2^2.

30. Find the diameter of a cylindric iron rod which has a cross-section area of 9.3 sq. in.

31. What is the cross-section area of a cylindric iron rod which has a diameter of $4.7''$? a radius of $1.5''$?

Using the slide rule, find the value of each of the following:

32. $7 \sin 41°$. **35.** $15 \sin 45°$. **38.** $12 \sin 17° 24'$.
33. $9 \tan 25°$. **36.** $16 \tan 50°$. **39.** $2.6 \cos 36° 20'$.
34. $8 \cos 27°$. **37.** $2.8 \cos 35°$. **40.** $32.8 \tan 58° 5'$.

To find $7 \sin 41°$, first find $\sin 41° = 0.6561$ on page 144, and then multiply 0.6561 by 7. It will thus be seen that the slide rule is often convenient when using trigonometry.

41. Find the quotient of $2\sqrt{576}$ divided by $\sin 45°$.

42. Find the value of $\frac{1}{7} \cos 35°$; of $0.2 \div \sin 20°$.

CHAPTER VI

GENERAL APPLICATIONS

Exercises. Review

1. There are 48 rivets in a length of $17' \, 8\frac{3}{4}''$ measured along a riveted seam. Find the pitch of the rivets.

The *pitch* is the distance apart, measured between centers.

2. From a large can containing $4\frac{3}{4}$ gal. of oil the toolroom boy fills five cans which hold $\frac{3}{8}$ gal. each, and one which holds $1\frac{1}{2}$ gal. How much oil is left in the large can?

3. How many linear feet of molding will be needed for a room $23' \, 9'' \times 31' \, 8''$? for a room $32' \, 6'' \times 38' \, 9''$?

4. Find the cost of $2270'$ of pine lumber at $105 per M.

5. At $6\frac{3}{4}$¢ per pound, how many pounds of iron can be bought for $175? for $350? for $475?

Results of this kind should be given only to the nearest pound.

6. An electrician's helper works $7\frac{3}{4}$ hr. a day at $66\frac{2}{3}$¢ an hour. How much does he receive per day?

7. If 375 lb. of fire clay cost $2.50, how much will 1000 lb. cost? How much will $2\frac{1}{2}$ T. cost?

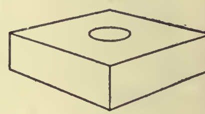
Results of this kind should be given only to the nearest cent.

8. How many planks $7''$ wide can be laid side by side across a beam $16' \, 11''$ long?

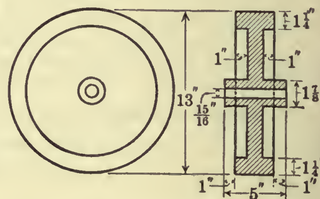
9. If it takes $1\frac{1}{8}$ lb. of Babbitt metal for one bearing, how many pounds will it take to babbitt 920 bearings?

10. A stream is 164′ wide and averages $6\frac{1}{2}'$ in depth. If it has a flow of $3\frac{1}{2}$ mi./hr., how many gallons of water flow by in 1 hr.? How many tons of water flow by in 24 hr.?

Water weighs $62\frac{1}{2}$ lb./cu. ft., and 1 cu. ft. contains $7\frac{1}{2}$ gal.



11. At 11¢ a pound, find the cost of 150 blank nuts made of cold-rolled steel weighing 0.2816 lb./cu. in., the blanks being $3\frac{7}{8}''$ square and $1\frac{3}{4}''$ thick, and the center hole, as shown in the figure above, being $2\frac{1}{6}\frac{1}{4}''$ in diameter.



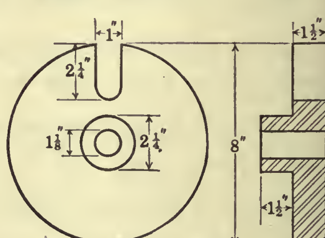
12. Find the weight of a cast-iron pulley of the dimensions here given. At $6\frac{3}{4}$¢ a pound, find the cost of 1675 pulleys of this type.

The student should use 0.26 lb. as the weight of 1 cu. in. of cast iron.

13. An automobile manufacturer makes 115 lb. of steel into ball bearings $\frac{3}{4}''$ in diameter. How many does he make?

Take 0.2816 lb. as the weight of 1 cu. in. of this kind of steel.

14. At 500 cu. ft. to the ton, how many tons of hay will fill a hay mow 27′ 9″ long, 19′ 3″ wide, and 11′ 6″ high?



15. At $7\frac{1}{2}$¢ a pound, what is the cost of 75 steel lathe face-plates of the kind here shown?

Use the weight of steel given in Ex. 13 above.

16. In Ex. 15 suppose that the price were $8\frac{1}{4}$¢ a pound.

17. If a mechanical trench digger removes earth at the rate of 1450 cu. yd./hr., how many days of 8 hr. each will it take to dig a trench $6\frac{1}{2}'$ wide, 7′ deep, and $8\frac{1}{2}$ mi. long?

18. If brass castings shrink $\frac{3}{16}''$ per foot of finished dimensions, find the length of the pattern for each of the following lengths of finished castings: $2'\,9''$, $9'\,4''$, $5'\,3''$, $1'\,11''$.

19. Find the length of the piece of stock necessary to make 25 machine bolts, each $1\frac{7}{8}''$ long, allowing $\frac{3}{4}''$ on each bolt for cutting and finishing.

20. If a motor makes 560 R.P.M., how many revolutions does it make in 8 hr.?

21. If carriage bolts of a certain size weigh 19.2 lb. per 100, what is the weight of 1975 bolts of this size? How many bolts will it take to weigh 384 lb.?

22. Neglecting any allowance for the seams, find the cost of the sheet iron required for a smokestack $25'$ high and $20''$ in diameter at 29¢ per square foot.

23. On the arbor of a milling machine a machinist places five collars measuring respectively $0.434''$, $0.968''$, $0.250''$, $0.625''$, $0.5156''$. Find the total length of the collars.

24. How many pieces, each $1.45''$ long and the width of the plank, can be sawed from a plank $16.75'$ long, making no allowance for waste? How long a piece is left over?

25. On a lathe making 175 R.P.M. the cutting tool advances $0.015''$ per revolution. How far does the tool travel in 1 min.? in 8 min.?

26. If a contractor receives \$973.08 for excavating a cellar at \$1.06 per cubic yard, how many cubic yards does he excavate?

27. In Ex. 26 suppose that the cellar were twice as long, twice as wide, and 50% deeper, how much would the contractor receive at the same rate?

28. In Ex. 27, if the rate were \$1.12½ per cubic yard, how much would the contractor receive?

29. A casting weighed $625\frac{1}{4}$ lb. before it was machined, and $598\frac{7}{8}$ lb. when finished. If it cost $37\frac{1}{2}$¢ per pound to produce the finished casting, including labor but not counting the value of the scrap, and the scrap was sold at $3\frac{1}{2}$¢ per pound, what was the actual cost of production after the value of the scrap was deducted?

30. How many tons of iron conduit weighing 1066 lb./100′ are needed in a building for which the specifications require 6270′ of the conduit?

31. How many pieces, each 3.15″ long, can be sawed from a steel bar 27.9′ long, the thickness of the saw being 0.162″, and how long a piece will be left over?

32. How many shingles does a carpenter use in shingling a barn if he lays 88 rows with 128 shingles to a row? If when buying the shingles he gave his order to the next higher $\frac{1}{2}$ M required, find the cost of the shingles at $9.87 per M.

33. The width of a street was 54.6′ after improvements were made by which the street was widened 5%. How wide was the street before the improvements were made?

34. If soft-steel bars are selling at $3.57 per 100 lb., and if they sold a year ago at $4.20, what is the per cent of decrease in price?

35. Find the cost of laying a cement walk 4′ 11″ wide and 50′ 8″ long, at $33\frac{1}{3}$¢ per square foot; at $37\frac{1}{2}$¢ per square foot; at $37\frac{7}{8}$¢ per square foot.

36. A builder contracted to erect a house for $9450. When the house was completed he found that the actual cost was $8032.50. Find the rate of profit on the contract price; on the cost price.

37. How many feet of lumber will it take to make 3 doz. drawing boards, each $2' \, 3'' \times 3' \, 2'' \times \frac{3}{4}''$?

38. How many revolutions will an automobile wheel 32″ in diameter make while the car is traveling 1 mi.?

In such cases the slipping of the wheel and similar factors are not to be considered unless the contrary is expressly stated.

39. If a compositor earns $207 in 4 wk., how many weeks will it take him to earn $1242?

40. The stock for a certain job of printing cost $27.90, and the printing itself cost $11.25. If the printer figures 18% profit on the stock and 35% profit on the printing, these items of profit to cover his profit and all overhead and other charges, for what amount does he bill the job?

Such a bill would, in ordinary practice, be figured to the next higher 25¢ or 50¢, although on a larger order the bill might be figured to the next higher $1. In this case take the next higher 25¢.

41. Making no allowance for waste, how many sheets of blueprint paper, each 11″×15″, can be cut from a roll 30′ wide and 10 yd. long?

42. The diameters of a #10 wire are given on different wire gages as follows:

Brown & Sharpe	0.1019″
Birmingham	0.1340″
Washburn	0.1350″
Trenton	0.1300″
Imperial	0.1280″

Which two gages differ the most in size, and by how much do they differ?

43. If a dealer gains 20% on the cost of a battery when he sells it for 60¢, what per cent would he gain on the cost if he should sell it for 65¢?

44. If a train travels 369 mi. in 9 hr., how far does it travel at the same rate in 23 hr. 15 min.?

45. In a lot of 950 cast-iron pulleys $7\frac{1}{2}\%$ are rejected on account of defects. How many pulleys are rejected and how many are accepted?

Since $7\frac{1}{2}\%$ of 950 gives a number involving a decimal, the student should use his common sense and take the nearest whole number. The per cent rejected cannot then be exactly $7\frac{1}{2}$, but in practical work we should rarely say that $7\frac{9}{19}\%$ of the pulleys were rejected.

46. At 9 lb./ft., find the total weight of 6 pieces of pipe 3′ 6″ long, 7 pieces 4′ 2″ long, and 5 pieces 6′ 9″ long.

47. If a dealer buys zinc sheets at $215.50 per ton, at what price per pound must he sell them in order to gain 45% on the cost?

48. How long will it take a pump delivering 1.6 gal. of water at a stroke and making 75 strokes per minute to pump 1500 gal. of water?

49. An electrical-appliance dealer bought the following bill of goods: 45 cast-bronze push buttons @ 56¢ each; 5625′ #18 annunciator wire, 150′/lb., @ 48¢ a pound; 30 steel outlet boxes @ $21.65 per 100. Make out the bill and find the total cost of the goods.

50. If in Ex. 49 the dealer took advantage of a $2\frac{1}{2}\%$ discount for cash within 10 da., how much did he save? What was the net amount which he paid for the goods?

51. Allowing an average space $8' \times 12'$ for each car, this allowance covering the aisle space, how many cars can be stored in a garage 90′ square?

52. A carpenter's wages were increased 15%, the increase amounting to 75¢ a day. How much was he getting per day before the increase and how much was he getting thereafter? If his wages were later decreased 8%, how much was he then getting per day?

53. If lead is worth \$98 per ton, how much is 750 lb. worth? If a dealer bought lead at this rate and sold it at 8¢ a pound, what per cent of profit did he make on the cost?

As already stated, a ton is to be taken as 2000 lb. unless the long ton of 2240 lb. is specifically mentioned.

54. Find the weight in tons of the rails required for 1 mi. of double-track railway, the rails weighing 120 lb. per yard.

55. If the speed of an engine which is running at the rate of 98 R.P.M. is increased $8\frac{1}{2}\%$, what is the number of R.P.M. after the speed is increased?

In problems dealing with the speeds of engines, pulleys, and the like, the result should be given to the nearest whole number.

56. If the speed of an engine which is running at the rate of 400 R.P.M. is increased 10%, what is then the number of R.P.M.? If this new speed is decreased 10%, what is then the number of R.P.M.?

57. By selling a lathe for \$183 above cost a dealer gained 15% on the cost. How much did the lathe cost the dealer?

58. By selling a lathe for \$1840 a dealer gained 15% on the cost. How much did the lathe cost the dealer?

59. An agent bought three motors for \$120, \$160, and \$190 respectively. He sold the first at a loss of 8%, the second at cost, and the third at a profit of 6%. Find his total profit or loss on the motors.

60. A contractor bought 75 M bricks at \$11.50 per M. If he sold $\frac{2}{5}$ of the bricks at $\frac{3}{4}$ their cost, and sold the rest for \$500, how much did he lose?

61. If a piece of iron 8′ long, 6″ wide, and 4″ thick weighs 600 lb., what is the weight of a piece of iron that is 13′ long, 8″ wide, and 5″ thick?

62. Find the cross-section area of the cast-iron beam shown in the blueprint on page 173.

In such cases neglect entirely the rounding of the corners.

63. How many square feet are there in the floor of the machine shop, the plan of which is shown in the blueprint?

64. If there are 15 machines in the shop in Ex. 63, what is the average number of square feet of floor space per machine?

65. In the machine shop in Ex. 63 it is desired to install six new machines, each of which requires 275 sq. ft. of floor space. If the additional floor space is obtained by building an addition along the entire length of the right side of the shop, how wide is the addition?

66. Find the area of the boiler patch shown in the blueprint.

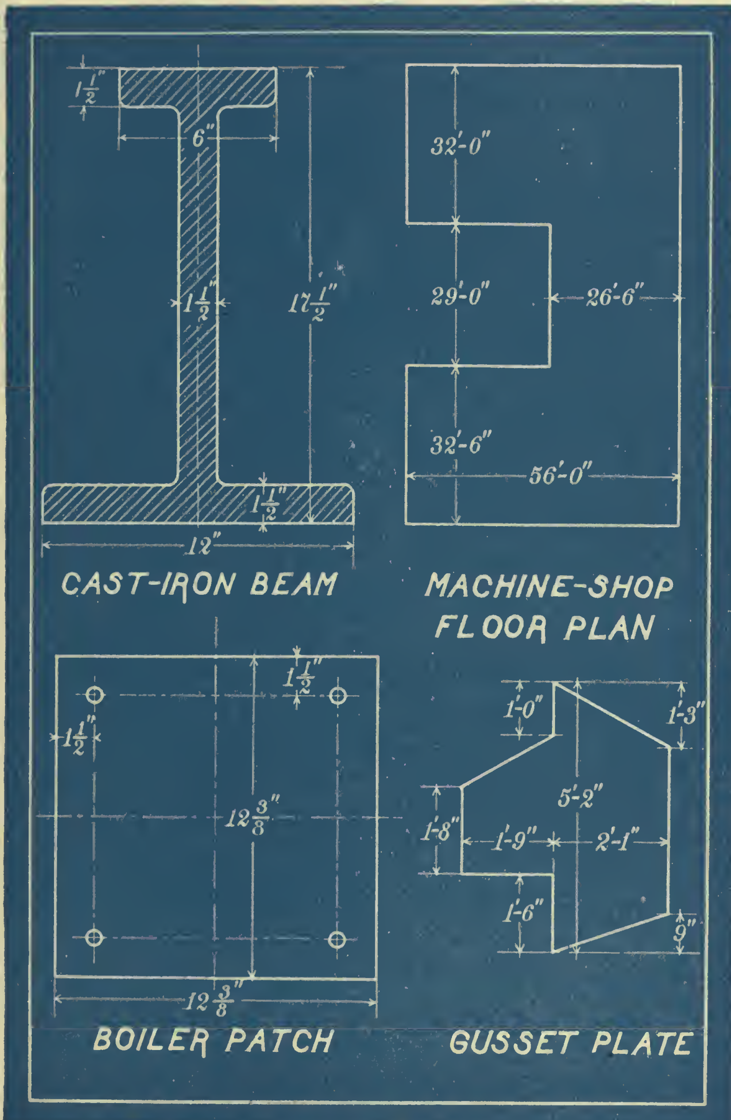
67. If the boiler has a pressure of 225 lb./sq. in., and the boiler patch covers an opening $8\frac{7}{8}''$ square, how much pressure is exerted against it?

68. The outside row of rivets on the boiler patch is to extend all the way around the patch as indicated in the blueprint. How many rivets are needed for this row if the rivets are to be spaced $1\frac{7}{8}''$ apart?

69. The gusset plate shown in the blueprint is made of #4 U.S. standard-gage steel weighing 9.525 lb./sq. ft. Find the weight of the plate.

70. If each side of the gusset plate were half as long again, what would then be the weight of the plate?

71. The outlet box of a heating system is rectangular in shape and is $4'\ 2''$ long and $14''$ wide. A pipe with a square cross section leads from this box, and the area of the cross section is the same as the area of the bottom of the outlet box. Find the length of a side of the square pipe.



1½"
6"
1½"
17½"
1½"
12"
CAST-IRON BEAM
32'-0"
29'-0"
26'-6"
32'-6"
56'-0"
MACHINE-SHOP FLOOR PLAN
1½"
1½"
12⅜"
12⅜"
BOILER PATCH
1'-0"
1'-3"
5'-2"
1'-8"
1'-9"
2'-1"
1'-6"
9"
GUSSET PLATE

72. If $\frac{1}{32}''$ is turned off the outside of the piston ring shown in the blueprint on page 175, by how many square inches is the area of the cross section of the ring reduced?

73. Find the cross-section area of the metal in the steam pipe shown in the blueprint.

74. Find the internal cross-section area of the steam pipe shown in the blueprint.

75. Find the number of square feet in the head of the boiler shown in the blueprint, without deducting the area taken up by any of the openings.

76. The inside diameter of each tube being $2.265''$, find the total area in square feet of the internal cross sections of the tubes. Find the area of the opening which is $16'' \times 22''$.

77. A cast-iron pulley weighing 0.26 lb./cu. in. has a cored center hole $3\frac{1}{2}''$ in diameter and $9''$ long. By how many pounds does this hole reduce the weight of the casting?

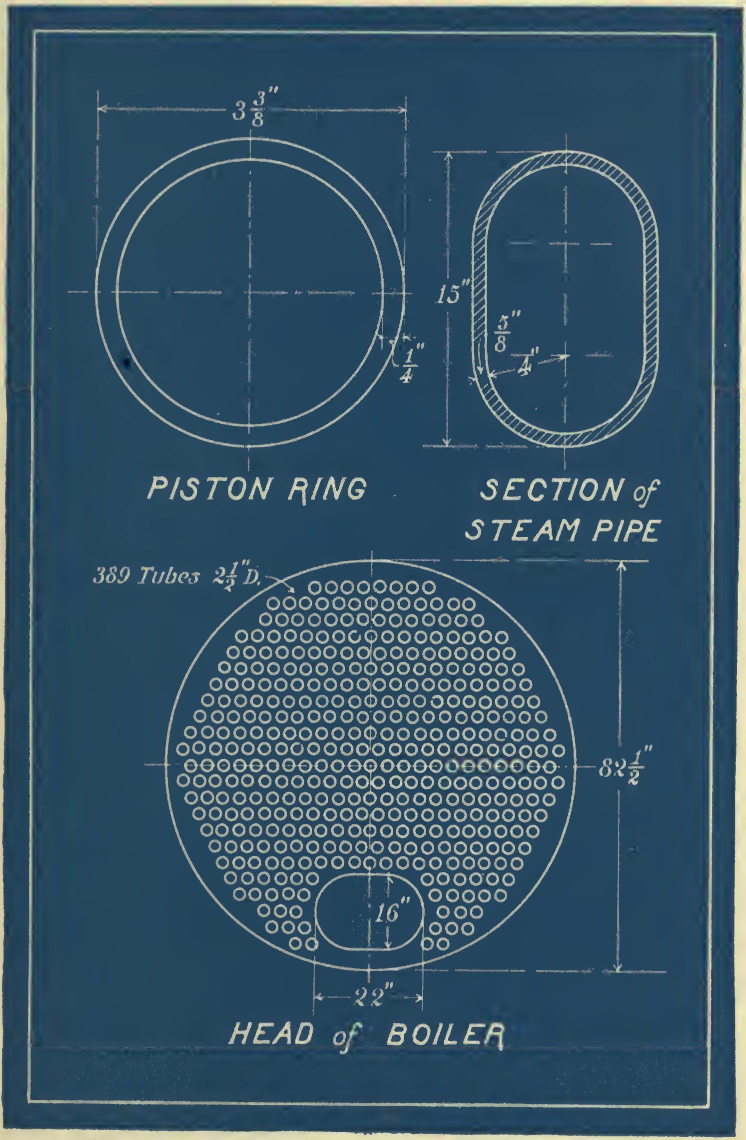
78. When the steam pressure is 95 lb./sq. in., what is the total pressure exerted on a $7\frac{1}{2}$-inch piston?

This means that the diameter of the piston is $7\frac{1}{2}''$.

79. A weight is supported by three rods, each of which has a diameter of $2\frac{3}{4}''$. Assuming that a single rod, with a cross-section area equal to the combined cross-section areas of the three rods, would support the same weight, what should be the diameter of that rod?

80. From a rectangular piece of sheet iron $8'\ 3''$ long and $3'\ 10''$ wide eight disks are cut, each disk being $22''$ in diameter. Making no allowance for waste, find the number of square inches of sheet iron left in the original piece.

81. A flywheel made of iron weighing 450 lb./cu. ft. has an outside diameter of $15'$. The rim is $12''$ wide and has a radial thickness of $7''$. Find the weight of the rim.



3 3/8"
15"
5/8"
4"
1/4"
PISTON RING
SECTION of STEAM PIPE
389 Tubes 2 1/2" D.
82 1/2"
16"
22"
HEAD of BOILER

Exercises. Automobile Transmission

1. The blueprint on page 177 shows part of the gear connections of a 4-speed selective automobile transmission. The power is transmitted from the engine through the sleeve A to the gear B and thence to the countershaft H by the gear C. The engagement of the gears is explained below. If A makes 1000 R.P.M., what is the speed of the countershaft H?

In the blueprint the gear connections of three of the forward speeds are shown. The main shaft G is a square shaft which runs independently of sleeve A, and the gears on this shaft are so arranged that they can be moved into position to engage the corresponding gears on the countershaft. Thus, D (64 T) can be brought into engagement with D (32 T) for low speed. The position of the gears shown in the blueprint is called "neutral" since no gear on the main shaft is engaged, and the countershaft only is being revolved by the engine. For high speed there are projections called "dogs" on the front of gear F on the main shaft and on the back of gear B which can be brought into engagement, thus driving the main shaft direct from the engine. For reverse an intermediate gear is thrown up from below between gears D, D, thus reversing the direction of rotation of the main shaft. The horizontal shaft driven by the bevel gears is connected through the differential (not shown) to the rear wheels.

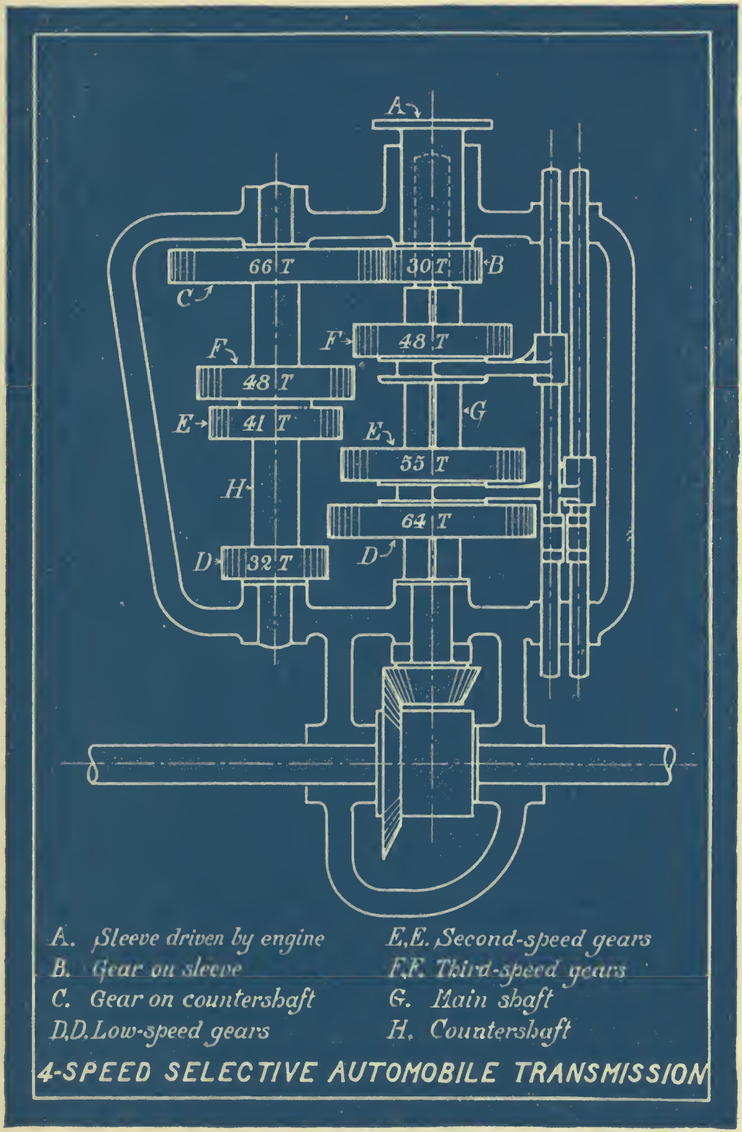
2. In Ex. 1 what is the speed of the main shaft G when the low-speed gears D, D are engaged? when the second-speed gears E, E are engaged? when the third-speed gears F, F are engaged?

3. Find the speed of the countershaft when the engine is running at 750 R.P.M.

4. In Ex. 3 find the speed of the main shaft for low speed; for second speed; for third speed.

5. Consider Ex. 4 when the engine runs at 1400 R.P.M.

6. In Ex. 5 what is the speed of the main shaft when the intermediate reverse gear is engaged between the gears D, D? when the dogs on gears B and F are engaged for high speed?



4-SPEED SELECTIVE AUTOMOBILE TRANSMISSION

Exercises. Apartment-House Structure

1. In excavating the cellar for the apartment house, the floor plan of which is shown on page 179, the excavation was carried 1′ beyond each outside wall and 8′ below the surface of the lot. Find the number of loads of earth removed.

Consider a load as 1 cu. yd. In making the excavation the space occupied by the two fire escapes in the court is not considered.

2. For the cellar floor the contractor used a layer of concrete $1\frac{1}{8}''$ thick extending to the inside of the outer foundation walls, which are 18″ thick. Find the number of cubic feet of concrete used in laying the cellar floor.

3. Find the number of feet of $\frac{3}{4}$-inch lumber required to floor all the bedrooms on this floor of the apartment house, allowing $\frac{1}{3}$ extra for waste.

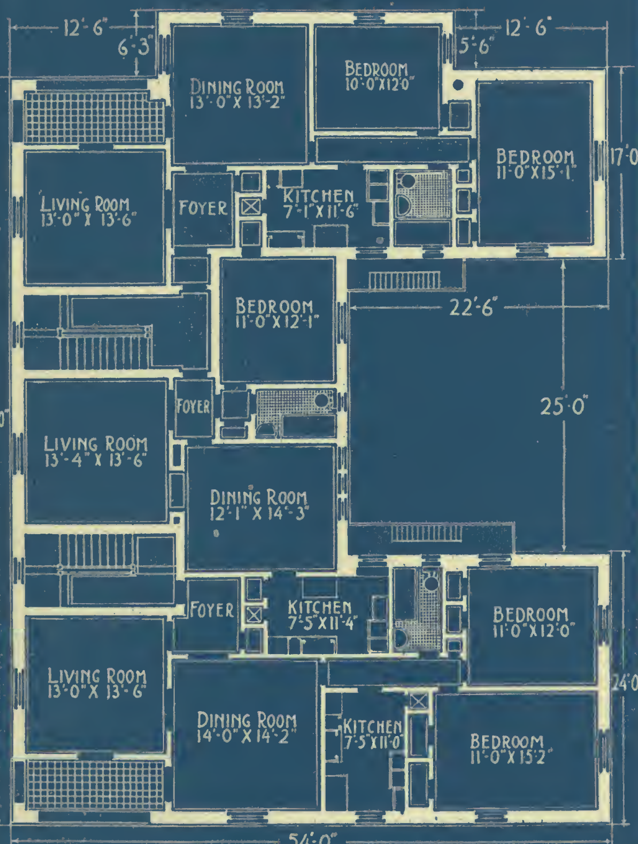
4. The kitchen floors are to be covered with linoleum costing $37\frac{1}{2}$¢ per square foot. Find the total cost of the linoleum for the three kitchens on this floor.

In this problem first calculate the amount of linoleum required for each kitchen, disregarding any allowance for dressers, cupboards, and the like, to offset the waste in cutting and laying. If these results contain fractions, use the next larger whole numbers.

5. If the rooms are 9′ high, find the total number of square yards of plastering required for all the living rooms and bedrooms, making no allowance for doors and windows.

6. The dining rooms are to have oak flooring. Allowing $\frac{1}{3}$ extra for waste, find the number of feet of $\frac{3}{4}$-inch flooring required for all three dining rooms. Find the cost of this flooring at $135 per M.

7. Each dining room is to have a plate rail around the walls. How many running feet will be needed for the three dining rooms, making an allowance of 10% of the length for each room for doors and windows?



APARTMENT—HOUSE FLOOR PLAN

Exercises. Pulley and Gear Trains

1. In the pulley train shown in the blueprint on page 181 find the number of R.P.M. of the emery wheel.

2. In the same pulley train find the number of R.P.M. of the hack saw.

3. If a new motor with a speed of 650 R.P.M., but having the same size of driving pulley as the old motor, were installed to drive the pulley train, what would then be the speed of the emery wheel?

4. With the new motor of Ex. 3 what would be the number of R.P.M. of the hack saw?

5. If it is desired to increase the speed of the hack saw in Ex. 4 by 25 R.P.M., what size pulley should replace the 18-inch pulley on the line shaft?

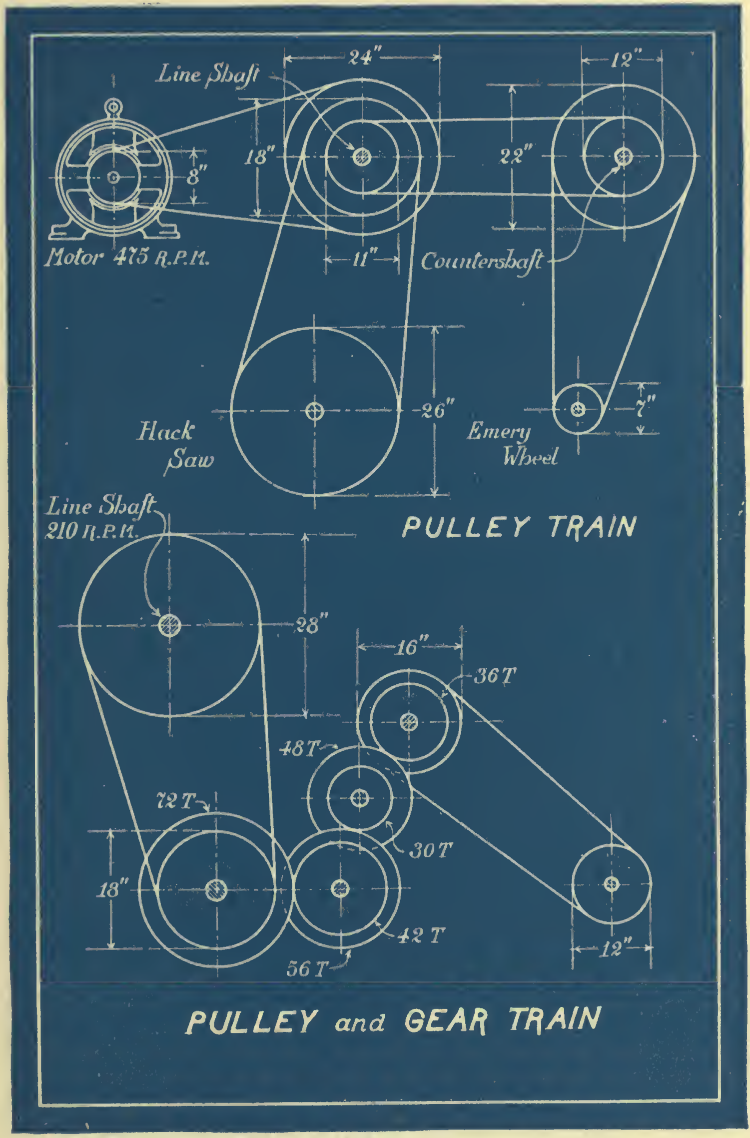
6. In Ex. 3 what size pulley to the nearest $\frac{1}{8}''$ should replace the 24-inch pulley on the line shaft in order to have the speed of the emery wheel the same as in Ex. 1?

7. Find the number of R.P.M. of the 12-inch pulley in the blueprint of the pulley and gear train.

8. If the speed of the line shaft in Ex. 7 were reduced to 150 R.P.M., what would then be the speed of the 12-inch pulley? of the 48-T gear?

9. If the pulley on the spindle of a lathe is $4\frac{1}{8}''$ in diameter, and the countershaft carrying a pulley $10\frac{3}{4}''$ in diameter makes 195 R.P.M., what is the speed of the lathe spindle?

10. A stepped-cone pulley on the driving spindle of a lathe has diameters of $4\frac{1}{8}''$, $6\frac{3}{4}''$, $7\frac{1}{8}''$, and $9\frac{3}{4}''$, and is belted to a similar pulley, the diameters of which are the same but in reverse order, on the countershaft. If the countershaft makes 165 R.P.M., what is the number of R.P.M. of the spindle at each of the four different speeds?


Line Shaft
24"
12"
18"
8"
22"
Motor 475 R.P.M.
11"
Countershaft
26"
7"
Hack Saw
Emery Wheel
Line Shaft 210 R.P.M.
PULLEY TRAIN
28"
16"
36 T
48 T
72 T
30 T
18"
42 T
56 T
12"
PULLEY and GEAR TRAIN

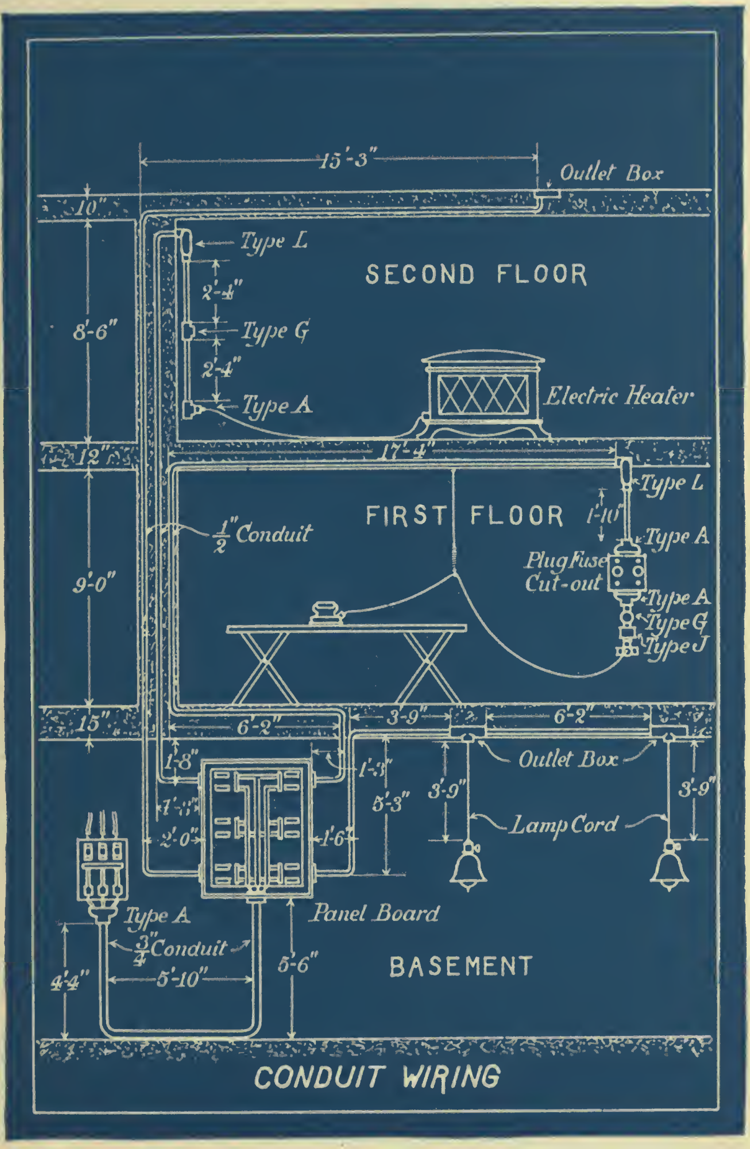
Exercises. Conduit Wiring

1. From the data in the blueprint on page 183 insert the quantities required in the following stock bill for wiring:

Quantity	Description	Price
	$\frac{3}{4}$-inch conduit	\$17.50 per 100′
	$\frac{1}{2}$-inch conduit	\$12.50 per 100′
	$\frac{3}{4}$-inch condulets, type A	\$49 per 100, less $2\frac{1}{2}\%$
	$\frac{1}{2}$-inch condulets, type A	\$46 per 100, less $2\frac{1}{2}\%$
	$\frac{1}{2}$-inch condulets, type G	\$64.50 per 100, less $2\frac{1}{2}\%$
	$\frac{1}{2}$-inch condulets, type L	\$55 per 100, less $2\frac{1}{2}\%$
	$\frac{1}{2}$-inch outlet boxes	34¢ each, less 6%
	Pilot light, type J	\$115 per 100, less $2\frac{1}{2}\%$
	Plug-fuse cut-out	\$72 per 100
	6-circuit panel board	\$15.20
	Incandescent-lamp cord	\$71.30 per 1000′
	#14 R.C. wire, 2 wires in all $\frac{1}{2}$-inch conduit	\$22.40 per 1000′
	#10 R.C. wire, 3 wires in all $\frac{3}{4}$-inch conduit	\$39.30 per 1000′

In figuring the length of the conduit and wire needed, compute as a whole foot any fraction of a foot in the total length required.

2. Find the total cost of all the items in the above bill.



15'-3"
Outlet Box
10"
Type L
SECOND FLOOR
2'-4"
8'-6"
Type G
2'-4"
Electric Heater
Type A
12"
17'-4"
Type L
FIRST FLOOR
1'-10"
1/2" Conduit
Type A
Plug Fuse Cut-out
9'-0"
Type A
Type G
Type J
15"
6'-2"
3'-9"
6'-2"
1'-8"
1'-3"
Outlet Box
1'-3"
5'-3"
3'-9"
3'-9"
2'-0"
1'-6"
Lamp Cord
Type A
Panel Board
3/4" Conduit
4'-4"
5'-10"
5'-6"
BASEMENT
CONDUIT WIRING

Exercises. Boiler Connections

1. From the data shown in the blueprint on page 185 insert the different quantities required in the following stock bill:

Quantity	Description	Price
	$\frac{1}{2}$-inch galvanized pipe	11¢ per foot
	$\frac{3}{4}$-inch galvanized pipe	13¢ per foot
	1-inch galvanized pipe	19¢ per foot
	$\frac{3}{4}$-inch galvanized elbows	$1.25 per dozen
	$\frac{3}{4}$-inch 45° galvanized elbows	$1.30 per dozen
	$1 \times \frac{3}{4}$-inch galvanized elbows	$1.70 per dozen
	1-inch galvanized unions	$42 per 100
	$1 \times \frac{3}{4}$-inch galvanized unions	$35 per 100
	$\frac{3}{4}$-inch unions	$30 per 100
	$\frac{3}{4}$-inch tees	$1.30 per dozen
	$\frac{3}{4} \times \frac{1}{2} \times \frac{3}{4}$-inch tees	$1.30 per dozen
	$\frac{1}{2}$-inch couplings	$1.10 per dozen
	$\frac{1}{2}$-inch sediment cock	90¢ each
	$\frac{3}{4}$-inch valve	$1.75 each
	Boiler	$19.20 each
	Stand	$1.25 each

In figuring the length of pipe needed, compute as the next half foot any fraction of a foot in the total length required.

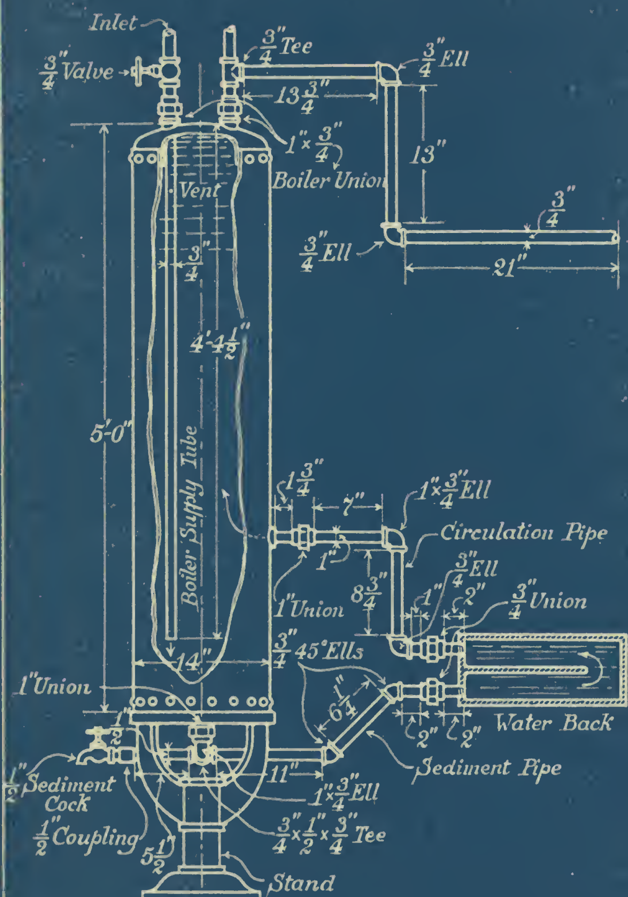
2. Find the total cost of all the items in the above bill.

3. Approximately, how many gallons does the boiler hold?

Consider the boiler as a cylinder 5′ in height and disregard the fact that the top is rounded.

4. Find the approximate weight of the empty boiler which is made of #7 gage sheet iron weighing $7\frac{1}{2}$ lb./sq. ft.

Disregard the overlapping seams and the weight of the rivets.



BOILER CONNECTIONS

Exercises. Section of Lockers

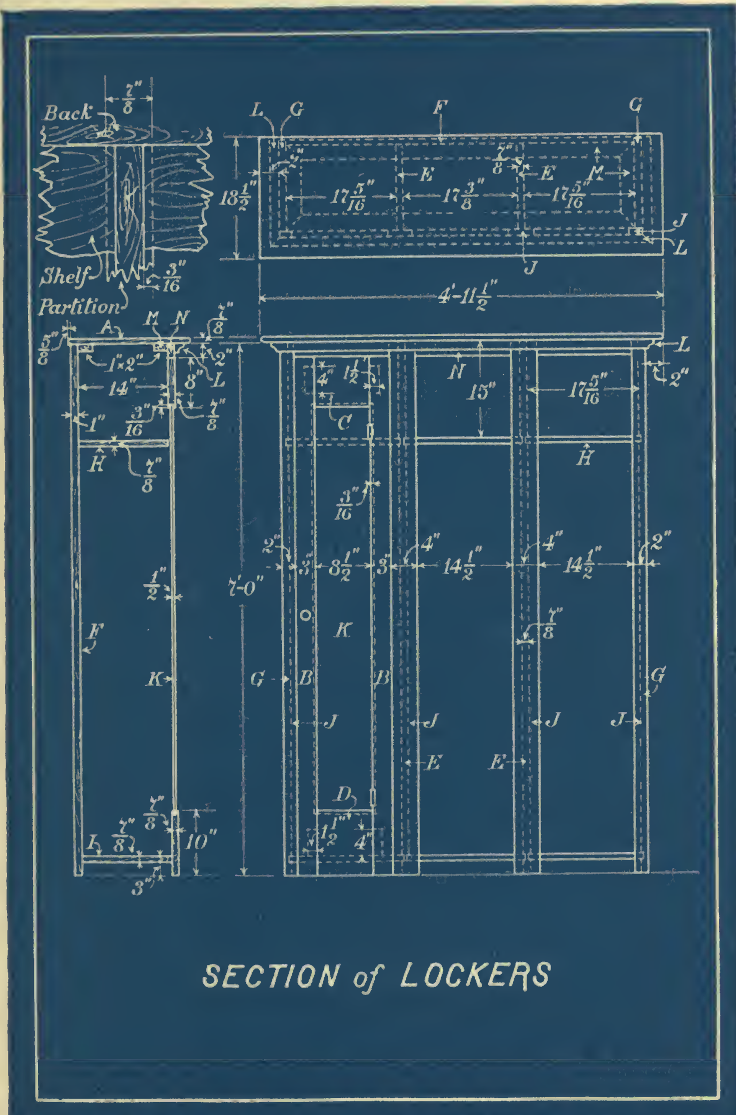
1. From the data given in the blueprint on page 187 fill out the following stock bill for the section of lockers:

Number of Pieces	Length	Width	Thickness	Description
				A, top
				B, door stiles
				C, door top rails
				D, door bottom rails
				E, partitions
				F, back
				G, sides
				H, upper shelf
				I, lower shelf
				J, frame
				K, door panel
				L, molding
				M, cleats
				N, upper part of frame

In computing the amount of molding, which is $1\frac{1}{2}''$ wide and $1''$ thick, figure the number of running feet required.

2. The lockers are to be made of chestnut, with the exception of the back, which is to be made of matched whitewood strips. The chestnut costs \$110 per M, the whitewood \$65 per M, and the molding 10¢ per running foot. Find the total cost of the lumber for the section.

3. Allowing 1 gal. of paint to 600 sq. ft. for each coat, find the amount of paint required for the section, giving one coat inside and two coats outside.



SECTION of LOCKERS

Exercises. Review

1. The turret-lathe cross slide shown in the blueprint on page 189 is made of cast iron weighing 450 lb./cu. ft. Find the weight of the slide before the slots were machined in it.

2. In Ex. 1 find the weight of the cross slide after all the slots are machined.

3. The dimensions shown in the blueprint of the shaper table, which is made of cast iron weighing 450 lb./cu. ft., represent the finished size. In finishing the table $\frac{3}{16}''$ was first machined off each outside surface, and then the holes and T-slots were cut. Find the weight of the casting before it was machined.

4. The six holes in each of the two sides were drilled and the T-slots were milled after the table in Ex. 3 was machined. By how many pounds was the weight of the table reduced in these two operations?

5. The concrete pipe-support shown in the blueprint is 875′ long. Find the number of cubic feet of concrete required.

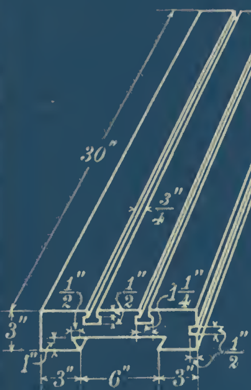
6. The web pulley shown in the blueprint is made of cast iron weighing 450 lb./cu. ft. Find the cost of 25 pulleys of this type at $6\frac{1}{2}$¢ a pound.

7. In the web pulley find the area to the nearest $\frac{1}{8}$ sq. in. of the 4-inch face of the rim R.

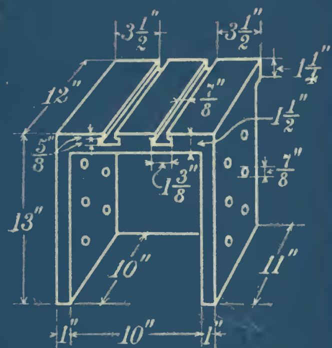
8. Find the reduction in weight of the web pulley if eight $1\frac{1}{4}$-inch holes are drilled through the web W.

9. Find the speed in F. P. M. (feet per minute) of a point on the rim of the web pulley when the shaft on which the pulley is fixed is making 85 R. P. M.

10. What is the weight of a pulley made of steel weighing 490 lb./cu. ft., if each dimension of the steel pulley is $\frac{3}{4}$ the length of the corresponding dimension in the blueprint?

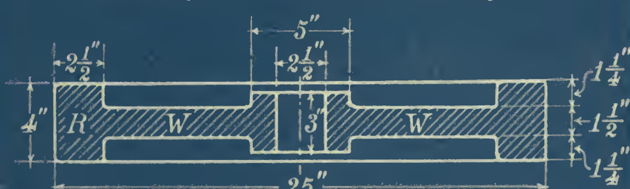


TURRET-LATHE CROSS SLIDE



SHAPER TABLE

CONCRETE PIPE-SUPPORT



CROSS SECTION of CAST-IRON WEB PULLEY

11. The flanged shaft-coupling shown in the blueprint on page 191 is made of cast iron weighing 450 lb./cu. ft., and there are eight of the $\frac{3}{4}$-inch holes through the flange. Find the weight of 25 couplings of this type.

12. Find the area of each of the two $1\frac{1}{4}$-inch faces of the cone pulley shown in the blueprint.

13. In the cone pulley, which is made of cast iron weighing 450 lb./cu. ft., the web A is $\frac{3}{4}''$ thick and is pierced by six 1-inch holes. Find the weight of the web.

14. In the cone pulley the cored hub B is $3\frac{1}{2}''$ long. Find the weight of the hub.

15. Find the total weight of the cone pulley.

16. The dimensions in the blueprint of the flanged pulley, which is made of cast iron weighing 450 lb./cu. ft., show the size of the pulley as cast. In finishing the pulley $\frac{1}{8}''$ is turned off each surface except the $\frac{7}{8}$-inch core. Find the cost of 1250 finished pulleys at $7\frac{1}{2}$¢ a pound.

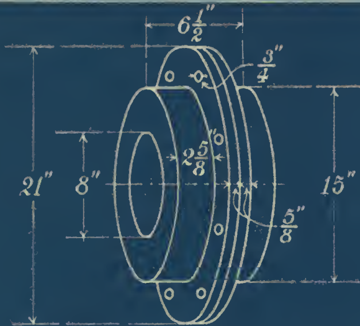
17. Find the area of the rim R of the flange on the rough flanged pulley as shown in the blueprint. Find this area when the pulley has been finished as described in Ex. 16.

18. The lathe faceplate is made of cast iron weighing 450 lb./cu. ft., and, while the cored slots all have the same width, there are three different lengths, as shown in the blueprint. Considering all the slots as rectangular openings, find, approximately, how much more the faceplate would have weighed if it had been cast solid.

19. Find the area of the rim of the lathe faceplate.

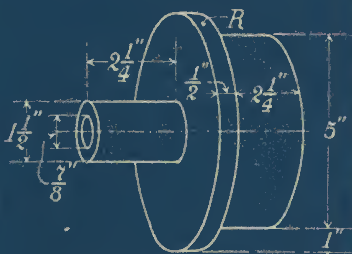
20. Find the weight of metal removed in boring the cored center hole in the lathe faceplate to a diameter of $2\frac{1}{2}''$.

21. Find the speed in F.P.M. of a point on the rim when the faceplate is being driven at 40 R.P.M.



FLANGED SHAFT-COUPLING

CONE PULLEY



FLANGED PULLEY

LATHE FACEPLATE

22. If a hack saw makes 96 strokes per minute, how many strokes does it make in $1\frac{1}{2}$ hr.? in 1 hr. 55 min.?

23. The driving pulley on a shaft is 44″ in diameter and makes 240 R.P.M. Find the speed in F.P.M. of a point on the circumference of the pulley. Find the rim speed of a 12-inch pulley which is belted to the large pulley.

24. If steel expands 0.000007 of its length for each degree Fahrenheit that it increases in temperature, computed on its length when it begins to expand, find the increase in the length of a 30-foot steel rail when heated from 10° F. to 120° F.

The abbreviation F. means Fahrenheit.

25. If the diameter of a piston is 28″ and the pressure of steam in the cylinder is 120 lb./sq. in., what is the total pressure of the steam upon the piston?

26. A countershaft has upon it two pulleys, each 10″ in diameter, and the speed of the countershaft is 500 R.P.M. Find the diameters of the pulleys of two machines which, when belted to the two pulleys mentioned, will have speeds of 200 R.P.M. and 800 R.P.M. respectively.

27. Experiment has shown that the maximum load L in tons, which can safely be fastened to an iron chain in which the diameter of the chain iron is d inches, is expressed by the formula $L = 6.4\,d^2$. Find the maximum safe load that can be lifted by a chain in which $d = 0.75''$.

28. The front sprocket wheel of a certain bicycle has 26 teeth and the rear sprocket wheel has 9 teeth. If the rear tire is 32″ in diameter, how many turns of the pedals will be made in riding the bicycle 1 mi. without coasting?

29. A pump 18″ in diameter, having a 24-inch stroke and making 25 strokes per minute, can pump how many cubic feet of water per hour? how many gallons?

TABLES AND RULES

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)
2240 pounds = 1 long ton

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Metric Length

1 kilometer (km.) = 1000 meters
Meter (m.)
1 centimeter (cm.) = 0.01 meter
1 millimeter (mm.) = 0.001 meter

For ordinary comparisons we usually think of 1 km. as 0.6 mi.; of 1 m. as $39\frac{1}{3}''$, or $3\frac{1}{4}'$; of 1 cm. as $0.4''$; and of 1 mm. as $0.04''$. In cases requiring greater accuracy the following approximate equivalents may be used: 1 km. = 0.62 mi., 1 m. = $39.37''$, 1 cm. = $0.394''$, and 1 mm. = $0.039''$.

Metric Weight

1 metric ton (t.) = 1000 kilograms
1 kilogram (kg.) = 1000 grams
Gram (g.)

The following approximate equivalents of metric weight may be used: 1 t. = 2204.6 lb., 1 kg. = 2.2 lb., and 1 g. = 15.43 grains.

Metric Capacity

1 hektoliter (hl.) = 100 liters
Liter (l.)
1 centiliter (cl.) = 0.01 liter

The equivalent of 1 l. is approximately 1 qt. The liter is the volume of a cube that is 0.1 m., or about $4''$, on an edge.

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)
12 gross, or 1728 units = 1 great gross

Common Equivalents

1 gal. contains 231 cu. in., or 0.134 cu. ft.

1 cu. ft. contains $7\frac{1}{2}$ gal.

1 bbl. contains $4\frac{1}{5}$ cu. ft., or $31\frac{1}{2}$ gal.

1 bu. contains 2150.42 cu. in. (approximately 2150 cu. in.).

1 bu. contains approximately $1\frac{1}{4}$ cu. ft.

1 cu. ft. of water weighs 62.425 lb. (approximately $62\frac{1}{2}$ lb.).

1 gal. of water weighs 8.345 lb. (approximately $8\frac{1}{3}$ lb.).

A ton of coal varies in volume according to kind and grade, but for general purposes the volume may be taken as 35 cu. ft.

Convenient Rules

The circumference of a circle is $\frac{22}{7}$ times the diameter.

For a higher degree of accuracy, $c = 3.1416\,d$.

The diameter of a circle is $\frac{7}{22}$ of the circumference.

For a higher degree of accuracy, $d = 0.3183\,c$.

The area of a circle is $\frac{11}{14}$ times the square of the diameter.

For a higher degree of accuracy, $A = 0.7854\,d^2$.

The height of an equilateral triangle is 0.866 of the side.

The diagonal of a square is 1.414 times the side.

In the shop the diagonal of a square is also called "long diameter" or "distance across the corners."

The "long diameter" of a regular hexagon is twice the side.

The "short diameter," or the perpendicular distance between parallel sides, of a regular hexagon is 1.732 times the side.

To convert Fahrenheit into centigrade subtract 32° from the Fahrenheit reading and take $\frac{5}{9}$ of the result.

Expressed as a formula, $C = \frac{5}{9}(F - 32)$.

To convert centigrade into Fahrenheit take $\frac{9}{5}$ of the centigrade reading and add 32° to the result.

Expressed as a formula, $F = \frac{9}{5}\,C + 32$.

DECIMAL EQUIVALENTS OF COMMON FRACTIONS

Fraction				Decimal	Fraction				Decimal
			1/64	0.016				33/64	0.516
		1/32		.031			17/32		.531
			3/64	.047				35/64	.547
	1/16			.063		9/16			.563
			5/64	.078				37/64	.578
		3/32		.094			19/32		.594
			7/64	.109				39/64	.609
1/8				**.125**	**5/8**				**.625**
			9/64	.141				41/64	.641
		5/32		.156			21/32		.656
			11/64	.172				43/64	.672
	3/16			.188		11/16			.688
			13/64	.203				45/64	.703
		7/32		.219			23/32		.719
			15/64	.234				47/64	.734
1/4				**.250**	**3/4**				**.750**
			17/64	.266				49/64	.766
		9/32		.281			25/32		.781
			19/64	.297				51/64	.797
	5/16			.313		13/16			.813
			21/64	.328				53/64	.828
		11/32		.344			27/32		.844
			23/64	.359				55/64	.859
3/8				**.375**	**7/8**				**.875**
			25/64	.391				57/64	.891
		13/32		.406			29/32		.906
			27/64	.422				59/64	.922
	7/16			.438		15/16			.938
			29/64	.453				61/64	.953
		15/32		.469			31/32		.969
			31/64	.484				63/64	.984
1/2				**.500**	**1**				**1.000**

DEFINITIONS

Terms Defined. Students who use a book of this nature will do so in schools which have shops in which the machines mentioned in this book either will be found or be described by the instructor. Formal definitions are therefore undesirable. In order to save the time of the instructor and the student, however, informal definitions of a few terms are given, particularly such as are not evident from the blueprints.

Angle clamp. A clamp for holding work together at an angle.

Arbor. A shaft to hold work during some machine operation, as on a lathe or milling machine.

Bearing cap. The upper half of a bearing.

Bearing support. A casting used to support a bearing.

Binding post. A post on an electric instrument to connect wires.

Blind collar. A collar in which the hole is drilled partly through.

Boring mill. A machine for boring, turning, or facing.

Bushing. A tube or shell for reducing the diameter of a hole.

Cartridge fuse. A fuse used to protect electric circuits.

Change gears. Gears used on a lathe to drive the lead screw.

Collar pin. A pin having a collar and carrying a roll, gear, or other part at the outer end.

Conduit. Sherardized pipe used as protection to electric wires.

Cone pulley. A pulley made up of several pulleys of successively increasing sizes cast in one piece.

Countershaft. A shaft carrying tight and loose pulleys for starting and stopping machines or reversing their motion.

Crankshaft. The main shaft in a gas engine.

Diameter. The distance across a circle measured through the center; also, in workshop practice, the diagonal of a square.

Drill socket. A device for driving drills having a taper shank.

Emery wheel. A grinding wheel made of emery or carborundum.

Faceplate. A disk on the nose of a lathe for driving the work.

Feed screw. A threaded screw which gives movement to the cross carriage of a lathe.

Flanged coupling. A form of coupling used to connect two shafts.

Flanged pulley. A pulley in which the rim has a flange.

Gear. A wheel with teeth on the rim for transmitting power.

Gear train. A series of gears in which the teeth mesh.

Gusset plate. A plate for strengthening or holding two or more pieces at an angle.

Hack saw. A saw for cutting metals.

I-beam. A beam with a cross section shaped like the letter "I".

Jig bushing. A hardened tool-steel collar.

Lathe spindle. The shaft which drives the work on a lathe.

Line shaft. The shafting driving the machinery in a shop by means of pulleys and belting.

Milling machine. A machine for removing metal by means of revolving cutters.

Piston. A hollow piece sliding in a cylinder and transmitting power through a rod to the crankshaft of a gas engine.

Piston ring. A ring snapped into the grooves of a piston to prevent the escape of gas.

Planer. A machine for producing plane surfaces on metals.

Pulley. A wheel for transmitting power by means of a belt.

Shaper. A machine for planing straight and angle surfaces.

Spark plug. A device for exploding the gas in the cylinder of a gas engine.

Spur gear. A toothed wheel in which the teeth are cut parallel to the axis of revolution.

Taper spindle. A tapered round shaft.

Tool post. A post used on a lathe to hold the cutting tool.

Transmission case. The covering inclosing the gears for controlling the speed of an automobile.

Turret lathe. A lathe with special fixtures for production work.

Web pulley. A pulley having a web, or ring of metal, connecting the hub with the rim.

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