

D 140191

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Name.....

Reg. No.....

**SIXTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
APRIL 2026**

Mathematics

MTS6B10—REAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. State discontinuity criterion for a function at a point.
2. Give an example of a continuous function which is not bounded.
3. Define Lipschitz function.
4. Define norm of a partition. Let $I = [0, 4]$. Calculate the norm of the partition $\mathcal{P} = (0, 1, 2, 4)$.
5. Prove that any step function is integrable.
6. State and prove product theorem.
7. Define null set and give an example of a null set.
8. State first form of Fundamental theorem of calculus.
9. Define uniform convergence of a sequence of functions.
10. Evaluate $\lim_{x \rightarrow \infty} \frac{nx}{1 + nx}$ for $x \in \mathbb{R}, x \geq 0$.
11. State Cauchy criterion for uniform convergence of a series of functions.
12. Evaluate $\int_{-1}^0 \frac{dx}{\sqrt[3]{x}}$.

Turn over

13. Define Cauchy principal value of an improper integral.
14. Define integral test.
15. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. State and prove boundedness theorem.
17. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . If $\epsilon > 0$, prove that there exists a step function $s_\epsilon : I \rightarrow \mathbb{R}$ such that $|f(x) - s_\epsilon(x)| < \epsilon$ for all $x \in I$.
18. State and prove squeeze theorem for Riemann integrals.
19. If $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$, then prove that $f \in \mathcal{R} [a, b]$.
20. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.
21. Prove that all $q \in \mathbb{R}$, $\int_1^\infty x^q e^{-x} dx$ converges.
22. Evaluate $\int_0^\infty \sqrt[3]{ye^{-y^2}} dy$.
23. Prove that $\Gamma(2p) = \frac{2^{2p-1/2}}{\sqrt{2\pi}} \Gamma(p) \Gamma(p+1/2)$.

Section C

Answer any two questions.

Each question carries 10 marks.

Maximum 20 marks.

24. (a) State and prove uniform continuity theorem.
- (b) Prove that $g(x) = \sqrt{x}$ is uniformly continuous on $A = [0, \infty)$.
25. Let $f : [a, b] \rightarrow \mathbb{R}$ and let $c \in (a, b)$. Prove that $f \in \mathcal{R}[a, b]$ if and only if its restriction to $[a, c]$ and $[c, b]$ are both Riemann integrable and in this case $\int_a^b f = \int_a^c f + \int_c^b f$.
26. Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f . Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$.
27. State Cauchy test. Prove that $\int_0^\infty \frac{\sin x}{x} dx$ exists.

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Name.....

Reg. No.....

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, MARCH 2025

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. Prove that polynomial functions are continuous on \mathbb{R} .
2. Give an example of uniformly continuous function which is not a Lipschitz function.
3. State Weierstrass approximation theorem.
4. Prove that every constant function on $[a, b]$ is Reimann integrable on $[a, b]$.
5. Suppose f and g are in $\mathcal{R}[a, b]$. If $f(x) \leq g(x)$ for all $x \in [a, b]$, prove that $\int_a^b f \leq \int_a^b g$.
6. State substitution theorem and use this to evaluate $\int_1^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$.
7. Prove that rational numbers in $[0, 1]$ is a null set.
8. If $f \in \mathcal{R}[a, b]$, Prove that $|f| \in \mathcal{R}[a, b]$ and $\left| \int_a^b f \right| \leq \int_a^b |f|$.
9. Define uniform norm on a set of bounded functions.
10. Evaluate $\lim (e^{-nx})$ for $x \in \mathbb{R}, x \geq 0$.

Turn over

11. State and prove Weierstrass M test.

12. Evaluate $\int_1^{\infty} \frac{dx}{x^2+1}$.

13. Find P.V. $\int_{-\infty}^{\infty} x dx$.

14. Prove that $\int_2^{\infty} \frac{1}{\ln(x)} dx = \infty$.

15. Prove that $\Gamma(p)$ is strictly convex for $0 < p < \infty$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. Prove that Dirichlet's function is discontinuous on \mathbb{R} .

17. State and prove preservation of intervals theorem.

18. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then prove that $f \in \mathcal{R}[a, b]$.

19. State and prove Taylor's theorem with the remainder.

20. Let (f_n) be sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Prove that f is continuous on A .

21. Prove that for all $q \leq -1$, $\int_0^1 x^q e^{-x} dx$ diverges.

22. Prove that $\int_0^{\infty} \sqrt{y} e^{-y^2} dy = \frac{1}{2} \Gamma(3/4)$.

23. Prove that $B(p, q) = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$ for all $p > 0$ and $q > 0$.

Section C

*Answer any two questions.
Each question carries 10 marks.
Maximum 20 marks.*

24. State and prove location of roots theorem.
25. State and prove second form of Fundamental theorem of calculus.
26. State and prove Cauchy criterion for uniform convergence.
27. For any $p > 0$, fixed, Prove that $\lim_{0 < q \rightarrow \infty} q^p B(p, q) = \Gamma(p)$.

(2 × 10 = 20 marks)

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Name.....

Reg. No.....

**SIXTH SEMESTER UG (CBCSS-UG) DEGREE
EXAMINATION, MARCH 2024**

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions onwards)

Time : Two Hours and a Half

Maximum Marks : 80

Section A

Questions 1—15. Answer any number of questions.

Each carry 2 marks. Maximum marks 20.

1. State discontinuity criterion. Hence show that the signum function is not continuous at $x = 0$.
2. State maximum-minimum theorem.
3. Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.
4. Define Riemann integral of a function f on an interval $[a, b]$.
5. If f and g are in $R[a, b]$ and if $f(x) \leq g(x)$ for all x in $[a, b]$ then show that $\int_a^b f \leq \int_a^b g$.
6. State Lebesgue's integrability criterion.
7. If f and g belong to $R[a, b]$ then the product fg belongs to $R[a, b]$.
8. Show that $\lim_{n \rightarrow \infty} \frac{\sin(nx + n)}{n} = 0$ for $x \in \mathbb{R}$.
9. Discuss the uniform convergence of $f_n(x) = \frac{x}{n}$ on $A = [0, 1]$.
10. Evaluate $\lim_{x \rightarrow \infty} (e^{-nx})$ for $x \in \mathbb{R}, x \geq 0$.
11. Define absolute convergence of series of functions.
12. Evaluate $\int_{-\infty}^0 e^x dx$.
13. Find the principal value of $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$.

Turn over

14. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
15. Define Beta function. St $B(p, q) = B(q, p)$.

Section B

Questions 16—23. Answer any number of questions.
Each carry 5 marks. Maximum marks 35.

16. Let $A = \{x \in \mathbb{R} | x > 0\}$. Define h on A by $h(x) = 0$ if $x \in A$ is irrational and $h(x) = \frac{1}{n}$ if $x \in A$ is rational with $x = \frac{m}{n}$, $m, n \in \mathbb{N}$ have no common factor except 1. Then show that h is continuous at every irrational number in A and discontinuous at every rational number in A .
17. Let I be an interval and $f : I \rightarrow \mathbb{R}$ be a continuous function on I then show that $f(I)$ is an interval.
18. If $f \in \mathbb{R}[a, b]$ then show that f is bounded on $[a, b]$.
19. Show that if $\phi : [a, b] \rightarrow \mathbb{R}$ is a step function then $\phi \in \mathbb{R}[a, b]$.
20. Evaluate $\lim_{n \rightarrow \infty} \frac{x^2 + nx}{n}$, $x \in \mathbb{R}$. Is the convergence uniform on \mathbb{R} ?
21. Let (f_n) be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Then show that (f_n) converges uniformly on A to a bounded function f iff for each $\varepsilon > 0$ there is a number $H(\varepsilon)$ in \mathbb{N} such that for all $m, n \geq H(\varepsilon)$ then $\|f_m - f_n\|_A \leq \varepsilon$.
22. Discuss the convergence of $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$.
23. Define Beta function and show that $\forall p > 0, q > 0, B(p, q) = 2 \int_0^{\pi/2} \sin^{2p-1} \theta \cos^{2q-1} \theta d\theta$.

Section C

Questions 24—27. Answer any two questions.
Each carry 10 marks.

24. (a) Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and they are bounded on A then their product fg is also uniformly continuous.
- (b) Show that $f(x) = \sqrt{x}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.

25. Suppose f and g are in $\mathcal{R}[a, b]$. Then

(a) if $k \in \mathbb{R}$, show that $kf \in \mathcal{R}[a, b]$ and $\int_a^b kf = k \int_a^b f$.

(b) $f + g \in \mathcal{R}[a, b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

26. Discuss the pointwise and uniform convergence of :

(a) $f_n(x) = \frac{\sin(nx + n)}{n}$ for $x \in \mathbb{R}$.

(b) $g_n(x) = \frac{x^2 + nx}{n}$ for $x \in \mathbb{R}$.

27. Show that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

(2 × 10 = 20 marks)

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Name.....

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SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Maximum marks 25.*

1. State sequential criterion for continuity.
2. Show that the sine function is continuous on \mathbb{R} .
3. Define Lipschitz function. If $f : A \rightarrow \mathbb{R}$ is a Lipschitz function then show that f is uniformly continuous on A .
4. Define tagged partition.
5. Show that every constant function on $[a, b]$ is in $\mathbb{R}[a, b]$.
6. State Cauchy's criterion for Riemann integrability.
7. Let F, G be differentiable on $[a, b]$ and let $f = F'$ and $g = G'$ belong to $\mathbb{R}[a, b]$, then show that

$$\int_a^b f G = [FG]_a^b - \int_a^b F g.$$

8. Show that $\lim \left(\frac{x}{n} \right) = 0$ for $x \in \mathbb{R}$.
9. Define uniform convergence of a sequence of functions.
10. State bounded convergence theorem.
11. State Weierstrass M-test for the uniform convergence of series of functions.

12. Evaluate $\int_1^{\infty} \frac{dx}{x^2 + 1}$.

Turn over

13. Find the principal value of $\int_{-2}^3 \frac{dx}{(x-1)^3}$.
14. Discuss the absolute convergence of $\int_0^{\infty} \frac{\sin x}{n+1} dx$ for $n\pi \leq x \leq (n+1)\pi$, $n = 0, 1, 2, \dots$
15. If $\int_0^b \frac{dx}{1+ax} = \frac{1}{a} \ln(1+ab)$. Evaluate $\int_0^b \frac{xdx}{(1+ax)^2}$.

Section B

Questions 16–23, answer any number of questions.

Each question carries 5 marks.

Maximum marks 35.

16. State and prove Boundedness theorem for continuous function.
17. Show that $f(x) = \frac{1}{1+x^2}$, $x \in \mathbb{R}$ is uniformly continuous in \mathbb{R} .
18. State and prove Squeeze theorem for Riemann integrable functions.
19. If $f \in \mathbb{R}[a, b]$ and f is continuous at a point $c \in [a, b]$. Then show that the indefinite integral $F(z) = \int_a^z f$ for $z \in [a, b]$ is differentiable at c and $F'(c) = f(c)$.
20. Show that a sequence (f_n) of bounded functions on $A \subset \mathbb{R}$ converges uniformly on A to f iff $\|f_n - f\|_n \rightarrow 0$.
21. Discuss the convergence of $f_n(x) = \frac{x^n}{n+x^n}$, $x \geq 0$. Is the convergence uniform on $[0, \infty]$.
22. Evaluate $\int_{-1}^1 \frac{dx}{x^2-1}$.
23. Show that $\forall q \in \mathbb{R}$, $\int_1^{\infty} x^q e^{-x} dx$ converges.

Section C

Questions 24–27, answer any **two** questions.
Each question carries 10 marks.

24. State and prove Maximum Minimum Theorem.
25. State and prove Cauchy's criterion of Riemann integrability.
26. Let (f_n) be a sequence of functions in $\mathbb{R}[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f . Then show that $f \in \mathbb{R}[a,b]$.
27. Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ exists and converges to a finite real value and that this integral does not converge absolutely.

(2 × 10 = 20 marks)

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Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

*Answer at least ten questions.
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.*

1. Define continuity of a function. Show that the constant function $f(x) = b$ is continuous on \mathbb{R} .
2. State Boundedness theorem. Is boundedness of the interval, a necessary condition in the theorem? Justify with an example.
3. If $f : A \rightarrow \mathbb{R}$ is uniformly continuous on $A \subseteq \mathbb{R}$ and (x_n) is a Cauchy sequence in A . Then show that $f(x_n)$ is a Cauchy sequence in \mathbb{R} .
4. Define Riemann sum of a function $f : [a, b] \rightarrow \mathbb{R}$.
5. Suppose f and g are in $\mathbb{R}[a, b]$ then show that $f + g$ is in $\mathbb{R}[a, b]$.
6. State squeeze theorem for Riemann integrable functions.
7. If f belong to $\mathbb{R}[a, b]$, then show that its absolute value $|f|$ is in $\mathbb{R}[a, b]$.
8. Define pointwise convergence of a sequence (f_n) of functions.
9. Discuss the uniform convergence of $f_n(x) = x^n$ on $(-1, 1]$.
10. If $h_n(x) = 2nxe^{-nx^2}$ for $x \in [0, 1], n \in \mathbb{N}$ and $h(x) = 0$ for all $x \in [0, 1]$, then show that :

$$\lim_{n \rightarrow \infty} \int_0^1 h_n(x) dx \neq \int_0^1 h(x) dx.$$
11. State Cauchy criteria for uniform convergence series of functions.

Turn over

12. Evaluate $\int_{-1}^0 \frac{dx}{\sqrt[3]{x}}$.

13. What is Cauchy principle value. Find the principal value of $\int_{-1}^1 \frac{dx}{x}$.

14. State Leibniz rule for differentiation of Ramann integrals.

15. State that $\Gamma(p+1) = p\Gamma(p)$ for $p > 0$.

(10 × 3 = 30 marks)

Section B

*Answer at least five questions.
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.*

16. Show that the Dirichlet's function :

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases} \text{ is not continuous at any point of } \mathbb{R}.$$

17. State and prove Bolzano intermediate value theorem.

18. Show that the following functions are not uniformly continuous on the given sets :

(a) $f(x) = x^2$ on $A = [0, \infty)$.

(b) $g(x) = \sin \frac{1}{x}$ on $B = (0, \infty)$.

19. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then show that $f \in \mathbb{R}[a, b]$.

20. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Then show that f is continuous on A .

21. Let $f_n : [0, 1] \rightarrow \mathbb{R}$ be defined for $n \geq 2$ by :

$$f_n(x) = \begin{cases} n^2 x & , 0 \leq x \leq \frac{1}{n} \\ -n^2(x - 2/n), \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & , \frac{2}{n} \leq x \leq 1. \end{cases}$$

Show that the limit function is Riemann integrable. Verify whether $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$.

22. Given $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)} dx dy = \pi$, find the value of $\int_0^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$.

23. Show that $\forall p > 0, q > 0$ $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

(5 × 6 = 30 marks)

Section C*Answer any two questions.**Each question carries 10 marks.*

24. State and prove Location of roots theorem.

25. State and prove Additivity theorem.

26. Evaluate (a) $\lim_{x \rightarrow \infty} \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}, x \geq 0$. (b) $\lim_{x \rightarrow \infty} \frac{\sin nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.

Discuss about their uniform convergence.

27. (a) Show that $\forall q > -1, \int_0^1 x^q e^{-x} dx$ converges.

(b) Show that $\forall q \leq -1, \int_0^1 x^q e^{-x} dx$ diverges.

(2 × 10 = 20 marks)