Q.P Code : D143772	Total Pages:2	Name 737118
		Register No.

FOURTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2026

MATHEMATICS

MAT4CJ203 Real Analysis I

2024 Admission Onwards

Maximum Time :2 Hours	**Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Show that the set of all integers $\mathbb{Z}$ is countable.
2	Define a) A Finite set b) Denumerable Set
3	Show that if $a \in R$, then $a.0 = 0$
4	Find all $x \in \mathbb{R}$ that satisfy $\lvert 4x - 5\rvert \leq 13$
5	Prove that: A sequence in $\mathbb{R}$ can have at most one limit
6	Give an example of two divergent sequences X and Y such that: a) their sum $X + Y$ converges, b) their product XY converges.
7	Give an example of an unbounded sequence that has a convergent subsequence.
8	Write a short note on Properly Divergent Sequences
9	Define Euler's Number
10	State Sequential Criterion for Limits

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Section B	
All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)	
11	Suppose that S and T are sets and that $T \subseteq S$. Prove that (a) If S is a finite set, then T is a finite set. (b) If T is an infinite set, then S is an infinite set.
12	State and prove Cantor's Theorem
13	Prove that "There does not exist a rational number r such that $r^2 = 2$"
14	Find all $x \in \mathbb{R}$ that satisfy the equation $\lvert x+1\rvert + \lvert x-2\rvert = 7$
15	Let $X = (x_n; n \in \mathbb{N})$ be a sequence of real numbers and let $m \in \mathbb{N}$. Then prove that the m-tail $X_m = (x_{n+m}; n \in \mathbb{N})$ of X converges if and only if X converges. In this case, $$\lim X = \lim X_m$$
16	Construct a sequence (s_n) of real numbers that converges to $\sqrt{5}$ and correct to within 4 decimals.
17	State and prove Monotone Subsequence Theorem
18	If $x_1 = 2$ and $x_{n+1} := 2 + \frac{1}{x_n}$ for all $n \in \mathbb{N}$ show that (x_n) is a contractive sequence. Find the limit.
Section C	
Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)	
19	Prove that the following statements are equivalent: 1. S is a countable set. 2. There exists a surjection of $\mathbb{N}$ onto S. 3. There exists an injection of S into $\mathbb{N}$.
20	State and prove the Density Theorem of rational numbers

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