

D 140195**(Pages : 3)****Name.....****Reg. No.....****SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION****APRIL 2026****Mathematics****MTS 6B 13—DIFFERENTIAL EQUATIONS****(2020 Admission onwards)****Time : Two Hours and a Half****Maximum : 80 Marks****Section A***Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. Solve the differential equation $(4 + t^2) \frac{dy}{dt} + 2ty = 4t$.
2. State existence and uniqueness theorem for first order non linear equations.
3. Verify that $u(x, y) = \cos x \cosh y$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$.
4. Show that if $y = \phi(t)$ is a solution of $y' + p(t)y = 0$, then $y = c\phi(t)$ is also a solution for any value of the constant c .
5. Find the general solution of $y'' + 5y' + 6y = 0$.
6. Find the wronskian of $y_1 = e^{2t}$ and $y_2 = e^{-3t/2}$.
7. Find the longest interval in which the initial value problem $ty'' + 3y = t$, $y(1) = 1$, $y'(1) = 2$ is certain to have a unique twice differentiable solution.
8. Find the solution of $y'' - 2y' + y = 0$.
9. Find a particular solution of $y'' - 2y' - 3y = 3e^{2t}$.

Turn over

10. Evaluate the improper integral $\int_1^{\infty} e^{ct} dt$. For what values of c does this improper integral converge.
11. Find the inverse Laplace transform of $F(s) = \frac{2s+2}{s^2+2s+5}$.
12. Prove that convolution integral is commutative.
13. State Fourier convergence theorem.
14. Prove that product of an even function and an odd function is odd.
15. Find the steady state solution of the heat conduction equation $\alpha^2 u_{xx} = u_t$ that satisfies the boundary conditions $u_x(0, t) = 10, u(L, t) = 0$.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. Solve $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$.
17. Find an integrating factor and solve the equation $(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0$.
18. Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0$. then show that $y = c_1t^2 + c_2t^{-1}$ is also a solution of this equation for any c_1 and c_2 .
19. State Abel's theorem and use this theorem to find the wronskian of two solutions of the equation $t^2y'' - t(t+2)y' + (t+2)y = 0$.
20. Find the general solution of $y'' - y' - 2y = -2t + 4t^2$.
21. Use the Laplace transforms to solve the initial value problem $y'' - y' - 6y = 0, y(0) = 1, y'(0) = -1$.

22. Solve the boundary value problem $y'' + 2y = 0$, $y(0) = 0$, $y(\pi) = 0$.
23. Find the Fourier series for $f(x) = \begin{cases} i & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$, $f(x+4) = f(x)$.

Section C

Answer any two questions.

Each question carries 10 marks.

Maximum 20 marks.

24. Let $\phi_0(t) = 0$ and use the method of successive approximations to solve the initial value problem $y' = ty + 1$, $y(0) = 0$.
25. Find a series solution of the equation $y'' + y = 0$, $-\infty < x < \infty$.
26. Find the solution of the initial value problem $y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 0$.
27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)

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Name.....

Reg. No.....

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, MARCH 2025

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 25.*

1. Give an example of a second order linear differential equation.
2. Find the general solution of the differential equation $\frac{dy}{dt} - 2y = 4 - t$.
3. State existence uniqueness theorem for first order linear equations.
4. Test exactness for the differential equation $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$.
5. Find the solution of the initial value problem $y'' + 5y' + 6y = 0$, $y(0) = 2$, $y'(0) = 3$.
6. Show that $y_1 = e^{-t}$ and $y_2 = e^{2t}$ form a fundamental set of solutions of the equation $y'' - y' - 2y = 0$.
7. Find the general solution of $16y'' - 8y' + 145y = 0$.
8. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t}$.
9. Determine a lower bound for the radius of convergence of series of solution of the differential equation $(1 + x^2)y'' + 2xy' + 4x^2y = 0$ about the point $x = 0$.
10. Find the Laplace transform of $f(t) = 5e^{-2t} - 3 \sin(4t)$, $t \geq 0$.

Turn over

11. Find the inverse Laplace transform of $F(s) = \frac{2s-3}{s^2-4}$.
12. State convolution theorem.
13. Define fundamental period. Find the fundamental period of $\sin\left(\frac{\pi x}{L}\right)$.
14. Define Fourier cosine series.
15. Give an example of a function which is neither odd nor even.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. Solve $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$.
17. Find the value of b for which $(ye^{2xy} + x) + bxe^{2xy}y' = 0$ is exact, and then solve it using that value of b .
18. Find the wronskian of two solutions of $(\cos t)y'' + (\sin t)y' - ty = 0$ without solving the equation.
19. Use the method of reduction of order to find a second solution of $t^2y'' - 4ty' + 6y = 0, t > 0$ whose one solution is $y_1(t) = t^2$.
20. Find the general solution of $y'' + y' - 6y = 12e^{3t} + 12e^{-2t}$.
21. Using Laplace transforms find the solution of the initial value problem $y'' + y = \sin(2t), y(0) = 2, y'(0) = 1$.
22. Solve the boundary value problem $y'' + y = 0, y(0) = 1, y(\pi) = a$, where a is a given number.
23. Find the Fourier series for $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \end{cases}, f(x+4) = f(x)$.

Section C

*Answer any two questions.
Each question carries 10 marks.
Maximum 20 marks.*

24. Solve the initial value problem $y' = 2t(1 + y)$, $y(0) = 0$ by the method of successive approximations.
25. Find a solution of $y'' - xy = 0$, $-\infty < x < \infty$ in powers of $(x - 1)$.
26. Find the solution of $y'' + 2y' + 2y = \delta(t - \pi)$; $y(0) = 1$, $y'(0) = 0$.
27. Find the Fourier series of $f(x) = \begin{cases} x + 1 & \text{if } -1 \leq x < 0 \\ 1 - x & \text{if } 0 \leq x < 1 \end{cases}$; $f(x + 2) = f(x)$.

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Name.....

Reg. No.....

**SIXTH SEMESTER UG (CBCSS-UG) DEGREE
EXAMINATION, MARCH 2024**

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum Marks : 80

Section A (Short Answer Type Questions)

Answer any number of questions.

Each carry 2 marks. Maximum marks 25.

1. State Existence and Uniqueness Theorem for First Order Linear Differential Equations.
2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.

3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.

4. Show that the given differential equation is exact :

$$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0.$$

5. Find the Wronskian of the functions $e^{\lambda_1 x}, e^{\lambda_2 x}$.

6. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}$.

7. Let $y = \phi(x)$ be a solution of the initial value problem

$$(1 + x^2)y'' + 2xy' + 4x^2y = 0, y(0) = 0, y'(0) = 1.$$

Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.
9. Find the Laplace transform of $2t + 6$.
10. Find the inverse Laplace transform of $\frac{s-4}{s^2+4}$.

Turn over

11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0$, and if c is a constant. Show that

$$\mathcal{L}(e^{ct} f(t)) = F(s - c), s > a + c.$$

12. If $\mathcal{L}(f)$ denote the Laplace transform of the function $f(x)$. Show that

$$\mathcal{L}(f_1 + f_2) = \mathcal{L}(f_1) + \mathcal{L}(f_2), \quad \mathcal{L}(cf) = c\mathcal{L}(f).$$

13. Solve the boundary value problem :

$$y'' + y = 0, y(0) = 1, y(\pi) = a.$$

14. Define an even function and show that if $f(x)$ is an even function then

$$\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx.$$

15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0$.

Section B (Paragraph/Problem)

*Answer any number of questions.
Each carry 5 marks. Maximum marks 35.*

16. Show that the equation $\frac{dy}{dx} = \frac{x^2}{1 - y^2}$ is separable, and then find an equation for its integral curves.
17. Find the value of b for which the following equation is exact, and then solve it using that value of b .

$$(xy^2 + bx^2y) + (x + y)x^2y' = 0.$$

18. Solve the initial value problem $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2$.
19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi/2$.
20. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0, y(0) = 3, y'(0) = -1.$$

21. Find the inverse Laplace transform of the following function using the convolution theorem :

$$F(s) = \frac{1}{(s+1)^2 (s^2 + 4)}.$$

22. Determine the coefficients in the Fourier series of the function

$$f(x) = \begin{cases} -x, & -2 \leq x \leq 0, \\ x, & 0 \leq x < 2 \end{cases}$$

with $f(x + 4) = f(x)$.

23. Find the solution of the following heat conduction problem :

$$\begin{aligned} 100u_{xx} &= u_t, & 0 < x < 1, t > 0; \\ u(0, t) &= 0, u(1, t) = 0, & t > 0; \\ u(x, 0) &= \sin(2\pi x) - \sin(5\pi x), & 0 \leq x \leq 1. \end{aligned}$$

Section C (Essay Type Questions)

Answer any **two** questions.

Each carry 10 marks.

24. Find the general solution of the following differential equation using the method of integrating factors

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0, 1).

25. Find a series solution of the differential equation :

$$y'' + y = 0, \quad -\infty < x < \infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau) \cos \tau d\tau$.

27. Find the Fourier series of the following periodic function $f(x)$ of period $p = 2L$ defined by

$$f(x) = 3x^2 - 1 < x < 1.$$

(2 × 10 = 20 marks)

C 40606

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Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Short Answer Type Questions.**Ceiling 25 Marks.*

1. Find the solution of the differential equation $\frac{dp}{dt} = 0.5p - 450$.
2. Solve the differential equation $(4 + t^2) \frac{dy}{dt} + 2ty = 4t$.
3. State Existence and Uniqueness Theorem for First-Order Linear Differential Equations.
4. Solve the initial value problem $y' = y^2$, $y(0) = 1$.
5. Find the general solution of $y'' + 5y' + 6y = 0$.
6. If y_1 and y_2 are two solutions of the differential equation, $y'' + p(x)y' + q(x)y = 0$.
Show that $c_1y_1 + c_2y_2$ is also a solution for any values of the constants c_1 and c_2 .
7. Let $y_1 = e^t \sin t$, $y_2 = e^t \cos t$. Find the Wronskian $W[y_1, y_2]$.
8. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}$.
9. Find the Laplace transform of e^{at} .

Turn over

10. Find $\mathcal{L}^{-1}\left(\frac{s}{s^2 - a^2}\right)$ for $s > |a|$.
11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0$, and if c is a constant, Show that
- $$\mathcal{L}(e^{ct} f(t)) = F(s - c), \quad s > a + c.$$
12. Solve the boundary value problem $y'' + y = 0$, $y(0) = 1$, $y(\pi) = a$, where a is a given number.
13. Find the fundamental period of the function $\sin(5x)$.
14. Define an odd function. Prove that if $f(x)$ is an odd function then
- $$\int_{-L}^L f(x) dx = 0.$$
15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0$.

(2 Marks each)

Section B*Paragraph / Problem Type Questions.**Ceiling 35 Marks.*

16. Show that the equation

$$\frac{dy}{dx} = \frac{x^2}{1 - y^2}.$$

is separable, and then find an equation for its integral curves.

17. Solve the differential equation

$$(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0.$$

18. Given that $y_1(t) = t^{-1}$ is a solution of

$$2t^2 y'' + 3ty' - y = 0, t > 0,$$

find a fundamental set of solutions.

19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi/2$.

20. Find the Laplace transform of the following function $f(t) = \int_0^t (t - \tau)^2 \cos(2\tau) d\tau$.

21. Find the inverse Laplace transform of the following function using the convolution theorem

$$F(s) = \frac{1}{(s+1)^2 (s^2 + 4)}.$$

22. Determine the co-efficients in the Fourier series of the function

$$f(x) = \begin{cases} -x, & -2 \leq x \leq 0, \\ x, & 0 \leq x \leq 2 \end{cases}$$

$$\text{with } f(x+4) = f(x).$$

23. Find the displacement $u(x, t)$ of the vibrating string of length $L = 30$ that satisfies the wave equation

$$4u_{xx} = u_{tt}, \quad 0 < x < 30, t > 0.$$

Assume that the ends of the string are fixed and that the string is set in motion with no initial velocity from the initial position

$$u(x, 0) = f(x) = \begin{cases} x/10, & 0 \leq x \leq 10, \\ (30-x)/20, & 10 < x \leq 30 \end{cases}$$

(5 marks each)

Turn over

Section C (Essay Type Questions)*two out of four.*

24. (a) Find the general solution of the differential equation $\frac{dy}{dt} - 2y = 4 - t$ by the method of integrating factors.
- (b) Find the value of b for which the following equation is exact, and then solve it using that value of b

$$(ye^{2xy} + x) + bxe^{2xy}y' = 0.$$

25. Find a series solution in powers of x of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

26. Use the Laplace transform and solve the following initial value problem

$$y'' + 3y' + 2y = 0; \quad y(0) = 1, y'(0) = 0.$$

27. Find the Fourier series of the following periodic function $f(x)$ of period $p = 2L$ defined by

$$f(x) = 3x^2 \quad -1 < x < 1.$$

(10 marks each)

C 20648

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

- Find the general solution of the differential equation $\frac{dy}{dt} = -ay + b$ where a,b are positive real numbers.
- Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.
- Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.
- Find the solution of the differential equation :

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, y(0) = -1.$$

- Find the Wronskian of the functions $\cos^2 \theta, 1 + \cos(2\theta)$.
- Find the general solution of the differential equation $y'' + 2y' + 2y = 0$.
- Let $y = \phi(x)$ be a solution of the initial value problem :
 $(1+x^2)y'' + 2xy' + 4x^2y = 0, y(0) = 0, y'(0) = 1$.
 Determine $\phi'''(0)$.
- Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.

Turn over

9. Find the Laplace transform of the function $\sin(at)$.
10. Find the inverse Laplace transform of $\frac{n!}{(s-a)^{n+1}}$ where $s > a$.
11. Let $u_c(t)$ be unit step function and $L(f(t)) = F(s)$. Show that :

$$L(u_c(t)f(t-c)) = e^{-cs}F(s).$$

12. Find the inverse Laplace transform of the following function by using the convolution theorem

$$\frac{1}{s^4(s^2+1)}.$$

13. Solve the boundary value problem :

$$y'' + y = 0, y(0) = 0, y(\pi) = 0.$$

14. Define an even function and show that if $f(x)$ is an even function then :

$$\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx.$$

15. Define the following partial differential equations :

- (a) heat conduction equation.
 (b) one-dimensional wave equation.

(10 × 3 = 30 marks)

Section B

*Answer at least five questions.
 Each question carries 6 marks.
 All questions can be attended.
 Overall Ceiling 30.*

16. Let $y_1(t)$ be a solution of $y' + p(t)y = 0$ and let $y_2(t)$ be a solution of $y' + p(t)y = g(t)$.
 Show that $y(t) = y_1(t) + y_2(t)$ is also a solution of equation $y' + p(t)y = g(t)$.
17. Find the value of b for which the following equation is exact, and then solve it using that value of b .
 $(xy^2 + bx^2y) + (x+y)x^2y' = 0.$
18. Solve the initial value problem
 $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2.$

19. Use method of variation of parameters find the general solution of :

$$y'' + 4y = 8 \tan t, -\pi/2 < t < \pi/2.$$

20. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5), y(0) = 0, y'(0) = 0.$$

here $\delta(t)$ denote the unit impulse function.

21. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0, y(0) = 3, y'(0) = -1.$$

22. Find the co-efficients in the Fourier series for f :

$$f(x) = \begin{cases} 0, & -3 < x < -1 \\ 1, & -1 < x < 1 \\ 0, & 1 < x < 3 \end{cases}$$

Also suppose that $f(x+6) = f(x)$.

23. Find the solution of the following heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1, t > 0$$

$$u(0, t) = 0, u(1, t) = 0, t > 0$$

$$u(x, 0) = \sin(2\pi x) - \sin(5\pi x), 0 \leq x \leq 1.$$

(5 × 6 = 30 marks)

Section C

*Answer any two questions.
Each question carries 10 marks.*

24. Find the general solution of the following differential equation using the method of integrating factors :

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0,1).

25. Find a series solution of the differential equation :

$$y'' + y = 0, -\infty < x < \infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau \, d\tau$

27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)