D 123853 (Pages : 12) Name.....

Reg. No.....

CALICUT UNIVERSITY CENTRALIZED ENTRANCE TEST (CU-CET)
APRIL 2025

M.Sc. MATHEMATICS

Time : Two Hours Maximum : 200 Marks

Symbols

$\mathbb{R}$: The set of all real numbers

$\mathbb{C}$: The set of complex numbers

$\mathbb{Z}$: The set of all Integres

$\mathbb{Q}$: The set of all rational numbers

det (A) : determinant of A.

Each question carries 4 marks.
1 mark will be deducted for each wrong answer.

1. Find the distance between the points $P(1,-4)$ and $P(4,-8)$ on the two dimensional plane :

   (A) $2\sqrt{5}$. (B) 5.

   (C) $\sqrt{5}$. (D) $5\sqrt{5}$.

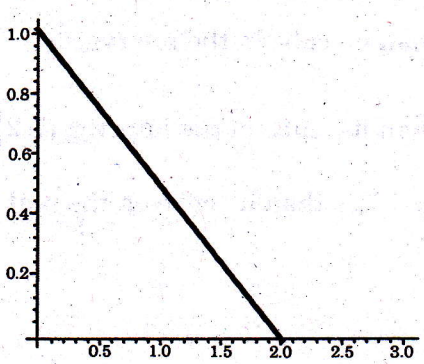
2. Which one among the following is a cube root of unity ?

   (A) $\frac{1+i\sqrt{3}}{2}$. (B) $1-i\sqrt{3}$.

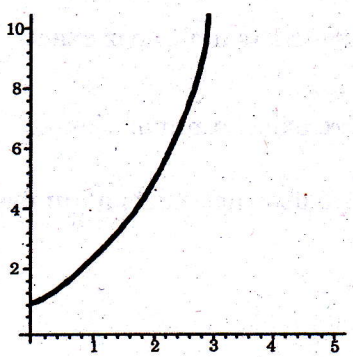
   (C) $\frac{-1+i\sqrt{3}}{2}$. (D) $-1+i\frac{\sqrt{3}}{2}$.

3. Identify the solution of the initial value problem $\frac{dy}{dx}=x-y,\ y(0)=1$ on the interval $[0,3]$.

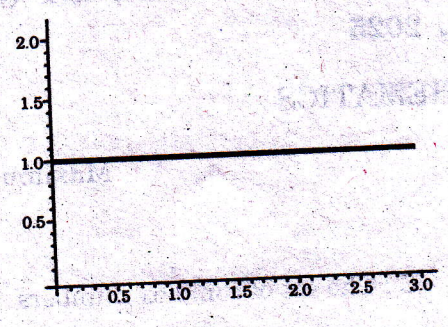
   (A)



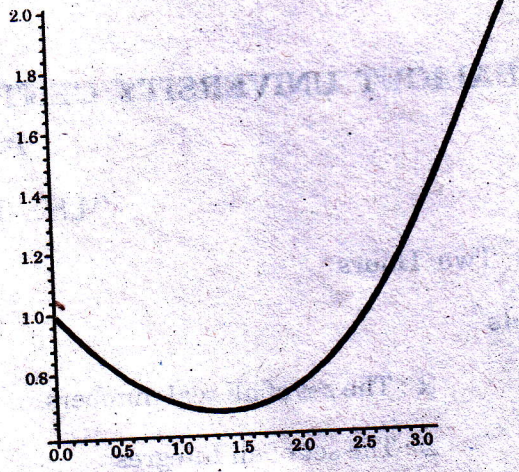
   (B)



(C)



(D)



4. What is the radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$?

(A) 4.

(B) 3.

(C) 6.

(D) 13.

5. Consider $f(x) = \dfrac{\sqrt{2+x} - \sqrt{2}}{x}$. What will be $\lim\limits_{x \to 0} f(x)$?

(A) $\dfrac{1}{2}$.

(B) $\sqrt{2}$.

(C) $\dfrac{1}{\sqrt{2}}$.

(D) $\dfrac{1}{2\sqrt{2}}$.

6. Which of the following statement is correct ?

(A) There exists a number exactly 1 less than its cube in the interval $[0, 1]$.

(B) There exists a number exactly 1 less than its cube in the interval $[-1, 0]$.

(C) There exists a number exactly 1 less than its cube in the interval $[1, 2]$.

(D) There does not exists a number exactly 1 less than its cube on the real line.

7. What is the rate of change in area of a circle with respect to the diameter when the radius is 5 m ?

(A) $-5\pi m^2/m$. (B) $10\pi m^2/m$.

(C) $-10\pi m^2/m$. (D) $5\pi m^2/m$.

8. What is the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $P(3, -4)$?

(A) $\frac{3}{4}$. (B) $-\frac{3}{4}$.

(C) $\frac{4}{3}$. (D) $-\frac{4}{3}$.

9. What is the largest area of a rectangle possible if inscribed in a semicircle of radius 2 units ?

(A) $2\sqrt{2}$ square units. (B) 4 square units.

(C) $8\sqrt{2}$ square units. (D) 16 square units.

10. What is the linearization of the function $f(x) = \sqrt{1+x}$ locally at $x = 0$?

(A) $1 + \frac{x}{4}$. (B) $\frac{1+x}{2}$.

(C) $1 + \frac{x}{2}$. (D) $1 - \frac{x}{4}$.

11. Which one among the following curves has slope of its tangent at the point (x, y) is $3x^2$ and passes through the point $P(1, -1)$?

(A) $x^3 + 2$. (B) $x^3 - 4$.

(C) $x^3 - 2$. (D) $x^3 + 4$.

12. What is the value of the definite integral $\int_{-1}^{1} 3x^2 \sqrt{x^3+1}\, dx$?

(A) $\frac{2\sqrt{2}}{3}$.

(B) $\frac{3\sqrt{2}}{4}$.

(C) $\frac{4\sqrt{2}}{3}$.

(D) $\frac{8\sqrt{2}}{3}$.

13. Choose correct statement from the following :

(A) If $\det(A) = 0$, then the equation $Ax = y$ has at least one solution for every $y \in \mathbb{R}^n$.

(B) If the equation $Ax = y$ has a solution for every $y \in \mathbb{R}^n$, then $\det(A) \neq 0$.

(C) If A and B are invertible $n \times n$ matrices, then $\det(A+B) \neq 0$.

(D) If $A^2 = A$, then -1 is an eigenvalue of A.

14. What is the area of the region enlosed by the parabola $y = 2 - x^2$ and the line $y = -x$?

(A) $\frac{7}{2}$.

(B) $\frac{9}{2}$.

(C) $\frac{11}{2}$.

(D) $\frac{13}{3}$.

15. What is the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$?

(A) $\frac{1}{27}\left(\sqrt{10} - 1\right)$.

(B) $\frac{2}{27}\left(\sqrt{11} - 10\right)$.

(C) $\frac{2}{27}\left(10\sqrt{10} - 1\right)$.

(D) $\frac{2}{27}\left(10\sqrt{11} - 1\right)$.

16. Choose correct statement from the following :

(A) $S(z)=\bar{z}$ satisfy Cauchy-Riemann Equations.

(B) $S(z)=\bar{z}$ is an analytic function.

(C) $S(z)=\bar{z}$ is a harmonic function.

(D) $S(z)=\bar{z}$ is not an analytic function.

17. Let $\phi:\mathcal{R}\to\mathcal{S}$ is a ring homomorphism. Which one of the following statements is correct ?

(A) ϕ is injective if and only if ker ϕ is empty.

(B) ker ϕ is not an ideal of $\mathcal{R}$.

(C) ker ϕ is an ideal of $\mathcal{R}$.

(D) The image $\phi(\mathcal{R})$ need not be a subring of $\mathcal{S}$.

18. If $f(z)$ is an entire function with $|f(z)|<2024$ for all z, then which of the following statement is correct ?

(A) $f(z)=0$ for all z.

(B) $f(z)=2023$ for all z.

(C) $f(z)$ is a constant function

(D) The equation $f(z)=2$ has only one solution.

19. The function $f(z)=\dfrac{\sin z}{z}$ has :

(A) A simple pole at 0.

(B) A removable singularity at 0.

(C) An essential singularity at 0.

(D) None of the above.

20. Choose the correct value of $\int x^2 e^x \, dx$.

(A) $x^2e^x + 2e^x + C.$ (B) $x^2e^x - 2xe^x + C.$

(C) $\left(x^2 - 2x + 2\right)e^x + C.$ (D) $\left(x^2 + 2x - 2\right)e^x + C.$

21. What is the sum of the infinite series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots.$?

(A) $\frac{1}{11}.$ (B) $\frac{1}{6}.$

(C) $\frac{1}{17}.$ (D) $\frac{1}{91}.$

22. $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents :

(A) An ellipse. (B) A circle.

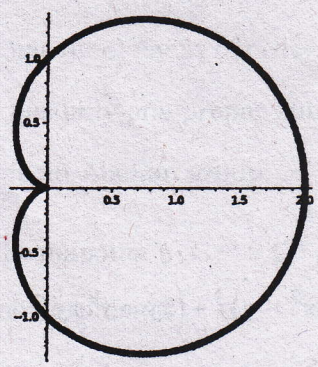
(C) A parabola. (D) A hyperbola.

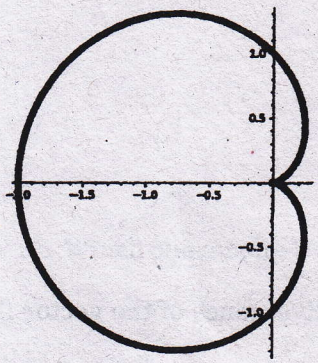
23. What is the angle between the vectors $\hat{i} - 2\hat{j} - 2\hat{k}$ and $6\hat{i} + 3\hat{j} + 2\hat{k}$?

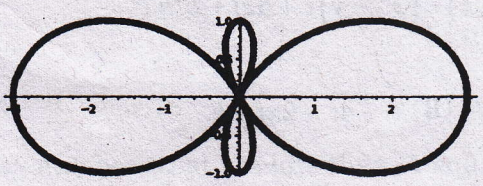
(A) $\cos^{-1}\left(\frac{4}{21}\right).$ (B) $\cos^{-1}\left(\frac{4}{23}\right).$

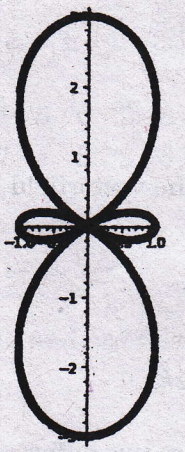
(C) $\cos^{-1}\left(-\frac{4}{21}\right).$ (D) $\cos^{-1}\left(-\frac{4}{23}\right).$

24. Which one among the following figure represents the function $r = 1 + \cos(\theta), 0 \leq \theta \leq 2\pi$?

(A) 

(B) 

(C) 

(D) 

25. What is the area of the triangle with vertices $P(1,-1,0), Q(2,1,-1)$ and $R(-1,1,2)$?

(A) $6\sqrt{2}$.

(B) $4\sqrt{2}$.

(C) $6\sqrt{3}$.

(D) $3\sqrt{2}$.

26. Which one among the following is the linearization of the function $f(x,y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point $(3,2)$?

(A) $4x - y - 2$.

(B) $2y - 2x + 2$.

(C) $4y + 2x - 2$.

(D) $3x + 4y - 2$.

27. What is the area of the region enclosed by the parabola $y = x^2$ and the line $= x + 2$.

(A) $\frac{7}{2}$.

(B) $\frac{9}{2}$.

(C) $\frac{11}{2}$.

(D) $\frac{11}{3}$.

28. What is the divergence of the vector field $\vec{f}(x, y) = (x^2 - y)\hat{i} + (xy - y^2)\hat{j}$?

(A) $2x + 3y$.

(B) $4x - 3y$.

(C) $3x - 2y$.

(D) $7x + 2y$.

29. What is the curl of the vectorfield $\vec{f}(x, y, z) = (x^2 - y)\hat{i} + 4z\hat{j} + x^2\hat{k}$?

(A) $3\hat{i} + 4x\hat{j} + \hat{k}$.

(B) $-4\hat{i} - 2x\hat{j} + \hat{k}$.

(C) $5x\hat{i} - 2y\hat{j} - z\hat{k}$.

(D) $7yz\hat{i} + 5xz\hat{j} + 6z\hat{k}$.

30. Suppose F_1 and F_2 are two differentiable vectorfields. Which one among the following is incorrect?

(A) $\nabla \cdot (F_1 \times F_2) = F_2 \cdot \nabla \times F_1 + F_1 \cdot \nabla \times F_2$.

(B) $\nabla \cdot (aF_1 + bF_2) = a\nabla \cdot F_1 + b\nabla \cdot F_2$.

(C) $\nabla \times (aF_1 + bF_2) = a\nabla \times F_1 + b\nabla \times F_2$.

(D) $\nabla \cdot (F_1 \times F_2) = F_2 \cdot \nabla \times F_1 - F_1 \cdot \nabla \times F_2$.

31. What is the determinat of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$?

(A) 35.

(B) 36.

(C) 37.

(D) 38.

32. Which of the following statements about the group G of order 12 is correct ?

(A) G has no proper subgroups.

(B) G has an element of order 2.

(C) G has only one proper subgroup.

(D) G is an Abelian group.

33. Consider the mapoing $\phi : \mathbb{C}^* \to \mathbb{C}^*$ given by $\phi(x) = x^4$. Which one among the following is the kernel of this homomorphism ?

(A) $\ker \phi = \{0\}$.

(B) $\ker \phi = \{1, i\}$.

(C) $\ker \phi = \{1, -1, i, -i\}$.

(D) $\ker \phi = \{1, -1, \sqrt{3}i\}$.

34. What is the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 4 & 7 & 1 & 8 & 3 & 6 \end{pmatrix}$?

(A) 6.

(B) 5.

(C) 4.

(D) 8.

35. How many numbers that are less than or equal to 670 and are relatively prime ?

(A) 245.

(B) 260.

(C) 264.

(D) 356.

36. Choose the solution of the system of equations $3x + y + z = 2, 6x + y + 2z = 1, 2x + 8y + 9z = -3$:

(A) $x = \frac{18}{25}, y = 33, z = \frac{19}{25}$.

(B) $x = \frac{18}{25}, y = 3, z = \frac{-79}{25}$.

(C) $x = \frac{81}{25}, y = 13, z = \frac{-19}{25}$.

(D) $x = \frac{81}{25}, y = 3, z = \frac{19}{25}$.

37. What is the rank of the given matrix $A = \begin{pmatrix} 1 & 6 & -5 \\ 4 & -6 & 7 \\ 1 & 4 & 2 \\ 3 & -2 & 4 \end{pmatrix}$?

(A) 1.

(B) 2.

(C) 3.

(D) 4.

38. Which of following is the characteristic polynomial for the given matrix $A = \begin{pmatrix} 1 & 2 & -6 \\ 8 & 6 & -7 \\ 3 & -2 & 4 \end{pmatrix}$?

(A) $x^2 + 11x^2 - 22x + 108.$ (B) $x^2 - 11x^2 + 22x - 108.$

(C) $2x^3 - 11x^2 - 22x + 108.$ (D) $-2x^3 + 11x^2 - 22x - 108.$

39. What are the characteristic values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 3 & 0 \\ -6 & 3 & 2 \end{bmatrix}$?

(A) $-2, 2, 3.$ (B) $3, 2, 1.$

(C) $0, 1, 3.$ (D) $4, -2, 6.$

40. What is the corrrect value of $(1+i)^{43}$?

(A) $2^{21} - 21i.$ (B) $2^{-21} + 2^{21}i.$

(C) $-2^{21} + 2^{21}i.$ (D) $21 + 2^{21}i.$

41. Which of the following statements is correct ?

(A) Pointwise convergence of a sequence of functions preserve boundedness.

(B) Pointwise convergence of a sequence of functions preserve continuity.

(C) Uniform convergence of a sequence of functions doesn't preserve continuity.

(D) Sequence of functions $f_n : \mathbb{R} \to \mathbb{R}$ defined by $f_n(x) = \frac{x}{1+nx^2}$ unifcmnly converges to 0.

42. What is the Laplace transform of the function $f(t) = e^{3t} + \cos(3t) - e^{3t}\cos(6t)$?

(A) $-\frac{s-3}{s^2-6s+45} + \frac{s}{s^2+36} + \frac{1}{s-3}.$ (B) $\frac{s-3}{s^2-6s+45} - \frac{s}{s^2+36} + \frac{1}{s+3}.$

(C) $-\frac{s+3}{s^2-6s+45} + \frac{s}{s^2-36} + \frac{1}{s-3}.$ (D) $-\frac{s-3}{s^2+6s+45} - \frac{s}{s^2+36} - \frac{1}{s-3}.$

43. Which of the following statements is incorrect ?

(A) If f is continuous on $[a, b]$, then f is Riemann integrable.

(B) All monotone functions are Riemann integrable.

(C) The function $f(x) = \begin{cases} 0 & \text{if } 0 < x \leq 1, \\ 1 & \text{if } x = 0 \end{cases}$ is Riemann integrable.

(D) The function $f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap \mathbb{Q} \\ 0 & \text{if } x \in [0,1] \setminus \mathbb{Q} \end{cases}$ is Riemann integrable.

44. A basket contains 30 blue balls and 70 pink balls. What is the probability of getting exactly 10 blue balls in a sample of size 20 if the sampling is done with replacement ?

(A) 0.0418.

(B) 0.0308.

(C) 0.0234.

(D) 0.2234.z

45. What is the average value of the function $f(x) = 36 - 12x$ over the interval $[4, 10]$?

(A) -44.

(B) -48.

(C) 44.

(D) 48.

46. Consider $F(x) = \int_x^{2x} t^3 \, dt$. What is $F'(x)$?

(A) $-12x^3$.

(B) $3x^2$.

(C) $15x^3$.

(D) $15x^4$.

47. Consider the field $\mathbb{Q}(\sqrt{2}) := \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. What is the inverse of the element $a + b\sqrt{2} \in \mathbb{Q}(\sqrt{2})$?

(A) $\frac{-a}{a^2+b^2} + \frac{b}{a^2+b^2}\sqrt{2}$.

(B) $\frac{-a}{a^2-2b^2} + \frac{b}{a^2+2b^2}\sqrt{2}$.

(C) $\frac{-a}{a^2+b^2} + \frac{b}{a^2-2b^2}\sqrt{2}$.

(D) $\frac{a}{a^2-2b^2} + \frac{b}{a^2-2b^2}\sqrt{2}$.

48. Which of the following matrix has the property $A^4 = A$?

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

(B) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

(C) $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$.

(D) $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$.

49. Which one among the following is an example of a harmonic function ?

(A) $f(x, y) = x^2 + y^2$.

(B) $f(x, y) = \sin(x) + \sin(y)$.

(C) $f(x, y) = x^2 - y^2$.

(D) $f(x, y) = \cos(x) + \cos(y)$.

50. What is the remainder if $2\,2^{402}$ is divided by 11 ?

(A) 1.

(B) 4.

(C) 9.

(D) 7.

C 41697 (Pages : 11) Name.....

Reg. No.....

PG/INTEGRATED PG ENTRANCE EXAMINATION, APRIL 2023

MATHEMATICS

Time : Two Hours Maximum : 200 Marks

All questions carries 4 marks.
1 mark will be deducted for each wrong answer.

1. Let G be a non-abelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G :

   (a) Is 6.

   (b) Is 12.

   (c) Is of the form $12k$ for $k \geq 2$.

   (d) Need not be finite.

2. Which one of the following series is divergent ?

   (a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$.

   (b) $\sum_{n=1}^{\infty} \frac{1}{n} \log$.

   (c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$.

   (d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$.

3. The matrix equation

$$(x \quad y)\begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 15$$

   represents

   (a) A circle of radius $\sqrt{15}$.

   (b) An ellipse of semi major axis $\sqrt{5}$.

   (c) An ellipse of semi major axis 5.

   (d) A hyperbola.

4. In a H.P., p^{th} term is q and the q^{th} term is p. Then pq^{th} term is :

(a) 0. (b) 1.

(c) pq. (d) $pq(p+q)$.

5. Newton Raphson scheme for solving $x^2 - 153 = 0$, is :

(a) $x_{n+1} = 0.5\left[x_n + \frac{153}{x_n}\right]$. (b) $x_{n+1} = 0.5\left[x_n - \frac{153}{x_n}\right]$.

(c) $x_{n+1} = \left[x_n + \frac{153}{x_n}\right]$. (d) $x_{n+1} = \left[x_n - \frac{153}{x_n}\right]$.

6. The Jacobian $\frac{d(x,y)}{d(u,v)}$ of the transformation

$x = \frac{-u+v}{6}$

$y = \frac{u+2v}{3}$ is

(a) $\frac{1}{6}$. (b) $-\frac{1}{6}$.

(c) 6. (d) – 6.

7. An urn contains 3 balls numbered 1, 2 and 3 . The co-efficient of equation $px^2 + qx + c = 0$ is determined by drawing the numbered balls with replacement. What is the probability that the equation will have imaginary roots ?

(a) $\frac{4}{27}$. (b) $\frac{23}{27}$.

(c) $\frac{16}{27}$. (d) None of the above.

8. If $f(x) = x^a \log x$ and $f(0) = 0$, then the value of a for which Rolle's theorem can be applied to $[0, 1]$ is:

(a) -2. (b) -1.

(c) 0. (d) 1/2.

9. Which among the following is not possible ?

(a) $\exists A$ s.t $P(A) = \{\}$. (b) $\exists A$ s.t $P(A) = \{\phi\}$.

(c) $\exists A$ s.t $P(A)$ is countable. (d) $\exists A$ s.t $P(A)$ is uncountable.

10. The number of group homomorphisms from the A_5 to the symmetric group S_4 is :

(a) 1. (b) 12.

(c) 20. (d) 6.

11. Which of the following is not a definition of Gamma function ?

(a) $\Gamma(n) = n!$. (b) $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$.

(c) $\Gamma(n+1) = n\Gamma(n)$. (d) $\Gamma(n) = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1}$.

12. If a 3×3 matrix A has eigen values 1, -1 and 0 then which among the following is not an eigen value $A^{100} + I$:

(a) 2. (b) 1.

(c) -1. (d) All of above.

13. Let $T : R^7 \to R^7$ be a linear transformation with Nullity $(T) = 2$. Then, the minimum possible value of Rank (T^2) is:

(a) 2. (b) 1.

(c) 3. (d) 4.

14. Which among the following iteration technique is having non linear rate of convergence :

(a) Bisection. (b) Regula-falsi.

(c) Newton-Raphson method. (d) All of the above.

15. Which among the following functions is not uniformly continuous on the given domain :

(a) $f(x)=x^2$ on R. (b) $f(x)=\sin\frac{1}{x}$ on $(0,1)$.

(c) $f(x)=\ln(x)$ on $(0,\infty)$. (d) All of the above.

16. Second degree Maclaurin polynomial of $f(x)=\frac{1}{\sqrt{x+1}}$ is :

(a) $1-\frac{1}{2}x+\frac{3}{8}x^2$. (b) $1-\frac{1}{2}x+\frac{3}{4}x^2$.

(c) $1-\frac{1}{2}x-\frac{3}{8}x^2$. (d) $1+\frac{1}{2}x-\frac{3}{8}x^2$.

17. The product AB of any two real, symmetric matrices A and B is :

a) Symmetric for all A and B.

(b) Never symmetric.

(c) Symmetric, if AB = BA.

(d) Anti-symmetric for all A and B.

18. Order of the permutation $\begin{pmatrix}1&2&3&4&5&6&7&8\\3&2&1&8&7&6&4&5\end{pmatrix}$ is :

(a) 4. (b) 2.

(c) 6. (d) 5.

19. Which among the following is not a generator of Z_9 :

(a) 1. (b) 2.

(c) 3. (d) 4.

20. In the Fourier series of the periodic function

$$f(x) = |\sin x|$$
$$= \sum_{n=0}^{\infty} [\alpha_n \cos nx + \beta_n \sin nx]$$

Which of the following co-efficients are non-zero ?

(a) α_n for odd n.

(b) α_n for even n.

(c) β_n for odd n.

(d) β_n for even n.

21. In a restaurant, 17 men and 7 women are seated on 24 chairs at a round table. Find the total number of possible ways such that 17 men are always sitting next to each other.

(a) 23 !.

(b) $7! \times 17!$.

(c) $6! \times 16!$.

(d) $8! \times 17!$.

22. The number of real solutions of $2\sin\left(e^x\right) = 5^x + 5^{-x}$ in $[0,1]$ is / are :

(a) 0.

(b) 1.

(c) 2.

(d) Infinite.

23. Tangent line at $(1,1)$ to the curve $x^2 + xy + 5 = 0$ is:

(a) $y = 3x - 2$.

(b) $y = -3x + 4$.

(c) $x = 3y - 2$.

(d) $x = -3y + 4$.

24. The remainder when $2^{20} + 3^{30} + 4^{40} + 5^{50} + 11^7$ is divided by 7 is :

(a) 3.

(b) 0.

(c) 6.

(d) 5.

25. The Sieve of Eratosthenes is used for finding :

(a) All primes below a given integer.

(b) All even numbers below a given integer.

(c) All odd numbers below a given integer.

(d) All composite numbers below a given integer.

26. Which of the following can only be used in disproving the statements ?

(a) Direct proof. (b) Contrapositive proofs.

(c) Counter Example. (d) Mathematical Induction.

27. The argument of the Complex number $z = \left(1 + i\sqrt{3}\right)(1+i)(\cos\theta + i\sin\theta)$ is :

(a) $\frac{\pi}{3} + \theta$. (b) $\frac{\pi}{4} + \theta$.

(c) $\frac{7\pi}{12} + \theta$. (d) None

28. Polar form of Complex number $\frac{1+7i}{(2-i)^2}$ is :

(a) $\sqrt{2}e^{(3i\pi)/4}$. (b) $\sqrt{2}e^{-3i\pi/4}$.

(c) $\sqrt{2}e^{3i\pi/2}$. (d) $\sqrt{2}$.

29. The value of $\left| \int_0^{3+i} (\bar{z})^2 \, dz \right|^2$, along the line $3y = x$, where $z = x + iy$ is :

(a) $\frac{1000}{9}$. (b) $\frac{1000}{27}$.

(c) $\frac{100}{9}$. (d) $\frac{100}{3}$.

30. The value of $\int_C \frac{z^2+5z+6}{(z-2)}\,dz$, where C is $|z|=1$ is:

(a) $2\pi i$. (b) 1.

(c) 0. (d) $40\pi i$.

31. For $f(x, y)=x^2+xy-2y-2x+1, (2,-2)$ is a:

(a) Maximum. (b) Saddle point.

(c) Minimum. (d) None of these.

32. If u and v are harmonic functions then $f(z)=u+iv$ is:

(a) Analytic function.

(b) Need not be analytic function.

(c) Analytic function only at $z=0$.

(d) None of the above.

33. Fixed points for the transformation $w=\frac{z-1}{z+1}$ is:

(a) $Z=\pm i$. (b) $Z=\pm 2i$.

(c) $Z=\pm 1$. (d) $Z=\pm 2$.

34. Image of the circle $|z|=2$ under the transformation $w=3z$ is :

(a) $|w|=2$. (b) $|w|=3$.

(c) $|w|=6$. (d) $|w|=4$.

35. The bilinear transformation that maps the points $z = 0, 1, \infty$ into the points $w = -1, -2, -i, i$ respectively is :

(a) $w = \dfrac{(iz-2)}{z+2}$.

(b) $w = \dfrac{(iz-2)}{z-2}$.

(c) $w = \dfrac{(iz+2)}{z+2}$.

(d) $w = \dfrac{(-iz-2)}{z+2}$.

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} \left(\dfrac{n+2}{n}\right)^{n^2} x^n$ is :

(a) e^2.

(b) $\dfrac{1}{\sqrt{e}}$.

(c) $\dfrac{1}{e}$.

(d) $\dfrac{1}{e^2}$.

37. Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$. A point at winch the gradient of the function f is equal to zero is :

(a) $(-1, 1, -1)$.

(b) $(-1, -1, -1)$.

(c) $(-1, 1, 1)$.

(d) $(1, -1, 1)$.

38. Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$:

(a) Diverges.

(b) Converges to $3S - a_1 - a_2$.

(c) Converges to $3S - a_1 - 2a_2$.

(d) Converges to $3S - 2a_1 - a_2$.

39. Let $M=\begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of det $\left((8I-M)^3\right)$ is :

(a) -216.

(b) 116.

(c) 256.

(d) 216.

40. If vector field $\bar{v}=(x+3y)\,\bar{i}+(y-2z)\,\bar{j}+(x+az)\,\bar{k}$ is solenoidal then value of a is :

(a) 0.

(b) 3.

(c) 2.

(d) -2.

41. The n^{th} derivative of $y=\sin px+\cos px$ is :

(a) $\sin(px+n\pi)+\cos(px+n\pi)$.

(b) $\sin\left(px+\frac{n\pi}{2}\right)+\cos\left(px+\frac{n\pi}{2}\right)$.

(c) $p^n\sin(px+n\pi)+p^n\cos\left(px+\frac{n\pi}{2}\right)$.

(d) $p^n\sin\left(px+\frac{n\pi}{2}\right)+p^n\cos\left(px+\frac{n\pi}{2}\right)$.

42. Residue of

$\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z=1$ is :

(a) $\frac{100}{6}$.

(b) $\frac{101}{6}$.

(c) $\frac{101}{16}$.

(d) $\frac{1}{116}$.

43. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a/an :

(a) Circle.

(b) Parabola.

(c) Hyperbola.

(d) Ellipse.

44. The values of $P_n(1)$ and $P_n(-1)$, where $P_n(x)$ is Legendra polynomial of degree n respectively are :

(a) $(-1)^n, 1$.

(b) $1, -1$.

(c) $-1, 1$.

(d) $1, (-1)^n$.

45. Solution of the LP problem

Maximize $z = 2x + 6y$ subject to $-x + y \leq 1$, $2x + y \leq 2$ and $x \geq 0, y \geq 0$ is :

(a) $\frac{4}{3}$.

(b) $\frac{1}{3}$.

(c) $\frac{26}{3}$.

(d) No feasible region.

46. The orthogonal trajectories of the family of curve $y = ax^2$ is (a and c being constants) :

(a) $\frac{x^2}{1} + \frac{y^2}{2} = c$.

(b) $\frac{x^2}{2} + \frac{y^2}{3} = c$.

(c) $\frac{x^2}{2} + \frac{y^2}{1} = c$.

(d) $\frac{x^2}{3} + \frac{y^2}{2} = c$.

47. If the order of every element of a group is 2 , then this group :

(a) Is Abelian.

(b) Is cyclic.

(c) Is of infinite order.

(d) Is definitely non-abelian.

48. The sum of interior angles of a geodesic triangle on the surface of a sphere radius R is :

(a) Less than π.
(b) π.
(c) Greater than π.
(d) Not constant.

49. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric. Then :

(a) Heine-Borel theorem holds for (X, d).
(b) Heine-Borel theorem does not hold for (X, d).
(c) X is not bounded.
(d) X is compact.

50. Which of the following sets is in one-to-one correspondence with N

(I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\right\}$.

(II) $\{\ldots\ldots\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\ldots\ldots\}$.

(III) $\left\{\frac{p}{q} : p, q \in Z, q \neq 0\right\}$.

(IV) $\left\{\frac{p}{q} : p, q \in N\right\}$.

(a) (I) and (II).
(b) (I), (II) and (III)
(c) (I) and (IV).
(d) All of the above.